Agglomeration externalities and the dynamics of firm location choices*

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Abstract

We develop a new dynamic general equilibrium model of firm location choice that can explain the observed sorting of firms by productivity and is consistent with the observed entry, exit, and relocation decisions of firms within an urban economy. We discuss existence of equilibrium of and characterize the stationary distribution of firms in each location. The parameters of the model can be estimated using a nested fixed point algorithm. We implement the estimator using data collect by Dunn and Bradstreet for the Pittsburgh metropolitan area. The data suggest that firms located in the city are older and larger than firms located outside the urban core. As a consequence they use more land and labor in the production process. However, they face higher rental rates for land and office space which implies that they operate with a higher employee per land ratio. We find that our model explains these observed features of the data well. Finally, we consider the impact of different relocation policies that provide targeted subsidies to new start-ups and superstar firms.

1 Introduction

Cities and metropolitan areas play in an important role in the economy since economic proximity makes for more efficient production and trade.[1] These efficiency gains typically arise because of agglomeration externalities resulting from synergies between firms in the same industry. In particular, firms may benefit from lower transaction costs or sharing of a common labor pool. (Marshall, 1890). Alternatively, efficiencies may arise due to positive diversity externalities and synergies between different industries (Jacobs, 1969). Firms that operate in locations with high externalities, therefore, have a competitive advantage over firms that are located in less efficient locations.[2] Since firms will bid for the right to locate in areas with high agglomeration externalities, these locations have higher land values than locations that are less efficient.[3] As a consequence, firms with different productivity levels will sort in equilibrium with high productivity firms locating in areas with high agglomeration effects and high rents. Low productivity firms are forced to exit the economy or operate in cheaper locations.[4]

As the productivity of a firm changes over time, a firm's demand for land and labor changes as well. Moreover, productivity shocks create incentives to relocate within the city to exploit a better match with the agglomeration externalities. A firm that may have initially located in the suburbs may find it in its interest to move to a more densely populated central business district in order to grow and capture the

[1]The idea of geographic returns to scale was first introduced by von Thünen (1825).

[2]Krugman (1991) provides theoretical foundations for a two-location model of agglomeration. Ellison and Glaeser (1997) argued that agglomeration externalities are important to understand geographic concentration of manufacturing in the U.S. The literature of agglomeration theory is reviewed in Fujita and Thisse (2002) and Duranton and Puga (2004).

[3]Anas and Kim (1996) and Lucas and Rossi-Hansberg (2002) have developed equilibrium models of mono- and poly-centric urban land use with endogenous congestion and job agglomeration. Rossi-Hansberg (2004) studies optimal land use policies in a similar framework.

[4]While we use the terms "firm" and "establishment" interchangeably, our unit of analysis in the empirical section is an establishment.

full benefits of a persistent positive productivity shock. Similarly, a firm that has experienced a persistent negative shock may find it in its interest to downsize and move to the urban fringe where land and labor is cheaper than in the city. [5] The first objective of this paper is then to develop a new dynamic general equilibrium model of firm location choice that can explain the observed sorting of firms by productivity and is consistent with the observed entry, exit, and relocation decisions of firms in an urban economy.

We consider an urban economy with two distinct locations. We can interpret the two locations as the Central Business District and the rest of the metropolitan area.[6] In equilibrium these locations differ in the magnitude of their agglomeration externalities.[7] The latter increase with employment density and increase the productivity of firms. Firms are heterogeneous in their productivities. We model firm dynamics and industry equilibrium following Hopenhayn (1992). Firms enter our urban economy with an initial productivity and must pay an entry cost. Productivity then evolves according to stochastic first order Markov process. Each period firms compete in the product market, must pay a fixed cost of operating, and realize a profit. Entry,

[5]There is some evidence that shows that agglomeration effects are important to understand firm dynamics. Henderson, Kunkoro, and Turner (1995) show that agglomeration effects for mature industries are related to Marshall scale economies, while newer industries benefit from diversity akin to Jacobs economies. This work is important because it points to agglomeration as part of a dynamic process. Other research has continued to study the relevance of agglomeration in firm life-cycle dynamics. Duranton and Puga (2001) study the the effect of agglomeration externalities in innovation and the development of production processes, while Dumais, Ellison, and Glaeser (2002) examine the effect of firm dynamics (entry, exit, expansion, and contraction) on the concentration of economic activity.

[6]An alternative interpretation of our model is that we focus on two cities with different externalities.

[7]Deckle and Eaton (1999) find that geographic scale of agglomeration is mostly at the national level, while the financial sector is concentrated in specific metropolitan areas. Other work finds that agglomeration can occur on a much more local scale. In particular, Rosenthal and Strange (2001, 2003) establish the level and type of agglomeration at different geographic scales, and also the measure the attenuation of these externalities within metropolitan areas. Holmes and Stevens (2002) finds evidence of differences in plant scale in areas of high concentration, suggesting production externalities act on individual establishments. A review of empirical evidence of agglomeration economies is found in (Rosenthal and Strange, 2004).

exit and relocations are dynamic and based on expectations of future productivity shocks. Using recursive methods, we can characterize the optimal decision rules for firms in each location as well as those for potential entrants. Low productivity firms exit from the economy, while high productivity firms continue to operate. Relocation choices are driven by the interaction of agglomeration effects and firm productivity shocks. Due to a minimum land requirement in the production function, large firms with higher productivity shocks prefer locations with high agglomeration externalities relative to smaller, less productive firms. As a consequence, a high productivity firm that is located outside the central business district may have strong incentives to relocate to the city center.[8]

We define the stationary equilibrium of our model and characterize the stationary distribution of firms in equilibrium.[9]

T he second objective of the paper is to estimate the parameters of our model and determine whether our model can explain the observed sorting of firms in one metropolitan area. We focus on equilibria with entry in both locations since this is a common feature of the data. The parameters of the model can then be estimated using a nested fixed point algorithm. The inner loop computes the equilibrium for each parameter value, while the outer loop searches over feasible parameter values. Our simulated method of moments estimator matches the observed distribution of firms by age, size and land use by location to the one predicted by our model.[10]

[8]There are some similarities with the literature that studies the sorting pattern of household in urban areas which starts with the classic papers by Alonso (1964), Mills (1967), and Muth (1969).

[9]Related to our research is also work by Melitz (2003) who studies the impact of trade on intra-industry relocations. Rossi-Hansberg and Wright (2007) examine the relationship of establishment scale and entry and exit dynamics. Finally, Combes, Duranton, Gobillon, Puga, and Roux (2010) distinguish between selection effects and productivity externalities by estimating productivity distributions across cities.

[10]In related work, Davis et al. (2009) develop a growth model in which the total factor productivity of cities depends on the density of economic activity. They estimate the magnitude of this external effect and evaluate its importance for the growth rate of consumption per capita in the U.S.

We implement the estimator using data collected by Dunn and Bradstreet for the Pittsburgh metropolitan area. Since regional cities often act as a hub for services for a larger region, we focus on locational choices within that sector. The data suggest that firms located in the city are older and larger than firms located in the rest of the metro area. As a consequence they use more land and labor in the production process. However, they face higher rental rates for land and office space. Thus, they operate with a higher employee per land ratio. We find that our model explains these observed features of the data reasonably well. The parameter estimates have the expected sign and are highly significant. Using the estimate model, we perform a number of policy experiments. We consider the impact of different relocation policies that provide targeted relocation subsidies to firms. Since relocation costs are large in our baseline model, we find that policies that fully subsidize firm relocations have potentially large effects on economic growth and firm concentration in central business districts.

The rest of the paper is organized as follows. Section 2 provides some stylized facts that characterize firm location choices within U.S. cities. Section 3 develops our stochastic, dynamic equilibrium model and discusses its properties. Section 4 describes the estimation of the parameters of our model. Section 5 describes the data set used in our application. Section 6 presents the empirical results and discusses the policy experiments. Section 7 offers some conclusions that can be drawn from the analysis.

2 Firm Location Choices in U.S. Cities

To get some quantitative insights into firms sorting behavior, we collected Census data for a number of metro areas. We are mostly interested in characterizing the sorting of establishments by age, employment, and facility size. We define a business district within a metropolitan area as those zip codes within a city that have a high density of firms signifying local agglomeration. To make this concept operational, we use an employment density of at least 10,000 employees per square mile. These locations need not be contiguous, as some metropolitan areas exhibit multiple dense business districts.

We pay special attention to service industries, given that there is strong evidence that large U.S. cities have undergone a transformation over the past decades moving from centers of individual sectors toward becoming hubs for service industries. Duranton and Puga (2005), for example, show evidence that cities have become more functionally specialized, with larger cities, in particular, emerging as centers for headquarters and business services. They posit that this change is primarily related to industrial structure, and a decrease in remote management costs in particular. In addition, Davis and Henderson (2008) provide further evidence that services and headquarters are indeed more concentrated in large cities relative to the entire economy, and that headquarter concentration is linked to availability of diverse services.

We exclude wholesale and retail businesses from our analysis of services. Hotelling and others have shown that retail locational decisions are primarily driven by proximity to costumers (Hotelling, 1929).[11] For similar reasons, we also do not consider businesses in the entertainment sector. Finally, we omit businesses related to agriculture, forestry, mining and fishing for fairly obvious reasons. We thus define the service sector as consisting of businesses that operate in information, finance, real estate, professional services, management, administrative support, education, health care and related services. We find evidence that the concentration of services is also prevalent at a much more local level, with the service industry choosing to locate in dense business districts.

[11]See Bresnahan and Reiss (1991) and Holmes (2010) for some structural empirical studies of retail location.

Table 1: Concentration of employment in dense business districts

MSA	Total Emp. Outside CBD	Total Emp. in CBD	Avg. Emp. outside CBD	Avg. Emp. in CBD	% Services outside CBD*	% Services in CBD**
Atlanta	1,115,398	229,002	15.79	29.25	45.24%	63.31%
Boston	1,728,075	531,349	15.66	39.01	41.99%	59.90%
Chicago	3,070,387	528,529	15.86	24.47	41.85%	66.50%
Columbus	705,534	63,278	18.69	23.73	42.88%	58.64%
Hartford	499,718	18,783	17.26	26.95	40.31%	61.41%
Houston	1,720,625	286,574	16.38	28.47	42.86%	65.51%
Jacksonville	491,959	24,315	15.24	25.38	43.09%	66.28%
Los Angeles	4,257,269	974,693	15.02	19.39	44.16%	52.39%
Philadelphia	1,921,626	196,428	15.91	27.66	43.99%	55.74%
Phoenix	1,551,921	64,793	18.31	27.78	47.79%	71.01%
Pittsburgh	822,013	157,009	14.58	40.04	39.16%	60.90%
Salt Lake	440,239	53,086	15.22	21.08	45.64%	58.90%
San Antonio	655,740	26,572	17.21	20.49	43.22%	56.59%
Seattle	1,260,335	179,230	14.55	20.33	42.07%	58.97%
St Louis	1,253,959	84,034	16.38	42.57	41.41%	52.43%
Wash. DC	1,930,848	303,770	15.42	21.68	49.96%	60.05%

Source: 2006 Zip Code Business Patterns, U.S. Census
*Percentage of establishments outside the CBD that are in the service industries (NAICS 51-62)
**Percentage of establishments in the CBD that are in the service industries (NAICS 51-62

Table 1 shows the concentration of employment in dense business districts for a sample of U.S. cities. First, we report statistics using all firms that are located in the metro area. We find that there is a significant amount of heterogeneity among the cities in our sample. There are some cities such as Phoenix and Hartford where employment is not concentrated in dense business districts. Most larger cities in the U.S. such as New York, Los Angeles, Chicago, Boston, Washington, Philadelphia, and Houston have a significant fraction of firms located in high density central business districts. This finding is also true for a variety of mid-sized cities such as Pittsburgh and Seattle. Focusing on the differences between firms located in and out of the CBD, we find firms in the CBD are larger than the MSA average. This indicates that they have higher levels of productivity. This finding is common among all cities in our sample. In addition, firms in the service sector are more concentrated in the CBD compared to firms in general, suggesting that service oriented firms benefit more from local agglomeration than other sectors.

3 Theory

3.1 A Dynamic Model of Firm Location

We consider a model with two locations, denoted by $j = 1, 2$. There is a continuum of firms that produce a single output good and compete in the product market.[12] In each period a firm chooses to stay where it is, relocate to the other location, or shutdown. Firm are heterogeneous and productivity evolves according to a stochastic law of motion.

Assumption 1 *In each period a firm is subject to an exogenous probability of exiting. We denote by ξ the complement probability of a firm surviving into the next period. If the firm survives, it draws a new productivity shock, φ' each time period. The productivity shock evolves over time according to a Markov process with a conditional distribution $F(\varphi'|\varphi)$.*

In our parametrized model, we assume that the logarithm of the productivity shock follows an AR(1) process, i.e. $\log(\varphi)' = \rho \log(\varphi) + \varepsilon'$, where ρ is the correlation coefficient and ε is a normally distributed random variable with mean μ_ϵ and variance σ_ϵ^2.

[12]If there is only one location and agglomeration effects are irrelevant, our model is identical to the one studied in Hopenhayn (1992) and Hopenhayn and Rogerson (1993).

Each firm produces a single output good using labor and land as input factors. The technology that is available to the firms in the economy satisfies the following assumption.

Assumption 2 *The production function of a firm in location j can then be written as:*

$$q = f(\varphi, n, l; e_j) \tag{1}$$

where q is output, n is labor, l is land, and e_j is the agglomeration externality in location j. The production function satisfies standard regularity conditions.

Rosenthal and Strange (2003) suggest that the externality acts as a multiplier on the production function. We use a standard Cobb-Douglas function with parameters α and γ in our computational model, $q = \varphi\ e_j\ n^{\alpha}\ (l - \bar{l})^{\gamma}$. Note that $\bar{l}$ is a minimum amount of land required for production. Since $r_j\ \bar{l}_j$ can also be interpreted as fixed costs, this specification implies that fixed cost vary by location.

The agglomeration effects arise due to a high concentration of firms operating in the same location.

Assumption 3 *The agglomeration externality can be written as*

$$e_j = \Theta(L_j, N_j, S_j) \tag{2}$$

where N_j and L_j are aggregate measures of labor and land respectively, and S_j is a measure of the mass of firms in location j. The function Θ is such that $\Theta_L < 0$, $\Theta_N > 0$, and $\Theta_S > 0$.

Following Lucas and Rossi-Hansberg (2002), in our computational model we assume that

$$e_j \ = \ \left(\frac{N_j}{L_j - S_j\bar{l}}\right)^{\theta} \tag{3}$$

If $\theta > 0$, the externality is an increasing function of a measure of concentration of economic activity in a location j. This measure is represented by the ratio of the total number of workers and the amount of land used in production over and above the minimum land requirement.

The urban economy is part of a larger economic system which determines output prices and wages.

Assumption 4 *Output prices, p, and wages, w, are constant and determined exogenously.*

Rental prices, r_j, however, are equilibrium outcomes. The supply of land is determined by an inverse land supply function in each location.

Assumption 5 *The inverse land supply function is given by:*

$$r_j = r_j(L_j), \quad j = 1, 2 \tag{4}$$

The inverse supply function is increasing in the amount of land denoted by L_j.

In the computational analysis, We adopt an iso-elastic functional form: $r_j = A_j L_j^{\delta}$, $j = 1, 2$, where A_j and δ are parameters. Our model thus captures the fundamental trade-off faced by all firms in the urban economy. The benefits of the agglomeration externality are at least partially offset by higher rents.

We can break down the decision problem of firms into a static and a dynamic problem. First, consider the static part of the decision problem that a firm has to solve each period. This problem arises because firm compete in the product market each period.

Assumption 6 *The product market is competitive and firms behave as price takers. Firms make decisions on land and labor usage after they have observed their productivity shock, φ, for that period.*

Let π_j denote a firm's one period profit in location j. The static profit maximization problem can be written as:

$$\{n, l\} = \underset{\{n,l\}}{\arg\max}\ \pi_j(n, l; \varphi), \tag{5}$$

where the profit function is given by:

$$\pi_j(n, l; \varphi) \ = \ p\ f(\varphi, n, l; e_j) - w\ n - r_j l - c_f. \tag{6}$$

The parameter c_f denotes a fixed cost of operation independent of location. Solving this problem we obtain the demand for inputs as a function of φ, denoted by $n_j(\varphi)$ and $l_j(\varphi)$, as well as an indirect profit function, denoted by $\pi_j(\varphi)$. [13]

Let μ_j denote the measure of firms located in j, then the mass of firms located located in j, denoted by S_j, is given by the following expression:

$$S_j = \int \mu_j(d\varphi) \tag{7}$$

Given the static choices for land and labor use for each firm, we can also calculate the aggregate levels of land and labor:

$$L_j \ = \ \int l_j(\varphi)\mu_j(d\varphi), \tag{8}$$

$$N_j \ = \ \int n_j(\varphi)\mu_j(d\varphi) \tag{9}$$

[13]Note that the sub-index j summarizes the dependence of the profit and input demand functions on location j's rent and externality.

After choosing labor and land inputs, each firm faces the (dynamic) decision of whether to stay in its current location, move to the other location, or shut down. The following Bellman equations formalize the decision problem of a firm that begins the period in location j with a productivity shock φ:

$$\begin{aligned} V_1(\varphi) &= \pi_1(\varphi) + \beta\xi \max\left\{0, \int V_1(\varphi')\, F(d\varphi'|\varphi), \int V_2(\varphi')\, F(d\varphi'|\varphi) - c_r\right\} \quad (10) \\ V_2(\varphi) &= \pi_2(\varphi) + \beta\xi \max\left\{0, \int V_2(\varphi')\, F(d\varphi'|\varphi), \int V_1(\varphi')\, F(d\varphi'|\varphi) - c_r\right\} \end{aligned}$$

where β is the discount factor, c_r is the cost of relocating from one location to another.

Solving the dynamic decision problem above implies decision rules of the following form for firms currently in location j:

$$x_j(\varphi) = \begin{cases} 0 \; if \; firm \; exits \; in \; next \; period \\ 1 \; if \; firm \, chooses \, location \, 1 \; in \, next \, period \\ 2 \; if \; firm \, chooses \, location \, 2 \; in \, next \, period \end{cases} \tag{11}$$

To close the model, we need to specify the process of entry.

Assumption 7 *Firms can enter into both locations. All prospective entrants are ex-ante identical. Upon entering a new firm incurs a cost c_{ej} and draws a productivity shock φ from a distribution $\nu(\varphi)$.*

Note that we allow the entry cost to vary by location. In our parametrized model the entrant distribution is assumed to be log-normal with parameters μ_{ent} and σ^2_{ent}. These assumptions guarantee that the expected discounted profits of a prospective firm are always less or equal than the entry cost:

$$c_{ej} \geq \int V_j(\varphi)\, \nu\,(d\varphi)\,, j = 1, 2 \tag{12}$$

If there is positive entry of firms, then this condition holds with equality.

We are now in a position to define the equilibrium to our economy.

Definition 1 *A stationary equilibrium for this economy consists of rents, r_j^* , masses of entrants, M_j^*, stationary distributions of firms, $\mu_j^*(\varphi)$, externalities, e_j^*, land demand functions , $l_j^*(\varphi)$, labor demand functions , $n_j^*(\varphi)$, value functions, $V_j^*(\varphi)$, and decision rules, $x_j^*(\varphi)$, for each location $j = 1, 2$, such that:*

1. *The decision rules (11) for a firm's location are optimal, in the sense that they maximize the right-hand side of equations (10).*
2. *The decision rules for labor and land inputs solve the firm's static problem in (5).*
3. *The free entry conditions (12) are satisfied in each location, with equality if $M_j^* > 0$.*
4. *The market for land clears in each location consistent with equation (4).*
5. *The mass of firms in each location is given by equation (**??**).*
6. *The externalities are consistent with (2)*
7. *The distributions of firms μ_j^* are stationary in each location and consistent with firms' decision rules.*

3.2 Existence and Computation of Equilibrium

An equilibrium to our model is characterized by vector of equilibrium values for rents, mass of entrants, and externalities in each location $(r_1, r_2, M_1, M_2, e_1, e_2)$. Finding an equilibrium for this model is equivalent to the problem of finding the root of a non-linear system of equations with size equations. For any vector $(r_1, r_2, M_1, M_2, e_1, e_2)$, we can

1. solve the firms' static profit maximization problem and obtain land demand, labor demand, and the indirect profit functions for each location;
2. solve the dynamic programming problem in equations (10) and obtain the optimal decision rules;
3. chose an initial mass of entrants in each location and simulate the economy forward until the distribution of firms, μ_j, converges to a stationary distribution;
4. calculate the aggregates land, labor demands and supplies the economy;
5. check whether market clearing conditions and the equations that define the mass of firms and the externalities in each location are satisfied.

If they are not, we update the vector of scalars and repeat the process until all of the conditions for equilibrium are satisfied. If this algorithm converges, we have computed an equilibrium of the model.

The task of computing an equilibrium can be simplified by exploiting some properties the parametrization used in our computational model. The static first order condition that determines that ratio of land and labor inputs is given by:

$$\frac{n}{l - \bar{l}} = \frac{\alpha}{\gamma} \frac{r_j}{w} \tag{13}$$

Notice that the ratio in this equation is the same for all firms in the same location j. Aggregating over all firms in such location, we obtain that:

$$\frac{N_j}{L_j - S_j \bar{l}} = \frac{\alpha}{\gamma} \frac{r_j}{w} \tag{14}$$

Thus, equation (14) then implies that we obtain an expression linking the externality, e_j in each location to that location's rent, r_j. We can, therefore, solve the Bellman equations without knowing the aggregate levels of land and labor. As a consequence we can characterize equilibrium rent values solely based on the free entry conditions.

To see how this works, we adopt the following simplified notation for the expected values functions:

$$\begin{aligned} EV_1(r_1, r_2) &= \int V_1(\varphi) d\nu(\varphi) \\ EV_2(r_1, r_2) &= \int V_2(\varphi) d\nu(\varphi) \end{aligned} \tag{15}$$

The entry condition for location one then defines a mapping $r_1 = \Gamma_1(r_2)$, i.e. for given r_2, $\Gamma_1(r_2)$ is the value of r_1 such that $EV_1(r_1, r_2) = c_e$. Similarly, we can define a mapping $r_1 = \Gamma_2(r_2)$ for location two. These two mappings then effectively define the set of rent pairs $\{r_1, r_2\}$, such that the two free entry conditions are satisfied with equality.

The non-linearity of the model implies that $\Gamma_1(r_2)$ and $\Gamma_2(r_2)$ can intersect multiple times. As a consequence, there may be more than one possible candidate values for equilibria with entry in both locations.[14]

Next, define the ratio of entrants in the two locations as $m = \frac{M_1}{M_2}$ and the distribution of firms standardized by the mass of entrants in locations 2 as,

$$\hat{\mu}_j = \frac{\mu_j}{M_2}. \tag{16}$$

[14] In addition to equilibria with entry in both locations, it is also possible to have equilibria in which entry only occurs in one of the two locations.

The standardized stationary distributions satisfy

$$\begin{aligned}\int_0^{\varphi'} \hat{\mu}_1(dx) &= \xi \int F(\varphi'|\varphi)\,1\{x_1(\varphi)=1\}\,\hat{\mu}_1(d\varphi) \\ &+ \xi \int F(\varphi'|\varphi)\,1\{x_2(\varphi)=1\}\,\hat{\mu}_2(d\varphi) + m\int_0^{\varphi'} \nu(dx) \\ \int_0^{\varphi'} \hat{\mu}_2(dx) &= \xi \int F(\varphi'|\varphi)\,1\{x_1(\varphi)=2\}\,\hat{\mu}_1(d\varphi) \\ &+ \xi \int F(\varphi'|\varphi)\,1\{x_2(\varphi)=2\}\,\hat{\mu}_2(d\varphi) + \int_0^{\varphi'} \nu(dx) \qquad (17)\end{aligned}$$

where $1\{x_j(\varphi)=j\}$ is an indicator function equal to 1 if x_j equals j and 0 otherwise. Given m forward iteration on these two equations yields the equilibrium standardized stationary distributions $\widehat{\mu}_j$, $j=1,2$.

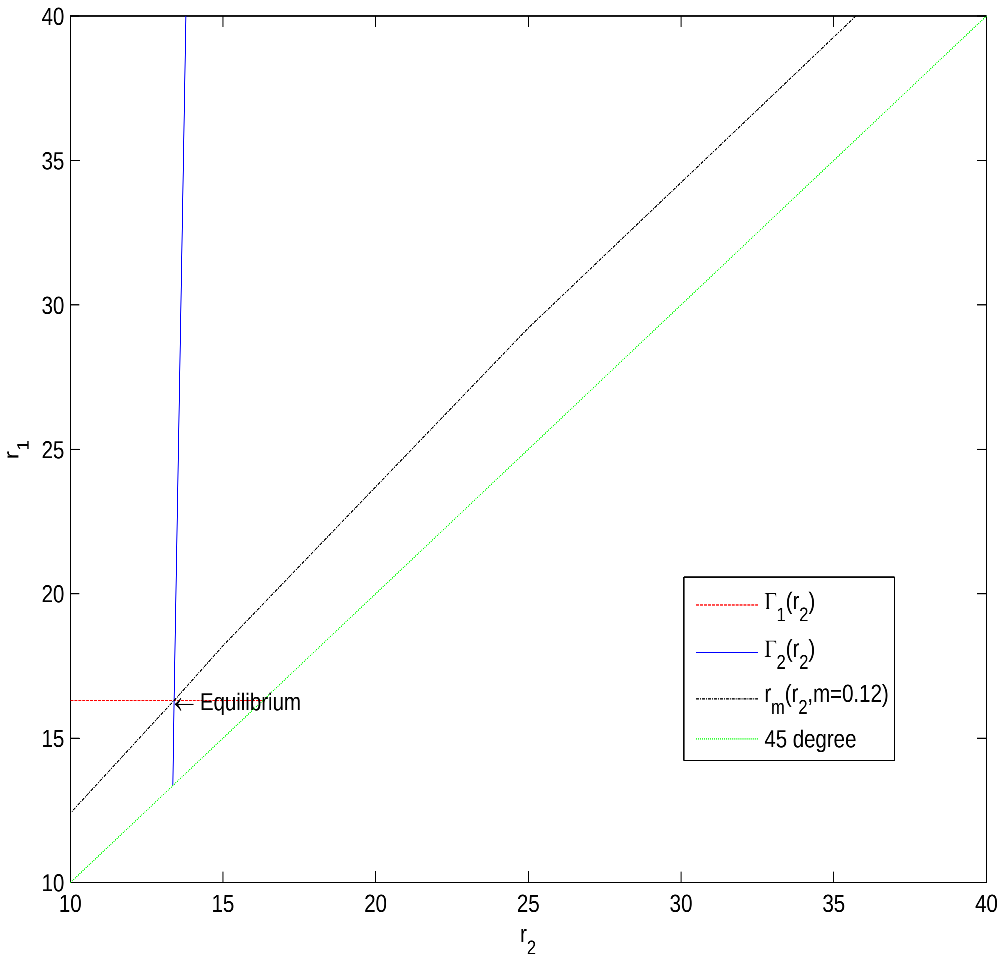
To find the equilibrium m, substitute the aggregate demands for land in the two locations into the inverse land supply functions and take their ratios. Given that the inverse elasticity, δ, is the same in both locations we then obtain:

$$\frac{r_1}{r_2} = \frac{A_1}{A_2}\left[\frac{\int l_1(\varphi)\hat{\mu}_1(d\varphi)}{\int l_2(\varphi)\hat{\mu}_2(d\varphi)}\right]^{\delta}. \qquad (18)$$

Let $r_1 = r_m(r_2;m)$ be the value of r_1 that clears the relative land markets given r_2 and m, keeping in mind that both the labor demand functions $l_j(\varphi)$ and the masses of firms $\hat{\mu}_j$ depend on r_1 and r_2.

We can thus conclude that all rent pairs $\{r_1, r_2\}$ that are consistent with entry in both locations are characterized by the intersection of the two functions $\Gamma^j(r_2)$. In addition, we have characterized the set of rent pairs consistent with land market clearing condition, $r_m(r_2;m)$, corresponding to different values of m. By analyzing these functions all together, we can completely characterize the set of triplets $\{r_1, r_2, m\}$ consistent with equilibrium in the economy. Figure 1 illustrates the (locally unique)

Figure 1: Graphical Representation of Equilibrium



equilibrium that arises in our model using our estimated parameter values.[15]

Finally, the mass of entrants in location 2, M_2, is determined by the market clearing condition for land:

$$\left(\frac{r_2}{A_2}\right)^{\frac{1}{\delta}} = M_2 \int l_2\ (\varphi)\hat{\mu}_2(d\varphi), \tag{19}$$

Note that M_2 can be solved for analytically.

3.3 Analytical Properties of Equilibrium

To get some additional insights into the properties of our model it is useful to simplify the structure of the model and shut down the future productivity shocks. We can then characterize the equilibrium of the model almost in closed-form.[16] Let us impose the following additional assumptions.

Assumption 8

1. *The shock is drawn upon entry once and for all from a uniform distribution in* $[0,1]$*:*

$$\nu\left(\varphi\right) = 1 \text{ for } \varphi \in \left[0,1\right]. \tag{20}$$

2. *There are no fixed cost of operation:* $c_f = 0$.

3. *Importance of externality:* $\theta = 1 - \alpha > \gamma$

Let 1 denote the high rent location and 2 the low rent one (1=city, 2=suburb). We show how to construct a unique equilibrium in which $r_1 > r_2$ and firms move

[15] A more careful discussion of the properties of this equilibrium is provided in Section 6.

[16] The model cannot be entirely solved in closed form because the equilibrium r_2 has to satisfy a highly non-linear equation. Sufficient conditions on the model's parameters for r_2 to exist and be unique are imposed instead. Conditional on r_2, everything else can be solved for analytically.

from location 2 to location 1, but not vice versa. Firms who enter in location 1 stay there all the time or exit.

First note that under assumptions 2 and 3 above the indirect profit functions can be written as:

$$\pi_j(\varphi) = r_j\left(\Delta\varphi^{\eta} - \bar{l}\right), \ j = 1, 2, \tag{21}$$

where $\Delta > 0$ and $\eta > 1$ are known functions of the parameters of the model. Consider location in the city. We have the following result.

Proposition 1 *If* $r_1 > r_2$*,*

a) then firms in location 1 follow a simple cut-off rule. Firms below a threshold φ_l *exit while firms above the threshold stay in location 1 forever. The cut-off is defined as:*

$$\varphi_l = \left(\frac{\bar{l}}{\Delta}\right)^{\frac{1}{\eta}}. \tag{22}$$

b) then firms in location 2 follow a simple cut-off rule. Firms below the threshold φ_l *exit, firms with shocks between* φ_l *and* φ_h *stay in location 2, and firms with shocks larger than* φ_h *move to location 1. The cut-off* φ_h *is defined as:*

$$\varphi_h = \left(\frac{\bar{l}}{\Delta} + \frac{c_r(1-\beta\xi)}{\Delta(r_1 - r_2)}\right)^{\frac{1}{\eta}}. \tag{23}$$

Proof:

a) Note that static firm profits are monotonically increasing in φ. Define φ_l such that $\pi_1(\varphi_l) = 0$. Then firms with $\varphi < \varphi_l$ exit immediately. It is straight forward to show that

$$V_1(\varphi_l) = \pi_1(\varphi_l) + \beta\xi \max\{0, V_1(\varphi_l), V_2(\varphi_l) - c_r\} \ = \ \pi_1(\varphi_l) = 0 \tag{24}$$

where the second equality follows from the fact that the productivity cut-off for switching to location 2 is less then cut-off for exit if $r_1 > r_2$ as assumed. Firms with $\varphi > \varphi_l$ stay in location 1 as long as they survive the exogenous destruction shock ξ. Their payoffs are:

$$V_1(\varphi) = \frac{\pi_1(\varphi)}{1-\beta\xi} > 0 \tag{25}$$

b) Next consider the decision rule of firms located in the suburb. Firms with $\varphi < \varphi_l$ exit immediately:[17]

$$V_2(\varphi_l) = 0 \tag{26}$$

Firms with shocks in (φ_l, φ_h) stay in 2 forever (as long as they survive the exogenous destruction shock). Firms with high shock move to 1. The indifference condition for staying vs moving is:

$$\frac{\pi_2(\varphi_h)}{1-\beta\xi} = \pi_2(\varphi_h) + \beta\xi\left(V_1(\varphi_h) - c_r\right). \tag{27}$$

This equation defines the cut-off value φ_h. The lemma then follows from the result that benefits of switching to location 1 monotonically increase with φ. Q.E.D.

Next we consider the free entry conditions and show that these conditions determine the rents in both locations. We have the following result:

Proposition 2 *There is at most one set of rental rates* (r_1, r_2) *that are consistent with the entry in both locations. Conditions on the parameter values guarantee existence of* (r_1, r_2).

[17]Note that equation (21) implies that z_l does not depend on the location.

Proof (of uniqueness):

First consider the free entry condition in location 1 which is given by

$$\int V_1(\varphi)\,\nu(\varphi)\,d\varphi = c_e. \tag{28}$$

Substituting in our optimal decision rule and simplifying we obtain the equilibrium rent in location 1:

$$r_1 = \frac{c_e(1-\beta\xi)(\eta+1)}{\Delta\left(1-\beta\xi\varphi_l^{\eta+1}\right) - \bar{l}(1-\beta\xi\varphi_l)(\eta+1)}. \tag{29}$$

Free entry in location 2 requires:

$$\int V_2(\varphi)\,\nu(\varphi)\,d\varphi = c_e. \tag{30}$$

Replacing the value function in location 2 and taking into account the definition of φ_h in (23) this equation simplifies to:

$$\eta{\varphi_h}^{1+\eta} - (1+\eta)\varphi_h^{\eta} - K = 0, \tag{31}$$

where K represents a non-positive combination of the parameters and is defined in the appendix. The left hand side of this equation is positive when $K = 0$ and has a negative first derivative. Thus, if a solution for φ_h exists it must be unique. In turn, φ_h is monotonically related to r_2 by equation (23):

$$r_2 = r_1 - \frac{c_r(1-\beta\xi)}{\Delta\varphi_h^{\eta} - \bar{l}}. \tag{32}$$

Thus, if the solution φ_h to equation (31) is unique, the equilibrium value of r_2 is also unique. The appendix provides sufficient conditions on the parameters for this solution to exist. Q.E.D.

Next we characterize the equilibrium distribution of firms in each location.

Proposition 3 *For each value of M_2, there exists a unique stationary equilibrium distributions of firms in each location.*

Proof:

Without loss of generality, let us normalize the model so that entry in location 2 is always equal to $M_2 = 1$. This implies a specific choice of A_2. Given this the mass of firms in location 2 is $\hat{\mu}_2(\varphi)$:

$$\hat{\mu}_2(\varphi) = \begin{cases} z_l \text{ if } \varphi < z_l \\ \frac{1}{1-\xi}(z_h - z_l) \text{ if } \varphi \in [z_l, z_h] \\ 1 - z_h \text{ if } \varphi > z_h \end{cases}. \tag{33}$$

Note that firms in location 2 with $\varphi < z_l$ exit and there is a measure z_l of them. Firms with $\varphi > z_h$ move to 1, and there is a measure $1 - z_h$ of them. Firms in the middle group $\varphi \in [z_l, z_h]$ remain in 2 forever subject to surviving the death shock ξ.

Let m denote entry in location 1. The mass of firms in location 1 is:

$$\hat{\mu}_1(\varphi) = \begin{cases} m\, z_l \text{ if } \varphi < z_l \\ \frac{m}{1-\xi}(z_h - z_l) \text{ if } \varphi \in [z_l, z_h] \\ \frac{(\xi+m)}{1-\xi}(1 - z_h) \text{ if } \varphi > z_h \end{cases}. \tag{34}$$

Firms in the first group exit immediately. Firms in the middle group stay in 1 forever. Firms with $\varphi > z_h$ come from 2 sources: 1. firms who entered in 1 and stayed there forever subject to death shock $m(1 - z_h)/(1 - \xi)$ plus firms who entered in location 2 last period, survived the shock and moved to 1 where they remain forever: $\xi(1 - z_h)/(1 - \xi)$. Q.E.D.

Finally, we have the following result:

Proposition 4 *There is at most one value of m such that the relative demand for land equals the relative supply of land. Under conditions on the parameters, m is shown to exist.*

Proof:
Given the equilibrium distributions, we can solve for equilibrium value for entry, denoted by m. Note that given the assumptions the demand for labor is:

$$l_j(\varphi) = \bar{l} + \left(\frac{\alpha}{w}\right)^{\eta} \varphi^{\frac{1}{1-\alpha-\gamma}} r_j^{\frac{1-\alpha}{1-\alpha-\gamma}}. \tag{35}$$

The equilibrium value of m is such that it solves the relative land equilibrium condition which can be written as

$$\int l_1(\varphi)\,\hat{\mu}_1(d\varphi) = \frac{A_2}{A_1}\frac{r_1}{r_2}\int l_2(\varphi)\,\hat{\mu}_2(d\varphi) \tag{36}$$

where the right hand side does not depend on m. The left-hand side depends linearly in m through the mass $\hat{\mu}_1(\varphi)$ in an increasing way. This means that if m exists it is unique.

For $m \to \infty$ the left hand side of (36) goes to infinity. For $m \to 0$ the left hand side is strictly positive. To show that it is less than the right hand side A_1 must be sufficiently small. Since the rest of the equilibrium is independent of A_1 one can always choose A_1 small enough in order to guarantee existence. Thus, there exists a unique value of m. Q.E.D.

In what follows we present the equilibrium of the model in a numerical example.

Result 1 *Consider the following parameter values:* $\beta = 0.5$, $\alpha = 0.65$, $\theta = 0.35$, $\xi = 0.9$, $\bar{l} = 0.01$, $\gamma = 0.01$, $\eta = 2.94$, $w = 1$, $\Delta = 0.0367$, $A_1 = 0.5$, $A_2 = 1.0$, $c_e = 0.1$, $c_r = 0.01$, $\delta = 1$. *Then, the unique equilibrium of the model is characterized by the following:* $\varphi_l = 0.64$, $\varphi_h = 0.69$, $r_1 = 37.18$, $r_2 = 34.98$, $m = 0.21$.

The analysis of this section shows that there exists a unique (up to scale) equilibrium with entry in both locations. Our analysis in the previous section reinforces the notion that equilibria with entry in both locations are often locally unique. These results suggest that we can design a full solution estimation algorithm for the parameters of the model that is based on equilibrium with entry in both locations.

4 Estimation

Let θ denote the parameter vector of the structural model to be estimated. The equilibrium of our dynamic equilibrium model defines a nonlinear mapping from the parameter vector θ to the distribution of observed equilibrium outcomes. This mapping is implicitly defined by the equilibrium of the model analyzed in the previous section. Since the econometrician does not necessarily observe all relevant variables, the empirical strategy relies on latent-variable simulation methods to estimate the parameters of the model. The structural model is assumed to be true in the sense that there is a particular value θ_0 of the structural model and a realization of the exogenous variables such that the observed data will correspond to the simulated data. The estimation strategy relies on the idea that the structural model should be able to replicate the empirical regularities observed in the data.

More precisely, the observed data are completely described by the joint empirical distribution function of age, facility size, and employment conditional on location choice. The basic idea is to match these distributions with those generated by our model. Instead of focusing on the joint empirical distribution of the observed variables, one can restrict the attention to a number of select moments which one can try to match along the lines suggested by Hansen (1982). We then match selected observed moments with their simulated counterparts generated by the structural model.[18]

The moments we use are based on the joint distribution of age and employment, and the distribution of facility size. These are calculated separately by location. In addition, we use the total percentage of firms in the city relative to the entire county. In practice, these moments are constructed by placing establishments into categories, (e.g. 5 to 8 employees, 11 to 20 years old, located in the city). This is analogous to creating a histogram of the distribution. The moments then are calculated as the percentage of firms in a given category relative to the number of establishments in the entire county.[19]

Combine all moments used in the estimation procedure into one vector m_N and denote with $m_S(\theta)$ their simulate counterparts where S denotes the number of simulations. The orthogonality conditions are then given by

$$g_{N,S}(\theta) \;=\; m_N \;-\; m_S(\theta) \tag{37}$$

Following Hansen (1982), θ can be estimated using the following moments estimator:

$$\theta_N \;=\; \arg\min_{\theta \in \Theta} \; g_{S,N}(\theta)' \, A_N \, g_{S,N}(\theta) \tag{38}$$

for some positive semi-definite matrix A_N which converges in probability to A_0. Using standard asymptotic arguments, it follows that the estimator θ_N is a consistent estimator of θ_0 and that:

$$N^{1/2}\,(\theta_N \;-\; \theta_0) \xrightarrow{d} N(0, (\tilde{A}_0 D_0)^{-1} \tilde{A}_0 \; V \; \tilde{A}_0' \; (\tilde{A}_0 D_0)^{-1\prime}) \tag{39}$$

[18]Alternatively, one could estimate an auxiliary model using semi-nonparametric estimation as developed by Gallant and Nychka (1987) and then match the scoring functions of the auxiliary and the structural model (Gallant and Tauchen, 1989). This approach is more appealing for a time series application since there are much more potential moments to chose.

[19]The complete set of moments is described in Appendix B, along with the computational equivalents.

where $\tilde{A}_0 = D_0' A_0$, $D_0 = E[\partial m(\theta) / \partial \theta_0]$ and V is asymptotic covariance matrix of the vector of sample moments. In addition, the most efficient estimator is obtained by setting $A_N = V_N^{-1}$. In this case:

$$N^{1/2} (\theta_N - \theta_0) \xrightarrow{d} N(0, (D_0' V^{-1} D_0)^{-1}) \tag{40}$$

Furthermore, standard J-statistics can be used to do hypothesis and specification tests.[20]

5 Data

Our empirical application focuses on firm location choices in the City of Pittsburgh and Allegheny County.[21] To illustrate the spatial distribution of economic activity in Allegheny County, Figure 2 plots the employment concentration in the county using data from the U.S. Census. Figure 2 shows that over 20 percent of employment is concentrated in three zip codes in the center of Pittsburgh which include the downtown business district and the Oakland neighborhood of Pittsburgh which are the two significant dense commercial areas of Pittsburgh.[22] The second location or the suburbs include the rest of Allegheny County.

To provide a more detailed analysis of spatial activity of firms, we obtained firm level data from Dun and Bradstreet's Million Dollar Database.[23] The data cover establishments in Allegheny county in 2008. The database provides detailed informa-

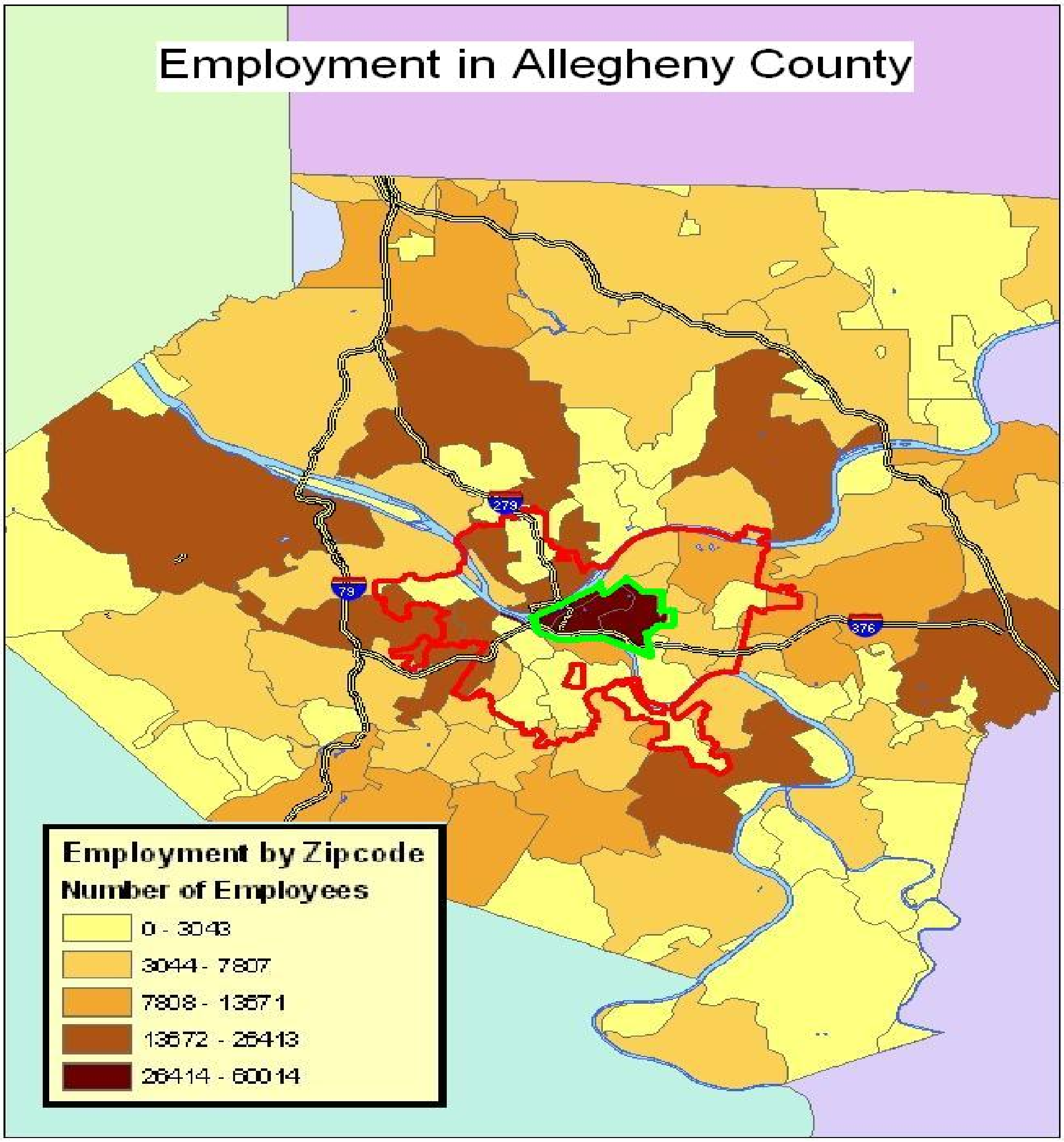
[20]Strictly speaking, one would need to correct for the sampling error induced into the estimation procedure by the simulations. However, if the number of simulations is large, these errors will be negligible. (For a more careful discussion see Gourieroux and Monfort (1993).)

[21]Appendix A of the paper compares to Pittsburgh to other metropolitan areas and finds that similar firm location patterns can be found in other cities in the US.

[22]The zip codes are 15222, 15219, and 15213.

[23]Information on Dun and Bradstreet data is available on-line at http://www.dnbmdd.com/

Figure 2: Allegheny County and the City of Pittsburgh



tion on establishments and the coverage is near universal compared to Census counts of establishments in the county. Of particular relevance to the current research, the database provides data on location, facility size, total employment, industry, and year established. While most of the data is complete, the year established field, which is used to determine age of establishments, is only available for 52.5 percent of the observations. However, there is little evidence that the missing data field is strongly or systematically correlated with other observable data, and therefore entries missing the "year established" field are dropped from the analysis.[24]

We analyze the employment and facility size characteristics for different industries in the Pittsburgh area. Table 2 reports the total employment, the average employment and the facility space per employee for firms in the city and the suburbs for the service industries that are the focus of the empirical analysis.

Table 2: Employment and facility size by industry

NAICS	Total Emp.	% emp CBD	Avg. Emp. Suburbs	Avg. Emp. CBD	sq. ft. per employee Suburbs	sq. ft. per employee CBD
Information	16,975	25.15%	13.52	31.16	336.88	214.44
Finance	42,960	53.51%	8.55	55.66	318.28	193.59
Real Estate	18,459	17.97%	7.51	12.43	743.36	1190.21
Professional Services	64,076	32.85%	6.99	13.29	334.83	309.60
Management	2,062	11.30%	19.46	14.56	272.88	360.52
Administrative Support	41,830	14.97%	11.01	20.67	240.89	352.14
Education	52,995	42.69%	30.46	205.66	316.70	121.27
Health Care	115,048	18.12%	16.53	24.01	293.39	291.46
Total	354,405	28.66%	11.78	27.47	326.81	265.19

We find that the average employment size of establishments is larger in the central business district reflecting an increase in productivity, and the facility space per employee is lower in the CBD. The high concentration of economic activity in central

[24]In the appendix we provide details of the comparison between the total data set and the data set with 'year established' data based on observed data.

business districts is driven by the fact that regional cities act as a hub for services for a larger region. The service industry concentrates in the city to reap the benefits of agglomeration externalities available at an urban district scale.

Table 3 provides some descriptive statistics for firms located in the city and the suburbs for these industries using our Dun and Bradstreet sample.[25] Note that 13.43 percent of all our firms are located in the city. However, the three zip codes that comprise the city account for a less than one percent of all the land in Allegheny county.

Table 3: Percentiles of data by location

	CBD				Suburbs			
percentile	age	employment	facility	sales/emp	age	employment	facility	sales/emp
10th	6	1	890	42,500	4	1	780	28,000
25th	12	2	1,400	60,000	10	1	1,000	44,444
50th	20	5	2,500	75,000	18	3	1,700	67,647
75th	34	14	4,700	93,750	28	7	3,000	85,000
90th	53	40	11,100	185,714	41	20	5,800	127,500
95th	72	78	23,000	309,524	53	39	10,000	210,000
99th	119	460	230,000	2,000,000	96	150	50,000	892,870

A few clean patterns arise from the analysis of Table 3. Firms in the city are older, employ more workers and operate in larger facilities. This is especially true for the right tail of the age distribution. In addition, sales per employee are considerable higher in the city, suggesting higher productivity per worker.

We also consider the dynamics of firm exit, entry, and relocations. U.S. Census data for Allegheny County reports the number of new establishment and the number of firm death per period. We collected this data for the period from 1999 through 2003. We find that the economy appears to have been in a steady state for a number of years. Take, for example, the year 2003. The Census reports 2,871 establishments

[25] Sales data are estimated by Dun and Bradstreet, and may be subject to errors. Hence, these data are not used in formal estimation of the model.

births and 2,989 deaths. Unfortunately, we do not have these data by firm sector and thus do not use these statistics in estimation.

6 Empirical Results

6.1 Parameter Estimates

To facilitate the estimation of the model we use several parameter estimates from previous studies. Estimates about the land share are reported in Deckle and Eaton (1993), Adsera (2000), and Caselli and Coleman (2001). We set α, the labor share of to be equal to 0.65. In addition, we fix two key parameters. First, we set the relocation cost parameter c_r equal to 200,000. In the absence of reliable data on relocations it is hard to identify this parameter.[26] Second, we set the facility supply elasticity, denoted by δ, equal to 0.2. [27]We also set the exogenous exit probability to 0, or $\xi = 1$. and the discount factor equal to 0.95. Last, we set the exogenous exit probability to 0 (or $\xi = 1$) and the discount factor equal to 0.95, taking a year as the relevant unit of time. The remaining parameters of the model are the estimated using the GMM estimator discussed above. Table 4 reports the parameter estimates and the estimated standard.

We find that the estimate of the land share parameter is 0.0884 which is less than the parameter estimate of the agglomeration externality which is equal to 0.102. We have seen before that the restriction that $\theta > \gamma$ is often necessary to get an equilibrium sorting pattern in which high productivity firms prefer locations with high

[26]Parameter estimates for other choices of relocation costs are available upon request from the authors.

[27]This value is to be interpreted as the inverse supply elasticity. Estimates vary for this rent elasticity of supply for office space, but are generally accepted to be significantly greater than unity. See Wheaton 1999, Hekman 1985, Henderschott et al 1999 for estimates

Table 4: Parameter Estimates

Description	Parameter	Point Estimate	Standard Errors
shock correlation parameter	ρ	0.9773	0.00000213
shock variance parameter	σ_ϵ	0.1371	0.0000883
shock mean parameter	μ_ϵ	0.2385	0.0000182
entrant (log) mean	μ_{ent}	11.67	0.000408
entrant (log) variance	σ_{ent}	0.23	0.000420
minimum land	$\bar{l}$	1210	344
fixed cost	c_f	52,000	2880
externality	θ	0.102	0.000985
production land share	γ	0.0884	0.000848
entry cost - location 1	$c_{e,1}$	763,954	12900
entry cost - location 2	$c_{e,2}$	786,331	13000
land supply ratio	$\frac{A_1}{A_2}$	1.6327	0.189

agglomeration externalities. The fixed costs of operation are $52,000, or a quarter of the costs of relocating to a different community. Entry costs differ by location and vary between $764,000 and $786,000. It is only slightly more expensive to enter into the suburbs. The minimum land requirement is approximately 1210 square foot with an estimate standard error of 344. The productivity shocks are highly correlated across time. The point estimate of 0.977 is consistent with previous estimates in the literature and points to a high a degree of persistence.

To obtain additional insights into the sorting process of firms and to evaluate the fit of our model we consider the distribution of employment by age in both the city and suburbs. Table 5 reports the distribution of firms by age and employment size for our sample and the one predicted by our model. The data suggest that firms located in the city are older and larger than firms located in the rest of the metro area. Moreover, the firms in the city have more employees holding age constant. Overall, we find that our model fits this feature of the data reasonably well. The main drawback of the model is that it has problem matching the age distribution of small firms that are located in the suburbs. The vast majority of these firms have only one to three employees. A large number of the small firms have been in business

Table 5: Age-Employment distributions of establishments by location, as a percentage of total establishments in the entire county (computational moments in parenthesis)

City

		Age			
		1 to 10	11 to 20	21 to 30	> 30
Employment	1 to 3	1.48	1.71	0.96	1.17
		(2.42)	(1.13)	(0.36)	(1.68)
	4 to 8	0.59	1.10	0.56	0.93
		(0.94)	(0.56)	(0.36)	(1.07)
	9 to 16	0.33	0.56	0.48	0.64
		(0.27)	(0.25)	(0.18)	(0.62)
	> 16	0.41	0.64	0.63	1.24
		(0.12)	(0.22)	(0.25)	(1.18)

Suburbs

		Age			
		1 to 10	11 to 20	21 to 30	> 30
Employment	1 to 3	16.25	16.00	9.48	8.17
		(20.99)	(10.25)	(5.97)	(11.77)
	4 to 8	3.59	5.89	4.64	4.44
		(7.86)	(4.82)	(3.11)	(6.56)
	9 to 16	1.44	2.69	1.81	2.31
		(2.25)	(2.07)	(1.50)	(3.38)
	> 16	1.44	2.84	2.04	3.50
		(0.95)	(1.61)	(1.42)	(3.57)

for a long time.

Table 6: Age-Facility Size establishment distributions by location as percentage of total establishments in the county, (computational moments in parenthesis

City

		Age			
		1 to 10	11 to 20	21 to 30	> 30
Facility (sq ft)	1 to 1250	0.82 %	0.73 %	0.44 %	0.65 %
		(2.06 %)	(0.95 %)	(0.54 %)	(1.38 %)
	1251 to 2150	0.64 %	1.18 %	0.59 %	0.88 %
		(0.82 %)	(0.43 %)	(0.26 %)	(0.43 %)
	2151 to 3850	0.75 %	1.13 %	0.70 %	0.82 %
		(0.48 %)	(0.32 %)	(0.21 %)	(0.64 %)
	> 3850	0.60 %	0.98 %	0.91 %	1.63 %
		(0.38 %)	(0.47 %)	(0.43 %)	(1.81 %)

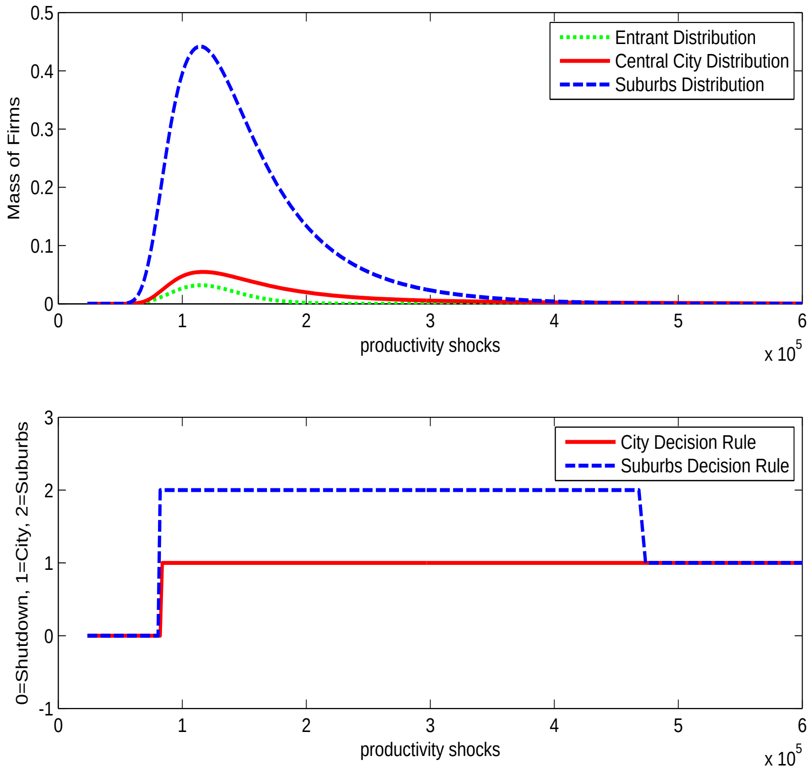
Suburbs

		Age			
		1 to 10	11 to 20	21 to 30	> 30
Facility (sq ft)	1 to 1250	9.57 %	9.76 %	5.51 %	4.20 %
		(15.39 %)	(7.42 %)	(4.26 %)	(8.31 %)
	1251 to 2150	7.08 %	7.56 %	4.86 %	4.74 %
		(6.99 %)	(3.58 %)	(2.17 %)	(3.40 %)
	2151 to 3850	3.81 %	6.01 %	4.14 %	4.18 %
		(5.53 %)	(3.38 %)	(2.18 %)	(4.58 %)
	> 3850	2.30 %	4.09 %	3.45 %	5.13 %
		(4.14 %)	(4.38 %)	(3.40 %)	(7.97 %)

Table 6 focuses on the age-facility size establishment distributions. In our model, employment and facility size are highly correlated since they are both nonlinear functions of the underlying productivity shock. A similar high correlation can be observed in the data. As a consequence, Table 6 reinforces our previous findings about model fit. Firms in the city face higher rental rates for land and office space. As a consequence they operate with a higher employee per land ratio. Overall, the model fits this feature of the data reasonably well.

To obtain some additional insight into the properties of the equilibrium implied by

Figure 3: Distributions and decision rules for estimated parameter set



the parameter estimates we plot the stationary distribution of firms in both locations as well as the distribution of entrants in the upper panel of Figure 3. Note that neither of these distributions must integrate to one since the mass of firms and entrants are equilibrium outcomes. The lower panel of Figure 3 plots the optimal decision rules. Note that the equilibrium implies that firm with high productivity shocks will leave the suburbs to relocate to the central city to reap the benefits of the higher externality in the city.

Finally, some other characteristics of the model can be compared to additional data sources and moments. These data were not specifically used in the estimation, and may contain some measurement error, but nonetheless, are useful in approximate comparison of the model. Table 7 presents data for rent ratios[28], entry[29], relocation[30], average employment, average facility size, and average age[31], along with equivalent moments from the model. Note that the rent ratio along with the estimate of the ext ernality parameter, θ, tells us that the firms located in the CBD receive a 2.02 percent productivity gain over firms in the suburbs due to the local agglomeration externality.

6.2 Policy Analysis

We consider a policy experiment which fully subsidizes potential relocations of firms. Given the high relocation costs of $200,000 this policy is very costly. Nevertheless

[28]The Building Owners and Managers Association collects information on expenses and income for office space throughout North America. The reported rent data is the ratio of suburban and CBD office space for the United States and comes from the 2006 Experience Exchange Report. See www.boma.org for more information.

[29]Entry data comes from the U.S. Census dynamics data, 2006, and is reported as a percentage of total establishments in Allegheny County

[30]Relocation is difficult to measure, and therefore little data is available to precisely capture relocation. We constructed a measure of relocation by tracking movement of the largest 10,000 establishments between 2005 and 2007 using the Dunn and Bradstreet data.

[31]Average employment, facility size, and age come from the Dunn and Bradstreet data.

Table 7: Other characteristics of the economy: Comparison of computational results and data

Description	model	data
rent ratio CBD/suburbs	1.22	1.21
entry	4.61%	8.65%
relocation	0.031%	0.056%
avg. employment CBD	16.52	38.82
avg. employment Suburbs	7.07	13.16
avg. facility size CBD	7,268.03	11,360.31
avg. facility size Suburbs	3,781.95	4758.64
avg. age CBD	32.85	25.62
avg. age Suburbs	24.74	20.71

it is useful to consider this case since it provides some additional insights into the properties of the model.

Implementing the full relocation subsidy, we find that this policy has relative large effects on economic activity. The size of the economy increases dramatically, with an increase of 238 percent employment. This increase is due to the fact that many firms will now relocate from the suburbs to the city to reap the benefits of higher externalities. Not surprisingly, relocation increases significantly, with 7.89 percent of establishments, and 10.9 percent of employment relocating in every period (compared to 0.03 percent and 2.05 percent of the baseline specification). These migration effects act as a multiplier since they reinforce the high densities in the city. Rents increase in both locations, with rent in the suburbs going from 13.4 to 15.9, and in the city 16.3 to 22.5. This results in a larger rent ratio moving from 1.22 to 1.42. While the percentage of establishments in the city remains relatively constant, the percentage of employment in the city increases from 24 percent to 49 percent.

The growth in the economy obviously comes at a cost. We can characterize this subsidy in several ways. First, we can simply calculate the subsidy as a percentage of total wages. Assuming a wage rate of $\$48,661$ and a relocation cost of $\$200,000$ per move, the subsidy corresponds to a wage tax of 3.84 percent on wages. We can also characterize the subsidy as a percentage of rent akin to a property tax. The total subsidy is then equivalent to a 26 percent tax on rent on commercial property.

While these measures are interesting, they do not give a sense of how welfare is effected under this subsidy. In the model, establishments have zero expected profits and workers' preferences are exogenous. Hence, the only useful measure of welfare in this economy is surplus to land owners. Surplus in this sense can be measured as the area between the rent and the land supply curve, which can be calculated analytically through the following formula:

$$Surplus = \int_0^{L_1^*} \left(r_1^* - A_1 L^\delta\right) dL + \int_0^{L_2^*} \left(r_2^* - A_2 L^\delta\right) dL. \quad (41)$$

Calculated in this manner, the subsidy accounts for 27.3 percent of the surplus gained. This suggests that net improvements can be made by subsidizing relocation costs.

7 Conclusions

We have provided a quantitative analysis of locational decisions of within a given metropolitan area using a dynamic general equilibrium framework. We have documented a number of stylized facts about firm location decisions that are common for a large set of metro areas in the U.S. The data suggest that firms located in the city are older and larger than firms located outside the urban core. As a consequence they use more land and labor in the production process. However, they face higher rental rates for land and office space which implies that they operate with a higher employee per land ratio. We have developed a new stochastic dynamic equilibrium model that can not only explain the observed sorting of firms by productivity, but is also consistent with the observed entry, exit, and relocation decisions of firms within an urban economy. Our estimation procedure provides reasonable estimates for the underlying structural parameters of the model.

Our analysis has some important policy implications. Relocation costs prevent establishments from moving because the gains for the individual firm are not worth the moving cost. However, this decision is not efficient since firms are not considering the external benefits to other firms. A relocation subsidy allows firms to sort more efficiently in the economy. This result has implications in metropolitan areas where there is competition for businesses between central cities and suburbs. A targeted relocation subsidy can encourage a better distribution of firms, and lead to welfare gains due to the existence of the production externality.

We view the findings of this paper as promising for future research. We have focused on firm locations within a metropolitan area. However, there is also a lot of relocations of firms across metropolitan area. Firms often decided to relocate to larger metro areas such as New York or Chicago to repeat the special benefits that these large cities provide. Headquarters of large firms tend to be disproportionately located in a few large cities. Our framework can be also be used to model and analyze these type of locational decisions.

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