# Dollar Shortage, Central Bank Actions, and the Cross Currency Basis ◊

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**Abstract**

In our model, cross-currency basis, which captures the deviations from covered interest rate parity, reflects the relative value of the scarcer currency as collateral in funding constraints. Our empirical evidence shows that measures of dollar shortage derived from ECB tenders, and actions to move to fixed-rate tenders with full allotment and to expand the eligible collateral by the ECB have significant power in explaining the cross currency basis. We show that the relaxation of euro funding constraints through 3-year Long Term Re-financing Operations (LTROs) does not contribute to the narrowing of the cross-currency basis, consistent with the theory developed in the paper.

**JEL Classification:** D52, D53, G12, G14, G15. G18.

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# 1 Introduction

We offer a theory of cross-currency basis (between US dollars and euro), and relate it to the US dollar shortage and the actions of central banks during the period 2008-2012. Cross-currency basis is a measure of the deviation of the market forward exchange rate from the one implied by the covered interest rate parity (CIP) arguments. Such deviations observed in the market are measured in units of interest rates. Our theory provides a link between the basis and the shadow price of funding constraints and central bank actions. We offer some new empirical evidence, using ECB tender history on its US dollar operations, and its policies on full allotment and menu of eligible collateral that shows that a significant part of the cross currency basis can be explained by US dollar shortage. This is broadly consistent with the theory proposed in the paper.

We begin by formally defining the cross-currency basis, as it is the central object of our paper. We illustrate the basis with a standard covered interest rate parity argument: consider a spot transaction in which we exchange one euro in the spot market for $X$ US dollars. We immediately invest these US dollars into a one-year risk-free US dollar rate $r_U$, which produces a US dollar income of $X \times (1 + r_U)$ after a year. Simultaneous to these transaction, we can invest one euro into a one-year risk-free euro rate $r_E$ and sell the euro proceeds forward at a one-year forward rate $\chi_T$ to get after a year the amount $(1 + r_E) \times \chi_T$. By covered interest rate parity (CIP) we must have $X \times (1 + r_U) = (1 + r_E) \times \chi_T$. This implies that the theoretical forward rate must be related to the spot rate and the interest rates in the two currencies as: $\chi_T = X \frac{1+r_U}{1+r_E}$. The market forward rate, $\chi$, typically differs from the one implied by CIP. The cross-currency basis is the adjustment, $\alpha$, to the euro rate of interest, which brings the market forward rate in alignment with the spot exchange rate and the interest rates in the two currencies as shown below:

$$\chi = X \frac{1 + r_U}{1 + r_E + \alpha}$$

If $\alpha$ is negative, the basis measures the amount of euro interest rate that must be given up in order to get US dollar in a forward agreement, relative to possessing US dollars in the spot market. In a completely analogous manner, we can express the basis in units of US dollar interest rates as follows:

$$\chi = X \frac{1 + r_U + \beta}{1 + r_E}$$

The economic interpretation of $\beta$ is also intuitive: if dollar is the currency in shortage, then the convenience yield associated with the physical ownership of dollars is reflected by the fact that $\beta > 0$. The owners of dollars at date 1 will only part with their physical holdings of dollars and agree to a forward transaction if they are compensated at date 2 with the effective interest rate $r_U + \beta > r_U$.

The market convention is more in terms of placing the spread on the foreign leg ($\alpha$). The approaches are equivalent as $\alpha = -\beta \frac{1+r_E}{1+r_U+\beta}$. In the empirical results and evidence that we present, we use the market convention $\alpha$. In the theoretical developments we use the measure $\beta$. To empirically construct the basis, we use the spot exchange rates, and the forward rates from the market.

In this paper, we develop a theory of funding constraints that will derive an endogenous expression for the cross-currency basis, $\beta$, as a function of underlying constraints, central bank actions and frictions in the markets. The resulting empirical implications are then tested using data on cross-currency basis, central bank interventions and exploiting the differences in different types of interventions.

## 1.1 CIP Violations and Limits to Arbitrage

Two natural questions arise in the context of cross-currency basis. Why is the cross-currency basis not close to zero? Does its continued presence present an arbitrage opportunity?

A non-zero cross-currency basis goes against the typical cash-flow replication arbitrage argument or CIP. For instance, a positive basis ($\beta > 0$) should not survive an arbitrage consisting in borrowing dollars at the rate $r_U$, exchanging them for euros at the spot rate ($X^{-1}$), investing at the rate $r_E$, and later exchanging back into dollars at the forward rate ($\chi$). Such combination of borrowing dollars at a rate and lending them through an FX swap would give a profit of $\chi X^{-1}(1 + r_E) > (1 + r_U)$ if $\beta > 0$. In the absence of borrowing or funding constraints, it would be scaled up arbitrarily. *In reality, there are impediments to enforcing arbitrage. Besides credit risk, transactions costs and commissions, a key impediment is scalability: arbitrage directly commits funding capability in the scarcer currency.* Such a capability may not be unbounded and may be shared by many other bank activities. Lack of scalability may arise from the inability of some banks to raise dollar funding. High quality collateral (UST) is scarce, so using it to fund other assets (high yield bond) is hard. On the contrary, the risk premium on high-yield assets (which is not scarce) is high, but its ability to be used as a collateral is limited.

The inability of one bank to precisely identify the default risk associated with the bank with which it must engage in forward rate agreements may be another factor. We, however, present evidence later in the paper that it is the funding shortage that accounts for much of the cross-currency basis and its variations. In part, this is due to the fact that the counter-parties can be chosen to trade the basis at market level. We show that the *aggregate* demand for funding in one currency was, in aggregate, very high. Therefore, agents who possess that currency will be price sensitive when allocating their scarce resource. The basis can be seen as the market clearing price for exchanging funding ability in one currency versus another.

We will show that in 2008, right after the failure of Lehman Brothers, and later in 2012, during the Sovereign debt crisis, it was extremely costly (if not impossible) to carry out the arbitrage described above.[1] Banks were extremely reluctant to lend the scarcer currency for any term for the arguments presented earlier: since this is a pre-requisite for enforcing arbitrage, the basis did not converge to zero in the immediate aftermath of the credit crisis. To sum up, we make two observations: first, a premium for the physical possession of the scarcer currency leads to the existence of cross-currency basis. Second, the resulting reluctance to lend the scarcer currency can perpetuate the continuation of cross-currency basis. It stands to reason that any intervention by central banks to reduce this reluctance should alleviate the cross-currency basis and push it towards zero.

What went wrong with the theory of CIP in the case of euro versus dollar in 2008 and 2012? The problem is that for the CIP argument to work, it must be possible to start with arbitrarily large long positions in dollar – *the ability to enter into physical possession of the dollar is the difference.* To see this, let us consider a "buy-sell" transaction, or FX swap. This is a contract that simultaneously agrees to buy (sell) an amount of currency at an agreed rate and to resell (re-purchase) the same amount of currency for a later date to (from) the same counter-party, also at an agreed rate. Note that in the buy-sell, even though a holder of dollars gets the same cash flow holding dollars or doing a

[1] This episode has been pointed out by Goldberg, Kennedy, and Miu (2009), but their arguments rely only on unsecured borrowing.

buy-sell, in terms of physical possession of asset in his account things are quite different. The financial statement will show a dollar asset balance all along if the dollar is managed according to a policy of physical possession, while in the buy-sell the agent will be in possession of euros. This difference lies at the heart of our analysis. In our theory, the basis is the value attributed by the market to the possession desirability of one currency relative to the other. We shall show how this premium, not captured by naive foreign exchange (FX) "arbitrage" formulas (with no funding constraint), can be directly linked to the prevailing shadow values of binding dollar funding constraints.

The remainder of the paper is structured as follows. Section 2 presents some motivating empirical evidence on cross-currency basis, and the nature of central bank interventions that took place during the sample period March 2008 to April 30, 2012. Also, in this section we review the related literature. Section 3 explains why a possession approach is needed to understand deviations from the standard Covered Interest rate Parity (CIP) formula. In Section 4 we provide a simple model that is useful for understanding the before and during the financial crisis that started in 2007 and the effects of central banks' interventions to alleviate the dollar squeeze. In section 5 we develop some testable implications of our theory and provide formal empirical tests. Section 6 shows that the shortage during 2010-2012 is primarily driven by the penalty rates charged by the ECB as opposed to quantity rationing. Section 7 concludes.

## 2 Evidence on Cross-Currency Basis

We present in this section some motivating evidence on cross-currency basis, and the coordinated interventions of the Federal Reserve and the ECB. Figure 1 plots the cross-currency basis for 1-week, 1-month and 3-months tenors, together with the spot exchange rates for the period October 2006 to October 2012. It uses the market convention and measures the basis by $\alpha$ in annualized spreads applied to the risk-free (OIS) rates in euro.

Our main sample is from March 2008 to April 2012. In Figure 1, however, we display the basis for a slightly longer period, going back to 2006, to emphasize the fact that the basis prior to the onset of credit crisis was broadly consistent with CIP. Several interesting facts emerge from Figure 1. First, the cross-currency basis, $\alpha$, is relatively small until the middle of 2007. After the middle of 2007,

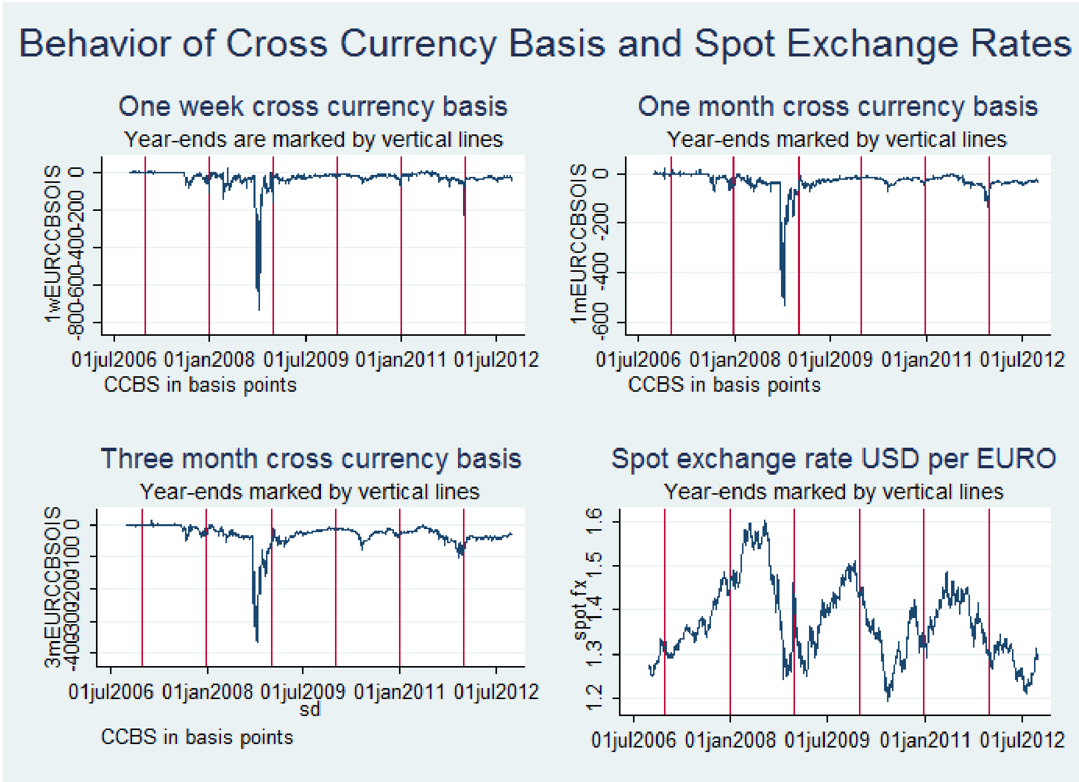


Figure 1: Cross Basis Currency History (Source: JP Morgan). The basis is computed as a annualized spread on the euro interest rate received in basis points (bps) using the respective overnight interest swap (OIS) rates (close to secured borrowing rates).

the basis becomes significantly negative and remains that way throughout the rest of the sample. Summary statistics for the cross-currency basis are presented in Table 1 for the period March 2008 to April 2012. Figure 1 also shows that the cross-currency basis tended to widen around some year-ends (especially in the 2011 year-end), a behavior well documented in money market rates. The spot exchange rates plotted in Figure 1 show relative strengthening of US dollar relative to euros around year-ends, again notably in 2011 year-end. Table 1 shows that the mean of the cross-currency basis for all tenors is significantly negative with the average around $-40$ basis points. This finding was not heavily influenced by extreme outliers in the data, given the large number (over 1,000) of daily observations. It is therefore clear that the cross-currency basis was significantly different from zero for extended periods of time. For the overall sample, we can reject at conventional levels of significance that the mean of the basis during the sample period is zero. Table 1 reports the 95% confidence

Table 1 Summary Statistics on cross-currency Basis

| Tenor | Mean | Std.Error | 95% Confidence interval |
|---|---|---|---|
| 1 week ccbs | -38.498 | 1.916 | [-42.258 to -34.737 ] |
| 1 month ccbs | -40.595 | 1.573 | [-43.682 to -37.509 ] |
| 3 months ccbs | -39.718 | 1.306 | [-42.281 to -37.156 ] |

intervals, and it is clear that throughout this sample period the cross-currency basis was significantly negative.

The basis widened dramatically at two stages in the sample period. The first widening of the basis occurred in 2008-2009 shortly after Lehman Brothers filed for bankruptcy on September 15, 2008. The second widening occurred later in the sample, around late 2011 when the European sovereign debt crisis escalated. We will show that the nature of dollar shortage during the Lehman crisis period was qualitatively different from the nature of dollar shortage in late 2011 and in early 2012.

In this paper we provide an explanation as to why "standard CIP arbitrage" was not possible in the euro-dollar foreign exchange (FX) market, and, in particular, what happened in those two critical subsamples. The main thrust of our argument is that during these two subsample periods, the possession of dollar denominated collateral was relatively more desirable than the possession of euro denominated collateral. This is what we mean by the *dollar squeeze* of the financial crisis. In particular, we examine how the coordinated actions of central banks successfully provided dollar funding and reduced this dollar squeeze.

## 2.1 Potential Explanatory Factors

The literature has identified some important underlying economic factors that can drive the basis, and the inability of market participants to arbitrage away the basis. These factors typically include a) transactions costs, b) counter-party credit risk, c) lack of liquidity in secondary markets, d) lack of funding in one or more currencies due to systemic withdrawal of lending by short-term lenders in a currency. While all these factors might have had a role to play in explaining the basis, we argue that the funding shortage (in US dollars) is the prime cause of basis, after controlling for credit risk and liquidity using empirically observable proxies.

We now review the main features of the 2008-2012 dollar squeeze, in order to provide a perspective for modeling this crisis. Between 2001 and 2008 European banks increased their holdings of US dollar denominated asset-backed security (ABS) - mostly residential and commercial mortgages.McGuire and von Peter (2009) and Fender and McGuire (2010) provide a discussion of this development. Shin (2012) discusses related issues in the context of a global banking glut. Originally, such dollar funding was raised, primarily, through the following avenues: 1) Asset-backed commercial paper (ABCP), 2) short-term repo financing, where ABS is a security with a good repo market; or 3) the bank raises euros and lends them against US dollars. McGuire and von Peter (2009) estimate that, "until the onset of the crisis, European banks had met their dollar funding needs through the inter-bank market ($400 billion), borrowing from central banks ($800 billion), and using FX swaps ($800 billion) to convert domestic currency funding into dollars". However, these three avenues came under significant stress during the 2008 crisis.

As the financial crisis began to unfold from the summer of 2007 and up into the Lehman crisis, lenders (such as money market mutual funds) became risk averse, and began to reject many of those assets as collateral, forcing European banks to bring US assets back on to their balance sheet. By the end of 2010, the amount of the inclusive exposure of European banks is revealed to have been 3.2 trillion dollars.[2] Among those US dollar denominated assets held by the European banks, a vast amount had dropped out of the pool of collateral accepted by the lenders, and European banks found themselves with euro funding capabilities and acute US dollar funding needs. Some European banks, which had been relying on ABCP and repo to fund these assets, sometimes through special purpose vehicles, such as structured investment vehicles or SIVs, were forced to take these US dollar denominated assets back on to their balance-sheets. This left European banks with the third funding option described above: raise euros and lend them to get US dollars. This, however, proved difficult after the credit crisis: private counter-parties were reluctant to lend their dollars because they were worried about the financial health of their counter-parties and they themselves were hoarding dollars. In fact, as shown by Imakubo, Kimura, and Nagano (2008), the LIBOR-OIS spread (an indicator of credit risk and liquidity premium)

[2]See "Recent Developments in Securitization", February 2011, European Central Bank.

significantly increased between August 2007 and April 2008[3]. European banks could not afford to suffer the loss of selling their American ABS long positions at fire-sale prices ("distress selling"). Sometimes the market for the ABS that they held did not even exist any more. The bottom line was that they needed to maintain their US dollar funding or face bankruptcy or significant equity capital infusions. So the situation in the crisis was one of acute dollar funding need. We want our framework to capture such a pressure and be able to model the policy response of central banks in this new situation. In our empirical tests we will control for other factors that might have contributed to the basis.

## 2.2 Coordinated Actions of Central Banks: Dollar Swap Lines

In view of the market developments described above, central banks had to step in and coordinate to try to expand the supply of dollars wherever needed. The Fed could not do it alone because in the end the most critical need for dollars was outside its jurisdiction, i.e., the marginal class of players facing the dollar shortage were European banks. The foreign central banks could not do it alone either as they cannot create dollars. This meant that central banks had to coordinate and channel those dollars, and against these the foreign central bank had to accept a collateral in its own currency. The way the Fed worked with the ECB to handle such a dollar demand was to do a spot foreign exchange, giving them dollars at the spot rate, combined with a forward sale of the same amount of euros.[4]

The dollars that the ECB borrowed were given to European banks through cross-currency repo. Such repos were actually quite different from repos on government bonds. European banks issued new euro denominated bonds, backed by non-related European assets, for which the ECB has some expertise.[5] The

[3]LIBOR is the rate on unsecured inter-bank lending, whereas the OIS, in practice, is used as a GC rate proxy and seen as the expected overnight interest rate, with limited credit and liquidity risk. See Ibakubo, Kimura, and Nagano (2008) for the relationship of LIBOR and OIS with credit and liquidity risk.

[4]The amount of dollar swaps that the Federal Reserve made with the ECB had a peak in December 2008. See Golberg, Kennedy, and Miu (2010) on other months and also on swaps with other central banks.

[5]See ECB document "The implementation of monetary policy in the Euro area", February 2011: http://www.ecb.int/pub/pdf/other/gendoc2011en.pdf, page 50.

ECB took those euro covered bonds together with already existing bonds as collateral against dollar loans.

The holdings of American assets (e.g., subprime ABS, newly illiquid MBS and CMBS) that needed funding could not directly be pledged unless the bank had access to the American Term Auction Facility (TAF) program through its US affiliates. However, as Goldberg, Kennedy, and Miu (2010) point out, the TAF facility was not enough, by itself, to ease the strains in money markets after the Lehman Brothers bankruptcy episode. The central banks' swap facilities were crucial for the "normalization" of the LIBOR. See also McAndrews, Sarkar, and Wang (2008) on the effectiveness of the TAF program on the LIBOR rate during the crisis period.

In a coordinated action following the financial crisis of 2007, the Federal Reserve (Fed) and the European Central Bank (ECB) swapped dollars for euros in order to let the ECB meet some of the high demand for dollars by the European banks. The ECB was then able to provide dollar funding to the member banks by accepting as collateral euro denominated covered bonds.[6] In addition, on July 31, 2008 ECB announced that it would conduct, in conjunction with the Federal Reserve, Term Auction Facilities to inject US dollars. Starting on August 8, the ECB started to conduct 84-day operations under the Term Auction Facility, while continuing to conduct operations with a maturity of 28-days. The ECB conducted bi-weekly operations, alternating between operations of USD 20 billion of 28-days maturity and operations of USD 10 billion of 84-days maturity.

*October 15 2008*: Perhaps the most significant development occurred when ECB announced that effective from October 15 2008, a fixed-rate, full allotment policy would be used in all its refinancing operations for the different maturities. Under fixed rate full allotment counterparties have their bids fully satisfied, against adequate collateral, and on the condition of financial soundness. This allowed the counterparties to control the amount of liquidity they demand. Thus, a falling demand for liquidity can be seen as a sign of normalization. ECB also committed to maintaining the fixed-rate full allotment policy until the middle of July 2012. In addition, ECB expanded significantly the menu of collateral on October 15 2008, and permitted dollar collateral in its dollar financing operations.

[6] Such an action, with more aggressive pricing of the swap lines, was also taken in December 2011 by the Fed.

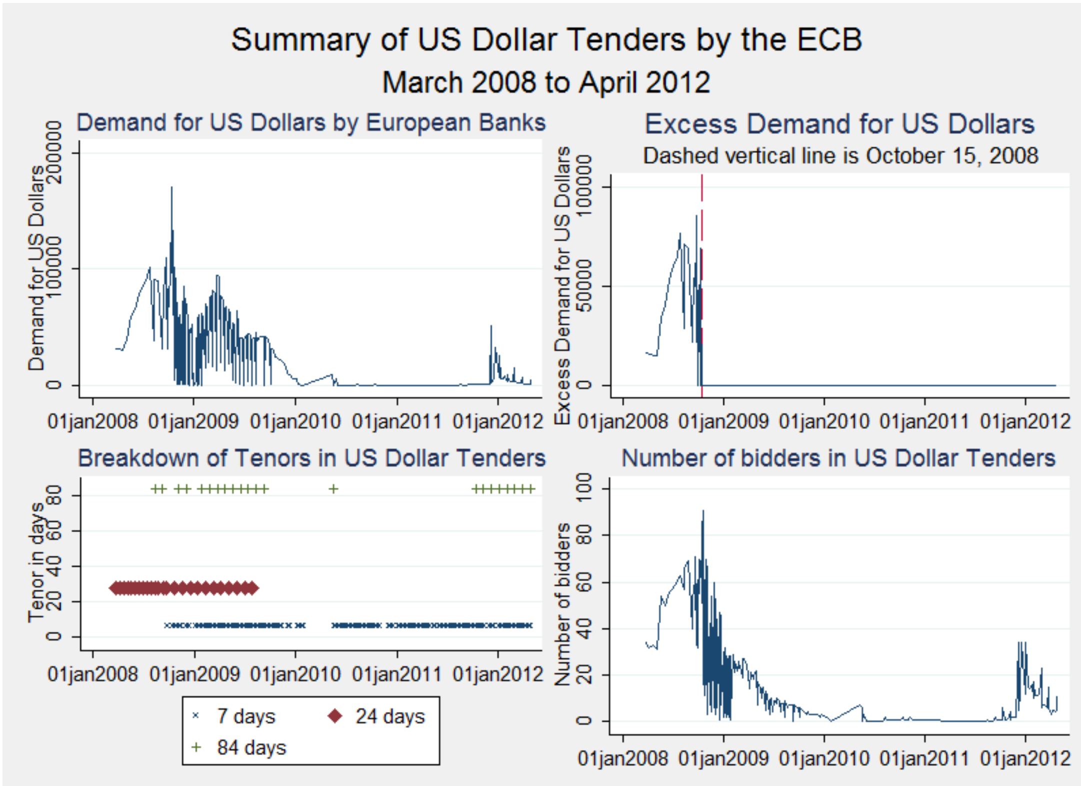


Figure 2: Summary of US dollar operations in ECB Tenders

During the period 18 March 2008 to 30 April 2012, a total of 259 US dollar operations occurred. Thus, on average, there was a dollar injection once every 5.8 days during this period! Of these the Term Auction Facilities or TAF accounted for 129 interventions. During the same period, there were 510 euro currency tenders, including 134 LTROs, and 214 MROs.

We begin by providing in Figure 2 a summary of all ECB tenders injecting US dollars in Figure 2 to motivate our theory and empirical work. The graph in the top left panel shows the demand for US dollars in each of the ECB tenders. The demand is measured by the volume of bids submitted by the European banks (in millions of US dollars) in the US dollar tenders of ECB. Note that the demand was quite elevated in the immediate aftermath of Lehman brothers bankruptcy and only settled down almost a year later. There is a smaller spurt in demand later in the sample period during the sovereign debt crisis. The top right panel shows the *excess demand* for US dollars from the perspective of European banks. This is measured by the difference between the volume of bids submitted minus the volume allotted by the ECB. As noted earlier, effective from October 15 2008 there was no caps on the dollar supplied in ECB tenders. Consequently, our measure of excess demand becomes zero after that date. It is clear that there was considerable excess demand for US dollars (or dollar squeeze) during the aftermath of Lehman bankruptcy. Thereafter, the demand for US dollars was fully met by the ECB tenders. We will show in Section 5 that after October 15 2008, while ECB was prepared to meet all the demand for dollars, it was setting a fixed rate that tended to be higher than the cross currency swap rates until late 2011.

The bottom left panel shows the time-series pattern of the tenor of US dollar ECB tenders. Although ECB supplied US dollars with several variations in the tenor, more than 80% of the supply of dollars was concentrated in three tenors: 7-days, 28-days and 84-days. Note the increased frequency of 3-months tenders after the Lehman bankruptcy and during the European sovereign debt crisis periods. Access to such term US dollar funding might have also contributed to reduce the shadow price of US dollar funding constraints. The graph on the bottom right plots the number of bidders who participated in the ECB tenders. Note the steep increase in the number of bidders as Lehman brothers bankruptcy became imminent. The number of bidders shows a smaller spurt later in the sample period. Even though the demand for US dollars were fully met by the ECB after October 15, 2008, it is clear from the top left panel and the bottom right panel, that the demand for US dollars was strong during the aftermath of Lehman Brothers bankruptcy and later in 2012. To the extent that there were banks with US dollar demand who could not post eligible collateral to ECB there could have existed excess demand for US dollars.

## 2.3 Further Relationship With The Literature

To the best of our knowledge, this paper is the first to analyze the cross currency basis from the point of view of relative funding needs of the same agent in different currencies. Following Bottazzi, Luque, and Pascoa (2012) we relate the underlying determinants of this financial crisis and the "convenience yield" of physically possessing the scarcer currency. The present paper provides a direct link between the cross-currency basis puzzle and the literature on collateral value of securities - see Duffie (1996) on "repo specialness", Brunnermeier and Pedersen (2009) on collateral margins and market liquidity, and Bottazzi, Luque, and Pascoa (2012) for a general equilibrium model of leverage with securities as collateral.[7]

Our work is also related to the recent literature on "limits of arbitrage" - see Gromb and Vayanos (2010) for an extensive survey of this literature. We offer a novel view on the role of funding constraints in the foreign exchange (FX) markets: the impediments to enforce the arbitrage in the FX market prevents the currency basis from disappearing.

Our paper is closely related to the empirical studies that link dollar funding costs to tensions in the FX swap market. Our model shows that, when dollars are immediately needed, the pressure is translated to the cross-currency basis. This is consistent with Baba and Packer (2009) and Goldberg, Kennedy, and Miu (2010), who find evidence of how the premium paid for dollars in the FX swap market, which rose dramatically (up to 400 basis points) in October 2008, is linked to the high dollar funding costs in terms of LIBOR rates that non-US banks experienced at the end of year 2008. Griffoli and Ranaldo (2010), on the other hand, execute a similar analysis considering instead dollar funding costs in terms of OIS rates and GC rates, and find that excess returns from secured arbitrage are nearly exactly equal to those from unsecured arbitrage. Also, Hrung and Sarkar (2012) provide an extensive empirical analysis of cross-currency basis and find out that the basis is higher for European banks following an unanticipated decrease in repo funding amounts, implying that dollar funding constraints were binding for European banks during the crisis.

A hallmark of the present crisis is the active interventions by central banks in private capital markets. While a theoretical model seems to be missing, many empirical papers have documented the effects of the central banks' actions on asset prices. Krishnamurthy, Nagel, and Orgov (2011) and Hrung and Sarkar (2012) provide important empirical analyses on the relationship between central banks interventions and the funding liquidity needs of major banks in the economy. On the one hand, Krishnamurthy, Nagel, and Orgov (2011) examine funding of global banks in private markets and in central bank facilities, and find that the Fed's Primary Dealer Credit Facility is highly significant for easing funding constraints. Hrung and Sarkar (2012), on the other hand, find evidence that the basis is lower the day after successful borrowing at the Fed's dollar

[7] Empirical support can be found in Jordan and Jordan (1997), Longstaff (2004) and Bartolini, Hilton, Sundaresan, and Tonetti (2011). Also, see Krishnamurthy, Nagel, and Orlov (2011) for evidence of how the Fed liquidity lowered funding costs and eased strains in repo markets.

liquidity facilities.

Our contribution differs from the above in the following important respects: first, we construct a theoretical framework to link the cross-currency basis with the shadow prices of US dollar funding constraints relative to euro funding constraints. We provide a "convenience yield" interpretation of the basis, and identify the components driving that convenience yield. Second, we show that actions of central banks that relax the euro funding constraints can exacerbate the *relative scarcity value of dollars* even higher and hence can potentially drive the basis even higher. Third, we estimate the excess demand for dollars from the US dollar tenders conducted by the ECB and find that this variable can be very useful in explaining the time-series variations in basis. Finally, we find evidence that the 3-year LTROs of ECB in supplying over 1 trillion euros did not contribute to pushing the basis towards zero.

# 3 A Theory of Currency Possession

## 3.1 CIP Violations

In our setting there are two dates and two assets. The two assets have the same value at date 1, which implies that buying one asset and selling the other is a self-financing or zero-investment strategy. In what we call a *possession swap,* owners of assets of similar value can agree to exchange their physical possession for a while with the agreement to pass back cash flows to original owners. Because the arrangement does not alter cash-flows received, doing it should have no pricing impact under the cash flow replication theory. We will later show that it can. Let us look at the special case of currencies.

We will provide two interpretations: first in the context of an FX swap, and then in the context of the Covered Interest rate Parity (CIP). Let us first consider the FX swap as it closely follows the scheme above: agent 1 can invest the domestic currency by himself and earn interest, or get the same cash flow from agent 2 but by exchanging the domestic currency for foreign currency at the beginning of the period at date 1, and then claiming back the domestic currency from agent 2 at date 2 through FX swap. This is the same amount of currency both in the front leg of the trade (at date 1) and in the back leg (at date 2). In both cases agent 1 has exactly the same currency that he started with plus interest earned over the period. The difference lies in the fact that agent 1 did not have possession of domestic currency between the two dates in the FX swap. So we have an implementation of a possession swap for currencies.

We also have the CIP variation: the owner of the first asset can invest the asset (say, domestic currency), earn the spot rate of interest (in domestic currency) and receive the principal plus interest at date 2. Alternatively, the owner can sell asset 1 in the spot market and thus give up the physical ownership, invest in the alternative asset (foreign currency), which can be sold at date 1 in the forward markets so that the proceeds at date 2 are converted back in the original asset (in domestic currency). With the latter, a canonical buy-sell *(that is, buying one asset and selling the other at zero costs)*, the owner of asset 1 is generating cash flows in asset 1 (domestic currency) at date 2, but does not physically own asset 1 between the two dates: instead, the original owner is relying on the counterparty in the forward market to deliver asset 1 at date 2.

The canonical buy-sell entails the *exchange at date 2 of the FX proceeds and the domestic currency.* This means that for an original holder of asset 1 he gets the same cash flows at date 2 in either of the two strategies. Thus, according to *the* cash-flow replication theory, the value of being long in one strategy and short in the other should be self-financing. The only difference between holding asset 1 and engaging in the buy-sell is that the asset is physically possessed between the two periods in the former strategy.[8]

## 3.2 Analogy Between FX Swap and Repo

We think that the FX swaps (combined short sale with forward purchase of currency) are to currencies what repos are to securities. A repo transaction exchanges possession of a security against possession of a currency for the duration of the repo transaction. An FX swap exchanges the possession of one currency against the possession of another currency for the duration of the FX swap. Such transactions are naturally collateralized. This feature makes it natural for us to examine the cross-currency basis in terms of the scarcity value of a currency, using as a theoretical basis for such an interpretation the analogue of what happens

[8]Note that the front exchange of the FX swap can easily be neutralized using a simple spot transaction. Note also that each currency earns a different interest when invested in debt, but we shall see that this approach (i.e., CIP) alone is not enough to explain the difference in value between $X$ and $\chi$.

in securities markets.

In repo, an important concept is the *specialness* of a security, which occurs when the security repo rate is below the General Collateral (GC) rate, the highest repo rate for those securities with similar maturity and asset class. There is an equivalent to specialness that we will introduce in this section: *cross-currency basis*. That such a basis may not be trivial is evident from Figure 1. *Specialness* for security is the correction that lowers the loan rate as a compensation for the value of possession of the security compared to the currency. Cross-currency basis is such a correction to the value of interest rate earned by each currency. Most of the time possession of currency is a trivial matter for the domestic player, and it is much more common for a security to attract possession value through specialness than for a currency. Nevertheless, and especially from an international perspective, currencies also can be in relative scarcity. This is mostly because, for some of those international agents (such as European banks), the link between some of their assets and the capacity to raise the currency they need (such as US dollars) can become tenuous in a crisis, as they sometimes cannot pledge their assets to raise foreign funding.

# 4 A Simple Model of Currency Desirability

## 4.1 Scenarios during the crisis

We develop a simple model with two dates, $t = 1, 2$. To understand what drives the value of physically holding a currency in the different stages of the crisis, we consider three different scenarios:

1. **Before the crisis with no central bank intervention**. Here we consider the period before the crisis, when markets functioned normally and liquidity was provided. We consider that the representative bank had access to the following six liquid markets:

- *The market for the american and european good.* We denote by $\omega_{U,1}$, $p_{U,1}$, and $x_{U,1}$ the initial endowments, price, and demand of the US good, respectively. We replace subscript $U$ by $E$ to denote the analogous European variables. We ignore international trade in commodities. Trade thus occurs within the two areas, US and EU, and not between them.

- *The uncollateralized borrowing market*: Access to unsecured funding in dollars and euros is represented by action variables $a_U$ and $a_E$ (actually depos), respectively, subject to the following constraints $a_U + A_U \geq 0$ and $a_E + A_E \geq 0$. The credit lines are represented by initial endowments $A_U \geq 0$ and $A_E \geq 0$ in depos that could possibly be issued. The associated uncollateralized borrowing rate at date 2 is $(1 + i_U)$ and $(1 + i_E)$. LIBOR and EURIBOR are used as proxies for the unsecured borrowing of american and european banks, respectively. Several points should be noted here. LIBOR is "fixed" from the quotes supplied by the panel of banks. LIBOR fixing involves "refreshing" of the panel whereby weak banks are replaced by strong banks to keep the credit quality of the banks in the panel AA. In addition, only inter-quartile range is used in averaging to fix LIBOR. In all their arbitrage transactions, however, banks must use their own funding costs, which can be different from the "fixed" LIBOR.

- *The american and european bond markets*: The corresponding action variables are $b_U$ and $b_E$, respectively.[9] Their respective rates correspond to relevant funding rates of each bank through its central bank. American banks with deposits at their Fed funds system lend to each other at the federal funds rate, here denoted by $r_U$. For the euroarea, banks lend to eah other unsecured at the EONIA rate, $r_E$. Banks typically hedge the risks in these rates by engaging in interest rate swaps. Thus, policy rates $r_U$ and $r_E$ are effectively tracked by the corresponding Overnight Index Swap (OIS) rate. OIS is very close to the repo rate in domestic repo operation of the Fed (and ECB) using high grade domestic collateral, but such rates can of course differ.[10] See Bech, Klee, and Stebunovs (2010) for a characterization of the relationship between the Treasury GC repo rate and the federal funds rate in three different periods: before the crisis (from 2002 to 2007), the early stage of the crisis (from August 2007 to December 2008), and after the Fed intervention. As shown by these authors, the federal funds rate communicated policy to the repo market quite well in the pre-crisis period, but after December 2008 the relationship deteriorated.

[9]We will deal later with a version were the collateral is lent to raise funding, which is probably the most relevant funding market. In such a case, action variables are the respective traded amounts $z_U$ and $z_E$ in the repo market of such collateral.

[10]In fact, funding can be thought of as happening in the repo market of central bank eligible collateral. Mancini-Grioffoli and Ranaldo (2010) show that CIP deviations are nearly exactly as equal in terms of both OIS rates and GC rates.

- The *repo market*: Alternatively, banks can secure their funding at repo rates, which can differ from OIS rates, especially during a crisis period. In a repo collateral is lent to raise funding. This is probably the most relevant funding market. Action variables are the repo trades $z_U$ and $z_E$ using american and european collateral, respectively. The corresponding repo rates are denoted by $\rho_U$ and $\rho_E$. To simplify matters, we assume that the haircuts $1 - h_U$ and $1 - h_E$ are specified exogenously. We will review the implications of this assumption on our results later. The representative agent has a funding capacity in both dollar and euro, expressed by $e_U$ and $e_E$ respectively - modeled here as short term bond holdings that can be sold or lent as collateral.[11] In our set-up, each European (American) bank starts with a relatively large amount of euro (dollar) collateral $e_E$ ($e_U$).

- *The FX spot market*: action variable $s$ denotes the amount of euros at date 1 that can be exchanged against $Xs$ amount of dollars at date 1, where $X$ is the spot exchange rate.

- *The FX swap market*: action variable $f$ denotes the amount of euro sold against $Xf$ dollars at date 1. Then, at date 2 the same amount of euros $f$ is bought back against $\chi f$ dollars, where $\chi$ is the rate that can be locked in at date 1 to trade euros against dollars at date 2 (the forward FX rate).

2. **Central banks intervention when the ECB only accepts collateral denominated in euros**. Here we consider the period following the financial crisis of 2007, when the Federal Reserve (Fed) and the European Central Bank (ECB) swapped dollars for euros. During this period, all uncollateralized and collateralized markets were under significant stress and this situation was reflected in the market for the basis, which rose up to 400 basis points after the Lehman Brothers bankruptcy episode in October 2008. The ECB was then able to provide dollar funding to the member banks by accepting as collateral euro denominated covered bonds. This period corresponds goes until October 15 2008, when a new policy of collateral relaxation was introduced, and starts again on January 1 2010,

[11]The credit reputation of the agent, capital adequacy, and access to markets such as credit lines are some of the variables, which will inform on the endowments $e_U$ and $e_E$, and reflect his funding capacity. Also, deposits base will be a key factor for funding capacity of banks. For simplicity and abstraction, we sum up such funding capacity as an initial supply of bond in the given currency.

when dollar denominated collateral was no longer accepted. In addition to the ECB's repo facility only acceting euro collateral, the representative european bank can get access to the FX spot and swap markets.

3. **Central banks intervention when the ECB accepts collateral denominated in dollars**. This period goes from the date when the ECB announced unortodox measures to allivate the demand for dollars, on October 15 2008, to the end of 2009, when dollar collateral was no longer accpted. As in case 2, in addition to the ECB's repo facility, now accepting only dollar collateral, the representative european bank can get access to the FX spot and swap markets.

Next, we describe in detail the constraints that the representative European bank faced during each of these three scenarios, and provide, for each scenario, a proposition that related the basis to the underlying frictions in these markets.

## 4.2 Before the crisis with no central bank intervention

In what follows, we consider a simple setting where a representative European bank maximizes a utility function defined on the consumption of the European and american goods at dates 1 and 2, $(x_{E,1}, x_{U,1}, x_{E,2}, x_{U,2})$, subject to natural constraints that apply to the amount of dollars, euros, American bond, European bond, uncollateralized borrowing at dates 1 and 2, and spot and forward FX trades. We now proceed to present the box constraints for this first scenario. The term "box constraint", which follows from Bottazzi, Luque, and Pascoa (2012), means that each agent can possess currencies and securities in non-negative quantities, but overdrafts are not allowed in currencies and security balances. Securities can be shorted and loans in each currency can be arranged, but non-negative possession of such securities and currencies have to be monitored and enforced all along. Each agent has a box constraint for each currency and asset.[12] The box "no overdraft" constraint for euros can be thought as a standard budget constraint. We will now present these box constraints.

The dollar no-overdraft box at date 1 is:

$$p_{U,1}(\omega_{U,1} - x_{U,1}) + X(s + f) - a_U - b_U - h_U z_U \geq 0, \qquad (\$.1)$$

[12]Position and possession should not be confused. The former can be negative, the latter cannot (negative positions have to be compensated in some way, as will be seen in detail).

We replace subscript $U$ by $E$ to denote the analogous European variables and write similarly the euro no-overdraft box at date 1:

$$p_{E,1}(\omega_{E,1} - x_{E,1}) - (s + f) - a_E - b_E - h_E z_E \geq 0 \tag{e.1}$$

The uncollateralized borrowing is subject to the following constraints:

$$a_U + A_U \geq 0 \tag{i.o.u. U}$$

$$a_E + A_E \geq 0 \tag{i.o.u. E}$$

We also have the two additional funding constraints. The boxes for US and euro bonds are, respectively:

$$z_U + b_U + e_U \geq 0 \tag{Funding.U}$$

$$z_E + b_E + e_E \geq 0 \tag{Funding.E}$$

It is worth observing the interaction between box constraints (\$.1) and (Funding.U). If dollars are needed in period 1, the agent can borrow uncollateralized ($a_U < 0$) subject to constraint (i.o.u. U). But also this agent can either sell American bonds ($b_U < 0$) or lend American bonds as collateral through repo ($z_U < 0$). In both cases, the agent is constrained by its box (Funding.U): the amount of the American bond that the agent purchases ($b_U > 0$) minus what he sells ($b_U < 0$), plus what he borrows ($z_U > 0$) minus what he lends ($z_U < 0$), plus his initial endowments, must be non-negative. The interpretation of the box constraint (Funding.E) for European bonds is analogous.

Finally, we introduce the date 2 dollar and euro no-overdraft boxes:

$$p_{U,2}(\omega_{U,2} - x_{U,2}) - \chi f + (1 + i_U)a_U + (1 + r_U)(b_U + e_U) + (1 + \rho_U)h_U z_U \geq 0 \tag{\$.2}$$

$$p_{E,2}(\omega_{E,2} - x_{E,2}) + f + (1 + i_E)a_E + (1 + r_E)(b_E + e_E) + (1 + \rho_E)h_E z_E \geq 0 \tag{e.2}$$

The agent can obtain dollars at date 1 by (i) exchanging euros for dollars at spot rate $X$, (ii) swapping euros by dollars at $X$ and giving them back at date 2 at the FX forward rate $\chi$, (iii) selling American bonds or the American good, (iv) pledging the bond as collateral through repo, and (v) borrowing uncollateralized at interest rate $i_U$.

Hereafter, we use the notation $\gamma_U$ for the Lagrange multipler of the constraint (i.o.u. U). Similarly, we use $\gamma_E$ for the multiplier on constraint (i.o.u. E). We use $\mu$ for the Lagrange multiplier of the corresponding box constraint. For instance, we write $\mu_U$ for (Funding.U), $\mu_E$ for (Funding.E), $\mu_{\$,1}$ for (\$.1), $\mu_{\$,2}$ for (\$.2), $\mu_{e,1}$ for (e.1), and $\mu_{e,2}$ for (e.2).[13]

An equilibrium for an economy with a set of traders $\mathbf{I} = \{1, ..., i, ..., I\}$ is defined by an allocation $(x^i_U, x^i_e, a^i_U, a^i_E, b^i_U, b^i_E, z^i_U, z^i_E, , s^i, f^i)$ such that: (a) each trader maximizes his utility function (or profits - see Remark A in the Appendix), subject to his box constraints (\$.1), (e.1), (i.o.u. U), (i.o.u. E), (Funding.U), (Funding.E), (\$.2), and (e.2); and (b) all markets (goods, uncollateralized borrowing and lending, bonds, repo, sopt FX, and forward FX) clear. We leave the details of each of the market clearing conditions for the Appendix.

While the setting that proposed here is quite simple, our results can be shown to be fairly robust. Before preseting our first proposition, we make two important remarks about our results below.

First, for all our purposes, as we will see, private agents could be maximizing utility or the present value of discounted profits. In the Appendix we derive similar results for the case when agents are profit maximizers (see Remark A).

Second, it is important to notice that, since all constraints introduced above are linear, Kuhn-Tucker conditions for the constrained maximization of utility (or of present value profit as in Remark A in the Appendix) hold as *necessary conditions* for any equilibrium. Roughly speaking, the results derived below must hold whatever is the equilibrium outcome.

Proposition 1 below presents a formula for the basis derived from the optimality conditions on bond trading. In the Appendix, we derive formulas for the basis from the optimality conditions on repo borrowing (Proposition 1.2), and on uncollatearlized borrowing (Proposition 1.3). These are just equivalent ways, all interesting, to express the *same* basis in terms of different interest rates, haircuts and shadow prices. Explanations of each item in Proposition 1 are provided after the statement.

[13]More formally, let $V$ be a differentiable utility function and $\hat{V}$ be the indirect utility resulting from maximization of $V$ under the above box constraints. Then $\mu_U = \partial\hat{V}/\partial e_{U,1}$ and $\mu_{\$2} = \partial\hat{V}/\partial(p_{U,2}w_{U2})$. These derivatives can be written in terms of marginal utilities as observed in the Appendix, Remark A.

**Proposition 1 (Before the crisis with no central bank intervention):** *Optimal funding by trading bonds requires the cross currency basis to be driven by the relative difference in shadow costs between pledging dollar collateral and pledging euro collateral, i.e.,*

$$\beta = \frac{\mu_U - \frac{\mu_E}{X}}{\mu_{\$,2}}$$

$\square$

From Proposition 1 we get the following insights. First, a positive multiplier $\mu_U$ driving the currency basis signals that European banks have an immediate funding need in dollars. In a trade-only set up, the higher is $\mu_U$, the more they wish to issue debt at the dollar denominated bonds rate $r_U$. If we were to allow for both bond trades and bond loans the multiplier signals the desire to obtain dollar funding by short-selling or by lending the bond. In any case, there is a scarcity of the bond. When $\mu_E > 0$, we see that our model predicts that the cross-currency basis should narrow. This prediction is also intuitive from an economic perspective: if euros also become scarce (and thus $\mu_E > 0$), the cross-currency basis should decline as there are now convenience yields in *both* currencies. That is, the dollar funding needs relative to euro funding needs are ameliorated, and therefore, the basis shrinks. To see why a possession value comes in the forward, one has to compare a spot transaction with forward transaction. For example, selling euros at date 1 versus locking in this sale at date 1 to be executed at date 2. The difference is not only the prevailing interest rate on both currencies. In the case of spot transaction for the interim period, one possesses dollars instead of euros. The value is thus adjusted for the relative possession value of both currencies.

Second, in a set-up with several bond categories, the basis would be driven by positive multipliers $\mu_U$ of any scarce bond - any bond whose box constraint is binding and with a positive shadow value for some agent. Following Lehman Brothers' bankruptcy and until the Fed increased the supply of Treasuries in October 2008, scarcity coexisted with specialness (repo rate below GC) in many bond categories, and with a peak in repo fails, due to the difficulty in getting a hold on them. However, after the Fed stepped in, specialness was gone but the currency basis persisted. That is, $\mu_U$ remained positive, which is equivalent to saying that the general collateral rate remained below OIS.

Third, the funding problem signaled by $\mu_U > 0$ is also related to the solvency problem signaled by a another shadow price, the multiplier $\mu_{\$,1}$ of the no-overdraft constraint (\$.1) in dollars at date 1. In fact, the first order condition on bond trades requires

$$\mu_{\$,1} = \mu_U + \mu_{\$,2}(1 + r_U) \tag{1}$$

and, therefore, the marginal rates $\mu_{\$,1}/\mu_{\$,2}$ and $\mu_U/\mu_{\$,2}$ move together, as long as $r_U$ stays the same. Hence, a funding need is concomitant with a solvency difficulty. See Brunnermeier and Pedersen (2008) for a model that relates market liquidity and funding liquidity.[14]

### 4.2.1 Quantifying the shadow prices

Shadow prices $\tilde{\mu}_{\$,1} \equiv \mu_{\$,1}/\mu_{\$,2}$, $\tilde{\gamma}_U \equiv \gamma_U/\mu_{\$,2}$ and $\tilde{\mu}_U \equiv \mu_U/\mu_{\$,2}$ can be quantified and measured in terms of the different interest rates. For this, we just need to take the first order conditions with respect to $a_U$, $b_U$ and $z_U$, respectively:

$$1 + i_U = \tilde{\mu}_{\$,1} - \tilde{\gamma}_U \ , \ 1 + r_U = \tilde{\mu}_{\$,1} - \tilde{\mu}_U, \text{ and } (1 + \rho_U)h_U = h_U\tilde{\mu}_{\$,1} - \tilde{\mu}_U$$

The three formulas above constitute a system of three equations and three unknowns, and therefore, we can get a solution:

$$\tilde{\mu}_U = \frac{(r_U - \rho_U)h_U}{1 - h_U}, \ \tilde{\mu}_{\$,1} = 1 + r_U + \frac{(r_U - \rho_U)h_U}{1 - h_U}, \text{ and } \tilde{\gamma}_U = \frac{r_U - h_U i_U - (1 - h_U)\rho_U}{1 - h_U}$$

These formulas deserve two remarks. First, the non-negative of Lagrange multipliers implies that $r_U \geq \rho_U$ and $r_U \geq h_U i_U + (1 - h_U)\rho_U$. The former naturally satisfies as OIS rate is never below the repo rate. The latter also holds by choosing an appropriate haricut $(1 - h_U)$ as $i_U \geq r_U \geq \rho_U$. This is a natural assumption that our exogenous haircut parameter should satisfy in this model.

Second, in our model we have only one class of dollar denominated bonds, so we cannot distinguish the repo rates of different bonds and talk about specialness. In a model with several bonds, if the general collateral rate coincides with OIS, the OIS-repo differential is the repo specialness of the bond. However, a positive $\mu_U$ is compatible with no specialness if the general collateral rate $\rho_U$ is already below OIS rate $r_U$. That is, a positive currency basis $\beta$ is driven by a funding

[14]To avoid expanding our model and losing the focus on the basis issue, we chose not to model defaults, but the potential for default is captured by $\mu_{\$,1}$.

difficulty in dollar denominated bonds, but can coexist with these bonds not being on special in repo markets. This is because General Collateral (GC) is defined as the highest repo rate within a class of bonds - but the overall class itself can become scarce (it would be reflected in GC Vs. OIS).

Finally, notice that the expressions derived above for the multipliers immediately imply that CIP violations in terms of secured funding rates, written as $\beta_\gamma \equiv \tilde{\gamma}_U - (\tilde{\gamma}_E/X)$, do not necessarily coincide with CIP violations in terms of unsecured funding rates, written as $\beta_\mu \equiv \tilde{\mu}_U - (\tilde{\mu}_E/X)$. What it remains to identify are the conditions under which unsecured and/or secured funding constraints bind. There are three possibilities:

1. *Unsecured funding unconstrained* ($\tilde{\gamma}_U = 0$)*, secured funding constrained* ($\tilde{\mu}_U > 0$) : This case occurs when the OIS rate is above the repo rate (i.e., $r_U > \rho_U$, a necessary condition for $\tilde{\mu}_U > 0$) and the haircut is such that $r_U = h_U i_U + (1 - h_U)\rho_U$. Also, by looking at $\beta_\gamma$ and $\beta_\mu$, we can assert that case 1 happens when the cross currency basis is 0 for LIBOR, but positive for OIS rate.

2. *Unsecured funding constrained* ($\tilde{\gamma}_U > 0$)*, secured funding unconstrained* ($\tilde{\mu}_U = 0$): The former requires that the OIS rate and the repo rate coincide (i.e., $r_U = \rho_U$), while the latter requires $r_U > h_U i_U + (1 - h_U)\rho_U$. However, these two conditions imply $r_U > i_U$, an impossibility result as empirically we know that OIS rate cannot be greater than the LIBOR. In terms of CIP violations, we can assert that whenever $\beta_\gamma > 0$, we have $\beta_\mu > 0$.

3. *Unsecured and secured funding constrained* ($\tilde{\gamma}_U > 0$, $\tilde{\mu}_U > 0$): This case occurs when $r_U > \rho_U$. CIP violations occur in terms of both secured and unsecured funding since $\gamma_U > 0$ and $\mu_U > 0$. This implies that $i_U > r_U$ and $r_U = h_U i_U + (1 - h_U)\rho_U$. Thus, Case 3 must happen when the cross currency basis is positive for both OIS and LIBOR.

The unsecured version of the basis can be often found in the literature - see Baba and Packer (2009), Genberg, Hui, Wong, and Chung (2009), and Jones (2009). However, recently, the provision of funding by central banks with secured rates came to dominate the funding by banks and hence our focus on secured repo rates and OIS.[15] Also, notice that it is easy to relate LIBOR-based basis

[15]See Mancini-Grioffoli and Ranaldo (2010) for an analysis of CIP deviations in terms of both OIS rates and GC rates.

and OIS-based basis. LIBOR rates are given by the following $R_U = r_U + s_U$ (for dollar) and $R_E = r_E + s_E$ (for euro), where $s_U$ and $s_E$ are the spreads between forward rate agreements (FRA) and the OIS rates, often referred to as the FRA-OIS spread in dollar and euro respectively. If $A$ is the cross currency basis with respect to LIBOR, one can relate it to the OIS based equivalent market basis $\alpha$ as follows: $A = \alpha + [s_U \frac{1+r_E}{1+r_U} - s_E] + \frac{s_U \alpha}{1+r_U}$. Empirically, in a crisis period, the value of treasury as a collateral raises the most relative to the collateral value of other securities. This in turn influences General Collateral rate (GC) versus LIBOR and FRA-OIS. This is why the most sensitive spread from the previous formula, and the one often used by practitioners in the short end of the term structure, is basis versus OIS. See Mancini-Grioffoli and Ranaldo (2010) for other arguments against computing the basis in terms of LIBOR rates, and also for the important result that excess returns from secured funding using GC rates are nearly as equal to those from OIS rates.

## 4.3 Central banks intervention when the ECB only accepts collateral denominated in euros

Let us now consider a setting with two central banks, the ECB and the Fed, and the representative European and American banks. We denote by $\rho$ the repo rate chosen by the ECB. As before, we assume to the same effect that European banks have a large endowment in the European government bond. To simplify, we ignore all differences between eligible collateral, in particular the differences among government bonds for different euro-members.[16] Thus, in this simple model we identify covered bonds with other regular eligible euro denominated bonds, and look at the introduction of cross-currency repo by the ECB (using dollars from the Fed), using all eligible bonds as an abstract representation of the overall funding capability of the European banks in euros.

In a context of ECB's intervention, the cross-currency dollar repo rate for euro covered bond was the best funding option for the representative European bank

[16] In this paper we shall not make the distinction among the different flavors of collateral accepted by the central banks, which we think is the anchor of funding relevant for the FX market. Our framework is readily extended to deal with different collateral - different repo rates are obtained pledging them. See Bartolini, Hilton, Sundaresan, and Tonelli (2011) for a comparison of repo rates of different collateral (Treasuries, mortgage-based and Federal agency).

at the onset of the financial crisis, as the cross-currency basis in the free market reached levels of several hundred basis points.[17] This assumption is according to the empirical evidence. In particular, as pointed out by Baba, McCaunley, and Ramaswamy (2009) and Coffey, Hrung, and Sarkar (2009), the cost of borrowing euros in unsecured markets (at the euro LIBOR) and swapping these euros for dollars was higher than borrowing dollars directly in the unsecured market (at the dollar LIBOR), in turn higher than borrowing dollars using the ECB repo facility. Also, Hrung and Sarkar (2012) show that anticipated reductions in repo funding compel banks to go to the FX swap market and obtain dollars at a higher price.

The main departure from the previous scenario with no central banks' intervention is that now the dollar funding market is frozen for the European banks and the best option for European banks is to raise dollars using the ECB's repo facility, i.e., European banks turn to the ECB to borrow dollars through repo in exchange of euro covered bonds. This implies that the collateral has to be taken into account in the box constraint of the euro covered bond, whereas the cash loans will appear in the dollar no-overdraft box constraints of dates 1 and 2 multiplied in both cases by the spot rate $X$. The box constraint of the European bank at date 1 with dollar liabilities $L^i$ should be re-written as follows:[18]

$$X(s_1^i + f^i - hz_E^i) - b_U^i - L^i \geq 0$$

We formulate in the Appendix the details of the the box constraints for the ECB and the Fed.[19] We directly move to results below.

**Proposition 2 (Central banks intervention when the ECB only accepts collateral denominated in euros):** *In the presence of central bank dollar operations, if the best option to raise dollars is to use the ECB's dollar repo facility, then the then the basis widens the higher is the spread $\rho - r_U$, the*

[17]Such a cross-currency dollar repo rate for euro covered bond has disadvantages: notably a big haircut to reflect FX risk.

[18]For simplicity, we omit the trading of American goods for the dollar no-overdraft constraint of the European bank at date 1. Also, notice that the box constraint for the American bond does not include repo operations, reflecting high illiquidity in repo markets by that time. Finally, the no-overdraft dollar box constraint in date 2 should now be written as follows:

$$-\chi f^i + (1 + i_U)a_U^i + (1 + r_U)(b_U^i + e_U^i) + (1 + \rho)Xhz_E^i \geq 0$$

[19]We have decided to omit in the Appendix the constraints of American banks as they do not play a role in our results.

*higher is the haircut* $(1-h)$*, and the higher is the shadow cost of pledging euro collateral, i.e.,*

$$\beta = \rho - r_U + \frac{(1-h)\mu^i_E}{hX\mu^i_{\$2}}. \tag{2}$$

The cross-currency basis can be decomposed into a spread $\rho - r_U$ that is in direct control of the coordinating central banks' policy and an additional term. This spread has been recently lowered to promote supply of dollar. Our model clearly implies that such an action will serve to lower the basis. But also clearly, the second term shows the importance of the pool of eligible collateral and the haircut, which is yet another policy lever available to the central banks. Our model suggests the following: if the euro collateral is scarce for the European bank, then $\mu^i_E > 0$ increases the dollar basis even in the presence of dollar supply operations. It is also clear that requiring a higher haircut increases the basis when the eligible collateral is scarce.[20]

**Corollary 1 (Plenty of euro):** *In the presence of central bank dollar operations, assuming that there is plenty of euro collateral and that the best option to raise dollars is to use the dollar repo facility of the ECB, then the basis* $\beta$ *becomes, to the first order, the difference between the ECB repo rate and the OIS rate,* $\rho - r_U$*.*

□

Corollary 1 makes it clear why it is effective for the central bank to make the pool of eligible collateral as wide as possible – in the limit we will look at the case of a collateral that is abundant for users of the ECB's dollar operations. In this case $\mu^i_E = 0$, and therefore, to a first approximation, the cross-currency basis is in fact equal to the spread between the general collateral repo rate and the OIS rate, from the perspective of the marginal agent. Thus, $\rho - r_U$ is the differential cost between raising dollars from the ECB or directly in the US market. Observe that if the European bank were a member of the Fed, then it could raise dollars at the rate $r_U$. But this is not a feasible possibility, and therefore, it has to pay the differential $\rho - r_U$ which is then reflected in the basis.

Before presenting our empirical analysis, it is useful to examine the implications of Corollary 1. Propositions 2 above implies that, in the absence of excess

[20]Observe in Proposition 2 that when the haircut $(1-h)$ increases, then $h$ decreases, $\chi$ increases. This is because the friction of borrowing dollars through the ECB facility is larger the higher is the haircut.

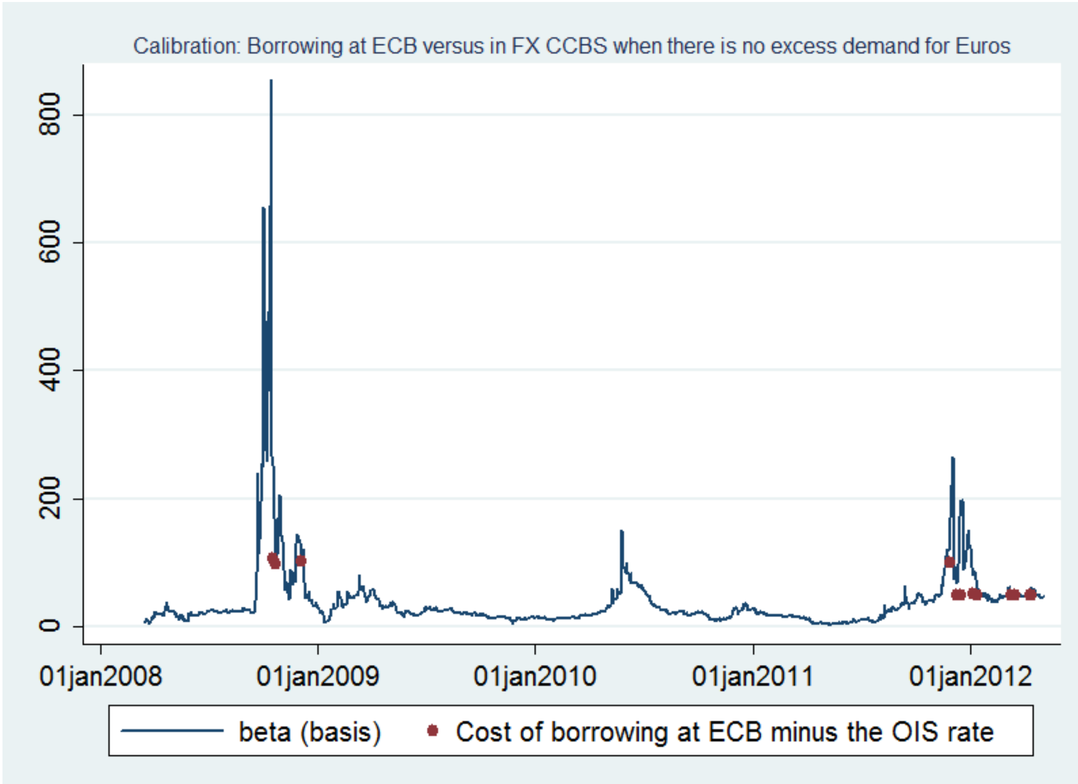


Figure 3: Cost of borrowing at the ECB versus borrowing through cross-currency basis when there is no excess demand.

demand for euros ($\mu_E = 0$), if the best option to raise US dollars is the ECB facility, then the basis, $\beta$, to a first order approximation, should be simply $\rho - r_U$. The rate $r_U$ is taken here to be the 3-months OIS rate. The repo rate $\rho$ at which the ECB lent dollars was obtained from the ECB website.[21] Using the ECB tender data, we examine auctions where the excess demand for euros is zero. For these auctions, we examine cases where the best option for European banks was to raise dollars through the ECB's repo facility (i.e., 3-month OIS $+\beta >$ ECB's repo rate for lending USD).

We compute $\beta$ as defined in the paper in Section 1. We plot $\beta$ in basis points along with $\rho - OIS_{3m}$ in Figure 3. For all the ECB tenders with zero excess demand, when the best option is borrowing in the ECB tender, we do find evidence consistent with Corollary 1. Namely, the cost of borrowing at the ECB lines up exactly as predicted by our theory!

[21]Notice that this rate is significantly higher than the TAF rate at which US banks could obtain dollars (see Goldberg, Kennedy, and Miu (2010) for a comparison between TAF stop-out rates to OIS and LIBOR for one month term).

## 4.4 Central banks intervention when the ECB accepts collateral denominated in dollars

This period goes from October 15 2008 to the end of 2009. As we discussed earlier, the funding options available on the euro side are important to insure that we have a "euro-plenty" situation with $\mu_E = 0$. Expanding eligible collateral on the euro side has certainly helped in this front. ECB's recent LTROs are also a step in this direction: as European banks hoard funding of euro assets through LTRO, the less precious collateral for the dollar operations is tied in euro operations. Moreover, recent policy action shows the central bank going back to the root of the problem. Originally, the dollar funding pressure has been created because European banks could not fund a dollar asset (asset backed) in the market. A natural idea is for the ECB to provide such funds accepting the dollar collateral on repo when the market ceases to accept it. Essentially, the ECB is doing a dollar repo better than the market could provide.

We now briefly explore this scenario using a similar approach as before. Let us denote by $(1-\tilde{h}_u)$ the haircut chosen by the ECB, and by $\tilde{\mu}_U^i$ the European bank's box multiplier for this closed repo operation. Now, the European box constraint at date 1 has the collateral denominated in dollars, and therefore, $X_1$ does not multiply the haircuted repo trade $\tilde{h}_U z_U^i$.

**Proposition 3:** *If the ECB accepts dollar collateral when lending dollars and if this is the best dollar funding option for a European bank, then the basis depends on the ECB's policy repo rate $\tilde{\rho}_u$ in the following way:*

$$\beta = \tilde{\rho}_U - r_U + \frac{\frac{\tilde{\mu}_U^i}{\tilde{h}_U} - \frac{\mu_E^i}{X}}{\mu_{\$,2}^i} \qquad \text{(Basis Result 3)}$$

*where $(1-\tilde{h}_u)$ is the haircut and has effect when the collateral becomes scarce.*

$\square$

Proposition 3 is important as it explains why a policy of collateral relaxation by the ECB can narrow the basis, since now the multiplier associated to collateral, $\tilde{\mu}_U^i$, is low or even zero in a context where eligible collateral is abundant among European banks. In the next section we will give validity to our theory that a policy of collateral relaxation by the ECB contributes to pushing the basis towards 0. Overall, if there is abundant unused eligible collateral and European banks have plenty of euro (as in the Lehman period), the main driver of the basis becomes the spread, here $\tilde{\rho}_u - r_u$. Moreover the same principles apply for the impact of the haircut: a higher required haircut $1 - \tilde{h}_U$ will decrease the policy impact of the operation unless unpledged dollar collateral is abundant.

# 5 Empirical Implications and Tests

Our model of cross-currency basis places the burden of explaining negative basis on dollar funding constraints. In particular, the model links the basis to the shadow prices of funding constraints in euro and in US dollar. Specifically, we show that any actions taken by central banks to alleviate the US dollar funding constraints will serve to push the basis towards zero, and any action to alleviate funding shortage in euro will widen the funding gap between the two currencies, and hence will result in a widening of the basis, in the short run. It might be argued that easing the euro funding constraint might allow European banks to then exchange euro for dollars in the spot exchange market and thus reduce dollar shortage. But as discussed by Mancini-Grifolli and Ranaldo (2011) the holders of dollars demand a very attractive exchange rate for doing this, reflecting risk and liquidity considerations, and in turn causes this channel to be very expensive.

To be concrete, we enumerate the following empirical implications of our theory. First, an implication of our paper is that an effective provision of US dollars by ECB will push the basis towards zero. Second, our model predicts that the menu of acceptable collateral can also influence the basis: wider the menu of collateral, and lower the haircuts, lower should be the basis. Likewise, any move towards reducing the excess demand for US dollars will also push the basis to zero. Hence we would expect the actions of ECB on October 15, 2008 to have a strong effect on cross currency basis. Finally, actions which increase the funding gap between euro financing and US dollar financing will widen the basis, ceteris paribus. On the pricing front, our model links the convenience yield $\beta$ explicitly to observable quantities such as the general collateral repo rates, OIS rates, haircuts and shadow costs of funding constraints. Our calibration exercise in the previous section provides support for Corollary 1.

## 5.1 Basis & US Dollar Shortage: Some Diagnostic Test

We conducted the following simple diagnostic tests to establish the link between US dollar shortage and basis. In the first specification, we regressed the cross-currency basis against the total bids submitted in ECB tenders for US dollar injections. The specification is shown next:

$$CCBS_t = a_0 + b_0 USD_t + \varepsilon_t \tag{3}$$

In the regression specification above, $CCBS_t$ refers to the basis in terms of $\alpha$ at date $t$, and $USD_t$ refers to an empirical measure of the US dollar demand. We examine several measures: a) the actual demand for dollars as measured by the volume of bids submitted by the European banks, b) the number of bidders in auctions of US dollar injections, and c) the excess demand as measured by the difference between the bids submitted and the amounts allotted. Table 2 below shows the results for three-months basis. Results are qualitatively similar for all other tenors, and are not shown to conserve space. In our empirical work, we do not make any distinction between the ECB tenders of different tenors: we focus on the aggregate demand or aggregate excess demand. We provide below our preliminary empirical results.

Table 2: Dollar demand and cross-currency basis

| | **Specification 1** | **Specification 2** | **Specification 3** |
|---|---|---|---|
| Demand for dollars | $-.0009932^{***}$ | | |
| Number of bidders | | -1.757509 $^{***}$ | |
| Excess demand | | | $-0019042^{***}$ |
| Intercept | $-32.94748^{***}$ | $-30.14984$ $^{***}$ | $-37.06142^{***}$ |
| $R^2$ | 0.3016 | 0.4325 | 0.1711 |
| $p$ | 0.0000 | 0.0000 | 0.0000 |
| $N$ | 1024 | 483 | 1024 |

The regression specifications above are diagnostic in the sense that we have not used controls for credit risk, and other funding costs, but these results are very strongly suggestive of the link between the dollar demand/shortage and the cross-currency basis. They support the basic proposition that the US dollar shortage as manifested by the measures that we have used has a significant negative loading on cross-currency basis: greater the demand for US dollars, the wider will be the basis. From an economic standpoint, an increase in excess demand for US dollars by 10 billion will result in a decrease of $-0.0019042 \times 10,000 = 19.04$

basis points. This is a very preliminary estimate as it ignores the interventions by the central banks during this period as well as other factors that might have contributed to the evolution of the basis. Note that our specifications can explain in the range of 17% to 43% of the variations in the basis over time.

Our diagnostic regressions results lend support to the main result of our paper, that US dollar shortage should serve to increase the basis. We have not controlled for central bank interventions in this specification. Nor have we controlled for other factors, which might influence the cross currency basis. We proceed to do that next.

## 5.2 Basis, Central Bank Interventions & Demand

The Federal Reserve and ECB coordinated and made several key announcements during the sample period. These announcements pertain to the FX swap lines whereby the Fed will provide US dollars to ECB, so that ECB can auction these US dollars to European banks who could post eligible collateral. First, the Fed announced on September 29, 2008 the following: "In response to continued strains in short-term funding markets, central banks today are announcing further coordinated actions to expand significantly the capacity to provide U.S. dollar liquidity." As a part of this announcement, the Fed made the following specific commitments: (1) an increase in the size of the 84-day maturity Term Auction Facility (TAF) auctions to $75 billion per auction from $25 billion beginning with the October 6 auction, (2) two forward TAF auctions totaling $150 billion that will be conducted in November to provide term funding over year-end, and (3) an increase in swap authorization limits with the Bank of Canada, Bank of England, Bank of Japan, Danmarks Nationalbank (National Bank of Denmark), European Central Bank (ECB), Norges Bank (Bank of Norway), Reserve Bank of Australia, Sveriges Riksbank (Bank of Sweden), and Swiss National Bank to a total of $620 billion, from $290 billion previously.

The ECB followed this with an announcement on October 15, 2008 in which it a) moved to fixed rate tender with full allocation, and b) significantly broadened the collateral that it would accept for providing US dollars to European banks. These were extremely strong moves to alleviate dollar shortage. Specifically ECB added the following instruments: a) Marketable debt instruments denominated in other currencies than the euro, namely the US dollar, the British pound and the Japanese yen, and issued in the euro area. b) Debt instruments issued by credit institutions, which are traded on the accepted non-regulated markets that are mentioned on the ECB website; this measure implies inter alia that certificates of deposits (CDs) will also be eligible when traded on one of these accepted non-regulated markets, and c) Subordinated debt instruments when they are protected by an acceptable guarantee as specified in section 6.3.2 of the General Documentation on Eurosystem monetary policy instruments and procedures. In addition, ECB announced that the Eurosystem will lower the credit threshold for marketable and non-marketable assets from A- to BBB-, with the exception of asset-backed securities (ABS), and impose a haircut add-on of 5% on all assets rated BBB-. ECB announced that these measures will remain in force until the end of 2009.

These announcements have an important implication for empirically examining the extent to which US dollar shortage was perceived as a major problem by the European banks in the month following the Lehman bankruptcy.

For our empirical work, we introduced a binary variable, *collateral*, which takes a value of zero before October 15, 2008 and takes a value of 1 until the end of 2009. This variable captures the effects of ECB's move to fixed-rate tender with full allotment and its broadening of collateral on cross-currency basis swap spreads. The spreads between forward rate agreements FRA (which settle to LIBOR) and the OIS rates is a measure of dollar funding costs in the respective currencies, which we use as a control variable. In addition, we use the measures of dollar funding shortage and euro funding shortage as before. We specifically use dummy variables for the 3-year LTRO announcement date by ECB in December 2011 and the one conducted on March 2012: our theory predicts that reducing the shadow price of euro constraint can actually lead to a widening of the cross-currency basis. We examine this hypothesis empirically below. As additional control variables, we have used the CDS swap rates of BNP to proxy for the credit risk of the European banks.[22] The spot exchange rate is also used as an additional control variable.

In order to examine the effects (if any) of central bank interventions on basis, we posited the following specification:

$$\mathrm{CCBS_t} = \mathrm{a_2} + \mathrm{b_2}\ \ \mathrm{eurofraois} + \mathrm{b_3}\ \ \mathrm{fraois} + \mathrm{b_4}\ \ \mathrm{edemand_{eur}} + \mathrm{b_5}\ \ \mathrm{edemand_{usd}} + \mathrm{b_6}\ \ \mathrm{bnp_{cds}} + \mathrm{b_7}\ \ \mathrm{spotfx} + \mathrm{b_8}\ \ \mathrm{first_{3yLTRO}} + \mathrm{b_9}\ \ \mathrm{second_{3yLTRO}} + \mathrm{b_{10}}\ \ \mathrm{collateral} + \varepsilon_\mathrm{t}$$

[22] We also used the CDS spreads of Societie Generale - there was no qualitative change in our results.

The empirical results from this specification are reported below.

Table 3: Dollar Shortage and CCBS

| Variable | Coefficient | (Std. Err.) | t | p | 95% C.I |
|---|---|---|---|---|---|
| $edemand_{eur}$ | -6.50e-06 | .0000205 | -0.32 | 0.751 | $[-.0000467\ \ .0000337]$ |
| $edemand_{usd}$ | $-.0007476^{***}$ | .0000679 | -11.00 | 0.000 | $[-.000881\ \ -.0006143]$ |
| $eurofraois$ | $.3397684^{***}$ | .0433045 | 7.85 | 0.000 | $[.2547917\ \ .424745]$ |
| $fraois$ | $-.8446317^{***}$ | .0283127 | -29.83 | 0.000 | $[-.9001899\ \ -.7890735]$ |
| $bnp_{cds}$ | $-.2046916^{***}$ | .0141912 | -14.42 | 0.000 | $[-.2325392\ \ -.1768441]$ |
| $spotfx$ | $52.56359^{***}$ | 7.101843 | 7.40 | 0.000 | $[38.6276\ \ 66.49958]$ |
| $first_{3yLTRO}$ | $8.707019^{**}$ | 2.894721 | 3.01 | 0.003 | $[3.026689\ \ 14.38735]$ |
| $second_{3yLTRO}$ | $7.523416^{*}$ | 3.831075 | 1.96 | 0.050 | $[.0056731\ \ 15.04116]$ |
| $collateral$ | $5.074015^{***}$ | 1.525418 | 3.33 | 0.001 | $[2.080679\ \ 8.067352]$ |
| $constant$ | $-70.36949^{***}$ | 10.44435 | -6.74 | 0.000 | $[-90.8645\ \ -49.87449]$ |

The $R^2 = 0.8227$ with a total of 1024 observations. The results reported in Table 3 are strongly supportive of our theory: first, the excess demand for US dollars emerges as an important explanatory variable after controlling for credit risk (through CDS spreads), spot exchange rates, and FRA-OIS spreads. An excess demand of dollars translates into a difficulty to raise dollars by pledging collateral, which according to our theory will increase the shadow value attached to the box constraint of that collateral. The coefficient of excess demand for US dollars is of similar magnitude to the one we estimated in our diagnostic regression. If banks borrow dollars in the market, they have to pledge USD denominated collateral in order to borrow dollars, and the difficulty to get that collateral drives up the multiplier $\mu_U$, widening in turn the basis, as our theory predicts.[23]

Second, the two three-year LTRO injections of euro (December 2011 and March 2012) have positive coefficients. While the coefficients are positive, they are not significant at the 1% level. We will re-examine this question in the next section in greater detail where we take into account the endogeneity of the FRA-OIS spreads, and show that the euro LTROs do not narrow the basis.

Third, the move to fixed-rate tender with full allotments and the broadening

[23] The beauty of the first dollar operation, conducted by the ECB in 2008, was to actually recognize the situation and accept euro collateral to inject dollars.

of collateral by the ECB on October 15, 2008 had a strong effect in pushing the basis towards zero. This is consistent with our results, as the excess demand for US dollars is driven to zero in ECB tenders, and especially Proposition 3, which claims that a policy of collateral relaxation by the ECB will narrow the basis by decreasing the shadow price of providing the collateral ($\tilde{\mu}_U^i$).

## 5.3 Endogeneity of FRA-OIS Spreads

In our empirical analysis above, we have not controlled for the likelihood that the FRA-OIS spreads, which reflect the funding costs of banks, might themselves be endogenously affected by the variables that influence the cross-currency basis. In this section, we treat the US dollar FRA-OIS spreads as an endogenous variable, and examine the empirical results under this specification.

The three-stage least-squares regression, with 1024 observations yielded an $R^2 = 0.6912$ in the regression between the cross-currency basis and FRA-OIS spreads. In the regression between FRA-OIS spreads and the underlying regressors, the $R^2 = 0.8692$.

Table 4: Dollar Shortage and CCBS

| Variable | Coefficient | (Std. Err.) | z | $P >\mid z \mid$ | 95% C.I |
|---|---|---|---|---|---|
| $eurccbsois3m$ | | | | | |
| $fraois$ | $-.6095516^{***}$ | .014303 | -42.62 | 0.000 | $[-.637585 \quad -.5815182]$ |
| $constant$ | $-10.57641^{***}$ | .9949179 | -10.63 | 0.000 | $[-12.52642 \quad -8.626409]$ |
| $fraois$ | | | | | |
| $edemand_{eur}$ | .0000262 | .0000221 | 1.18 | 0.237 | $[-.0000172 \quad .0000696]$ |
| $edemand_{usd}$ | $.0006483^{***}$ | .0000693 | 9.36 | 0.000 | $[.0005125 \quad .0007841]$ |
| $eurofraois$ | $1.395586^{***}$ | .0195922 | 71.23 | 0.000 | $[1.357186 \quad 1.433987]$ |
| $bnp_{cds}$ | $-.3047648^{***}$ | .0127163 | -23.97 | 0.000 | $[-.3296883 \quad -.2798412]$ |
| $spotfx$ | $-48.8533^{***}$ | 7.463047 | -6.55 | 0.000 | $[-63.48061 \quad -34.226]$ |
| $collateral$ | $-8.835926^{***}$ | 1.621574 | -5.45 | 0.000 | $[-12.01415 \quad -5.6577]$ |
| $first_{3yLTRO}$ | $-7.905158^{*}$ | 3.117936 | -2.54 | 0.011 | $[-14.0162 \quad -1.794116]$ |
| $second_{3yLTRO}$ | $33.92474^{***}$ | 4.012996 | 8.45 | 0.000 | $[26.05942 \quad 41.79007]$ |
| $constant$ | $70.80051^{***}$ | 11.00478 | 6.43 | 0.000 | $[49.23154 \quad 92.36949]$ |

The results, controlling for endogeneity of FRA-OIS spreads, are qualitatively very similar in most respects: The excess demand for US dollars is still an important variable determining the basis. Moreover, we see that an increase in excess demand of dollars, which increases the funding shadow value of dollar assets ($\mu_U$), increases the credit risk, measured here in terms of the FRA-OIS spread. This is according with our equation (1), which asserts that a funding need is concomitant with a solvency difficulty.

Also, notice that the October 15, 2008 binary variable reflecting the full allotment, fixed-rate tender and the collateral expansion decisions of ECB enters again as a significant variable. The control variables such as the CDS spreads are still significant.

The results pertaining to the 3-year LTROs convey the following: the first 3-year LTRO appears to help to narrow the FRA-OIS spreads and hence the cross-currency basis, although the significance is lower than the earlier specification. We attribute this negative coefficient to the year-end effect, which as we argued before, was very important for year 2011.[24] The second 3-year LTRO, in March 1, 2012, enters clearly as a significant variable in widening the FRA-OIS spreads and hence the cross-currency basis, at a very high level of significance. In terms of our theory (Proposition 2), euro injections help decrease the shadow value of funding euro denominated assets, $\mu_E$, in turn widening the basis.

# 6 Nature of Dollar Shortage After October 15, 2008

As noted earlier, full allotment tenders were put in place after October 15, 2008 so that European banks in need of dollars could get all their demands filled by the ECB: there was no quantity rationing. Then, what was the nature of dollar shortage after October 15, 2008? This is especially an important question as in several of the US dollar operations conducted by the ECB during 2010-2011 period, there were no bids submitted by European banks. To shed some light on this question, we examined the relative attractiveness of obtaining dollars from two sources: using the cross currency basis swaps, or using the ECB dollar operations. In Figure 4 we show the costs of accessing these two sources, from the perspective of European banks. The cost of using the cross currency basis swap is $\beta$ plus the dollar OIS rate. The cost of accessing the ECB facilities is

[24] Year-end dummy variable for 2011, removes the significance of the coefficient of the first 3-year LTRO.

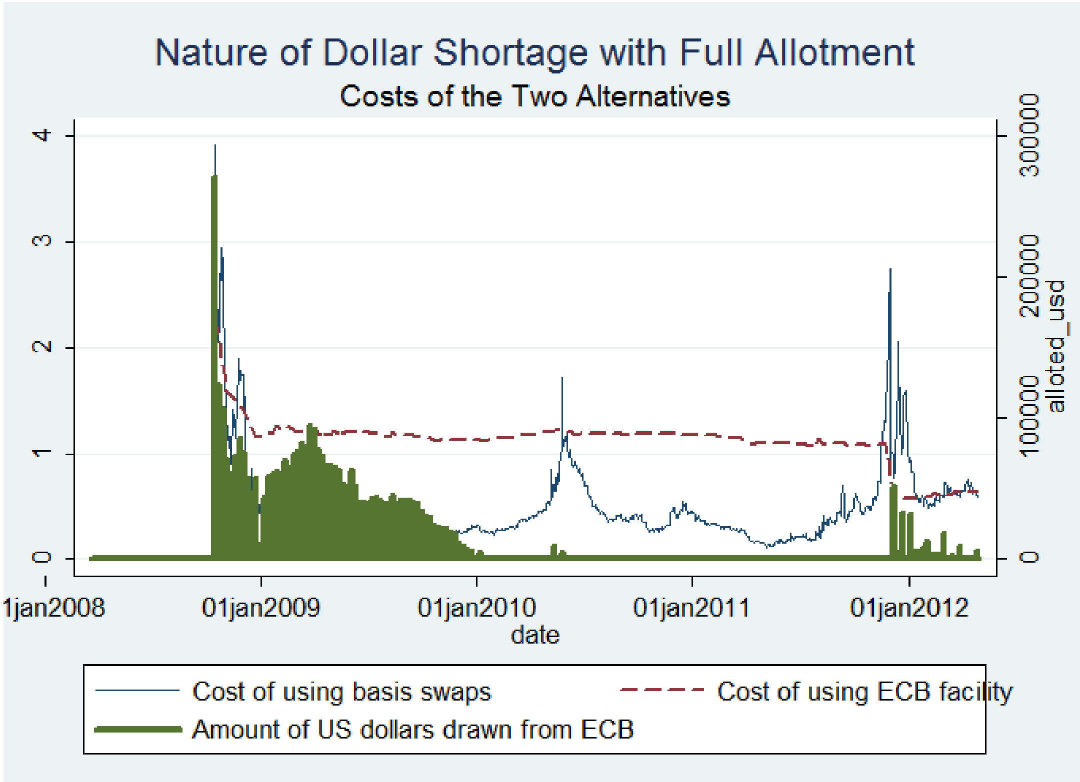


Figure 4: Costs of Accessing ECB versus Using Swaps

the fixed rate charged by the ECB in its dollar operations. Note that during December 2011 and in early 2012 it was much more cost effective to access ECB facilities and the evidence in Figure 4 shows that there were significant bidding by European banks in ECB dollar operations. On the other hand, during much of 2010 and 2011, the ECB dollar operations were more expensive and the European banks did not choose to bid in ECB dollar operations. We presume that they must have used the swap markets to meet their dollar needs. This shows that the nature of dollar shortage faced by the European banks during 2011-2012 was one of penalty rates charged by the ECB relative to the swap markets. Unlike the swap markets, wherein the depth is somewhat limited, the ECB was prepared to fill any amount of demand, but at a penalty rate. Note that the banks used the ECB tenders in 2009, despite the fact that the rates were higher than the swap markets. We attribute this to the fact that the magnitude of the dollar demand during that period was so great that ECB tenders was the most credible option for the banks. Our analysis is important in assessing the nature of and the extent to which dollar shortage in 2010-2012 might have caused banks to cut back on their US dollar lending as discussed in Ivashina, Scharfstein and Stein (2012).

The shortage during 2010-2012 is primarily driven by the penalty rates charged by the ECB as opposed to quantity rationing. The resumption of bidding by European banks at ECB tenders in the first quarter of 2012 suggests that they were price-sensitive in their strategies, but had access to US dollars from ECB.

# 7 Conclusion

Our paper offers a theoretical framework to understand huge departures of the cross-currency basis from what the standard CIP would predict, such as those occurred during the credit crisis of 2008 and the European sovereign debt crisis of 2011-2012. The main insight of the paper is that there was a significant "convenience yield" or physical possession value in the scarcer currency, namely, the dollar. Our theory shows that this is priced into the cross-currency basis. The model proposed here links the cross-currency basis to shadow costs associated with funding constraints in the spot and forward markets. Cross-currency basis is the rent paid for possession of the scarce currency for one period as a fraction of the long term value of the scarce currency. In this way, we establish a conceptual link between the "repo specialness" in securities markets and the cross-currency basis in FX swap markets. The model allows us to examine in a quantitative manner the activation of dollar swap lines by the Federal Reserve in coordination with other central banks, and also the ECB's special repo facilities targeted at the funding needs of European banks. We construct measures of excess demand for dollars from ECB's US dollar operations, and show that they have significant explanatory power in describing the cross currency basis. We show that the ECB's decision to move to full allotment tenders and to broaden the menu of collateral pushed the basis closer to zero. ECB's decision to use price as a rationing tool for auctioning dollars after October 15, 2008 led to selective utilization of ECB dollar operations by the European banks. We find that ECB's massive euro injections through LTROs increase the basis in general, while ECB's dollar injections decrease it. These results are consistent with our theory of relative funding pressure of the two currencies. Finally, our model offers a framework for examining the channels through which the tools of central banks, such as a) price of dollar liquidity, b) volume of liquidity, c) menu of collateral, and d) haircuts, influence the cross-currency basis.

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# A Online Appendix (Not for publication)

## A.1 Proposition 1 extended

In Section 4 we derived a formula for the basis from the optimality conditions on bond trading. We refer here to that result as Proposition 1.1. Here, we derive formulas for the basis from the optimality conditions on repo borrowing (Proposition 1.2), and on uncollatearlized borrowing (Proposition 1.3).

**Proposition 1 (Before the crisis with no central bank intervention):**

- **1.1)** *Optimal funding by trading bonds requires that the forward FX rate and the cross-currency basis must be, respectively,* $\chi = X\frac{1+r_U+\beta}{1+r_E}$ *and* $\beta = \frac{\mu_U - \frac{\mu_E}{X}}{\mu_{\$,2}}$.[25]
- **1.2)** *Optimal funding through repo requires that the forward FX rate and the cross-currency basis must be, respectively,* $\chi = X\frac{1+\rho_U+\tilde{\beta}}{1+\rho_E}$ *and* $\tilde{\beta} = \frac{\frac{\mu_U}{h_U} - \frac{\mu_E}{Xh_E}}{\mu_{\$,2}}$, *and therefore* $\beta = \frac{1+r_E}{1+\rho_E}(1+\rho_U+\tilde{\beta}) - (1+r_U)$.[26]
- **1.3)** *Optimal funding by borrowing uncollateralized requires that the forward FX rate and the cross-currency basis must be, respectively,* $\chi = X\frac{1+i_U+\beta}{1+i_E}$ *and* $\hat{\beta} = \frac{\gamma_U - \frac{\gamma_E}{X}}{\mu_{\$,2}}$.

□

**Proof of Proposition 1.1:** The necessary first order optimality conditions (FOC) with respect to $b_U$ (funding dollar), $b_E$ (funding euro), $s$ (spot), and $f$ (FX swap) are, respectively,

$$\mu_{\$,1} = \mu_U + \mu_{\$,2}(1+r_U) \tag{4}$$

$$\mu_{e,1} = \mu_E + \mu_{e,2}(1+r_E) \tag{5}$$

$$X\mu_{\$,1} = \mu_{e,1} \tag{6}$$

$$\chi\mu_{\$,2} + \mu_{e,1} = \mu_{e,2} + X\mu_{\$,1} \tag{7}$$

By using (6) we can simplify the initial form of (7) and get:

$$\chi\mu_{\$,2} = \mu_{e,2} \tag{8}$$

[25]Recall that we defined the basis $\beta$ such that $\chi = X\frac{1+r_U+\beta}{1+r_E}$, and therefore one can obtain the cross-currency basis from the forward FX rate formula.

[26]Equivalently, $\beta = \tilde{\beta} - (h_U^{-1} - 1)\frac{\mu_U}{\mu_{\$,2}} + (h_E^{-1} - 1)\frac{\mu_E}{X\mu_{\$,2}}$.

Then,

$$\chi \overset{(8)}{=} \frac{\mu_{e,2}}{\mu_{\$,2}} \overset{(5)}{=} \frac{\mu_{e,1}-\mu_E}{(1+r_E)\mu_{\$,2}} \overset{(6)}{=} \frac{X\mu_{\$,1}-\mu_E}{(1+r_E)\mu_{\$,2}} \overset{(4)}{=} X\frac{1+r_U}{1+r_E}\left(1+\frac{X\mu_U-\mu_E}{X\mu_{\$,2}(1+r_U))}\right)$$

Finally, recall that we defined the basis $\beta$ such that $\chi = X\frac{1+r_U+\beta}{1+r_E}$, and therefore one can obtain the cross-currency basis from the forward FX rate formula.

$\square$

**Proof of Proposition 1.2:** In this case the FOC with respect to $z_U$ (repo dollar funding), and $z_E$ (repo euro funding) are, respectively,

$$h_U\mu_{\$,1} = \mu_U + \mu_{\$,2}h_U(1+\rho_U) \tag{9}$$

$$h_E\mu_{e,1} = \mu_E + \mu_{e,2}h_E(1+\rho_E) \tag{10}$$

The first order conditions with respect to $s$ and $f$ are as (6) and (7). Here again, we can simplify the initial form of (7) and get (8). Now,

$$\begin{aligned}\chi &\overset{(7)}{=} \frac{\mu_{e,2}}{\mu_{\$,2}} \overset{(10)}{=} \frac{h_E\mu_{e,1}-\mu_E}{h_E(1+\rho_E)\mu_{\$,2}} \overset{(6)}{=} \frac{Xh_E\mu_{\$,1}-\mu_E}{h_E(1+\rho_E)\mu_{\$,2}} \\ &\overset{(9)}{=} X\frac{1+\rho_U}{1+\rho_E}(1+\frac{\mu_U}{h_U\mu_{\$,2}(1+\rho_U)}-\frac{\mu_E}{h_EX\mu_{\$,2}(1+\rho_U)})\end{aligned}$$

$\square$

### A.1.1 Further discussion of Propostion 1

While Propositions 1.1 and 1.3 show in a very simple way how the basis depends on shadow prices of binding constraints, Proposition 1.2 is also useful as it shows how the *same* basis depends on repo haircuts ($h_U$ and $h_E$) and on repo rates ($\rho_U$ and $\rho_E$). Note that in the case of a risk free rate $r_U$ ($r_E$) equal to $\rho_U$ ($\rho_E$) we have $\beta = \tilde{\beta}$. Combining Propositions 1.1 and 1.2, as the basis $\beta$ must be the same, we get $h_U = h_E = 1$. Thus, the riskless case can be seen as a shorthand of secured borrowing using a riskless asset.

In Section 4 we discussed in detail Proposition 1.1. Here we provide further discussion of Proposition 1.1 and also discuss the Propositions 1.2 and 1.3 presented in this Appendix.

**Comments on Proposition 1.1:** We first set $\mu_E = 0$ in the basis equation in Proposition 1.1. That is, we assume that there is no shadow value for euro funding constraints (i.e., a situation with plenty of euros and no difficulty in acquiring European bonds). With this simplification, we see that $\beta = \frac{\mu_U}{\mu_{\$,2}}$. Thus, in the absence of a binding funding constraint in euros, cross currency basis is the marginal value of possessing the dollar denominated bond today, relative to tomorrow's solvency shadow value ($\mu_{\$,2}$ measures how the bank values a marginal increase in its dollars holdings at the second date). This result is very intuitive and says that in an economy with no euro funding constraints, cross-currency basis is a pricing mechanism for intertemporal transfer of the scarcer currency. Furthermore, the impact on the basis is the highest in the presence of agents having short term difficulty to raise dollars while clearly staying long term solvent.

The basis in Proposition 1.1 can also be seen as the one that would occur in *a hypothetical no-repo framework* where funding would be done only by trading bonds and, therefore, without short-selling, as $z_U = z_E = 0$ implies $b_U + e_U \geq 0$ and $b_E + e_E \geq 0$. In this scenario, the dollar possession value, driven by the multiplier on the box of the dollar denominated bond $\mu_U$, would corresponds to the desirability to issue debt in dollars immediately at date 1 (relative to the desirability of date 2 dollars and discounted back to date 1). Notice that in this case, $\mu_U$ is the shadow value of increasing $e_U$, that is, of being able to issue bonds. Equivalently, the same additional debt could be incurred by taking a negative position on the bond, doing a short-sale (in a naked way, as repo would not be possible), so $\mu_U$ can also be seen as the value that would be attached to violating the no-short-sales constraint. Comparing the role of bond box multipliers here and in Bottazzi, Luque and Pascoa (2012), we see that even in such a no-repo scenario the multipliers still capture the desire to possess the bond. In a repo context such a desire is linked to repo specialness, whereas in a no-repo environment it is just the desire to issue more debt.

*In a trade and repo context*, where both Propositions 1.1 and 1.2 must hold, the box constraint is $b_U + e_U + z_U \geq 0$ and $\mu_U$ captures the desire of doing a short sale not covered by borrowing the bond, or of lending the bond without having it. More precisely, it measures the marginal value of being in possession of an additional amount of the bond that could then be short sold (without having to borrow it through reverse repo) or lent in repo. *Hence, a positive $\mu_U$ means that the agent wants to have immediate funding in dollars and attaches a value to possessing the bond that provides this funding.* Such scarcity of the bond brings the repo rate on the bond below the risk-free rate, which can be taken to be OIS.

Finally, to get some intuition for the 2008 crisis, one can simplify the above formulas by assuming there existed plenty of European bonds. As a consequence, the euro denominated bonds box constraint was non binding (as $e_E^i$ is very big). In this case, the possession value of a euro goes down to 0 and one gets:

$$\chi = X\frac{1+r_U}{1+r_E}\left(1+\frac{\mu_U^i}{\mu_{\$,2}^i(1+r_U))}\right).$$

Thus, the cross-currency basis is driven by the possession value of dollars between date 1 and date 2.

**Comments on Proposition 1.2:** The previous result followed solely from optimal bond trading. In practice, however, central bank eligible collateral repo market dominates the short end market in volume and position. Here we examine what optimal repo funding implies, thus exploring the impact of haircut on the basis.

As in Proposition 1.1, the currency basis in Proposition 1.2 is driven by the multiplier $\mu_U$ of the box on bonds denominated in dollars. This shadow value should now be interpreted as the desire to violate this box constraint by lending more of the bond than one is endowed with. Equivalently, $\mu_U$ is the marginal value of possessing more of the bond.

Observe that when there is no haircut ($h_U = h_E = 1$) Proposition 1.2 does not tell us anything about the basis beyond what Proposition 1.1 did. In fact, in this case, the general collateral rate coincides with the yield rate of the bond (i.e., $\rho_E = r_E$ and $\rho_U = r_U$, for the European and American bond, respectively).[27] This implication also rationalizes why investors prefer to quote the basis relative to OIS, which is closer to General Collateral repo rates than relative to LIBOR. Also notice that if the haircut $1 - h_U$ increases, so that $h_U$ decreases (a higher borrowing friction), the basis increases (as dollars become more scarce). The effect is however opposite for euros – so that the basis represents the relative funding pressure in both currencies.

Notice that, fixing the reference basis quoting rate $r_E$ to OIS, *if access to domestic funding is very good* (low haircut and low $\mu_E$), *then the cross currency basis is pushed higher, as it captures a higher relative funding pressure.*[28] This

[27]The first order condition (FOC) on repo is $\mu_{\$,1} = \mu_U h_U^{-1} + \mu_{\$,2}(1+\rho_U)$, whereas the FOC on bond trading is $\mu_{\$,1} = \mu_U + \mu_{\$,2}(1+r_U)$. Notice that the term of the repo funding and the term of the bond are identical in our setup.

[28]This was generally the case at the end of 2011 for German and to a lesser extent for French

insight tends to counter the credit-risk interpretation of the basis. In fact, it is when the relative funding pressure *difference* is highest that the basis impact is the highest: a good domestic credit increases the basis.

Again, the basis in Proposition 1.2 can be interpreted as the one prevailing in a hypothetical framework with fixed bond positions (i.e., $b_U = b_E = 0$) where the only source of funding is collateralized borrowing or lending. As we will see in the next section, this scenario is close to describing the repo facility that a central bank opens to the private banks of its jurisdiction.

**Comments on Proposition 1.3:** Formulas analogous to the ones in Proposition 1.1 would follow, but with essentially LIBOR rates. The interpretation of Proposition 1.3 is similar to that of Proposition 1.1, but now the constraints are the standard i.o.u. with an exogenous lower bound on the uncollateralized borrowing capacity. A positive $\gamma_U$ translates into a desire to violate the i.o.u. constraints.

## A.2 Remark A

**Remark A:** The shadow prices of box constraints can be written in terms of marginal utilities using equations (18) through (21). For instance, noticing that $\mu_{\$,1} p_{U,1} = \frac{\partial V}{\partial x_{U,1}}$ (and similarly for $\mu_{\$,2}$) we get $\mu_U = \frac{1}{p_{U,1}} \frac{\partial V}{\partial x_{U,1}} - \frac{(1+r_U)}{p_{U,2}} \frac{\partial V}{\partial x_{U,2}}$.

Also notice that if the agents are *profit maximizers* instead of *utility maximizers*, Proposition 1 still holds. For instance, let the present value profit function in euro be as follows for a certain discount factor $\delta$ :[29]

$$p_{E,1}(\omega_{E,1} - x_{E,1}) - (s + f) - b_E + \delta(p_{E,2}(\omega_{E,2} - x_{E,2}) + f + (1 + r_E)(b_E + e_E)) \quad (11)$$

Let $\tilde{\mu}_{e,1} = \mu_{e,1} + 1$ and $\tilde{\mu}_{e,2} = \mu_{e,1} + \delta$. Then it is easy to see that the first order conditions on $b_E$, $s$, and $f$ (see equations (5), (6), and (7) in the Appendix), which held for utility maximization, will now hold for profit maximization, once we replace $\mu_{e,1}$ by $\tilde{\mu}_{e,1}$, and $\mu_{e,2}$ by $\tilde{\mu}_{e,2}$. Hence, Proposition 1.1 also applies to the case of profit maximization. The proof is exactly the same as the proof of Proposition 1.1.

banks. Anecdotal evidence shows the involvement of such banks in cross border holdings has been actually very high, and this feeds through the relative importance of dollar operations done by the ECB compared to other central banks.

[29]We assume that European banks were the marginal agents under funding pressure in the 2008 crisis, which we address later. Hence looking ahead, we look at profit maximization in euro.

$\square$

## A.3 Equilibrium

Consider the framework of Section 4.2, where all markets are active. An equilibrium for an economy with $\mathbf{I} = \{1, ..., i, ..., I\}$ traders is then defined by an allocation $(x_U^i, x_e^i, a_U^i, a_E^i, b_U^i, b_E^i, z_U^i, z_E^i, , s^i, f^i)$ such that: (a) each trader maximizes his utility function (or profits - see Remark A), subject to his box constraints (\$.1), (e.1), (i.o.u. U), (i.o.u. E), (Funding.U), (Funding.E), (\$.2), and (e.2); and (b) good markets clear ($\sum_i x_{U,t}^i = \sum_i w_{U,t}^i$, $\sum_i x_{e,t}^i = \sum_i w_{e,t}^i$, for $t = 1, 2$), uncollateralized borrowing and lending markets clear ($\sum_i a_U^i = 0$ and $\sum_i a_E^i = 0$), bond markets clear ($\sum_i b_U^i = 0$ and $\sum_i b_E^i = 0$), repo markets clear ($\sum_i z_U^i = 0$ and $\sum_i z_E^i = 0$), spot FX markets clear ($\sum_i s^i = 0$), and forward FX markets clear ($\sum_i f^i = 0$). The proof of equilibrium follows the same lines as in Bottazzi, Luque and Pascoa (2012), and is therefore omitted.

## A.4 Central Banks

### A.4.1 ECB's constraints when accepted collateral is denominated in euros

Here we revisit the ECB's box constraints when accepted collateral is denominated in euros (Section 4.2). Recall that the repo facility provided by the ECB is a cross-currency repo: the collateral (whose positions are $z_E^{ECB}$ and $z_E^i$ for the ECB and European bank $i$, respectively) is denominated in euros but the cash loan is in dollars, so the haircuted cash loans appear in the dollar box constraints, but the collateral has to be taken into account in the boxes for euro denominated bonds. ECB dollar no-overdraft box constraint at date 1 is:

$$X(s^{ECB} + f^{ECB} - hz_E^{ECB}) - b_U^{ECB} \geq 0 \qquad \text{(ECB.\$.1)}$$

The haircut $1 - h$ ranged from 10% for 1 day to 17 % for one month, significantly above TAF haircuts that never exceeded 5 %. In equilibrium we expect $z^{ECB} > 0$ (long in repo) and $f^{ECB} > 0$, as the ECB only accepts collateral, and is expected to buy-sell dollars (purchaser of dollar in the first leg of the swap).

The ECB's box for euros is:

$$W_{e,1}^{ECB} \equiv W_{e,0}^{ECB} - (s^{ECB} + f_{ECB}) + I_{e,1}^{ECB} - b_E^{ECB} \geq 0 \qquad \text{(ECB.e.1)}$$

where $W_{e,0}^{ECB} \geq 0$ represents the ECB initial euro balance, whereas $I_{e,1}^{ECB}$ stands for the variation of economy-wide euros balances printed or withdrawn at date 1. Notice that the central bank may have to issue currency in order to have non-negative currency balances. Also the ECB may have to inject euro when it purchases foreign currency or European government bonds.

The ECB's boxes for American and European bonds are, respectively:

$$b_U^{ECB} + e_U^{ECB} \geq 0 \tag{ECB.U}$$

$$b_E^{ECB} + e_E^{ECB} + z_E^{ECB} \geq 0 \tag{ECB.E}$$

For simplicity, we assume that the ECB only does dollar repo on euro bonds.

At date 2, ECB's dollar and euro box constraints are, respectively:

$$-\chi f^{ECB} + (1+\rho)Xhz_E^{ECB} + (1+r_U)(b_U^{ECB} + e_U^{ECB}) \geq 0 \tag{ECB.\$.2}$$

$$W_{e,2}^{ECB} \equiv W_{e,1}^{ECB} + f^{ECB} + I_{e,2}^{ECB} + (1+r_E)(b_E^{ECB} + e_E^{ECB}) \geq 0 \tag{ECB.e.2}$$

### A.4.2 Fed

At the initial date the Fed has the following box constraint for dollars:

$$W_{\$,1}^{Fed} \equiv W_{\$,0}^{Fed} + I_{\$,1}^{Fed} + X(s^{Fed} + f^{Fed}) - b_U^{Fed} \geq 0 \tag{12}$$

where we expect $s^{Fed} < 0$ in equilibrium (as the Fed is a spot seller of dollars), compensated by issuing currency ($I_{\$,1}^{Fed} > 0$) or by making $b_U$ sufficiently negative (possibly issuing debt at the same time, i.e., increasing $e_U^i$), so that $W_{U,1}^{Fed} = W_{U,0}^{Fed}$. The Fed no overdraft box constraint for euros is:

$$-(s^{Fed} + f^{Fed}) - b_E^{Fed} \geq 0 \tag{13}$$

Box constraints of American and European bonds are, respectively:

$$b_U^{Fed} + e_U^{Fed} \geq 0 \tag{14}$$

$$b_E^{Fed} + e_E^{Fed} \geq 0 \tag{15}$$

At the following date the Fed's box constraints of dollars and euros are, respectively:

$$W_{\$,2}^{Fed} \equiv W_{\$,1}^{Fed} + I_{\$,2}^{Fed} - \chi f^{Fed} + (1+r_U)(b_U^{Fed} + e_U^{Fed}) \geq 0 \tag{16}$$

$$f^{Fed} + (1+r_E)(b_E^{Fed} + e_E^{Fed}) \geq 0 \tag{17}$$

where $f^{Fed} > 0$ in equilibrium.

## A.5 Proofs of Propositions 2 and 3

**Proof of Proposition 2:** The first order conditions with respect to $s$ and $f$ are again (6) and (7), respectively. Also here, we can simplify the initial form of (7) and get (8). Now, using the FOC with respect to $b^i_E$, we have

$$\chi = \frac{\mu^i_{e,1} - \mu^i_E}{(1+r_E)\mu^i_{\$,2}}$$

Using the FOC with respect to $s$, we have

$$\chi = \frac{X\mu^i_{\$,1} - \mu^i_E}{(1+r_E)\mu^i_{\$,2}}$$

The first order condition with respect to $z^i_E$ is

$$\mu^i_{\$,1} X h = \mu^i_E + X\mu^i_{\$,2}(1+\rho)h \tag{18}$$

Finally, using (18) we get

$$\chi = X\frac{1+\rho}{1+r_E}\left(1 + \frac{(1-h)\mu^i_E}{h\mu^i_{\$,2}(1+\rho)X}\right)$$

In the post-intervention the repo facility is used, so the FX forward rate $\chi$ can be obtained according to both Propositions 1 and 2, which implies equation (2).

$\square$

**Proof of Proposition 3:** The case under consideration if one where collateral accepted by the ECB is denominated in USD. The box constraints for the European bank are similar to the ones introduced in Section 4.2, with the exception that here the box constraint for the American bond excludes repo (as by the time of crisis repo markets were extremely illiquid). Now, similar to the proof of Proposition 1.1, we can use the FOC with respect to $s$, $f$, and $b_E$, to get:

$$\chi \overset{(8)}{=} \frac{\mu^i_{e,2}}{\mu^i_{\$,2}} \overset{(5)}{=} \frac{\mu^i_{e,1} - \mu^i_E}{(1+r_E)\mu^i_{\$,2}} \overset{(6)}{=} \frac{X\mu^i_{\$,1} - \mu^i_E}{(1+r_E)\mu^i_{\$,2}}$$

Recall that now the collateral is in dollars. Hence, in the dollar no-overdraft constraints, $X_1$ does not multiply the haircuted repo trade $\tilde{h}_U z^i_U$. So, the FOC with respect to $z^i_U$ is:

$$\mu^i_{\$,1}\tilde{h}_U = \tilde{\mu}^i_U + \mu^i_{\$,2}(1+\tilde{\rho}_U)\tilde{h}_U$$

Then

$$\chi = \frac{X\frac{\tilde{\mu}_U^i}{\tilde{h}_U} + X\mu_{\$,2}^i(1+\tilde{\rho}_U) - \mu_E^i}{(1+r_E)\mu_{\$,2}^i}$$

and therefore

$$\chi = X\frac{1+\tilde{\rho}_U}{1+r_E}\left(1 + \frac{\tilde{\mu}_U^i}{\mu_{\$,2}^i\tilde{h}_U(1+\tilde{\rho}_U)} - \frac{\mu_E^i}{\mu_{\$,2}^i X(1+\tilde{\rho}_U)}\right)$$

Comparing the above expression with

$$\chi = X\frac{1+r_U+\beta}{1+r_E}$$

we get

$$\beta = \tilde{\rho}_U - r_U + \frac{\frac{\tilde{\mu}_U^i}{\tilde{h}_U} - \frac{\mu_E^i}{X}}{\mu_{\$,2}^i}$$

$\square$