Financial Business Cycles

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Abstract

I construct a dynamic general equilibrium model where a recession is initiated by losses suffered by financial institutions, and exacerbated by their inability to extend credit to the real economy. The event that triggers the recession is similar to a redistribution shock: a small sector of the economy – borrowers who use their home as collateral – defaults on their loans (that is, they pay back less than contractually agreed). When banks hold little equity in excess of regulatory requirements, their portfolio losses require them to react immediately, either by recapitalizing or by deleveraging. By deleveraging, banks transform the initial redistribution shock into a credit crunch, and, to the extent that some firms depend on bank credit, amplify and propagate the financial shock to the real economy. In my benchmark experiment aimed at replicating key features of the Great Recession, credit losses (that is, a redistribution shock) of about 5 percent of total GDP lead to a 3 percent drop in output, whereas they would have little effect on economic activity in a model without financing frictions where banks are just a veil.

KEYWORDS: Banks, DSGE Models, Collateral Constraints, Housing.

JEL CODES: E32, E44, E47.

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1. Introduction

I construct a dynamic stochastic general equilibrium model where leveraged banks amplify the effects on economic activity of given financial shocks. The main questions that I want to address are: (1) To what extent can arbitrary redistributions of wealth disrupt the credit intermediation process that channels funds from savers to borrowers? (2) To what extent can a disruption of the credit intermediation process cause business cycles?

The motivation for these questions comes from the empirical observation that at least two of the last three recessions in the United States (the 1990-91 recession and the 2007-2009 recession) can be ascribed to situations that involved non-repayment on part of some borrowers on the one hand, and loan losses affecting financial institutions on the other. Under this interpretation, it is hard to classify the impulse of these recessions as something that can be easily inserted or found in standard equilibrium macroeconomic models. These models either abstract from financial frictions or, when they address them, they abstract from financial intermediation. Even when financial intermediation is modeled,\textsuperscript{1} the shock that hits the economy in these models often involves an exogenous decline in the net worth of financial intermediaries, thus being very similar to shocks that destroy the economy’s capital stock.

This paper goes one step further and addresses this gap. My objective is to develop a tractable framework that studies how disruptions to the flow of resources between agents can act as an exogenous impulse to business fluctuations. To do so, I construct a simple DSGE model where financial intermediaries (banks, for short) amplify and propagate business cycles that are “financial” in nature; that is, they are originated not by changes in technology, but by disruptions in the flow of funds between different group of agents. When one group of agents pays banks back less than expected, the resulting effect is a loan loss for the bank which causes a reduction in bank capital. As a consequence, the bank can either raise new capital or restrict asset growth by cutting back on lending. If raising capital is difficult, banks reduce lending. To the extent that some sectors of the economy depend on credit, the reduction in bank credit propagates a recession.

2. The Model

The model features two household types, entrepreneurs, bankers and a representative firm. A summary of the model setup in Figure 1.

\textsuperscript{1}Notable examples in this literature are the recent papers by Gertler and Kiyotaki (2010), Curdia and Woodford (2009), and Angeloni and Faia (2009).

\textsuperscript{2}Except for the introduction of the banking sector, the model structure closely follows a flexible price version of the model in Iacoviello (2005). Except for the introduction of the banking sector, the main difference is that I allow household savers to also accumulate productive capital, so that I can nest the RBC model as a special case. I also distinguish between entrepreneurs (who produce capital) and good producers (who rent capital from entrepreneurs and labor from households), but this distinction is only a matter of expositional convenience, and bears no implications for the results.
fashion, they are divided into patient savers and impatient borrowers. To fix ideas, I interpret the impatient borrowers as subprime people (subprimers from now on): the idea that I want to explore is that the original shock that hits the system starts from the decision of these agents not to repay their loans. As a whole, the household sector is a net supplier of savings to the rest of the economy.

Entrepreneurs accumulate physical capital (which they rent to a representative firm) and borrow from the bank, subject to a collateral constraint. In order to control the extent to which credit constraints bind for the entrepreneurial sector, I also allow patient households to accumulate part of the economywide capital stock. This way, to the extent that the size of impatient households and entrepreneurs becomes arbitrarily small, the model boils down to the familiar neoclassical growth model.

Bankers intermediate funds between patient savers on the one hand, and entrepreneurs and subprimers on the other. The nature of the banking activity implies that bankers are borrowers when it comes to their relationship with households, and are lenders when it comes to their relationship with the credit-dependent sectors (entrepreneurs and subprimers) of the economy. I design preferences in a way that two frictions coexist and interact in the model’s equilibrium: first, bankers’ are credit constrained in how much they can borrow from the patient savers; second, entrepreneurs and subprimers...
are credit constrained in how much they can borrow from bankers. My interest is in understanding how these two frictions interact with and reinforce each other.

Finally, the representative firm converts entrepreneurial capital and household labor into the final good using a constant-returns-to-scale technology.

**Patient Households.** There is a continuum of measure 1 of savers (indexed by $H$). I measure their economic size by controlling their wage share in production (either patient households or impatient households work) and their capital share in production (either patient households or entrepreneurs accumulate physical capital).

They choose consumption $C$, housing $H$ (which has zero depreciation), physical capital $K$ (which depreciates at rate $\delta$ and incurs a standard convex adjustment cost) and time spent working $N$ to solve the following intertemporal problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta_H^t \left( \log C_{H,t} + j \log H_{H,t} + \tau \log (1 - N_{H,t}) \right)$$

where $\beta_H$ is the discount factor, subject to the following flow-of-funds constraint:

$$C_{H,t} + K_{H,t} + \phi_{KH,t} + D_t + q_t (H_{H,t} - H_{H,t-1}) = (R_{M,t} + 1 - \delta) K_{H,t-1} + R_{H,t-1} D_{t-1} + W_{H,t} N_{H,t} \quad (2.1)$$

where $D$ denotes bank deposits (earning a gross return $R_H$), $q$ is the price of housing in units of consumption/final good, $W_H$ is the wage rate, and $R_K$ is the rental rate for capital. Housing does not depreciate. The capital adjustment cost function takes the form $\phi_{KH,t} = \phi_{KH} (K_{H,t} - K_{H,t-1})^2$. The optimality conditions yield standard first-order conditions for consumption/deposits, housing demand, capital supply, and labor supply.

$$\frac{1}{C_{H,t}} = \beta_H E_t \left( \frac{1}{C_{H,t+1}} R_{H,t} \right) \quad (2.2)$$

$$\frac{q_t}{C_{H,t}} = \frac{j}{H_{H,t}} + \beta_H E_t \left( \frac{q_{t+1}}{C_{H,t+1}} \right) \quad (2.3)$$

$$\frac{1}{C_{H,t}} \left( 1 + \frac{\partial \phi_{KH,t}}{\partial K_{H,t}} \right) = \beta_H E_t \left( \frac{1}{C_{H,t+1}} \left( R_{M,t+1} + 1 - \delta + \frac{\partial \phi_{KH,t+1}}{\partial K_{H,t}} \right) \right) \quad (2.4)$$

$$\frac{W_{H,t}}{C_{H,t}} = \frac{\tau}{1 - N_{H,t}}. \quad (2.5)$$

**Subprimers (Impatient Households, Borrowers).** Subprimers (indexed by $S$) do not save and borrow up to a fraction of the value of their house. They solve:

$$\max \sum_{t=0}^{\infty} \beta_S^t \left( \log C_{S,t} + j \log H_{S,t} + \tau \log (1 - N_{S,t}) \right)$$
subject to the flow-of-funds constraint and the borrowing constraint:

$$C_{S,t} + q_t (H_{S,t} - H_{S,t-1}) + R_{S,t-1} L_{S,t-1} - \varepsilon_t = L_{S,t} + W_{S,t} N_{S,t}$$  \hspace{1cm} (2.6)

$$L_{S,t} \leq E_t \left( \frac{1}{R_{S,t}} m_{q_{t+1}} H_{S,t} \right).$$  \hspace{1cm} (2.7)

The borrowing constraint limits borrowing to the present discounted value of their housing holdings. Below, I will show that the constraint binds in a neighborhood of the non-stochastic steady state if $\beta_S$ is lower than a weighted average of the discount factors of patient households and bankers. The term $L_S$ denotes (one-period) loans made to subprimers, paying a gross interest rate $R_S$. Finally the term $\varepsilon_t$ in the budget constraint denotes an exogenous repayment shock: I assume that subprimers can pay back less (more) than agreed on their contractual obligations if $\varepsilon$ is greater (smaller) than zero; from their point of view, this shock represents a positive shock to wealth, since it allows them to spend more than previously anticipated.

The optimality conditions for debt, housing demand and labor supply will be:

$$\frac{1 - \lambda_{S,t}}{C_{S,t}} = \beta_S E_t \left( \frac{R_{S,t}}{C_{S,t+1}} \right)$$  \hspace{1cm} (2.8)

$$E_t \left( \frac{1}{C_{S,t}} \left( q_t - \lambda_{S,t} m_{q_{t+1}} \frac{q_{t+1}}{R_{S,t}} \right) \right) = \frac{j}{H_{S,t}} + \beta_S E_t \left( \frac{q_{t+1}}{C_{S,t+1}} \right)$$  \hspace{1cm} (2.9)

$$W_{S,t} \frac{C_{S,t}}{C_{H,t}} = \frac{\tau_S}{1 - N_{S,t}}.$$  \hspace{1cm} (2.10)

Above, $\lambda_{S,t}$ denotes the Lagrange multiplier on the borrowing constraint. A positive value of the multiplier works to reduce the current user cost of housing, denoted by $q_t - \lambda_{S,t} m_{q_{t+1}} q_{t+1} R_{S,t}$, and tilts preferences towards housing goods (relative to consumption goods), as emphasized for instance in Monacelli (2009).

Note that one could endogenize the repayment shock in other ways: for instance, one could assume that if house prices fall below some value, borrowers could find it optimal to default rather than roll their debt over: defaulting would be equivalent to choosing a value for $R_{S,t} L_{S,t-1}$ lower than previously agreed, which would generate the same effect as a positive shock to $\varepsilon_t$.

**Entrepreneurs.** A continuum of unit measure entrepreneurs solve the following problem:

$$\max \sum_{t=0}^{\infty} \beta_E^t \log C_{E,t}$$
subject to:

\[ C_{E,t} + K_{E,t} + \phi_{KE,t} + q_t (H_{E,t} - H_{E,t-1}) + R_{E,t} L_{E,t-1} = L_{E,t} + (R_{K,t} + 1 - \delta) K_{E,t-1} + R_{V,t} H_{E,t-1} \]  

(2.11) \[ L_{E,t} \leq m_H E_t \left( \frac{q_{t+1}}{R_{E,t}} H_{E,t} \right) + m_E K_{E,t} \]  

(2.12)

Here, \( L_E \) are loans that banks extend to entrepreneurs (yielding a gross return \( R_E \)), \( K_E \) is capital that entrepreneurs rent to a goods producing firm at the rate \( R_K \), and \( \delta \) is the capital depreciation rate. Entrepreneurs also accumulate housing (commercial real estate) which they rent to the final good firm at the rate \( R_V \). Entrepreneurs cannot borrow more than a fraction \( m_E \) of their hard assets \( K_E \), and a fraction \( m_H \) of their real estate capital \( qH_E \). As for the case of impatient borrowers, this constraint will be binding near the non-stochastic steady state, provided that entrepreneurs are impatient enough. The term \( \phi_{KE,t} \) denotes convex capital adjustment costs, denoted by \( \phi_{KE} = (K_{E,t} - K_{E,t-1})^2 \).

The entrepreneur’s first-order conditions can be written as:

\[ \frac{1 - \lambda_{E,t}}{C_{E,t}} = \beta_E E_t \left( \frac{R_{E,t+1}}{C_{E,t+1}} \right) \]  

(2.13) \[ \frac{1}{C_{E,t}} \left( 1 - \lambda_{E,t} m_K + \frac{\partial \phi_{KE,t}}{\partial K_{E,t}} \right) \right) = \beta_E E_t \left( \frac{1}{C_{E,t+1}} \left( 1 - \delta + R_{K,t+1} + \frac{\partial \phi_{KE,t+1}}{\partial K_{E,t}} \right) \right) \]  

(2.14) \[ E_t \left( \frac{1}{C_{E,t}} \left( q_t - \lambda_{E,t} m_H \frac{q_{t+1}}{R_{E,t}} \right) \right) = \beta_E E_t \left( \frac{1 + R_{V,t+1}}{C_{E,t+1}} \right) \]  

(2.15)

**Bankers.** A continuum of unit measure bankers solve the following problem:

\[ \max \sum_{t=0}^{\infty} \beta_B^t \log C_{B,t} \]

subject to:

\[ C_{B,t} + R_{H,t-1} D_{t-1} + L_{E,t} + L_{S,t} = D_{t} + R_{E,t} L_{E,t-1} + R_{S,t-1} L_{S,t-1} - \varepsilon_t \]  

(2.16)

where the right-hand side measures the sources of funds for the bank (net of adjustment costs and loan losses): \( D \) are household deposits, and \( R_{E,t} L_E \) and \( R_{S,t-1} L_{S,t-1} \) are repayments from entrepreneurs and subprimers on previous period loans. The funds can be used by the bank to pay back depositors and to extend new loans, or can be used for banker’s consumption. Note that this formulation is analogous to a formulation where bankers maximize a convex function of dividends (discounted at rate \( \beta_B \)), once \( C_B \) is reinterpreted as the residual income of the banker after depositors have been repaid and loans have been issued.

In a frictionless model, one implicitly assumes that deposits can be costlessly converted into loans. Here instead I assume that the bank is constrained in its ability to issue liabilities by the amount of equity capital (assets less liabilities) in its portfolio. This constraint can be motivated by regulatory
concerns or by standard moral hazard problems: for instance, typical regulatory requirements (such as those agreed by the Basel Committee on Banking Supervision) posit that banks hold a capital to assets ratio greater than or equal to some predetermined ratio. Letting $K_{Bt} = L_{E,t} + L_{S,t} - \varepsilon_t - D_t$ define bank capital at the end of the period (after loan losses have been realized), a capital requirement constraint can be reinterpreted as a standard borrowing constraint, such as:

$$D_t \leq \gamma_E L_{E,t} + \gamma_S (L_{S,t} - \varepsilon_t).$$

(2.17)

Above, the left-hand side denotes banks liabilities $D_t$, while the right-hand side denotes which fraction of each of the banks’ assets can be used as collateral.

Denote with $\lambda_{B,t}$ the multiplier on the bank’s borrowing constraint (later, I will show the conditions that ensure that the constraint is binding). Let $m_{B,t} = \beta_B E_t \left( \frac{C_{B,t+1}}{C_{B,t}} \right)$ denote the banker’s stochastic discount factor. The bank’s optimality conditions for deposits, loans to entrepreneurs and loans to subprimers are respectively:

$$1 - \lambda_{B,t} = E_t (m_{B,t} R_{H,t})$$

(2.18)

$$1 - \gamma_E \lambda_{B,t} = E_t (m_{B,t} R_{E,t+1})$$

(2.19)

$$1 - \gamma_S \lambda_{B,t} = E_t (m_{B,t} R_{S,t}).$$

(2.20)

The interpretation of these first-order condition is straightforward. It also illustrates why the different classes of assets pay different returns in equilibrium. Consider the ways a bank can increase its consumption by one extra unit today.

1. The banker can borrow from household, increasing $D$ by one unit today: in doing so, the bank reduces its equity by one unit too, thus tightening its borrowing constraint one–for–one and reducing the utility value of an extra deposit by $\lambda_B$. Next period, when the bank pays the deposit back, the cost is given by the stochastic discount factor times the interest rate $R_H$.

2. The banker can consume more today by decreasing loans to, say, entrepreneurs, by one unit. By lending less to the entrepreneurs, the bank tightens its borrowing constraint, (since it reduces its equity, loans minus deposits), thus incurring a utility cost equal to $\gamma_E \lambda_{B,t}$; hence the cost is larger the larger $\gamma_E$ is: intuitively, the more loans are useful as collateral for the bank activity, the larger the utility cost of not making loans.

For the bank to be indifferent between collecting deposits (borrowing) and making loans (saving), the returns on all assets must be equalized. Given that $R_H$ is determined from the household problem, the banker will be borrowing constrained, and $\lambda_B$ will be positive, so long as $m_{B,t}$ is sufficiently lower than the inverse of $R_H$. In turn, if $\lambda_B$ is positive, the returns on loans $R_E$ and $R_S$ will be lower, the

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3In the simple case where $\gamma_E = \gamma_S = \gamma < 1$, the fraction $\frac{L_E}{L} = 1 - \gamma$ can be interpreted as the bank’s capital-asset ratio, while $\frac{L_E}{E} = \frac{1}{1-\gamma}$ denotes the bank’s leverage ratio (the ratio of bank’s liabilities to its equity).
lower $\gamma_E$ and $\gamma_S$ are. Intuitively, the larger $\gamma$ is, the higher is the liquidity value of loans for bank in relaxing its borrowing constraint, and the smaller the compensation required for the bank to be indifferent between lending and borrowing. Moreover, loans will pay a return that is (near the steady state) higher than the cost of deposits, since, so long as $\gamma$ is lower than one, they are intrinsically less liquid than the deposits.

In the quantitative implementation of the model, I allow for a more flexible form of the bank capital constraint that allows the bank some flexibility in satisfying its capital requirement. To do so, I allow for a formulation that allows bank capital to temporarily deviate according to the following formulation

$$K_{B,t} > \rho_B K_{B,t-1} + (1 - \rho_B) \left( (1 - \gamma_E) L_{E,t} + (1 - \gamma_S) (L_{S,t} - \varepsilon_t) \right);$$

Assume for the sake of exposition that $\gamma_E = \gamma_S = \gamma$. In this formulation, the capital–asset ratio of the bank, defined as $K_{B,t}/(L_{E,t} + L_{S,t} - \varepsilon_t)$, can temporarily deviate from its long-run target, $\gamma$, so long as $\rho_B$ is not equal to zero. Such a formulation allows the bank to take corrective action to restore its capital–asset ratio beyond one period.

**Firms.** The problem of final good firms is standard and purely static. I assume that these firms operate a standard constant-returns-to-scale technology, so they make no profits in equilibrium. They rent capital and real estate from entrepreneurs and labor from households to solve:

$$\max_t \Pi_t = Y_t - (R_{K,t} K_{E,t-1} + R_{V,t} q_t H_{E,t-1} + W_{H,t} N_{H,t} + W_{S,t} N_{S,t})$$

$$Y_t = K_{H,t-1}^{\alpha(1-\mu)} K_{E,t-1}^{\alpha H} H_{E,t-1}^{\nu} N_{H,t}^{(1-\alpha-\nu)(1-\sigma)} N_{S,t}^{(1-\alpha-\nu)\sigma}. \quad (2.21)$$

The first–order conditions are standard.

**Equilibrium.** I normalize the total supply of housing to unity. The market clearing conditions for goods and houses are:

$$Y_t = C_{H,t} + C_{S,t} + C_{B,t} + C_{E,t} + K_{E,t} - (1 - \delta) K_{E,t-1} + \phi_{KE,t} + \phi_{KS,t} \quad (2.22)$$

$$H_{E,t} + H_{H,t} + H_{S,t} = 1. \quad (2.23)$$

The set of equations summarizing the equilibrium of the model is summarized in Appendix A.

**3. Discussion**

**3.1. Steady State Properties of the Model**

In the non-stochastic steady state of the model, the interest rate on deposits equals the inverse of the household discount factor. This can be seen immediately from equation 2.3 evaluated at steady state.
That is:

\[ R_H = \frac{1}{\beta_H}. \]

In addition, when evaluated at their non-stochastic steady state, equations 2.18, 2.19 and 2.20 imply that: (1) so long as \( \beta_B < \beta_H \) (bankers are impatient), the bankers will be credit constrained and; (2) so long as \( \gamma_E \) and \( \gamma_S \) are smaller than one, there will be a positive spread between the return on loans and the cost of deposits. The spread will be larger the tighter the capital requirement constraint for the bank. Formally:

\[
\begin{align*}
\lambda_B &= 1 - \beta_B R_H = 1 - \frac{\beta_B}{\beta_H} > 0 \quad (3.1) \\
R_E &= \gamma_E \frac{1}{\beta_H} + (1 - \gamma_E) \frac{1}{\beta_B} > R_H \quad (3.2) \\
R_S &= \gamma_S \frac{1}{\beta_H} + (1 - \gamma_S) \frac{1}{\beta_B} > R_H. \quad (3.3)
\end{align*}
\]

I turn now to entrepreneur and subprimers. Given the interest rates on loans \( R_E \) and \( R_S \), a necessary condition for entrepreneur and subprimers to be constrained is that their discount factor is lower than the inverse of the return on loans above. When this condition is satisfied (that is \( \beta_E R_E < 1 \) and \( \beta_S R_S < 1 \)), entrepreneurs and subprimers will be constrained in a neighborhood of the steady state. Alternatively, this condition requires that entrepreneurs’ and subprimers’ discount factors are lower than a weighted average (geometric mean) of the discount factors of households and banks.

\[
\beta_E < \frac{1}{\gamma_E \frac{1}{\beta_H} + (1 - \gamma_E) \frac{1}{\beta_B}}
\]

\[
\beta_S < \frac{1}{\gamma_S \frac{1}{\beta_H} + (1 - \gamma_S) \frac{1}{\beta_B}}
\]

It is also easy to show that both the bankers’ credit constraint and the entrepreneurs’ credit constraint create a positive wedge between the steady state output in absence of financial frictions and the output when financial frictions are present. The credit constraint on banks limits the amount of deposits (savings) that banks can transform into loans. Likewise, the credit constraint on entrepreneurs limits the amount of loans that can become physical capital. Both forces work to reduce the amount of savings that can be transformed into capital, thus lowering steady state output. The same forces are also at work for shocks that move the economy away from the steady state, to the extent that these shocks tighten or loosen the severity of the borrowing constraints.

3.2. Dynamic Properties of the Model

To gain some intution into the workings of the model, it is useful to consider how time-variation in the tightness of the bankers’ borrowing constraint can affect equilibrium dynamics. To do so, it is useful to focus both on the price and the quantity side of the story.

I begin with the price side. For the sake of argument, consider a perfect foresight version of the
model, so that variables are equal to their expected values. In this case, in the limiting case of no adjustment costs, the expression for the spread between the return on loans and the cost of deposits can be written as:

$$E_t(R_{E,t+1}) - R_{H,t} = \frac{\lambda_{B,t}}{m_{B,t}}(1 - \gamma_E).$$

According to this expression, the spread between the return on entrepreneurial loans and the cost of deposits gets larger whenever the banker’s borrowing constraint gets tighter (an analogous expression holds for the spread between $R_{S,t}$ and $R_{H,t}$). Intuitively, when the capital constraint gets tighter (for instance because bank net worth is lower), the bank requires a larger compensation on its loans in order to be indifferent between making loans and issuing deposits. This occurs because loans are intrinsically more illiquid than deposits: when the constraint is binding, a reduction in deposits of 1 dollar requires cutting back on loans by $\frac{1}{\gamma_E}$ dollars. All else equal, a rise in the spread will act as a drag on economic activity during periods of lower bank net worth.

Now I move to the quantity side: whenever a shock causes a reduction in bank capital, the logic of the balance sheet requires for the bank to contract its asset side by a multiple of its capital, in order for the bank to restore its leverage ratio. The bank could avoid this by raising new capital (reducing bankers’ consumption), but the bankers’ impatience motive and the weak economy make this route impractical or, at best, insufficient. As a consequence, the bank reduces its lending. If a substantial part of the economy depends on bank credit to run its activities, the contraction in bank credit causes in turn a decrease in economic activity.

The obvious test of the model is: can bank losses of the magnitude occurred in the last couple of years justify a sharp, large and protracted drop in economic activity? Before I assess the quantitative significance of this mechanism, I need to calibrate the model.

### 3.3. The Model without Banks

As a reference point, it is useful to illustrate the key differences between the model above and a model without banks, or, alternatively, a model where banks are a pure veil and frictionlessy intermediate funds between borrowers and savers. In such a model, all savings are converted into loans, so that equation 2.17 is replaced by the identity

$$D_t = L_{E,t} + L_{S,t}. \quad (3.4)$$

Moreover, bankers disappear from the model, so that $C_{B,t} = 0$ and $\lambda_{B,t} = 0$. In addition, the relevant discount factor to price loans is the patient household’s stochastic discount factor $m_{H,t} = \beta_H E_t \left( \frac{C_{H,t+1}}{C_{H,t}} \right)$. 
4. Calibration

I begin with standard preference and technology parameters. The patient household discount factor is set at $\beta_H = 0.9925$. The entrepreneurial discount factor is 0.96. The subprimers discount factors is set at 0.96. I set the capital share $\alpha = 0.35$ and its depreciation rate $\delta = 0.035$. These parameters imply a steady state 3 percent return annualized on deposits, a capital-output ratio of 1.85 (annualized), and an investment to output ratio of 26 percent. I assume that the share of patient households’ capital in production is $\mu = 0.5$, which results in 53 percent of the capital stock accruing to agents that are not credit constrained (and 47 accruing to the constrained entrepreneurs). As for real estate, I set the share of commercial real estate in production $\nu$ at 0.05, and the weight on housing in utility at $j = 0.08$; these numbers yield a ratio of real estate wealth to output of 2.1 (annualized), of which 0.8 is commercial real estate, 1.3 is residential real estate.

The weights on leisure in the utility function of both households, $\tau_H$ and $\tau_S$, are set at 2. This numbers implies that individuals work about one third of their time endowment, and that the Frisch labor supply elasticity is around 2.

For the parameters controlling leverage, I choose $m_E = 0.9$, $m_S = 0.9$, $m_K = 0.9$, $\gamma_S = 0.9$ and $\gamma_E = 0.9$. I also set the income share of impatient households/subprimers to $\sigma = 0.30$.

I choose the discount factors for the bankers at $\beta_B = 0.965$. Together with the bank leverage parameters, these values imply a spread of about 1 percent (on an annualized basis) between lending and deposit rates.

The capital adjustment cost parameters $\phi_{KE}$ and $\phi_{KH}$ are set at 2. The parameter determining the inertia of the equity requirement for the bank, $\rho_B$, is set equal at 0.5.

For the stochastic process of repayment shock, I assume it follows an AR(1) process with autocorrelation coefficient of 0.9. So, when the shock first hits, most of its effects work through the expectations of further losses in the future.

5. Properties of the Model

5.1. The Baseline Financial Shock

The thought experiment that I consider is the following. What are the consequences of a financial shock in this model? Of course, the experiment begs the question: what is a financial shock?

One possibility is that a financial shock is something that affects the ability of a bank to transform savings into loans. However, this shock is very similar to an investment-biased technology shock, and almost assumes the conclusions: moreover, we already know that this shock (see the discussion in Justiniano, Primiceri and Tambalotti, 2010) has a somewhat hard time (in absence of bells and whistles) in generating the joint comovement of consumption, investment and hours that is the essence of business cycles. Another possibility is that the financial shock captures an exogenous disturbance to the wedge between the cost of funds paid by borrowers and the return on funds received by lenders (see Hall, 2010, for such an interpretation): however, it is hard to give a general equilibrium interpretation.
of this shock, and one would like to believe that changes in spreads are the effect, not the cause, of financial shocks.

For my purposes, I want to give to the financial shock a different interpretation that starts by directly feeding into the model the losses that the shock causes. I want to think of this shock as purely redistributive in nature: for some unmodeled reason, the shock starts with one group of agents paying back less than initially agreed on their obligations. I assume these agents are the subprimers. Hence the shock I consider has a dual nature: from the lender’s (bank) point of view, it is equivalent to an exogenous destruction of the lender’s assets (a negative wealth shock); however, from the borrowers’ point of view, it is equivalent to a positive shock to wealth. Now, it is obvious that the financial shock was not exogenous: one could argue that the real trigger of the crisis was the decline in housing prices that led to defaults that led to non-repayments, but first-hand evidence suggests that the big fallout on economic activity from the decline in housing prices did not occur until banks were forced by loan losses to take dramatic measures to reduce the size of their balance sheets.

The next question to ask is: how big is the shock? I can use available data on quantities to get a sense of its size. Any unexpected non-repayment from the borrower causes a loss of the same amount for a lender. I use the estimates of bank losses following the financial crisis to gauge how big the shock is. In particular, I use estimated loan writedowns for the years between 2007 and 2010, as calculated by the IMF’s Global Financial Stability Report (in April 2009). The Global Financial Stability Report estimates loan losses for U.S. banks over the 2007-2010 period of 1.07 trillion dollars, that is, about 9 percent of private GDP; focusing on the 2007–2008 period, the losses are slightly smaller, about 6 percent of private GDP. Using this information, I calibrate the financial shock as a persistent repayment shock that results, after a two-year period, in cumulative losses of around 6 percent of private GDP. My maintained assumption is that banks do not react to the shock by charging higher interest rates (to make up for the losses or for the higher risk).

How does the financial shock work? Figure 2 plots the dynamic simulations for my model economy.

The negative repayment shock impairs the bank’s balance sheet, by reducing the value of the banks’ assets (total loans minus loan losses) relative to the liabilities (in the model, these are household deposits): at that point, in absence of any adjustment, the bank would have a capital asset ratio that is below target. The bank can restore its capital-asset ratio either by deleveraging (reducing its borrowing from households), or by reducing consumption in order to restore its equity cushion. In the baseline scenario, both forces kick in, and the bank simultaneously reduces both loans and deposits, thus propagating the credit crunch. In particular, the decline in all types of loans to the credit-dependent sectors of the economy (entrepreneurs and subprimers) acts a drag on both consumption and investment. Note that aggregate consumption initially rises reflecting the increased consumption of the borrowers who pay back less. And that spreads rise reflecting the high utility cost of making

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2 These numbers are roughly in line with estimates of loan losses published by the Federal Financial Institutions Examination Council based on Call Reports data. Between 2007 and 2010, net loan losses as a fraction of total loans for all U.S. Banks rose from about 0.4 to about 3 percent; and loan loss reserves rose from about 1 percent to about 4 percent.
a loan for the bank in a period in which banker’s consumption and equity are low. And note that in
absence of deleveraging (top right panel) the bank would find itself nearly insolvent (its capital asset
ratio would go to almost zero): the deleveraging, in turn, forces a reduction in loans.

By contrary, in a model where banks are not forced to restore their capital–asset ratio because
they are just a veil, the losses are much smaller. The financial shock works like a pure redistribution
shock that transfers wealth from agents from agents with a low marginal propensity to consume (the
households who deposit their savings into the bank) to agents with a high marginal propensity to
consume (the subprimers). The main effect of this shock works through labor supply. Subprimers
work less (because their wealth is higher), savers work more (because their wealth is lower). Because
subprimers have higher wealth, they can increase their borrowing and housing demand, thus crowding
out some of the household savings away from the entrepreneurial sector. In the aggregate, the effects
on economic activity are negative but small, only one third as large as those in the model with credit–
constrained banks. These effects are illustrated in Figure 3.

5.2. Breaking Down the Financial Shock

The effects through which the financial shock operates are broken down in Figure 4 (for output) and
Figure 5 (for consumption, investment and other model variables). Figure 4 plots the total loss in
output (as a percentage of its initial level) in various versions of the model where the various frictions
are shut off. Figure 5 offers a more comprehensive picture plotting deviations from baseline for the
various model variables.

To illustrate the strengths of the various channels in shaping output dynamics in response to a
financial shock, I start from the simplest model where labor supply is completely inelastic (so that
$\tau_H$ and $\tau_S$ both approach zero), the banking channel is shut down, and collateral effects on firms
are absent, so that constrained entrepreneurs are not allowed to borrow ($m_H$ and $m_K$ are zero) and
their shares in production ($\mu$ and $\nu$) approach zero. The only effect of the financial shock is that of
transferring wealth away from the savers towards the borrowers. On the one hand, borrowers consume
more. On the other hand, household savers consume less, but also save less in order to smooth their
consumption, so that the decline in their consumption does not fully offset the rise in borrowers’
consumption, and aggregate consumption rises. However, the decline in saving leads to a decline in
investment that more than offsets the rise in consumption, so that aggregate output, although the
total effects are very small. As a fraction of steady–state annual GDP, the total output losses are only
1.5 percent after 10 years.

Consider next what happens when labor supply is assumed to be elastic, as in the benchmark case,
but banking and collateral frictions are still shut off. There is now an additional effect that works
through the differential effects on labor supply induced by the financial shock. On the one hand,
borrowers consume more, and, as can be seen from their labor supply equation, they also work more.
However, because savers’ consumption does not rise to fully compensate the decline in borrowers’
consumption, their increase in hours does not fully offset the decline in hours of the borrowers. As
hours fall in the aggregate, capital – which is complementary to labor in the production function – also falls over time (over and above the effect caused by the decline in savers’ consumption), and the total output loss is now larger, at around 5 percent. Note that here the decline in output comes both from a decline in labor demand (as capital falls because of the reduced savings, the marginal product of labor falls) and from a decline in labor supply. The story here echoes Mulligan (2010)’s idea that labor market distortions were at the core of the recession: for instance, Mulligan (2010) argues that renegotiations of business debts, student loans, and tax debts presented debtors with disincentives to work. In the general equilibrium model presented here, the disincentives to work for the creditors are not fully offset by the larger incentives to work for the creditors, since debtors’ consumption is not determined by their consumption Euler equation (they are facing a borrowing constraints), while creditors’ consumption is. Hence the net effect of a financial shock is an aggregate decline in labor supply.

The above effects are further reinforced when the collateral channel is turned on. The decline in output leads to a decline in asset prices and in the collateral capacity of credit–constrained firms, which exacerbates the decline in output and investment relative to a model in which all productive capital is held by unconstrained households. The overall output loss is about 9 percent after 10 years.

Finally, the largest effects occur in the baseline model where both the banking channel and the collateral channel are at work. By putting direct pressure on the bank’s balance sheet, the financial shock further strengthens the drop in output, and the total output loss after 10 years is about 18 percent (it is around 9 percent after two years).

6. Concluding Remarks

In this paper I have presented a simple model where losses incurred by banks can produced sizeable, pronounced and long-lasting effects on business activity. The key ingredients of the model are regulatory constraints on the leverage of the banks and a productive sector that is bank–dependent for its operations.

A quick back–of–the–envelope calculation illustrates how these effects compare relative to the data. Based on the Congressional Budget Office estimates of the potential output, the output gap turned positive beginning in 2007, and has been negative ever since. In annualized terms, the total GDP loss between 2007Q1 and 2010Q3 has been around 14.8 percent, roughly 1.5 times as big as the model counterpart. A somewhat larger labor supply elasticity, higher intensity of financial frictions, and smaller capital adjustment cost could explain all of the decline in output in the data.
References


Appendix A. The Complete Model Equations.

I summarize here the equations describing the equilibrium of the model.

\[ C_{H,t} + K_{H,t} + D_t + q_t \Delta H_{H,t} = (R_{M,t} + 1 - \delta) K_{H,t-1} + R_{H,t-1} D_{t-1} + W_{H,t} N_{H,t} \]  
(6.1)

\[ 1/C_{H,t} = \beta_H E_t \left( R_{H,t}/C_{H,t+1} \right) \]  
(6.2)

\[ W_{H,t} (1 - N_{H,t}) = \tau_H C_{H,t} \]  
(6.3)

\[ 1/C_{H,t} = \beta_H E_t ((R_{M,t+1} + 1 - \delta)/C_{H,t+1}) \]  
(6.4)

\[ C_{S,t} + q_t \Delta H_{S,t} + R_{S,t-1} L_{S,t-1} - \varepsilon_t = L_{S,t} + W_{S,t} N_{S,t} \]  
(6.5)

\[ L_{S,t} = m_s E_t \left( q_{t+1} H_{S,t}/R_{S,t} \right) \]  
(6.6)

\[ (1 - \lambda_{S,t}) / C_{S,t} = \beta_S E_t \left( R_{S,t}/C_{S,t+1} \right) \]  
(6.7)

\[ W_{S,t} (1 - N_{S,t}) = \tau_S C_{S,t} \]  
(6.8)

\[ C_{B,t} + R_{H,t-1} D_{t-1} + L_{E,t} + L_{S,t} = D_t + R_{E,t} L_{E,t-1} + R_{S,t-1} L_{S,t-1} - \varepsilon_t \]  
(6.9)

\[ D_t = \gamma_E L_{E,t} + \gamma_S L_{S,t} \]  
(6.10)

\[ (1 - \lambda_{B,t}) / C_{B,t} = \beta_B E_t \left( R_{B,t}/C_{B,t+1} \right) \]  
(6.11)

\[ (1 - \gamma_E \lambda_{B,t}) / C_{B,t} = \beta_B E_t \left( R_{E,t+1}/C_{B,t+1} \right) \]  
(6.12)

\[ (1 - \gamma_S \lambda_{B,t}) / C_{B,t} = \beta_B E_t \left( R_{S,t}/C_{B,t+1} \right) \]  
(6.13)

\[ C_{E,t} + K_{E,t} + q_t \Delta H_{E,t} + R_{E,-1} L_{E,-1} = R_{V,t} q_t H_{E,t-1} + (R_{K,t} + 1 - \delta) K_{E,t-1} + L_{E,t} \]  
(6.14)

\[ L_{E,t} = m_H E_t \left( \frac{q_{t+1}}{R_{E,t}} H_{E,t} \right) + m_K K_{E,t} \]  
(6.15)

\[ (1 - \lambda_{E,t}) / C_{E,t} = \beta_E E_t \left( R_{E,t}/C_{E,t+1} \right) \]  
(6.16)

\[ (1 - \lambda_{E,t} m_K) / C_{E,t} = \beta_E E_t \left( (1 - \delta + R_{K,t+1}) / C_{E,t+1} \right) \]  
(6.17)

\[ Y_t = K_H^{(1-\mu)} K_{E,t-1}^{\alpha \mu} H_{E,t-1}^{\varepsilon^\mu} N_{H,t}^{(1-\alpha-\nu)(1-\sigma)} N_{S,t}^{(1-\alpha-\nu)\sigma} \]  
(6.18)

\[ \alpha \mu Y_t = R_{K,t} K_{E,t-1} \]  
(6.19)

\[ \alpha (1 - \mu) Y_t = R_{M,t} K_{H,t-1} \]  
(6.20)

\[ \nu Y_t = R_{V,t} q_t H_{E,t-1} \]  
(6.21)

\[ (1 - \alpha - \nu) (1 - \sigma) Y_t = W_{H,t} N_{H,t} \]  
(6.22)

\[ (1 - \alpha - \nu) \sigma Y_t = W_{S,t} N_{S,t} \]  
(6.23)

\[ E_t \left( \frac{q_t \lambda_{E,t} m_K q_{t+1} / R_{E,t}}{C_{E,t}} \right) = \beta_E E_t \left( q_{t+1} + 1 + R_{V,t+1} \right) / C_{E,t+1} \]  
(6.24)

\[ q_t / C_{H,t} = j / H_{H,t} + \beta_H E_t \left( q_{t+1} / C_{H,t+1} \right) \]  
(6.25)

\[ E_t \left( \frac{q_t \lambda_{S,t} m_S q_{t+1} / R_{S,t}}{C_{S,t}} \right) = j / H_{S,t} + \beta_S E_t \left( q_{t+1} / C_{S,t+1} \right) \]  
(6.26)

\[ H_{H,t} + H_{S,t} + H_{E,t} = 1 \]  
(6.27)
Figure 2: Dynamic Simulations in Responses to a sequence of Financial Shocks: baseline banking model.

The shock is repayment shock that leads to loan losses for banks of 5% of GDP after two years (eight quarters).

Note: Each model period is a quarter. The y-axis measures percent deviation from the steady state.
Figure 3: Dynamic Simulations in Responses to a sequence of Financial Shocks: Model without Banks.

The shock is repayment shock that leads to loan losses for banks of 5% of GDP after two years (eight quarters).

Note: Each model period is a quarter. The y-axis measures percent deviation from the steady state.
Figure 4: Breaking down the effects of the financial shock: effects on cumulated GDP.  

*Note:* Each model period is a quarter. The y-axis measures percent deviation from the steady state.
Figure 5: Breaking down the effects of the financial shock: effects on GDP and other model variables.

Note: Each model period is a quarter. The y-axis measures percent deviation from the steady state.