Housing and Liquidity*

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Abstract

We study economies where houses, in addition to providing utility, also facilitate transactions when credit is imperfect, because home equity can be used to collateralize loans. We document big increases in real home equity loans since 1999, coinciding with the start of the house price boom, and suggest an explanation. When it can be used as collateral, housing can bear a liquidity premium. Since liquidity is endogenous, depending at least partially on beliefs, as we show, even when fundamentals are constant and agents are fully rational, house prices can display complicated equilibrium paths resembling bubbles. This is so with exogenous or with endogenous supply. Some of these paths look very much like the data. The framework is still tractable, reducing in some cases to supply and demand analysis, extended to capture special features of housing, including its role in credit transactions. The role of monetary policy is also discussed.

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1 Introduction

We study economies in which housing serves two roles. First, houses provide utility, either directly, as durable consumption goods, or indirectly as inputs into home production. Second, houses are assets that can help facilitate transactions when credit markets are imperfect: in the presence of limited commitment/enforcement, it can be difficult to get unsecured loans, and this generates a role for home equity as collateral. We show that this implies equilibrium house prices can bear a liquidity premium – people are willing to pay more than the fundamental price, defined by the present value of the (marginal) utility from living in the house, because home ownership provides security in the event that one needs a loan. Once this is understood, it is not hard to see how equilibrium house prices can display a variety of interesting dynamic paths, some of which look like bubbles. Intuitively, liquidity is at least to some extent a self-fulfilling prophecy, which means that the price of a liquid asset is to some extent a matter of beliefs. In this sense houses are similar to, but not the same as, money – e.g., while both help ameliorate credit frictions, only houses can generate direct utility, and only houses can be produced by the private sector.

The goal of this paper is to make these ideas precise and study their implications. We think it is interesting to analyze the housing market from this perspective for several reasons – mainly, because it is consistent with experience since the turn of the millennium. It is commonly heard that there was a bubble in house prices during this period, which eventually burst, leading to all kinds of economic problems, and it has been suggested that this is not unrelated to the use of home equity loans. Reinhart and Rogoff (2009) argue that developments in financial markets allowed consumers "to turn their previously illiquid housing assets into ATM machines." Ferguson (2008) also contends that these developments "allowed borrowers to treat their homes as cash machines," and reports that between 1997 and 2006 "US consumers withdrew an estimated $9 trillion in cash from the equity in their homes." According to Greenspan and Kennedy (2007), home equity withdrawal financed about 3% of personal consumption from 2001 to 2005. And according to Ferraris and Watanabe (2008), "In 2004, 47.9 percent of the US households had home-secured debt, whereby their house was used as a guarantee of repayment."[1]

Figure 1 shows some more detailed data for the US over the relevant period (exact data definitions and sources are given below). Prices are deflated in two ways. One divides by the CPI to correct for the purely nominal impact of inflation. The other divides by an index of rental rates, to correct for inflation as well as changes in the demand for shelter relative to other goods and services, yielding the inverse of the rent-price ratio. Even before we define terms precisely, this illustrates what people have in mind when they talk about a housing price bubble: a dramatic run up, followed by collapse. Also shown are measures of home equity loans, this time normalized in three ways. The first again uses the CPI to correct for the nominal impact of inflation, showing a huge increase in real loans during the period. The second normalizes by all bank loans, and still shows a big run up, to establish that the increase in home equity loans is not merely part of an overall increase in bank lending. The third normalizes by all real estate loans, to make it clear that the increase in home equity loans is not an artifact of an increase in the value of real estate generally – e.g., it is not merely the case that housing prices, mortgage loans and home equity loans all go up by the same factor. Finally, we show two series on investment in housing, the quantity investment index and gross housing investment series, both normalized by GDP.[2]

[1] The data described by Ferraris and Watanabe (2008) are from the Federal Reserve Survey of Terms of Business Lending (September 2006). As they also point out, real estate is used for more than just consumer loans: "the value of all commercial and industrial loans secured by collateral made by US banks accounted for 46.9 percent of the total value of loans in the US. Especially for commercial loans, the typical asset used as collateral is real estate."

[2] Definitions of the series used in to construct Figure 1 are as follows: Home Prices uses the FHFA Purchase Only price index. To turn it into a real variable, it is divided by the CPI, or by the BLS Rent index, with these real series then normalized to 1 in 1991. Loan data are from the Federal Reserve and are for all commercial banks in the US. Home Equity Loans are similarly divided by nominal GDP, by Bank Loans and Leases, and by Real Estate Loans, with the resulting three series normalized to 0.3 in 1991. Bank Loans and Leases includes Commercial and Industrial Loans, Real Estate Loans, Consumer Loans and Other Loans and Leases. Real Estate Loans contains all loans secured by real estate, including Revolving Home Equity Loans, Closed-end Residential Loans and Commercial Real Estate Loans. Finally, the Residential Fixed Investment Index is a quantity index from BEA divided by real GDP, and then normalized to 0.7 in 1991.

The messages we take away from these data are these: coinciding with the start of the boom in house prices, there is a very large increase in the real value of home equity loans and a moderate increase in housing investment; then, when prices fall, home equity loans stay up while investment drops. This suggests to us that the role of home equity as collateral is a potentially important piece of the puzzle in trying to understand this episode. If one considers a house only as a durable consumption good, with its value determined by the utility it provides, the ratio of rent (which should measure the utility flow) to the house price (the value of owning) should be roughly the sum of the discount and depreciation rates. There can be other costs and benefits of owning, including tax implications, but while these may affect the level of the rent-price ratio, as long as they are approximately constant, this should not generate the time series in Figure 1. Our position, following Reinhart and Rogoff (2009) and Ferguson (2008), is that financial developments led to a bigger role for home equity in the credit market, this fueled an increase in the demand for housing, and that led to an increase in price in the shorter run and an increase in quantity in the longer run.[3]

As we said, many people seem to think that these data indicate a bubble, although it is not always obvious what they means. As Case and Shiller (2003) put it, "The term 'bubble' is widely used but rarely clearly defined. We believe that in its widespread use the term refers to a situation in which excessive public expectations of future price increases cause prices to be temporarily elevated." We find this preferable to a purely empirical definition, like that in Kindleberger (1978), where a bubble is simply "an upward price movement over an extended range that then implodes," since such price patterns can easily emerge purely from changes in fundamentals (preferences, technology and policy). Thus, an increase in house prices in a particular location where more and more people want to live, followed by a drop when supply eventually catches up, cannot be a bubble if the

[3] Others have considered the data discussed above. Harding, Rosenthal, and Sirmans (2007), e.g., estimate the depreciation rate on houses to be around 2.5 percent, so if the discount rate is around 3 percent, the rent-price ratio should be around 5. In Campbell, Davis, Gallin, and Martin (2009), from 1975 to 1995, this ratio is indeed around 5, but then declines to around 3.7 percent in 2007. This is very much consistent with our general position.

term is meant to be something interesting. Case and Shiller (2003) say bubbles have to do with "excessive public expectations," and Shiller (2011) more recently says "In my view, bubbles are social epidemics, fostered by a sort of interpersonal contagion. A bubble forms when the contagion rate goes up for ideas that support a bubble. But contagion rates depend on patterns of thinking, which are difficult to judge."

There may well be merit in these views, and it may be interesting to try to rigorously model phenomena like "excessive public expectations, social epidemics, interpersonal contagion and patterns of thinking" – which are nothing if not bewitching. We are after something different. We want to emphasize the role of liquidity. And we want to use rudimentary economics, a framework where agents are fully rational optimizers, as in standard general equilibrium theory, but obviously extended to capture the special features of housing, including the role of home equity in credit markets. In terms of definition, for this paper, a bubble means a price different from the fundamental or intrinsic price, given by the prevent value of holding the asset (we make this more precise below). This definition is consistent with standard views, e.g., Stiglitz (1990), who says that "if the reason that the price is high today is only because investors believe that the selling price is high tomorrow – when 'fundamental' factors do not seem to justify such a price – then a bubble exists." To appeal to an even higher authority than Stiglitz, *Wikipedia* defines a bubble as "trade in high volumes at prices that are considerably at variance with intrinsic values." We show that in general equilibrium housing prices can indeed be different from fundamental values due to the role of home equity in credit markets, consistent with the data.

The liquidity-based approach implies that the price and quantity of housing can vary over time even when fundamentals are deterministic and stationary. In emphasizing credit frictions, we follow a large literature summarized in Gertler and Kiyotaki (2010) and Holmström and Tirole (2011), as well as a related literature surveyed in Nosal and Rocheteau (2011) and Williamson and Wright

(2010a,b) that endeavors to be relatively explicit about the process of exchange by going into detail concerning how agents trade (bilateral, multilateral, intermediated etc.), using which instruments (barter, money, secured or unsecured credit etc.), and at what terms (price taking, posting, bargaining etc.). A direct antecedent to our approach, in spirit, is the body of work emanating from Kiyotaki and Moore (1997, 2005). In terms of technical details, the model is similar to Rocheteau and Wright (2005, 2010), although there are many technical differences that arise because we study housing as opposed to some generic asset. To illustrate, the typical assets generates a dividend stream that enters your budget equation; housing does this, too, but additionally enters your utility function directly, and this matters for results As one example, it can change the circumstances under which bubbles exist from one of low supply to one of either low or high supply depending on preferences. As another, it means welfare can decrease with an exogenous increase in the stock of houses, which typically does not happen with financial assets.

Our approach is related to a large body of work on bubbles and liquidity, in general, too large to survey here (see Farhi and Tirole (2011) or Rocheteau and Wright (2010) for references); suffice it to say that, as we mentioned above, there are several interesting differences between houses and generic assets. In terms of research on housing markets, there are several other papers that also try to take seriously the precautionary or collateral function of home equity. A technical difference from some of this work is that we focus on fully rational agents, with homogenous beliefs, and indeed we can generate bubble-like equilibria in perfect foresight equilibrium. We can also do this with an endogenous supply of housing, which seems relevant since it has been suggested by, e.g., Shiller (2011), that “The housing-price boom of the 2000’s was little more than a construction-supply bottleneck, an inability to satisfy investment demand fast enough, and was (or in some places will be) eliminated with massive increases in supply.” The housing literature is sufficiently voluminous that, at the risk of neglecting some relevant contributions, we can only cite a few example that influenced our thinking on the issues.[4]

We do single out a recent paper, by Liu, Wang, and Zha (2011), which is complementary to our work. They study a closely related model, where real estate can be used as collateral, and develop its quantitative macro implications. They are more interested in calibration results, while we emphasize general theoretical results, in part because for their parameter values equilibrium exhibits saddle-path stability. If one knew for sure that their exact specification and parameters were correct, one might argue that in the empirically relevant case exotic dynamics cannot arise. We think this would be hasty, and in any case one should want to know how models behave more generally, and to see just what it takes to generate bubble-like housing market equilibria. Finally, to conclude this Introduction, we emphasize this paper is *not* about imperfect housing markets: in our model, houses are traded in frictionless markets, the way other forms of capital are traded in standard growth theory. This is not because we think it is realistic or that search-based theories of housing are uninteresting; we simply want to focus clearly on the role of home equity in imperfect credit markets.[5]

The rest of the paper is organized as follows. Section 2 lays out the basic environment. Section 3 discusses steady state equilibrium. Section 4 discusses dynamics, presenting explicit examples to show how bubble-like outcomes may arise. Section 5 endogenizes the supply of housing, discusses the multiplicity of equilibria, and presents an example that looks somewhat like recent experience,

[4] Carroll, Dynan, and Krane (2003) study the impact of a precautionary (against job-loss risk) motive on the demand for housing and related assets, and find evidence for it at moderate and higher income households. Hurst and Stafford (2004) find that unemployment shocks and low asset positions increase households' likelihood of using home-equity loans. Campbell and Hercowitz (2005) study a growth model where demand for housing increases over time, and existing houses alleviate borrowing constraints in buying new ones. Glaeser, Gyourko, and Saiz (2008) study the housing supply and bubbles at the city level, finding rational bubbles only exist if supply is fixed. Arce and Lopez-Salido (2011) study a life-cycle model and argue that there can be equilibrium where some agents hold houses purely as a saving vehicle. Other work emphasizing the role of asset shortages includes Caballero and Krishnamurthy (2006). Fostel and Geanakoplos (2008) emphasize assets' use as collateral when agents hold different beliefs. Aruoba, Davis, and Wright (2011) study the interaction between housing and inflation. Other papers on housing and monetary policy and include Taylor (2007), Brunnermeier and Julliard (2008), Follain (1982), Kearl (1979), and Poterba, Weil, and Shiller (1991). Other recent work on housing market dynamics includes Burnside, Eichenbaum, and Rebelo (2011), Coulson and Fisher (2009), Ngai and Tenreyro (2009) , Novy-Marx (2009) and Piazzesi and Schneider (2009b,a).

[5] Some housing papers that emphasize search inlcude Wheaton (1990), Caplin and Leahy (2008), Smith (2009), Albrecht, Gautier, and Vroman (2009) Albrecht, Anderson, Smith, and Vroman (2007), Head and Lloyd-Ellis (2010), Head, Lloyd-Ellis, and Sun (2010) and Diaz and Jerez (2010).

in terms of price, quantity and the use of home equity loans. Section 6 presents a monetary version of the model to study interplay between housing and inflation. Section 7 concludes. Some technical results are relegated to the Appendix.

2 The Basic Environment

Each period in discrete time agents interact in two distinct markets. First, they participate a decentralized market, labeled DM, with explicit frictions detailed below; then they trade in a frictionless centralized market, labeled CM. At each date t, in addition to labor ℓ_t, there are two nonstorable consumption goods x_t and y_t, plus housing h_t. We assume ℓ_t, x_t and h_t are traded in the CM, while y_t is traded in the DM. The utility of a household is given by

$$\lim_{T\to\infty} \mathbb{E} \sum_{t=0}^{T} \beta^t \left[\mathcal{U}(x_t, y_t, h_t) - \ell_t\right], \tag{1}$$

where $\beta \in (0,1)$ and $\mathcal{U}(x_t, y_t, h_t)$ satisfies the usual properties.[6] To ease the presentation, we assume that utility is separable between CM and DM goods, $\mathcal{U}(x_t, y_t, h_t) = U(x_t, h_t) + u(y_t)$, where U and u satisfy the usual assumptions, including $u(0) = 0$.

For now there is a fixed stock of housing H. In terms of CM goods, ℓ_t can be converted one-for-one into x_t (the framework is easily extended to more general production functions). In terms of DM goods, some agents can produce y_t using a technology summarized by the cost function $v(y_t)$. In many related models, households produce for each other in the DM, and $v(y_t)$ is interpreted as a direct disutility; in other models, DM producers are retail firms. Although it does not matter much for the results, in this paper, we follow the latter approach, with households buying y_t from DM retailers. Our retail technology works as follows: by investing at $t-1$ a fixed amount, normalized to 1, of the CM numeraire x_{t-1}, a retailer can at t convert it into any amount $y_t \leq 1$ of the DM

[6] Since we are interested in dynamics, and not merely stationary or recursive equilibria, we need to be careful defining preferences. We assume here that the limit in (1) exists; if not, we can use more advanced optimization techniques (see the citations in Rocheteau and Wright (2010)). An assumption that yields anylitic tractability is quasi-linear utility, although this can be replaced with indivisible labor à la Rogerson (1987), which has the added advantage that it also generates endogenous unemployment (see Rocheteau, Rupert, Shell, and Wright (2008)).

good and some amount $x_t = F(1-y_t)$ of the CM good. The profit from this activity, conditional on selling y_t in the DM at t for revenue R_t, measured in period t numeraire, is $R_t + F(1-y_t) - (1+r)$, given the initial investment at $t-1$ is repaid in the CM at t at an interest rate $1+r = 1/\beta$.

Not all retailers earn the same payoff, since not all trade, in the DM. Let α_f be the probability a retailer trades, often interpreted as the probability of meeting a household, in the DM, and symmetrically let α_h be the probability a household trades. Also, assume $y \leq 1$ is not binding, as must be the case, e.g., if $F'(0) = \infty$. Then expected profit is

$$\begin{aligned} \Pi_t &= \alpha_f \left[R_t + F(1-y_t) \right] + (1-\alpha_f) F(1) - (1+r) \\ &= \alpha_f \left[R_t - v(y_t) \right] + F(1) - (1+r), \end{aligned}$$

where $v(y_t) \equiv F(1) - F(1-y_t)$ as the opportunity cost of selling y_t in the DM. In general, if there is a $[0,1]$ continuum of households and a $[0,N]$ continuum of retail firms, the trading probabilities can be endogenized by $\alpha_f = \alpha(n)/n$ and $\alpha_h = \alpha(n)$, where $\alpha(\cdot)$ comes from a standard matching technology and $n \leq N$ is the measure of firms in the DM. Firms may have to pay a DM participation cost, in addition to their initial investment in goods, and n can be determined by the usual free-entry condition. To make our main points, however, we can assume the cost is small and $1+r < F(1)$, so that $n = N$, and α_h and α_f are fixed constants.

Although it is common in these types of models, it is not necessary to invoke search or matching. An alternative story that is equivalent for our purposes is that households sometimes realize a demand for y_t due to preference or opportunity shocks. Nice examples include the possibility that one has an occasion to throw a party, or an opportunity to buy a boat at a good price; not-so-nice examples include the possibility that one has an emergency medical breakdown, or one's boat does, and the probability of such an event is α_h. Then we can assume that the DM clears in various ways, and could involve bilateral or multilateral matching. We consider various options, but to simplify notation, we always assume there are the same number of agents on each side of the market, so that $\alpha_h = N\alpha_f$.[7] More significantly, in the DM, households use *credit*, since they have nothing to offer by way of quid pro quo: y_t is acquired in exchange for a debt obligation d_t to be retired in the next CM, where one-period debt can is imposed without loss of generality.

Credit is limited, however, by lack of commitment/enforcement: households are free to renege on promises, albeit possibly at the risk of some punishment. At one extreme, punishment can be so severe that credit is effectively perfect. At the other extreme, we can assume no punishment, not even exclusion from future credit as in Kehoe and Levine (2001) or Alvarez and Jermann (2000), say, because borrowers are *anonymous*. This would completely rule out unsecured credit, and generate a role for home equity as collateral. In general, one can impose a debt limit $d \le D = D(e_t)$, where $e_t = \psi_t h_t$ and ψ_t is the price of h_t in terms of x_t. For our purposes, it suffices to focus on the linear case $D(e_t) = D_0 + D_1 e_t$, with $D_1 > 0$. If D_0 is big the limit never binds, and unsecured lending works well; if D_0 is small, however, the limit may bind without sufficient home equity. A simple specification is $D(e_t) = e_t$, but it also makes sense to consider $D(e_t) < e_t$ if it is the case that, when debtors renege, we can seize part but not all of the collateral (they may have to forfeit the house, but perhaps can run off with some of the fixtures or appliances).

Let $W_t(d_t, h_t)$ be a household's value function entering the CM at t, with debt d_t and house h_t brought in from $t-1$. Since d_t is paid off each period in the CM, without loss in generality, households start debt free in next period's DM, where $V_{t+1}(h_{t+1})$ is the value function. The CM problem is

$$W_t(d_t, h_t) = \max_{x_t, \ell_t, h_{t+1}} \{U(x_t, h_t) - \ell_t + \beta V_{t+1}(h_{t+1})\} \tag{2}$$

$$\text{st } x_t + \psi_t h_{t+1} = \ell_t + \psi_t h_t + T_t - d_t \text{ and } \ell_t \in [0, \bar{\ell}] \tag{3}$$

[7] This must be true if trade is bilateral, but not if trade is multilateral; we make the assumption so we can easily consider both cases.

where $\psi_t h_t$ is home equity and T_t is other wealth, including government transfers, returns from investments etc. – but since wealth does not affect anything except leisure, with our quasi-linear utility function, this is all left implicit.[8] Notice that h_t affects the problem in two ways: it enters the objective function through $U(x_t, h_t)$; and it enters the budget constraint through $e_t = \psi_t h_t$.

Assuming $\ell_t \in [0, \bar{\ell}]$ does not bind, we eliminate ℓ_t using the budget equation to write

$$W_t(d_t, h_t) = \psi_t h_t + T_t - d_t + \max_{x_t} \{U(x_t, h_t) - x_t\} + \max_{h_{t+1}} \{\beta V_{t+1}(h_{t+1}) - \psi_t h_{t+1}\}. \tag{4}$$

Immediately this implies that choices at t, and in particular h_{t+1}, are independent of (d_t, h_t), which simplifies the analysis because we do not have to keep track of distributions across agents.[9] Assuming interior solutions, we have the FOC

$$U_1(x_t, h_t) = 1 \text{ and } \psi_t = \beta \frac{\partial V_{t+1}}{\partial h_{t+1}}. \tag{5}$$

Notice also that

$$\frac{\partial W_t}{\partial d_t} = -1 \text{ and } \frac{\partial W_t}{\partial h_t} = U_2(x_t, h_t) + \psi_t, \tag{6}$$

so that, in particular, W is linear in debt (more generally, it is linear in net worth).

We now describe in detail what happens in the DM in a trading opportunity, which we recall arises with probability α_h for a household and α_f for a retail firm each period. For agents with such an opportunity, firms (or sellers) generally produce while households (or buyers) consume y_t, in return for which the former receive and the latter issue a promise of a payment $d_t \leq D(e_t)$ in the CM. The terms of trade (y_t, d_t) can be determined in many ways in this type of model, including bargaining, price taking, auctions etc., as discussed in the surveys mentioned in the Introduction. We begin by describing competitive Walrasian trade where, to motivate this, we imagine that agents

[8] Of course, this requires that other such wealth cannot be used to collaterize loans, perhaps because it is hard to seize; in the language of Holmström and Tirole (2011), h_t is and T_t is not *pledgeable*.

[9] To be clear, these results hold only if the constraint $\ell_t \in [0, \bar{\ell}]$ is slack. More generally, people with very low or high net wealth may be unable to set ℓ_t high or low enough to get to their preferred h_{t+1} in a given period, allhough they will get there in the longer run. The key point is that if we start with a distribution of h_t and d_t that is not too disperse relative to $[0, \bar{\ell}]$, all agents settle their debts and choose $h_{t+1} = H$ in the CM, which allows use to solve the model by hand, not only by computer.

with opportunities to trade meet in large groups, rather than bilaterally, in the DM. First, note that buyers get a trade surplus from any (y_t, d_t) given by $S_{bt} = u(y_t) + W(d_t, h_t) - W(h_t) = u(y_t) - d_t$, since $W(\cdot)$ is linear in d_t by (6). Similarly, sellers get $S_{st} = d_t - v(y_t)$.

Then sellers and buyers choose

$$\begin{aligned}(y_t, d_t) &= \arg\max S_{st} \text{ st } d_t = p_t y_t \\ (y_t, d_t) &= \arg\max S_{bt} \text{ st } d_t = p_t y_t \leq D_t,\end{aligned}$$

taking as given p_t, as in any Walrasian market, except buyers have a debt limit D_t.[10] The standard way to solve this is as follows: As a preliminary step, first find equilibrium ignoring the constraint, which is easily shown to be $y_t = y^*$, where $u'(y^*) = v'(y^*)$, and $p_t = p^* = v'(y^*)$. Define $d^* = p^* y^* = v'(y^*) y^*$. If $d^* \leq D_t$ then the DM equilibrium at t is what we found ignoring the constraint. But if $d^* > D_t$ then the DM equilibrium at t is instead given by $p_t = v'(y_t)$, from the sellers' FOC, and $y_t = D_t/p_t$, from the buyers' constraint. In other words, when $d^* > D_t$ we get a constrained equilibrium, with $d_t = D_t$, and y_t is the solution to $v'(y_t) y_t = D_t$. For future reference, define $g(y_t) = v'(y_t) y_t$, so that when $d^* > D_t$ the equilibrium output can be written $y_t = g^{-1}(D_t) < y^*$. Summarizing:

Lemma 1 *Let y^* and $p^* = v'(y^*)$ be the Walrasian equilibrium ignoring the constraint $d_t \leq D_t$, and let $d^* = g(y^*)$. Then, as shown in Figure 2, equilibrium in the DM at t is given by:*

$$y_t = \begin{cases} g^{-1}(D_t) & \text{if } D_t < d^* \\ y^* & \text{if } D_t \geq d^* \end{cases} \quad \text{and } d_t = \begin{cases} D_t & \text{if } D_t < e^* \\ d^* & \text{if } D_t \geq e^* \end{cases} \tag{7}$$

As we said, one can consider alternative mechanisms. Suppose we pair up all the buyers and sellers and have the bargain bilaterally. A standard approach is to use the generalized Nash solution

$$(y_t, d_t) = \arg\max S_{bt}^{\theta} S_{st}^{1-\theta} \text{ st } d_t \leq D_t.$$

[10] Here we use the assumption that there are the same number of buyers and sellers in the market, so that they each choose the same (y_t, d_t), but again this is mainly to reduce notation.

Exactly as in Lagos and Wright (2005), even though that model uses money rather than credit, it is easy to work out, the following: The outcome is exactly as described by (7), except we redefine

$$g(y) = \frac{\theta v(y)\, u'(y) + (1-\theta)\, u(y)\, v'(y)}{\theta u'(y) + (1-\theta)\, v'(y)} \tag{8}$$

instead of $g(y) = v'(y)\, y$. In particular, with Nash we have $d^* = g(y^*) = \theta v(y^*) + (1-\theta)\, u(y^*)$, while with Walras we have $d^* = v'(y^*)\, y^*$, but in either case the Lemma holds exactly as stated.

We mention Nash here, since it is obviously quite common, but for examples constructed here, we instead use Kalai's proportional bargaining solution, which has several advantages, as discussed in Aruoba, Rocheteau, and Waller (2007). The proportional solution in this context is given by

$$(y_t, d_t) = \arg\max S_{bt} \text{ st } S_{bt} = \theta\left[u(y_t) - v(y_t)\right] \text{ and } d_t \leq D_t.$$

It is easy to see that the outcome is again given by (7) except now we redefine

$$g(y) = \theta v(y) + (1-\theta)\, u(y)\,. \tag{9}$$

The Nash and Kalai mechanisms give the same outcome (y^*, d^*) when the constraint is slack, but generally give different outcomes when it binds, except in the extreme cases $\theta = 0$ or $\theta = 1$. In any case, if $d^* \leq D_t$ then output y_t is efficient and the payment d_t is determined by the mechanism – including bargaining power with Nash or Kalai but obviously not with Walras – and $d^* > D_t$ then buyers borrow to the limit $d_t = D_t$ and output is $y_t = g^{-1}(D_t) < y^*$.

Finally, we also present some examples using a simple extensive-form game:[11]

Stage 1: The seller offers (y_t, d_t).

Stage 2: The buyer responds by accepting or rejecting, where:

- accept implies trade at these terms;

[11] For a recent discussion, with references, about the desirability of explicit extensive-form games in dynamic search-and-bargaining models, especially when one wants to study dynamics and has when one assumes nonlinear utility, see Wong and Wright (2011).

- reject implies they go to stage 3.

Stage 3: Nature moves (a coin toss) with the property that:

- with probability θ, the buyer makes a take-it-or-leave-it offer;
- with probability $1-\theta$, the seller makes a take-it-or-leave-it offer.

It is assumed that any offer must satisfy $d_t \leq D_t$. As shown in Appendix A, there is a unique SPE, characterized by acceptance of the initial offer, given by

$$(y_t, d_t) = \arg\max S_{st} \text{ st } S_{bt} = \theta \left[u(\bar{y}_t) - \bar{d}_t \right] \text{ and } d \leq D_t, \tag{10}$$

where $\left(\bar{y}_t, \bar{d}_t\right)$ is the take-it-or-leave-it offer a buyer would make if (off the equilibrium path) we were to reach Stage 3.

For each of the mechanisms mentioned above one can write the equilibrium outcome as given by (7) for a particular choice of the function $g(\cdot)$. Then, we can write the DM value function as

$$V_t(h_t) = W(0, h_t) + \alpha_h \left\{ u\left[y(\psi_t h_t) \right] - d(\psi_t h_t) \right\}, \tag{11}$$

where it is understood that $y(\psi_t h_t)$ and $d(\psi_t h_t)$ are given by (7) with $D_t = D(\psi_t h_t)$. The first term is the value to not trading in the DM; the second is the expected surplus from trading in the DM. Since (6) tells us that $\partial W_t / \partial h_t = U_2(x_t, h_t) + \psi_t$, we have

$$\frac{\partial V_t}{\partial h_t} = U_2(x_t, h_t) + \psi_t + \alpha_h \psi_t \left[u'(y)\, y'(\psi_t h_t) - d'(\psi_t h_t) \right].$$

Differentiating y and d using (7), we have

$$\frac{\partial V_t}{\partial h_t} = U_2(x_t, h_t) + \psi_t + \alpha_h \psi_t \mathcal{L}(\psi_t h_t), \tag{12}$$

where

$$\mathcal{L}(e) \equiv D'(e) \begin{cases} u'(y)/g'(y) - 1 & \text{if } e < e^* \\ 0 & \text{if } e > e^* \end{cases}. \tag{13}$$

The function $\mathcal{L}(e)$ represents a *liquidity premium* on home equity. First, $D'(e) = D_1$ tell us home much e_t extends one's credit limit. Then, if on the one hand $e < e^*$, the upper branch tells us how much this is worth when said limit is binding, since $1/g'$ is the extra y one gets when the limit is extended, u' tells us the marginal utility of y, and -1 appears because the extended debt has to be repaid. If on the other hand $e > e^*$, the lower branch tells us that extending the credit limit s worth 0, because the constraint is slack. For now we assume $\mathcal{L}'(e) < 0$. This is useful, as it delivers three related results: 1) it guarantees the SOC is satisfied at any solution to the FOC in the CM problem; 2) it implies a unique steady state equilibrium; 3) it yields monotone comparative statics.

We do not actually need this assumption: one can show all these results without $\mathcal{L}' < 0$, using the method in Wright (2010), and it is only to avoid these technicalities that we focus on the case $\mathcal{L}' < 0$. Moreover, sometimes it comes for free. With Kalai bargaining, e.g., from (9) we have

$$\mathcal{L}(e) = \theta D'(e) \frac{u'(y) - v'(y)}{(1-\theta)\, u'(y) + \theta v'(y)} > 0$$

for $e < e^*$. From this we can derive

$$\mathcal{L}'(e) = \theta D'(e) \frac{v'(y)\, u''(y) - u'(y)\, v''(y)}{[(1-\theta)\, u'(y) + \theta v'(y)]^2} \frac{\partial y}{\partial e} = \theta D'(e)^2 \frac{v'(y)\, u''(y) - u'(y)\, v''(y)}{[(1-\theta)\, u'(y) + \theta v'(y)]^3} < 0.$$

One can show something similar using Walrasian pricing.[12]

In any case, using the definition of $\mathcal{L}(e)$, we rewrite (11) as

$$\frac{\partial V_t}{\partial h_t} = U_2(x_t, h_t) + \psi_t + \alpha_h \psi_t \mathcal{L}(\psi_t h_t). \tag{14}$$

The three terms in (14) describe three marginal benefits from home ownership: 1) you get more utility from living in a bigger house; 2) you are wealthier when you own a bigger house; and 3) your credit constraint is relaxed when you own a bigger house. This third effect is relevant when you want to make a DM purchase and $\psi_t h_t < e^*$; otherwise, if the credit constraint is slack, the last

[12] It does *not* hold automatically for generalized Nash bargaining, although it does when θ is clore to 1, or when $v(y)$ is linear and $u(y)$ displays decreasing absolute risk aversion (Lagos and Wright (2005)). This is related to the fact that S_b is not necessarily increasing in e_t with generalized Nash bargaining (Aruoba, Rocheteau, and Waller (2007)).

term in (14) vanishes. Updating (14) from t to $t+1$, substituting it into the FOC for h_{t+1} in (5) and using $1+r=1/\beta$, we arrive at the Euler equation

$$(1+r)\,\psi_t = U_2(x_{t+1},h_{t+1}) + \psi_{t+1} + \alpha_h \psi_{t+1} D'\left(\psi_t h_t\right) \mathcal{L}(\psi_t h_t). \tag{15}$$

Equating demand to supply $h_t = H$ for all t, and using the FOC $U_1(x_t, H) = 1$ to define $x = X(H)$, (15) yields a difference equation in the price of housing, say $\psi_t = \Psi\left(\psi_{t+1}\right)$. Equilibrium is defined by a nonnegative and bounded sequence $\{\psi_t\}$ solving this difference equation, where the sequence must be bounded to satisfy a standard transversality condition (see, e.g., the discussion in Rocheteau and Wright (2010)). Given $\{\psi_t\}$, we can easily recover all other variables of interest, including $e_t = \psi_t H$, $D_t = D\left(e_t\right)$, $y_t = y\left(e_t\right)$ and so on.

3 Steady State Equilibrium

A stationary equilibrium, which is also a steady state, is a constant solution to $\psi = \Psi\left(\psi\right)$. In steady state (15) simplifies to

$$r\psi = U_2\left[X(h), h\right] + \alpha_h \psi \mathcal{L}(e), \tag{16}$$

before imposing the equilibrium condition $h = H$. One can interpret (16) as the long-run demand for housing as a function of price. The slope is

$$\frac{\partial h}{\partial \psi} = \frac{U_{11}\left(U_2 - \alpha_h \psi^2 h \mathcal{L}'\right)}{\left(U_{11}U_{22} - U_{12}^2 + \alpha_h \psi^2 \mathcal{L}'\right)\psi} < 0, \tag{17}$$

after inserting $X'(h) = -U_{12}/U_{11}$ and using (15) to eliminate r. Demand is downward at least as long as $\mathcal{L}'\left(e\right) < 0$, although again this is only a matter of convenience, and one can also show $\partial h/\partial\psi < 0$ without $\mathcal{L}' < 0$ using the method in Wright (2010). Given supply H, there is a unique steady state.

If at the unique steady state $e = \psi H > e^*$, then $\mathcal{L}(e) = 0$ and (16) implies $\psi = \psi^* \equiv U_2\left[X(H), H\right]/r$, where ψ^* is *the fundamental price*, defined as the present value of the marginal utility of living in house H forever. In this case, in equilibrium, households have enough home equity that they are never liquidity constrained, and houses bear no liquidity premium. However, if $e < e^*$ then $\mathcal{L}(e) > 0$ and (16) implies $\psi > \psi^*$. In this case, home equity is scarce, and bears a liquidity premium, which means price is above the fundamental value. By definition, this constitutes a bubble. It happens to be a stationary bubble, of course, since so far we are focusing on steady states, but it is a bubble nonetheless, by the criterion discussed in the Introduction.[13] The simple idea behind the liquidity premium is this: if at the fundamental price $e = H\psi^* < e^*$, there will be excess demand at ψ^*, because agents not only get a utility flow from shelter they can also use it to collateralize loans. If the credit constraint binds, agents are willing to pay a premium for assets that relax it. The exact outcome depends on the mechanism – e.g., Nash versus Kalai versus Walras pricing – but we get a bubble whenever $e < e^*$.

There are related results in models with different assets, including fiat money, which constitutes a bubble whenever it is valued, but also including Lucas trees and neoclassical capital, all of which bear liquidity premia in some circumstances (see, e.g., Geromichalos, Licari, and Suárez-Lledó (2007), Lagos and Rocheteau (2008) or Lester, Postlewaite, and Wright (2008)). The economics is different with housing, however. With trees, e.g., there is a liquidity premium iff the exogenous supply is low; and with capital there is a liquidity premium iff the endogenous supply is low, but the endogenous supply without liquidity considerations is itself pinned down by productivity. With housing, there can be a liquidity premium when the supply is low or when it is high. From (17) we get

$$\frac{de}{dH} = \psi + H\frac{d\psi}{dH} = \frac{\psi H \left(U_{22}U_{11} - U_{21}^2\right) + \psi U_2 U_{11}}{U_{11}\left(U_2 - \alpha\psi^2 H \mathcal{L}'\right)} \simeq -H\left(U_{22}U_{11} - U_{21}^2\right) - U_2 U_{11},$$

where the notation $A \simeq B$ means that A and B have the same sign. As this can be positive or negative, e can go up or down with an increases in H, and hence a bubble can be less or more likely

[13] Recall Stiglitz (1990): "if the reason that the price is high today is only because investors believe that the selling price is high tomorrow – when 'fundamental' factors do not seem to justify such a price – then a bubble exists." That is precisely what is going on here.

to exist with an increases in H.

To verify this, by way of example, consider

$$U(x,h) = \tilde{U}(x) + \frac{h^{1-\sigma}}{1-\sigma}.$$

Figure 3 shows demand when e can be used as collateral by the solid curve, and shows demand when it cannot be used in this way, say because we set $\alpha_h = 0$, by the dotted curve. When the solid is above the dotted curve, home equity is scarce and bears a liquidity premium. For $\sigma < 1$ this happens when H is low, while for $\sigma > 1$ this happens when H is high. Hence, home equity can be scarce or plentifully when the stock of houses is large, depending in an obvious way on the demand elasticity. Perhaps less obviously, this means welfare W can fall as H increases. If one ignores liquidity, by shutting down the DM by setting $\alpha_h = 0$, then W must rise with H, but in general, when ψH falls with H, DM utility can fall by enough to reduce total utility. We show this in a parametric example in the next Section. For now, we summarize the main results in this Section as follows:

Proposition 2 *With $h = H$ fixed, there exists a unique steady state equilibrium. If $e > e^*$ then ψ is given by the fundamental price $\psi^* = U_2[X(H), H]/r$; if $e < e^*$ then $\psi > \psi^*$, which is stationary bubble. We can have $e < e^*$ and hence $\psi > \psi^*$ either when H is low or when H is high, depending on the elasticity of housing demand. It is possible for W to fall with H over some range.*

4 Dynamics: Cyclic, Chaotic and Stochastic Equilibria

In this Section, we are mainly interested in deterministic cycles, although we also discuss stochastic cycles, or sunspot equilibria. In general, deterministic dynamic equilibria are given by nonnegative and bounded solutions to the difference equation defined by (15), which we repeat here after inserting $x = x(H)$:

$$\psi_t = \Psi\left(\psi_{t+1}\right) = \beta U_2\left[x(H), H\right] + \beta\psi_{t+1} + \alpha_h\beta\psi_{t+1}\mathcal{L}(\psi_{t+1}H). \tag{18}$$

The first observation is that any interesting dynamics in house prices must emerge here from liquidity considerations, which show up in the nonlinear term $\mathcal{L}(\psi_{t+1}H)$. If we set $\alpha_h = 0$, then (18) is a linear difference equation, which can be rearranged as a mapping from the price today to the price tomorrow

$$\psi_{t+1} = -U_2\left[x\left(H\right), H\right] + \left(1+r\right)\psi_t.$$

This has a unique steady state at the fundamental price $\psi = \psi^*$, which is also the unique equilibrium, since any solution to (18) other than $\psi_t = \psi^* \ \forall t$ becomes unbounded or negative.[14]

When $\alpha_h > 0$, as long as $H\psi_{t+1} < e^*$, we have $\mathcal{L}(\psi_{t+1}H) > 0$, and the nonlinear part of the dynamical system (18) kicks in. We now analyze this system in $\left(\psi_{t+1}, \psi_t\right)$ space, where it is natural to think of ψ_t as a function of ψ_{t+1}, because for any ψ_{t+1} there is a unique choice for individual demand h_t given the current price ψ_t, and hence market clearing uniquely pins down ψ_t. However, as usual, there can be multiple values of ψ_{t+1} for which this mapping yields the same ψ_t, so that in general the inverse $\psi_{t+1} = \Psi^{-1}\left(\psi_t\right)$ is a correspondence. Of course Ψ and Ψ^{-1} cross on the 45^o line in $\left(\psi_{t+1}, \psi_t\right)$ space at the unique steady state. Textbook methods (e.g., Azariadis (1993)) tell us that whenever Ψ^{-1} and Ψ cross off the 45^o line there exists a cycle of period 2 – i.e., a solution $\left(\psi^1, \psi^2\right)$ to $\psi^2 = \Psi\left(\psi^1\right)$ and $\psi^1 = \Psi\left(\psi^2\right)$ other than the degenerate (steady state) solution where $\psi^1 = \psi^2$ – and that this happens whenever Ψ has a slope less than -1 on the 45^o line. In this 2-cycle equilibrium, even though fundamentals are constant, ψ oscillates between ψ^1 and ψ^2 simply as a self-fulfilling prophecy. This is a nonstationary bubble.

Before discussing economic intuition, we consider higher-order $n-$cycles – i.e., nondegenerate solutions to $\psi = \Psi^n\left(\psi\right)$. To reduce notation, normalize $H = 1$, and consider by way of example

[14] This is a standard no-bubble result, versions of which can be found in many places. Heuristically, the only way to have $\psi_t > \psi^*$ is for agents to believe ψ_{t+1} will be even higher, by at least enough to make up for discounting, and this means $\psi_t \to \infty$. Such a belief is inconsistent with rationality because $\psi_t \to \infty$ is inconsistent with equilibrium.

$v(y) = y$, $D(e) = e$, and[15]

$$U(x,h) = \tilde{U}(x) + \kappa \frac{h^{1-\sigma}}{1-\sigma} \text{ and } u(y) = \eta \frac{(y+\varepsilon)^{1-\gamma} - \varepsilon^{1-\gamma}}{1-\gamma}.$$

To show our results are robust to the choice of mechanism used to determine the terms of trade in the DM, we consider Walrasian pricing, Kalai bargaining and the extensive-form game in Section 2. These choices and the parameter values across examples are shown in Table 1.

	Example 1	Example 2	Example 3	Example 4	Example 5
Mechanism	Walras	Walras	Kalai	Game	Walras
α_h	0.5	0.5	0.9	0.5	0.5
β	0.8	0.6	0.6	0.8	0.9
θ	n/a	n/a	0.9	0.6	n/a
κ	0.125	0.3333	0.1	0.125	1
σ	n/a	n/a	n/a	n/a	7
η	1.5125	3.2479	0.5882	3.0368	1
γ	2	7	9	8	0.1
ε	0.1	0.1	0.5	0.1	0.01

Table 1: Parameter Values for the Examples

Example 1, with Walrasian pricing, is shown in Figure 4a, where the unique equilibrium is the steady state. Example 2, also with Walrasian pricing, but different parameters, is shown in Figure 4b. In addition to the steady state, this example exhibits a 2-cycle. In this case, the unconstrained value of y is $y^* = 1.0833$, the constraint binds iff $\psi < y^*$, and this happens in alternate periods. Figures 4c and 4d show similar results for Example 3 and 4, using Kalai bargaining and strategic bargaining, instead of Walrasian pricing. These example illustrate that the effects come not any particular pricing mechanism, but from liquidity considerations, in general. It is not hard to generate higher order cycles. Example 2, e.g., also has a 3-cycle, with $\psi^1 = 0.8680 < y^*$, $\psi^2 = 1.5223 > y^*$, $\psi^3 = 1.1134 > y^*$. As is standard (again see Azariadis (1993)), when a 3-cycle exists, by the Sarkovskii theorem and the Li-Yorke theorem there exist cycles of all orders and chaotic dynamics. Chaos is defined in the context of our model as a solution $\{\psi_t\}$ to the difference equation (18) with

[15] The form of $\tilde{U}$ is irrelevant for all the restults. The role of ε in $u(y)$ is simply to force $u(0) = 0$. Also, the value of σ is irrelevant, since it vanishes from $\partial U / \partial h$, given $h = H = 1$.

the property that $\psi_s \neq \psi_t \; \forall s \neq t$, and, of course, ψ_t is still bounded by the definition of equilibrium. Figure 5 shows an equilibrium price path with chaotic behavior. Figure 6 is not about dynamics, but depicts steady state welfare W as a function of H, for Example 5, to illustrate the possibility that W may decrease over some range of H, as mentioned above.

We conclude that this model, which is standard except that it takes into account the collateral use of home equity, as seen in the data, can generate equilibria where prices display deterministic cycles of arbitrary periodicity, even though fundamentals are stationary and all agents have perfect foresight. Prices not only can differ from their fundamental value, but can fluctuate up and down, as a self-fulfilling prophecy. This is because a house has liquidity value, as collateral, in the imperfect credit market, and as we said above liquidity can be at least partially a matter of beliefs. Heuristically, in a 2-cycle, if at t one believes ψ_{t+1} and hence e_{t+1} will be high then liquidity tomorrow will be plentiful, which lowers the price people are willing to pay for it today. Hence a low ψ_t can be consistent with a high ψ_{t+1}. And a high ψ_{t+1} can be consistent with a low ψ_{t+2}, and so on. Cycles of order $n > 2$ and chaotic paths are more complicated, but the same logic conveys the basic intuition. One might naively wonder why rational investors are willing to pay a lot for an asset when they know its price is about to drop, as in the above description, between $t = 1$ and $t + 2$. It is precisely *because* the price is about to drop that liquidity will soon become scarce and hence is currently in high demand. This is what generates cycles.

Moreover, following standard methods, we can also construct sunspot equilibria, or stochastic cycles. In such an equilibrium, e.g., the price can be ψ^1 and jump to $\psi^2 > \psi^1$ with some probability λ^1 each period; then when it is ψ^2 it can fall back to ψ^1 with some probability λ^2 each period.[16]

[16] There are different methods for constructing sunspot equilibria. Azariadis and Guesnerie (1986) note that in the limit when $\lambda^1 = \lambda^2 = 1$ the sunspot equilibrium described in the text reduces to a 2-cycle, which exists under conditions described above, and appeal to continuity. This method generates sunspot equilibria that are close to 2-cycles (λ^1 and λ^2 are close 1). One can alternatively note that in the limit when $\lambda^1 = \lambda^2 = 0$ the conditions for sunspot equilibrium reduce to the conditions for multiple steady states and again use continuity to get sunspot equilibria that are very persistent (λ^1 and λ^2 are close 0). Now, our model does not have multiple steady state, but simple extensions can change that (e.g.,in a similar model, Rocheteau and Wright (2010) introduce a free entry

Again, agents are rational and all know the stochastic structure of the extrinsic uncertainty – in the above example, they know λ^1 and λ^2. Yet we can still have random price fluctuations, in this case random, when fundamentals are deterministic and stationary. In rational expectations equilibrium, demand for liquid assets and hence ψ^2 are high not because the price is about to drop to ψ^1 for sure, as in a 2-cycle, but because it drops with some probability; the economic idea is similar. We are not sure if one should think of this in terms of "excessive public expectations," "social epidemics," "interpersonal contagion" or whatever other fancy terms one hears about bubbles; we are sure that the results come from modeling liquidity explicitly.

Of course this is related to monetary economics, especially the branch mentioned in the Introduction that tries to model explicitly the process of exchange, and in that endeavor, tries to make precise notions like liquidity. But H is also very different from fiat currency M. Any reasonable model of fiat currency has at least two steady states, one where M is valued and one where it is not. By contrast, we get a unique steady state, and obviously cannot have $\psi = 0$ when H has fundamental value as shelter. To give credit where it is due, part of our motivation for this work came from seeing an example of Kocherlakota, where something he called housing could exhibit somewhat interesting dynamics – not as interesting as we get here, but at least an equilibrium where price jumps stochastically from $\psi > 0$ to an absorbing state with $\psi = 0$ (a special case of the sunspot equilibria discussed above, special in that once ψ goes down it never comes back). But what he calls housing in his example is in fact a fiat object, with fundamental utility value 0, which is not the case here. Ruling out $\psi = 0$, by giving H intrinsic value, as any decent model of housing should, also rules out equilibria where $\psi_t \to 0$ either stochastically or deterministically, so our dynamics are not merely reinterpretations of standard results in monetary theory.

Having said that, on the down side, equilibria with cycles, chaos or sunspots do not resemble

condition for sellers in the DM to get multiple steady states).

very closely recent housing market experience – prices in the model tend to go up and down rather too regularly. While these outcomes are interesting, and illustrate clearly how liquidity effects can generate exotic dynamics in house prices in rational expectations equilibrium, they do not display the stereotypical bubble pattern of a boom followed by collapse. Perhaps in conjunction with slowly moving real changes to preferences or technology these effects could generate more realistic time series, and this is worth exploring; we want instead to generate bubbles that look more like the data with no changes in fundamentals. One could perhaps also adapt the approach in Rocheteau and Wright (2010), introducing free entry of sellers into the DM, which can lead to multiple steady states, and try to construct equilibria where ψ_t grows for an extended period before collapsing. In the next Section, we take a different approach, not by making H look more like M, but by making it look less like M, when we allow it to be produced by the private sector.

5 Endogenous Housing Supply

Assume now that in the CM, in addition to the technology for converting ℓ_t into x_t, we introduce a technology for construction, with cost function $x_t = c(\Delta h_t)$. Thus, Δh_t units of new housing require an input of $c(\Delta h_t)$ units of numeraire. Construction, like other CM activity, is perfectly competitive. Hence, profit maximization implies

$$\psi_t = c' \left[h_{t+1} - (1-\delta)h_t\right], \tag{19}$$

equating price to the marginal cost of augmenting the supply from $(1-\delta)h_t$ to h_{t+1}, where δ denotes the depreciation rate on the existing stock. The households' CM problem is unchanged, except now $e_t = \psi_t (1-\delta) h_t$, and (15) becomes

$$(1+r)\,\psi_t = \psi_{t+1}(1-\delta) + U_2\left[x(h_{t+1}), h_{t+1}\right] + \alpha_h (1-\delta)\,\psi_{t+1}\mathcal{L}\left[(1-\delta)\,\psi_{t+1}h_{t+1}\right], \tag{20}$$

where as always $x(h)$ solves $U_1(x,h) = 1$ for a given h.

Steady state is characterized by two conditions,

$$(r+\delta)\,\psi = U_2\left[x(h),h\right] + \alpha_h(1-\delta)\,\psi\mathcal{L}\left[(1-\delta)\,\psi h\right] \tag{21}$$

$$\psi = c'(\delta h), \tag{22}$$

where (21) is a straightforward generalization of long-run demand (16), while (22) is a long-run supply relation. It is easy to show they intersect uniquely.[17]. As with H fixed, we can get a liquidity premium when the now endogenous supply of housing is high or when it is low, depending on elasticities. Therefore, we have:

Proposition 3 *The results in Proposition 1 continue to hold with h endogenous.*

Moving beyond steady states, equilibrium generally is defined by a path for (h_t, ψ_t) satisfying the dynamical system (19)-(20). One should anticipate the existence of interesting dynamic equilibria in this system, given the economics in the previous section. The mathematics is somewhat different, however, in a bivariate discrete-time system. One can imagine taking the limit as the period length gets small, or building from the ground up a continuous-time version of the model, and analyzing the system using standard differential equation methods. We leave that for future work, and instead use the discrete-time model to organize a particular narrative concerning recent events. As the story goes, at the start of the episode in question, financial innovation gave households much easier access to home equity loans: this is what Reinhart and Rogoff (2009) and Ferguson (2008) mean when they say financial developments allowed households to turn their previously illiquid housing into cash machines. We now show that this can indeed lead to dramatic house prices increase followed by a crash.

[17] Combine (21) and (22) to get

$$r+\delta = \frac{U_2\left[x(h),h\right]}{c'(\delta h)} + \alpha_h(1-\delta)\,\mathcal{L}\left[(1-\delta)\,c'(\delta h)\,h\right].$$

It is easy to check that the RHS goes to ∞ as h goes to 0, and vice-versa, and that it is stirctly decreasing.

Suppose at $t = 1$ the economy is at the unique steady state where housing is hard to use as collateral, in the sense that $D(e) = D_1 e$ with D_1 small. To make the point stark, in the CM at $t = 2$, an unexpected financial innovation occurs that enables households to more easily use home equity as collateral, so D_1 jumps. The resulting transition path depends on parameters. For some values, we get the analog of saddle-path stability: there is a unique equilibrium where ψ jumps at the change in D_1, and monotonically declines to its new steady state value, as supply increases to its new steady state value. For other parameters, the system displays a classic indeterminacy associated with a stable steady state: there are many transition paths starting from the initial steady state leading to the new one. Hence, ψ can jump at the change in D_1 to any value in some range before transiting to the new steady state. In the latter case, a particular transition path is shown in Figure 7, constructed under Walrasian pricing, and assuming parameters are such that the constraint $y \le D(e)$ is binding.[18]

We contend that the generated series in Figure 7 look like the portion of the data in Figure 1 starting just after 1999, as discussed back in the Introduction. We don't mean they look exactly the same, of course, since presumably the actual data were generated by something less extreme than a one-time surprise increase in D_1 (it may have taken time for financial developments to evolve, for agents to understand the, etc.). But they qualitatively similar. In particular, over the transition path, housing prices first soar then tumble, whether we measure them simply by the relative price in terms of numeraire ψ_t, or by the price-rent ratio. Home equity loans also rise dramatically, and stay up, as households take advantage of financial developments to relax credit constraints. Investment in new housing also rises, then drops, as we approach the new steady state. Home equity loans rise quickly, even if the housing stock increases relatively slowly, since $e_t = p_t h_t$ rises faster than h_t due to the price effects. The message is that the model can indeed generate price dynamics with the

[18] We verified that both eigenvalues are real and less than 1 at the new steady state in this example. Therefore, the transition path is monotone in the example.

characteristic boom and burst of a price bubble and shown in Figure 1. What is perhaps less obvious ex ante is that welfare increases over the period. The financial development in question, formalized as an increase in D_1, is good for the economy because it relaxes credit constraints, even though it can set us off on a path that resembles a housing bubble.

6 Intermediated Collateralized Lending

In the model, so far, households put up their home equity as collateral for consumption loans. More typically, in reality, they use home equity to secure cash loans from a bank or related institution, then use the cash to buy consumption goods. Here we model this explicitly, not only for the sake of realism, but to investigate the interaction between money and the housing. Money is the only means of payment in the DM in this version of the model due to the fact that buyers and sellers meet anonymously, as is standard in modern monetary economics (again, see the surveys cited in the Introduction). Sellers in the DM do not know the identity of individual buyers, and if a seller were to offer a buyer a loan collateralized by equity in some house, the latter could offer the former a claim on a nonexistent house, the house of someone else, one that is under water, etc. Nevertheless, buyers have relationships with their bankers: bankers can keep records of agents and know their identities.[19]

To simplify the presentation, in this version of the model, we fix the housing stock at H and set $\delta = 0$. Also, we use explicit preference or opportunity shocks. At the beginning of each period households receive a shock: with probability π they want or need to consume y, and with probability $1-\pi$ they do not. Before participating in the DM, households have access to a competitive financial market, or FM. Intermediaries called banks also participate in the FM. Households that want to

[19] This Section is motivated by Ferraris and Watanabe (2008) and Li and Li (2010). These are also models where assets and money are complimentary – since assets are used to secure money loans, the demand for assets can go down when money becomes less valuable, say due to an increase in inflation. By contrast, in many models money and assets are substitutes, because they provide different ways to facilitate trade (see, e.g., Lester, Postlewaite, and Wright (2008)), which would be the case here if some DM sellers accepted only currency while others made consumption loans secured by home equity.

consume in the DM may want to borrow money from banks to increase their purchasing power, and so we call them borrowers. Households that do not want to consume in the DM have no need for money and hence lend it to banks, so we call them depositors. The FM operates as a perfectly competitive, frictionless, market except we still maintain limited commitment: households are free to renege on their bank loans. However, banks can seize the house of borrowers who do not pay off their loans in the CM. This setup is similar to the banking model in Berentsen, Camera, and Waller (2007), and, at a more general level, to the literature on following Diamond and Dybvig (1983), where the role of intermediaries is to channel liquidity from those who have more than they need to those who need more than they have; the difference here is that home equity is required to secure bank loans.

The CM value function at t is now written $W_t(b_t, h_t, m_t)$, where b_t denotes the bank loans that replace consumer debt d_t, and m_t is cash. The FM value function next period is $J_{t+1}(m_{t+1}, h_{t+1})$, given that all loans are paid off in the current CM, which is without loss of generality. Then the CM problem is

$$W_t(b_t, h_t, m_t) = \max_{x_t, \ell_t, h_{t+1}, m_{t+1}} \{U(x_t, h_t) - \ell_t + \beta J_{t+1}(m_{t+1}, h_{t+1})\} \tag{23}$$

$$\text{st } x_t + \psi_t h_{t+1} + \phi_t m_{t+1} = \ell_t + \psi_t h_t + T_t + \phi_t m_t - (1+\rho_t)\phi_t b_t \tag{24}$$

where ϕ_t is the value of a dollar in terms of numeraire (the inverse of the price level) in the CM and ρ_t the interest rate on bank loans, which should be compared to (2)-(3). As usual, we can eliminate ℓ_t to write

$$\begin{aligned} W_t(d_t, h_t, m_t) &= \phi_t m_t + \psi_t h_t + T_t - (1+\rho_t)\phi_t b_t + \max_{x_t}\{U(x_t, h_t) - x_t\} \\ &\quad + \max_{h_{t+1}, m_{t+1}} \{-\psi_t h_{t+1} - \phi_t m_{t+1} + \beta J_{t+1}(m_{t+1}, h_{t+1})\}, \end{aligned}$$

showing W_t is linear in wealth, and we can derive the FOC

$$\beta \frac{\partial}{\partial m_{t+1}} J_{t+1}(m_{t+1}, h_{t+1}) = \phi_t \tag{25}$$

$$\beta \frac{\partial}{\partial h_{t+1}} J_{t+1}(m_{t+1}, h_{t+1}) = \psi_t, \tag{26}$$

showing h_{t+1} and m_{t+1} are independent of (d_t, h_t, m_t).

The FM value function satisfies

$$J_t(m_t, h_t) = \pi J_t^b(m_t, h_t) + (1-\pi) J_t^s(m_t, h_t), \tag{27}$$

where J_t^b and J_t^s are value functions for borrowers and depositors. For borrowers

$$\begin{aligned} J_t^b(m_t, h_t) &= \max_{\hat{m}_t, b_t} \{\alpha_h [u(y_t) - \phi_t \hat{m}_t] + W_t(\hat{m}_t, b_t, h_t)\} \\ \text{st } m_t + b_t &= \hat{m}_t \text{ and } (1+\rho_t)\phi_t b_t \leq D(\psi_t h_t), \end{aligned} \tag{28}$$

where again α_h is the probability a borrower trades in the DM, y_t is determined by $g(y_t) = \phi_t \hat{m}_t$ and $D(\psi_t h_t)$ is the limit on borrowing from the bank based on home equity. Symmetrically, for depositors we have

$$\begin{gathered} J_t^s(m_t, h_t) = \max_{\hat{m}_t, b_t} W_t(b_t, h_t, \hat{m}_t) \\ \text{st } m_t + b_t = \hat{m}_t \text{ and } (1+\rho_t)\phi_t b_t \leq D(\psi_t h_t), \end{gathered} \tag{29}$$

but since $\rho_t \geq 0$ we can set $\hat{m}_t = 0$ and $b_t = -m_t$, implying $J_t^s(m_t, h_t) = W_t(-m_t, h_t, 0)$.

Deriving $\partial J_{t+1}/\partial m_{t+1}$ from the above equations and combining this we (25), we get (see the Appendix for more details)

$$\phi_t = \beta\pi\phi_{t+1} + \beta(1-\pi)\left(1+\rho_{t+1}\right)\phi_{t+1} + \beta\alpha_h\pi\mathcal{L}(y_{t+1})\phi_{t+1}, \tag{30}$$

which is similar to the standard Euler equation for money. As regards the Euler equation of housing, we distinguish two cases: If the borrowing constraint in FM is not binding, we have

$$\psi_t = \beta\psi_{t+1} + \beta U_2(x_{t+1}, h_{t+1}),$$

and housing must be priced fundamentally. If the borrowing constraint binds, however, the Euler equation is

$$\psi_t = \beta\psi_{t+1} + \beta U_2\left(x_{t+1}, h_{t+1}\right) + \beta\alpha_h\pi\left[\mathcal{L}\left(y_{t+1}\right) - \rho_{t+1}\right]\frac{\psi_{t+1}}{1+\rho_{t+1}}. \tag{31}$$

The price of housing today is set to the discounted value of three terms: the price tomorrow; the marginal utility of the house; and the expected liquidity value, which is the probability one makes a DM purchase $\alpha_h\pi$, times the liquidity premium $\mathcal{L}\left(y_{t+1}\right)$ because more housing relaxes the borrowing constraint at the bank, net of the bank interest rate.

We look for symmetric steady state equilibrium. Two conditions are relevant. One is the individual debt limit – can individuals in FM borrow as much as they want? If $D\left(e\right)$ is low, borrowers are constrained in the FM, and housing bears a liquidity premium. The other is the aggregate debt limit – are there enough deposits to satisfy borrowers? If there are then FM does not clear at $\rho > 0$, and so $\rho = 0$. We therefore have three cases according to which constraints bind: (1) The aggregate and individual debt limits both binding; (2) the individual debt limit binds but not the aggregate limit; and (3) both are not binding.[20] Since the outcome depends on $D\left(\psi_t h_t\right) = D_0 + D_1 e$, of course, we now partition $\left(D_0, D_1\right)$ space into regions according to which case obtains.

To this end, let $\tilde{y}$ and $\bar{y}$ satisfy $\alpha_h\pi\mathcal{L}\left(\tilde{y}\right) = i$ and $\alpha_h\mathcal{L}\left(\bar{y}\right) = i$,respectively, and define two functions B_1 and B_2 as

$$\begin{aligned} B_1\left(D_0\right) &= \begin{cases} \dfrac{r\left[g\left(\tilde{y}\right)\left(1-\pi\right) - D_0\right]}{ig\left(\tilde{y}\right)\left(1-\pi\right) - iD_0 + HU_2} & \text{if } D_0 < g\left(\tilde{y}\right)\left(1-\pi\right) \\ 0 & \text{if } D_0 > g\left(\tilde{y}\right)\left(1-\pi\right) \end{cases} \\ B_2\left(D_0\right) &= \max\left\{\frac{r\left[g\left(\bar{y}\right)\left(1-\pi\right)\left(1+i\right) - D_0\right]}{HU_2}, 0\right\}. \end{aligned}$$

As shown in Figure 8, $B_2 \geq B_1$, the functions are decreasing in D_0 and they partition $\left(D_0, D_1\right)$ space into 3 regions, each of which corresponds to one of the 3 cases described above. In particular, for small D_0 and D_1, we are in Case 1, where there is more cash that borrower can borrow given

[20] The case where the individual debt limit does not bind and the aggregate binds is not possible.

the constraints, so $\rho = 0$ in the FM. As D_0 and D_1 increase, we move to Case 2, where all deposits can be lent to borrowers, but borrowers are still constrained in the sense that they cannot borrow enough to purchase y^* in the DM. In this case, $\rho \in (0, i)$, which means bank deposits pay and bank loans charge positive interest ρ, but less than the nominal rate i defined above in terms of illiquid assets (nominal claims between one CM and the next CM). Also, in this case, as inflation increases house prices can either go up or down, depending on parameters. As D_0 and D_1 increase further, we get to Case 3, where there is enough liquidity so that borrowers are not constrained in FM – they can get all the cash they want.

We summarize these results as follows.

Proposition 4 *There is a unique steady state equilibrium, and it satisfies:*

1. *if* $B_1(D_0) > D_1$ *then* $\rho = 0$*, Also,* $\psi = U_2/(r - iD_1)$*, so* $\partial\psi/\partial i > 0$.
2. *if* $B_1(D_0) < D_1 < B_2(D_0)$ *then* $\rho \in (0, i)$*. Also,* $\partial\psi/\partial i$ *is ambiguous, although* $\partial\psi/\partial i < 0$ *if* $g(y)[\alpha_h \mathcal{L}(y) + 1]$ *increasing in* y.
3. *If* $B_2(D_0) < D_1$ *then* $\rho = i$*, Also,* $\psi = U_2/r$ *is the fundamental price, so* $\partial\psi/\partial i = 0$.

In Case 1, inflation drives up the house price ψ. The intuition is as follows. Inflation makes money less valuable, but because there is idle cash in the economy, ρ does not adjust. With the same amount of home equity, borrowers can borrow the same amount of real balances. Therefore, households want to hold home equity instead of money, leading to a higher housing price. In Case 2, the effect of inflation on ψ is ambiguous, but it decreases with inflation under the condition given in the Proposition, which is analogous to a result Li and Li (2010) for financial assets, not housing, generalized in the sense that they only consider Walrasian pricing. Housing is priced fundamentally in Case 3. Households have no incentive to carry money in this case, because they can always borrow enough in FM if they turn out to be borrowers, so $\rho = i$, to make households indifferent between holding money and not holding money.

Other results can be derived – e.g., one can also show that in all the three cases, DM consumption y decreases with inflation, and that the Friedman rule $i = 0$ is the optimal monetary policy, as in many models. One can derive similar results with an endogenous housing supply. We leave further exploration of this type of model to future work, since this project is more about nonlinear dynamics and housing price bubbles. The goal of this Section is simply to show that once we have a liquidity-based theory of the housing market, where home equity is used to collateralize loans, one can naturally analyze interactions between housing markets and money, and hence one can study connections between house prices and monetary policy in a different light – i.e., different from most existing research, including all the papers mentioned in the Introduction.

7 Conclusion

This object of this project was to study economies where housing, in addition to providing utility as shelter, can also be used as collateral to facilitate transactions in imperfect credit markets. This implies that in equilibrium houses can bear a liquidity premium, meaning that they are priced above the present value of the marginal utility from living in the house. This further implies that house prices can display a variety of interesting dynamic equilibrium paths, some of which look like bubbles, even though agents are fully rational. Intuitively, this follows from the idea that liquidity is at least to some extent a self-fulfilling prophecy, which makes houses somewhat similar to money. However, houses are also very different from fiat currency, because they generate direct utility, and they can be produced by the private sector. We though it would be interesting to analyze the housing market from this perspective, mainly because it is consistent with the data on house prices, home equity loans and housing investment.

In a sense, what did is simply formalize ideas in Reinhart and Rogoff (2009), Ferguson (2008) and others, about financial developments leading to a bigger role for home equity in credit markets, fueling an increase in housing demand and leading to a big increase in prices in the shorter run and quantity in the longer run. While there is much talk about bubbles in housing markets, in our view is that there are not a lot of fully satisfactory formal models. We provide a candidate that is based on received theory in macro and monetary economics, that is tractable, and that appears able to match at least the stylized facts of housing bubbles. It is easy to get equilibria where house prices are above their fundamental value, and fluctuate either cyclically, chaotically or stochastically, and it is easy to get dynamic transition paths that resemble qualitatively recent experience. We think this is useful. Perhaps future work can investigate whether a liquidity-based theory can generate equilibria that do well quantitatively.

Appendix A

Here we solve the bargaining game in Section 2. The first observation is that if (off the equilibrium path) bargaining were to go to Stage 3 and the buyer gets to make the final take-it-or-leave-it offer, he would offer $(\bar{y}, \bar{d})$ where:

$$\bar{y} = \begin{cases} v^{-1}(D_t) & \text{if } D_t < v(y^*) \\ y^* & \text{if } D_t \geq v(y^*) \end{cases} \quad \text{and } \bar{d} = \begin{cases} D_t & \text{if } D_t < v(y^*) \\ v(y^*) & \text{if } D_t \geq v(y^*) \end{cases}$$

Now there are four possible cases: 1) the constraint $d \leq D$ is slack at the initial and the final offer stage; 2) it binds in the initial but not the final offer stage; 3) it binds in both; and 4) it binds in the final but not the initial offer stage. It is easy to check that case 4 cannot arise, so we are left with three.

Case 1: In the final offer stage, if the buyer proposes, his problem is

$$\max_{y,d} \{u(y) - d\} \text{ st } d = v(y),$$

with solution $y = y^*$ and $d = v(y^*)$. If the seller proposes the buyer gets no surplus, so the buyer's expected surplus before the coin flip is $\theta [u(y^*) - v(y^*)]$. Therefore, in the initial offer stage, the seller's problem is

$$\max_{y,d} \{d - v(y)\} \text{ st } u(y) - d = \theta [u(y^*) - v(y^*)],$$

with solution $y = y^*$ and $d = d^* = (1-\theta) u(y^*) + \theta v(y^*)$. Since $d^* > v(y^*)$, this case occurs iff $D > d^*$.

Case 2: The buyer's expected payoff before the coin flip is again $\theta [u(y^*) - v(y^*)]$, but at the initial offer stage the constraint binds, so the seller solves

$$\max_{y} \{D - v(y)\} \text{ st } u(y) - D = \theta [u(y^*) - v(y^*)].$$

The solution satisfies $u(y) = D + \theta [u(y^*) - v(y^*)]$ and $d = D$. This case occurs iff $v(y^*) < D < d^*$.

Case 3: In the final offer stage, if the buyer proposes, his problem is

$$\max_{y} \{u(y) - D\} \text{ st } D = v(y).$$

This implies $y = v^{-1}(D)$, and his expected surplus before the coin flip is $\theta \left[u \circ v^{-1}(D) - D\right]$. At the initial offer stage, the seller's problem is

$$\max_{y} \{D - v(y)\} \text{ st } u(y) - D = \theta \left[u \circ v^{-1}(D) - D\right].$$

The solution satisfies $u(y) = \theta u \circ v^{-1}(D) + (1-\theta) D$ and $d = D$. This case occurs iff $D < v(y^*)$ and $D < u(y^*) - \theta u \circ v^{-1}(D) + \theta D$, the last inequality coming from the observation that, at the first stage, if the constraint is slack, the buyer pays $u(y^*) - \theta u \circ v^{-1}(D) + \theta D$ to get y^*. This last inequality is equivalent to $(1-\theta) D < u(y^*) - \theta u \circ v^{-1}(D)$, which always holds if $D < v(y^*)$.

To sum up, $d = D$ if $D < d^*$, and otherwise $d = d^*$; and y is given by

$$y = \begin{cases} u^{-1}\left[\theta u \circ v^{-1}(D) + (1-\theta) D\right] & \text{if } D < v(y^*) \\ u^{-1}\left[D + \theta\left[u(y^*) - v(y^*)\right]\right] & \text{if } v(y^*) < D < d^* \\ y^* & \text{if } D > d^* \end{cases} .$$

If we look at the derivative dy/dD, we have

$$\frac{dy}{dD} = \begin{cases} \frac{\theta u'\left[v^{-1}(D)\right] + (1-\theta) v'\left[v^{-1}(D)\right]}{u'(y) v'(v^{-1}(D))} & \text{if } D < v(y^*) \\ \frac{1}{u'(y)} & \text{if } v(y^*) < D < d^* \\ 0 & \text{if } D > d^* \end{cases} .$$

Therefore, we get $y = g^{-1}(D)$ as a differentiable and strictly increasing function of D for $D < d^*$.

Appendix B

Here we verify Proposition 4. The following characterize steady state equilibrium:

$$\begin{aligned} i &= \alpha_h \pi L(y) + (1-\pi)\rho \\ r\psi &= \alpha_h \pi L(y) \frac{\psi D_1}{1+\rho} - \pi \frac{\rho \psi D_1}{1+\rho} + U_h(x(H), H) \\ g(y) &= \phi_t M_t + \frac{D_0 + D_1 \psi H}{1+\rho}. \end{aligned}$$

(1) If $\rho > 0$, depositors are willing to put all their money in the bank. Since the money market clears and the borrowing constraint is binding, we must have $\pi(D_0 + D_1 \psi h) = (1-\pi)(1+\rho)\phi_t M_t$. Therefore, equilibrium is characterized by

$$i = \alpha_h \pi L(y) + (1-\pi)\rho, \tag{32}$$

$$r\psi = \alpha_h \pi L(y) \frac{\psi D_1}{1+\rho} - \pi \frac{\rho \psi D_1}{1+\rho} + U_2[x(H), H], \tag{33}$$

$$g(y) = \frac{D_0 + D_1 \psi H}{(1+\rho)(1-\pi)}. \tag{34}$$

Solving for ρ, using (32), we get $\rho = (i - \alpha_h \pi \mathcal{L}) / (1-\pi)$. Then (34) gives

$$\psi = \frac{g(y)\left[1 + i - \pi - \pi \alpha_h \mathcal{L}(y)\right] - D_0}{D_1 H}.$$

Substituting these into (33), we get

$$\frac{r}{D_1} = \frac{\pi\left[\alpha_h \mathcal{L}(y) - i\right]}{1 + i - \pi\left[1 + \alpha_h \mathcal{L}(y)\right]} + \frac{H U_2(x(H), H)}{g(y)\left[1 + i - \pi - \pi \alpha_h \mathcal{L}(y)\right] - D_0} = \Phi(y). \tag{35}$$

The RHS is decreasing in y, given $\mathcal{L}' < 0$, so there is at most one solution. Since $i > \rho > 0$, this equilibrium exists iff (35) has a solution in $(\tilde{y}, \bar{y})$, where $\mathcal{L}(\tilde{y}) = i/\alpha_h \pi$ and $\mathcal{L}(\bar{y}) = i/\alpha_h$. This requires $\Phi(\tilde{y}) > r/D_1$ and $\Phi(\bar{y}) < r/D_1$, or

$$\frac{HU_2}{g(\bar{y})(1-\pi)(1+i)-D_0} < \frac{r}{D_1} < i + \frac{HU_2}{g(\tilde{y})(1-\pi)-D_0}.$$

One can derive

$$\begin{aligned}
\frac{\partial y}{\partial i} &\approx -\frac{gD_1(\alpha_h\mathcal{L}(y)+1)\sigma}{\psi(1+\rho)^2} - \frac{gU_h}{(1+\rho)\psi^2} < 0, \\
\frac{\partial \rho}{\partial i} &\approx -\frac{\alpha_h\pi L' D_1 g}{(1+\rho)\psi} + \frac{U_2(x(H),H)g'}{\psi^2} > 0, \\
\frac{\partial \psi}{\partial i} &\approx -D_1\pi(g\alpha_h\mathcal{L}' + g(\alpha_h\mathcal{L}(y)+1)) \approx -\frac{\partial}{\partial y}[\alpha_h\mathcal{L}(y)+1]g(y).
\end{aligned}$$

Therefore, if $[\alpha_h\mathcal{L}(y)+1]g(y)$ then $\partial\psi/\partial i < 0$.

(2) If $\rho = 0$, the equilibrium is characterized by

$$\begin{aligned}
i &= \alpha_h\pi L(y) &(36)\\
r\psi &= \alpha_h\pi L(y)\psi D_1 + U_h(x(H),H) &(37)\\
g(y) &\geq \frac{D(\psi H)}{1-\pi}. &(38)
\end{aligned}$$

Following the same argument as above, this equilibrium exists iff

$$\frac{r}{D_1} \geq i + \frac{HU_h}{g(\tilde{y})(1-\pi)-D_0}.$$

It is obvious in this case that $\partial y/\partial i < 0$, $\partial\psi/\partial i > 0$.

(2) Suppose the borrowing constraint is not binding. The FOC from (28) gives $\rho_t = \alpha_h\mathcal{L}(y_t)$. In steady state,

$$\begin{aligned}
i &= \alpha_h\pi L(y) + (1-\pi)\rho \\
r\psi &= U_2[x(H),H] \\
g(y) &< \frac{D_0 + D_1\psi H}{1-\sigma}.
\end{aligned}$$

The last equation comes from two observations: when $\rho > 0$, to clear the money market we must have $g(y) = 1\phi_t M_t/1-\pi$; and when the borrowing constraint is not binding, $(1-\pi)\phi_t M_t < \sigma(D_0 + D_1\psi H)$. In addition, we have $i = \rho$ and $\alpha_h\sigma L(y) = i$. Therefore this equilibrium exists iff $g(\bar{y}) < (D_0 + D_1\psi H)/(1-\pi)$ with $\psi = U_h/r$, or

$$\frac{r}{D_1} < \frac{HU_h}{g(\bar{y})(1-\pi)(1+i)-D_0}.$$

It is obvious that $\partial\psi/\partial i = 0$ and $\partial y/\partial i < 0$.

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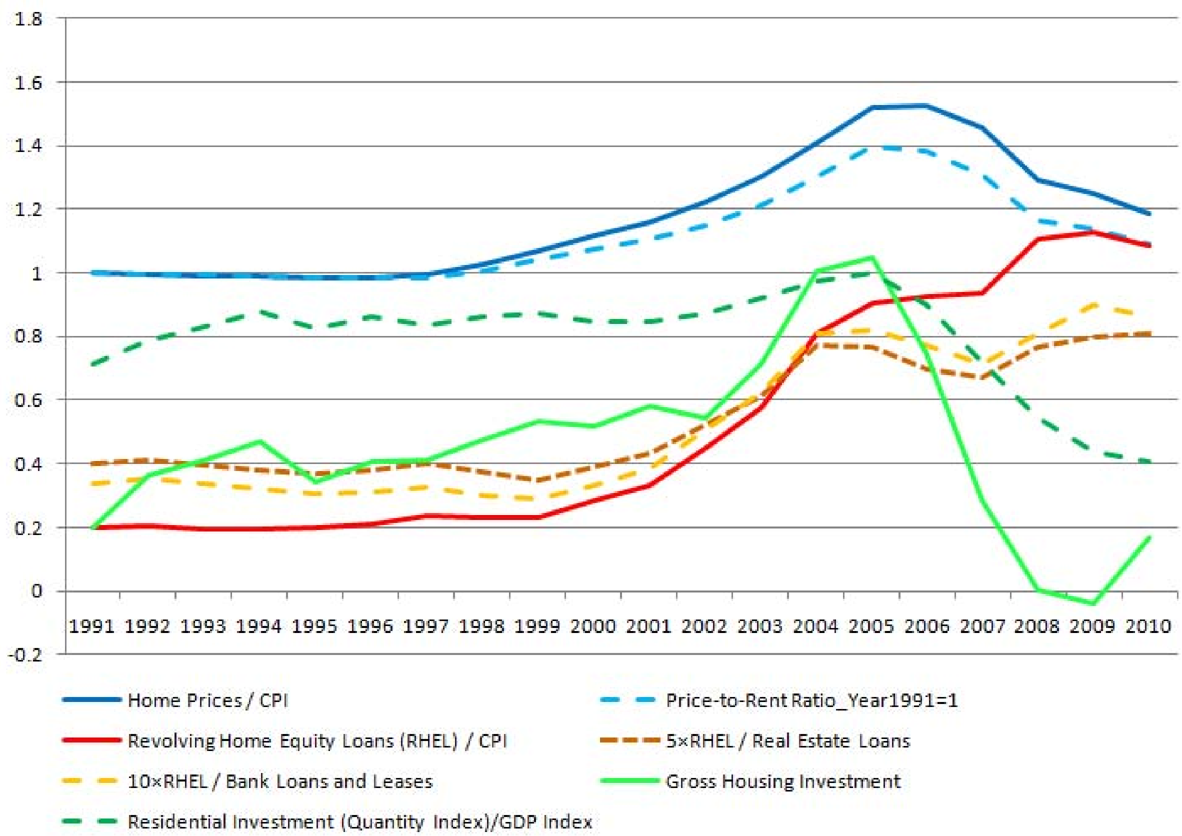
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
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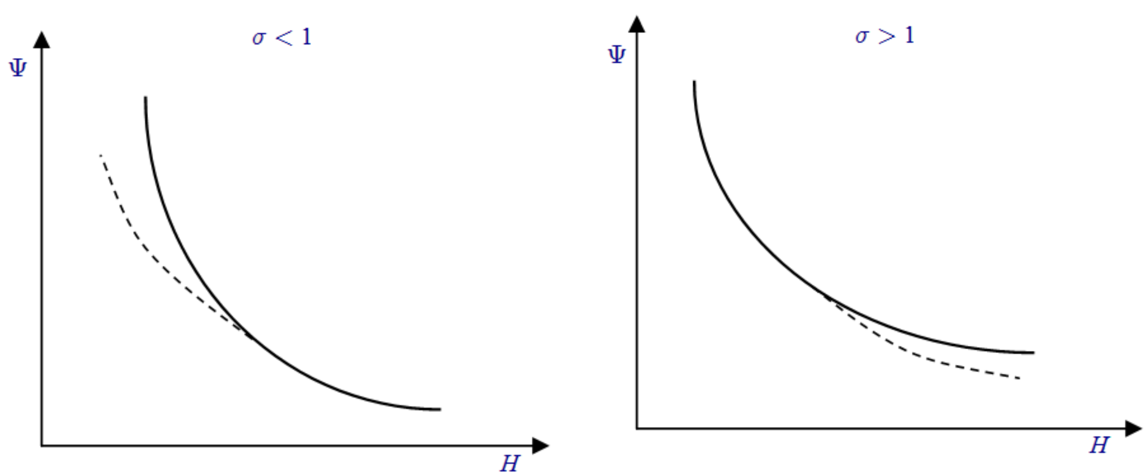
0
-0.2
1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010
Home Prices / CPI
Price-to-Rent Ratio_Year1991=1
Revolving Home Equity Loans (RHEL) / CPI
5×RHEL / Real Estate Loans
10×RHEL / Bank Loans and Leases
Gross Housing Investment
Residential Investment (Quantity Index)/GDP Index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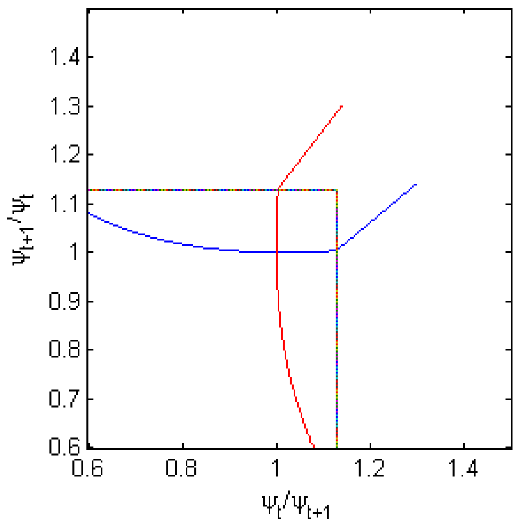
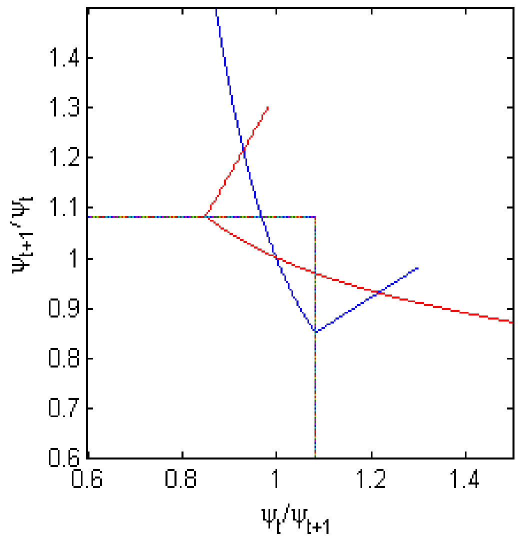
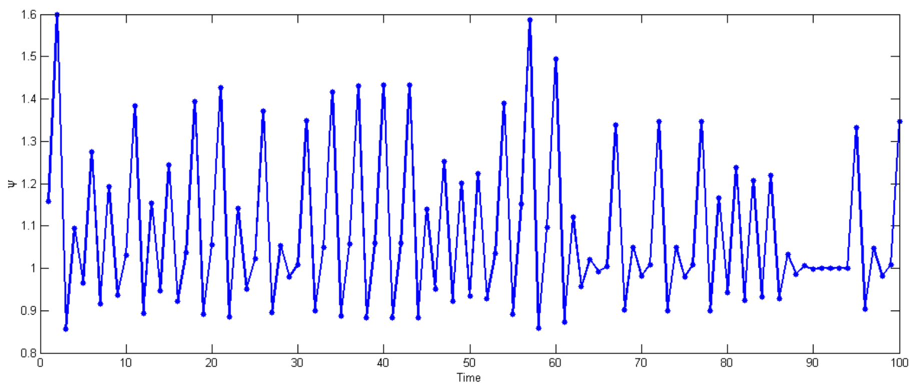


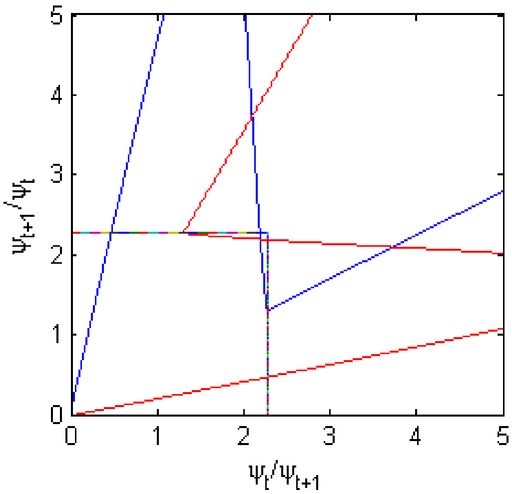
Example 1



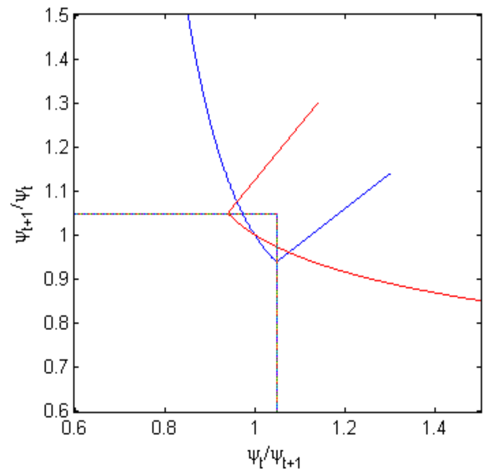
Example 2



Example 2: Chaotic Dynamics



Example 3



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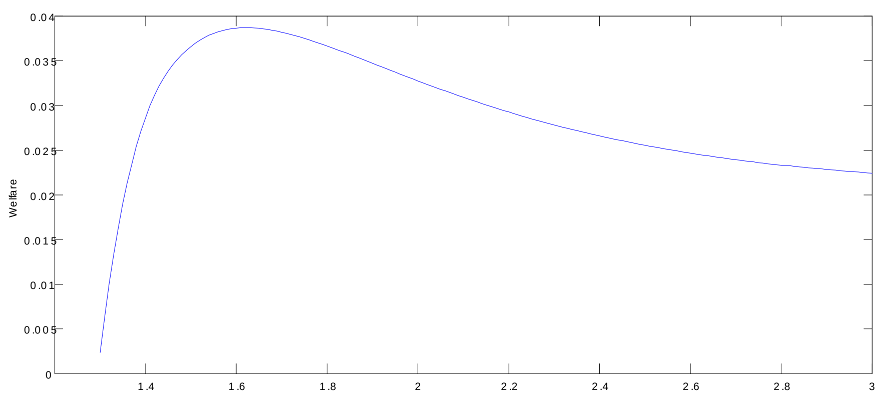
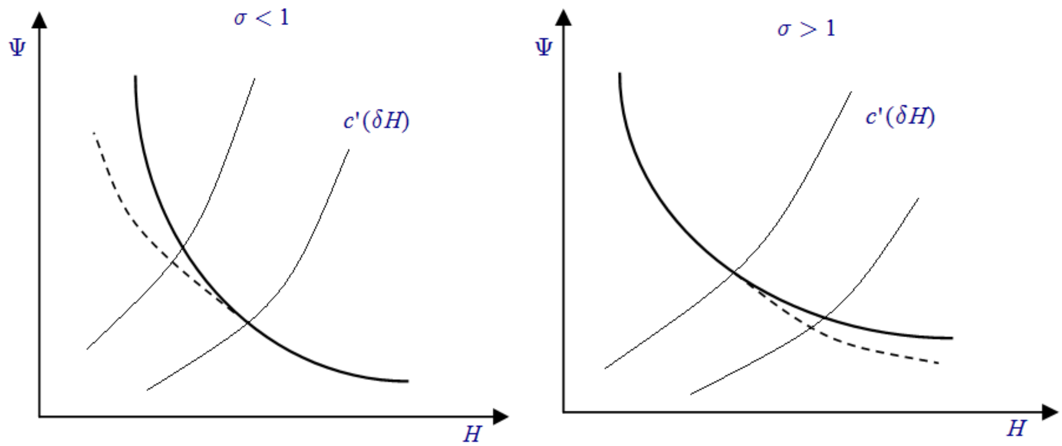
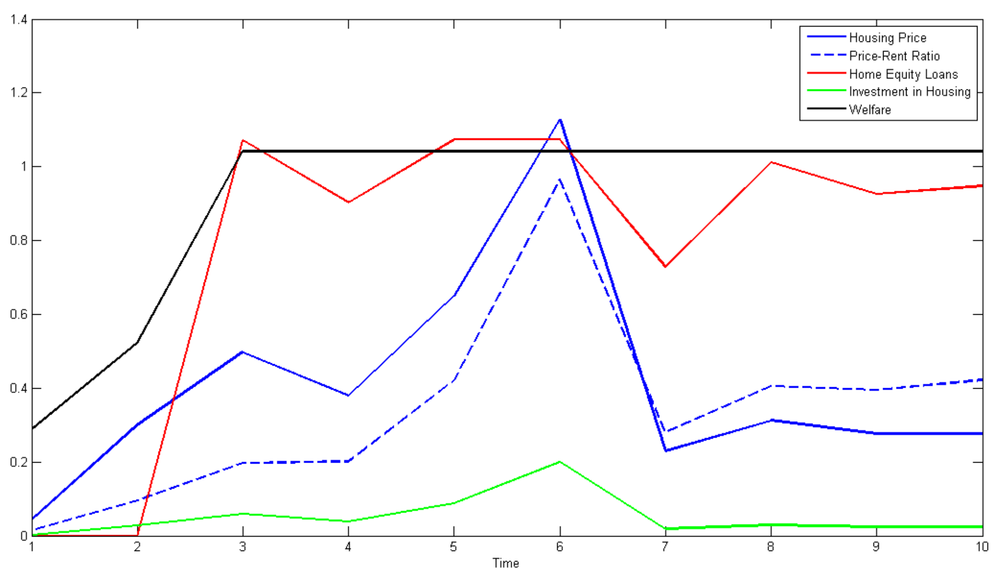


Example 5





$$\alpha_h = 0.5, \beta = 0.6, \gamma = 8, \sigma = 3, \eta = 3.52, \kappa = 1/3$$