# Do Interest Rates Smooth Investment?*

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**Abstract**

This paper studies the effects of interest rate movements on investment. The dependence of plant-level investment decisions on interest rates is important for understanding their smoothing effects. We introduce an interest rate process in the form of a state dependent discount factor that directly affects the decisions of the plant. Importantly, the stochastic discount factor is taken from the data rather than from the solution of a stochastic business cycle model. We find that that non-convexities at the plant level have aggregate implications when using an empirically consistent stochastic discount factor. The aggregate investment response to a shock to aggregate productivity depends on the nature of capital adjustment costs when the stochastic discount factor mimics the patterns in the data.

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# 1 Motivation

This paper studies the dependence of plant-level and aggregate investment on interest rates when adjustment costs are non-convex. This dependence is important both for understanding the smoothing effects of interest rate movements on investment and ultimately the transmission of monetary policy.

As is now understood from a number of studies at the establishment level, investment at the plant level is characterized by periods of only minimal changes in the capital stock coupled with intermittent periods of large capital adjustments. These patterns are difficult if not impossible to mimic in the standard quadratic adjustment cost model. Instead, these patterns are captured in models which rely on the presence of non-convex adjustment costs.[1]

Following the lead of Thomas (2002), Khan and Thomas (2003) and Khan and Thomas (2008), one might conjecture that the non-convexities at the plant-level are not important for aggregate investment. In those papers, state dependent interest rates are determined in equilibrium. A striking result from this literature is that the absence of aggregate effects of lumpy investment: the aggregate model with non-convexities at the micro-level is essentially indistinguishable from an aggregate model without non-convexities. The key to the result, as noted by Thomas (2002), is the response of the interest rate to aggregate shocks and the evolution of the capital stock.[2]

Those results, however, do not quite address the question we raise here. The issue is the interest rate process. Our focus is on the effects of interest rate movements found in the data, rather than those created in the underlying equilibrium of a stochastic business cycle model. It is entirely conceivable that the equilibrium smoothing of plant-level investment through interest rate movements does not arise in a model economy in which interest rates follow empirically consistent patterns.

The problem is that the interest rate process from the standard RBC model does not match the data well, as discussed, for example, in Beaudry and Guay (1996).[3] Thus, the smoothing of lumpy investment through interest rate movements produced in these models

[1] See, for example, the results reported in Caballero and Engel (1999) and Cooper and Haltiwanger (2006).

[2] Cooper and Haltiwanger (2006) find some, but not complete, smoothing by aggregation arising from idiosyncratic shocks when interest rates are held constant.

[3] This point appears in Thomas (2002) as well: Table 5 indicates a correlation of -0.385 between the real interest rate and output in the data but a correlation of 0.889 in the benchmark model.

might be both theoretically of interest and model-consistent, but not empirically based.

One way to see this difference is by studying aggregate investment in a model with two features. The first, as in Cooper and Haltiwanger (2006), is the presence of non-convexities in the capital adjustment process. The second is a stochastic discount factor which directly effects the decisions of the plant. But, in contrast to Thomas (2002) and the literature that has followed, we focus on representations of the stochastic factor that are empirically based rather than the outcome of a particular stochastic equilibrium model.

The results from this exercise indicate that non-convexities at the plant-level have aggregate implications for empirically consistent stochastic discount factors. Fluctuations in investment along the extensive and intensive margins are consistent with the empirical patterns reported in Gourio and Kashyap (2007). Interestingly, the estimation of parameters at the plant-level is robust to the choice of the interest rate process.

do we ever get back to this or to the lagged investment rate stuff?? we do get into this at the plant and aggregate levels. but not getting back to the data. not sure what GK actually present in terms of the data that is helpful since they focus on adjustment rate correlated with investment, not with driving stuff. instead, do the CHP pro cyclical spike stuff.

don't miss table 7 of thomas JPE. and recall that figure which shows a pro cyclical extensive margin. so where does the smoothing come from?

# 2 Dynamic Capital Demand

Our approach is to begin with a dynamic capital demand problem at the plant level. For this problem, we follow Cooper and Haltiwanger (2006) for the specification of the adjustment costs. But, in contrast to that analysis, we allow the discount factor to be stochastic. We parameterize this stochastic discount factor using the data. We then study the demand for capital at the plant level and in the aggregate.

## 2.1 Plant Level: Dynamic Optimization

Following Cooper and Haltiwanger (2006), the dynamic programming problem is specified as:

$$V(A,\varepsilon,K)=\max\{V^i(A,\varepsilon,K),V^a(A,\varepsilon,K)\},\qquad \forall(A,\varepsilon,K) \tag{1}$$

where $K$ represents the beginning of period capital stock, $A$ is the aggregate productivity shock, $\varepsilon$ is the idiosyncratic productivity shock. The superscripts refer to active investment "$a$," where the plant undertakes investment to obtain capital stock $K'$ in the next period, and inactivity "$i$," where no investment occurs. These options, in turn, are defined by:

$$V^i(A,\varepsilon,K)=\Pi(A,\varepsilon,K)+E_{A',\varepsilon'|A,\varepsilon}\left[\tilde{\beta}V(A',\varepsilon',K(1-\delta))\right] \tag{2}$$

and

$$V^a(A,\varepsilon,K) = \max_{K'}\left\{\Pi(A,\varepsilon,K)-C(A,\varepsilon,K,K')+E_{A',\varepsilon'|A,\varepsilon}\left[\tilde{\beta}V(A',\varepsilon',K')\right]\right\} \tag{3}$$

where $\tilde{\beta}$ is the state dependent discount rate for the establishment. The specification of this discount rate is a key to our analysis of this model.

The model includes three types of adjustment costs which, as reported in Cooper and Haltiwanger (2006), are the leading types of estimated adjustment costs.

$$\begin{aligned} C(A,\varepsilon,K,K') &= (1-\Lambda)\,\Pi(A,\varepsilon,K)-p_b(I>0)(K'+(1-\delta)K) \\ &\quad -p_s(I<0)((1-\delta)K-K')+\frac{\nu}{2}\left(\frac{K'-(1-\delta)K}{K}\right)^2 K \end{aligned} \tag{4}$$

The first is a disruption cost parameterized by $\Lambda$. If $\Lambda<1$, then any level of gross investment implies that a fraction of revenues is lost. The second is the quadratic adjustment cost parameterized by $\nu$. The third is a form of irreversibility in which there is a gap between the buying, $p_b$, and selling, $p_s$, prices of capital. These are included in (3) by the use of the indicator function for the buying ($I>0$) and selling of capital ($I<0$).

The profit function is

$$\Pi(A,\varepsilon,K)=A\varepsilon K^{\alpha}. \tag{5}$$

This is a reduced-form profit function which can be derived from an optimization problem over flexible factors of production (i.e. labor, materials, etc.). The parameter $\alpha$ will reflect factor shares as well as the elasticity of demand for the plant's output. Here $A$ is a aggregate productivity shock and $\varepsilon$ is the idiosyncratic productivity shock.

# 3 Finding the Stochastic Discount Factor

The optimization problem given in (1) includes a stochastic discount factor, $\tilde{\beta}$. As is customary within a stochastic equilibrium model in which households do not face costs of adjusting their portfolios, rates of return are linked to household preferences through an Euler equation

$$E_t\left[\tilde{\beta}_{t+1}R^j_{t+1}\right] = 1 \tag{6}$$

for any asset $j$. Here

$$\tilde{\beta}_{t+1} \equiv \frac{\beta u'(c_{t+1})}{u'(c_t)} \tag{7}$$

is the standard pricing kernel where $c_t$ is household consumption in period $t$ and $u(\cdot)$ represents utility for a representative household.

We study two specifications of $\tilde{\beta}_{t+1}$. We start with the process for $\tilde{\beta}_{t+1}$ that comes from the data. We then study, following Thomas (2002) and others, a stochastic general equilibrium version of the state dependent interest rates.

For both, we are interested in a state space (capital stock and productivity shocks) representation of this stochastic discount factor. This form of state dependence is immediate in versions of the stochastic growth model with capital and a technology shock in the state vector. Once there are nonlinearities in adjustment costs as well as plant-specific shocks, the state vector is more complex. But, when the results of Thomas (2002) hold, a specification of the stochastic discount factor as a function of the aggregate shocks and the aggregate capital stock should be sufficient. In this way, the state dependent discount factor can be used directly in (1).

## 3.1 Data-Based Approach

We study two data-based approaches to uncovering $\tilde{\beta}_{t+1}$. In the first, we use the Euler equation from (7). The second bypasses the household problem and looks directly at interest rates.

For the first approach, data on consumption and some assumptions on preferences, (7) can be used to generate a time series for $\tilde{\beta}_{t+1}$. For this exercise, assume $u(c) = \log(c)$ and set $\beta = 0.95$.[4]

[4] Future work will look at the robustness of our results to alternative specifications of preferences and the discount factor.

should we explore more here. like CRRA and then recursive. see Gallant and Hong and also the hansen-richard paper...and chapter in LS, pg 288

Once we solve for the stochastic discount factor, we compute the empirical relationship between the stochastic discount factor and observables corresponding to key state variables in our model. This relationship is estimated by regressing the stochastic discount factor, $\tilde{\beta}_{t+1}$, on measures of the capital stock, current and future productivity: $(A_t, A_{t+1}, K_t)$. The inclusion of both current period and future period values of the aggregate shock, $A_t$, are required since $\tilde{\beta}_{t+1}$ measures the realized real return between periods $t$ and $t+1$.

The appendix provides a detailed discussion of our data. For our analysis, we consider $A_t$ to be total factor productivity in period $t$. With a focus on business cycle dynamics, we model a stationary specification of the shock process, which we parameterize from the data after detrending using the H-P filter. As discussed in the appendix, our results depend on the choice of the parameter of the H-P filter, denoted $\lambda$. The choice of this parameter determines directly the serial correlation of the total factor productivity process and consequently the dependence of $\tilde{\beta}_{t+1}$ on the state vector $(A_t, A_{t+1}, K_t)$. The mapping from the values of $\lambda$ to the serial correlation of the aggregate shock, $\rho_A$, is given in Table 11 in the Appendix. We therefore report results for three distinct values of $\rho_A$.

resolve this by having the two extreme cases.

Our results are presented in Table 1. There are four different specifications and three different values of $\rho_A$ for each. For each of these empirical models, we report regression coefficients and goodness of fit measures.

for this table have the consumption based SDF ... eliminate the row without $A_{t+1}$. add in our new estimated model using the various FF portfolios and Shiller.... what about the last two? i think we should keep em...???

The first two blocks report two different specifications of the dependence of $\tilde{\beta}_{t+1}$ on different parts of the $(A_t, A_{t+1})$ state space. The model in which $\tilde{\beta}_{t+1}$ depends on $(A_t, A_{t+1})$ fits better than the model with $A_t$ alone. The results ignore the capital stock since the inclusion of $K_t$ adds very little to the empirical model.

As is made clear in the table, the magnitude of the coefficient depends on the choice of $\rho_A$. A lower value of this parameter leads to more dependence on $A_t$ and less, in absolute value, dependence on $A_{t+1}$ in the second block of results. This will be important later when we study how investment responds to productivity shocks.

Table 1: Empirical estimates of relationship between state variables and discount factors

| Specification | $\rho_A$ | Constant | $A_t$ | $A_{t+1}$ | $R^2$ | $\frac{dE_t[\tilde{\beta}(\cdot)]}{dA_t}$ |
|---|---|---|---|---|---|---|
| $\tilde{\beta}(A_t)$ | 0.14 | -0.08 | 0.12 | | 0.02 | 0.12 |
| | | (0.00) | (0.12) | | | |
| | 0.45 | -0.08 | 0.01 | | 0.00 | 0.01 |
| | | (0.00) | (0.09) | | | |
| | 0.84 | -0.08 | -0.04 | | 0.02 | -0.04 |
| | | (0.00) | (0.04) | | | |
| $\tilde{\beta}(A_t, A_{t+1})$ | 0.14 | -0.08 | 0.21 | -0.61 | 0.46 | 0.12 |
| | | (0.00) | (0.09) | (0.09) | | |
| | 0.45 | -0.08 | 0.24 | -0.52 | 0.49 | 0.01 |
| | | (0.00) | (0.07) | (0.07) | | |
| | 0.84 | -0.08 | 0.34 | -0.46 | 0.56 | -0.04 |
| | | (0.00) | (0.05) | (0.06) | | |
| $r(A_t)$ (30-day T-bill) | 0.14 | 0.02 | -0.37 | | 0.05 | 0.36 |
| | | (0.00) | (0.23) | | | |
| | 0.45 | 0.02 | -0.37 | | 0.10 | 0.36 |
| | | (0.00) | (0.16) | | | |
| | 0.84 | 0.02 | -0.25 | | 0.19 | 0.24 |
| | | (0.00) | (0.07) | | | |
| $r(A_t)$ (AAA LT Bond) | 0.14 | 0.03 | -0.27 | | 0.03 | 0.25 |
| | | (0.00) | (0.21) | | | |
| | 0.45 | 0.03 | -0.30 | | 0.07 | 0.28 |
| | | (0.00) | (0.15) | | | |
| | 0.84 | 0.04 | -0.40 | | 0.57 | 0.37 |
| | | (0.00) | (0.05) | | | |

Instead of working through the Euler equation, the last two blocks replace $\tilde{\beta}_{t+1}$ with observed returns on different assets. The argument for doing so is that the household asset pricing model has failed in a number of ways. Thus instead of going through the Euler equation, we can look directly at interest rates that a firm might use to discount profits.

We study two real interest rates, the 30-day T bill and a long term AAA bond. These are *ex ante* rates and thus depend on the current state, $A_t$. The real rates are constructed by subtracting realized inflation for the 30-day T bill and long-term inflation expectations from the Survey of Professional Forecasters for the AAA bond from the respective nominal rate. Interestingly, the response of the return to a variation in $A_t$ is generally negative and it is significant for the larger values of $\lambda$. Since the interest rate is inversely related to the stochastic discount factor, high realizations of productivity translate into higher discount factors.

## 3.2 Model-Based Approach

An alternative approach is to study the dependence of the stochastic discount factor on the state variables using a model. Table 2 reports results for three models. For each model, we use three different values of $\rho_A$ corresponding to those used in the empirical specification.

The first model, labeled RBC, is the standard real business cycle model, building from King, Plosser and Rebelo (1988). Here we see that the return responds positively to both the current productivity shock and the capital stock and negatively with future productivity for all values of $\rho_A$.

The second model, labeled Chat-Coop, comes from Chatterjee and Cooper (1993), which studies a stochastic real business cycle model with monopolistic competition. This environment is closer to the underlying market structure assumed in Cooper and Haltiwanger (2006).

The results for the Chat-Coop specification in Table 2 come from a model with no entry and exit and a markup of 25 percent.[5]. The results are quite similar to those in the standard RBC model.

recheck the GK stuff.... why so different ??

The third model, labeled Gourio-Kashyap, represents the Gourio and Kashyap (2007)

[5] The markup is based on a CES specification where the elasticity of substitution is set to 5 for both consumption and capital goods.

Table 2: Model-based relationship between state variables and discount factor

| Model | $\rho_A$ | $A_t$ | $A_{t+1}$ | $K_t$ | $\frac{dE_t[\tilde{\beta}(A_t,A_{t+1})]}{dA_t}$ |
|---|---|---|---|---|---|
| RBC | 0.14 | 0.08 | -0.39 | 0.09 | 0.031 |
| | 0.45 | 0.15 | -0.43 | 0.09 | -0.044 |
| | 0.84 | 0.37 | -0.59 | 0.09 | -0.121 |
| Chat-Coop | 0.14 | 0.09 | -0.41 | 0.09 | 0.032 |
| | 0.45 | 0.17 | -0.47 | 0.09 | -0.038 |
| | 0.84 | 0.43 | -0.64 | 0.09 | -0.110 |
| Gourio-Kashyap | 0.14 | 0.11 | -0.35 | 0.10 | 0.062 |
| | 0.45 | 0.18 | -0.40 | 0.10 | 0.004 |
| | 0.84 | 0.43 | -0.59 | 0.10 | -0.066 |

specification of the model proposed by Thomas (2002). However, the relationship in Table 2 is only an approximation of the true stochastic discount factor because the state space for that model includes the cross-sectional distribution of capital vintages as well as the capital stock and productivity shocks. Thus, the stochastic discount factor should, in principal, depend on the cross sectional distribution.

However, given that the real allocations from the Thomas (2002) model with lumpy investment are so close to the stochastic growth model, one would conjecture that the process for the stochastic discount factor would be close to that of the RBC and/or Chat-Coop specifications. Tables 4-6 of Thomas (2002) indicate that the interest rate process in the model with lumpy investment is extremely similar to that of the standard RBC model (benchmark in those tables). From Figure 4 of Thomas (2002), the interest rate response to a shock is only 3 basis points larger in the lumpy investment compared to the RBC benchmark. This suggests that plant-level investment decisions must be very sensitive to interest rate movements in the lumpy-investment model in order for such small movements in intertemporal prices to smooth investment.

Put in another way, the stochastic discount factor in the Thomas (2002) model can be well characterized in the same way as in the RBC and Chat-Coop specifications, using only aggregate states and ignoring underlying heterogeneity. This relationship, shown in

Table 2, is estimated based on a simulated data from the model using the Gourio-Kashyap specification of the underlying parameters. So while the model-based stochastic discount factor is a function of a larger set of state variables accounting for underlying heterogeneity, the simpler representation that includes only productivity and the capital stock captures 99.1 percent of the variation in the stochastic discount factor, as measured by the $R^2$ in the regression. The estimated relationships across all value of $\rho_a$ are very similar to the RBC and Chat-Coop specifications.

### 3.3 Response of the expected stochastic discount factor to productivity

It is instructive to compare the response of interest rates to the state variables in these models with the results from the data. The key is how the expected stochastic discount factor responds to variations in current productivity. Though the plant-level optimization problem does include a covariance of future values with the stochastic discount factor, the response of the expected discount factor to variations in current productivity will be a key element in the results that follow.

For the data-based stochastic discount factors reported in Table 1, we study how $E_t[\tilde{\beta}(A_t, A_{t+1})]$ varies with $A_t$. The results appear in the last column of that table.

A similar exercise is done with the results from the model based stochastic discount factors. These results are reported in the last column of Table 2.

In the first two cases of the $\tilde{\beta}(A_t, A_{t+1})$ estimates, corresponding to $\rho_A = 0.14$ and $\rho_A = 0.45$, the expected stochastic discount factor is increasing in current productivity. The response is slightly negative at $\rho_A = 0.84$ as the negative coefficient on $A_{t+1}$ is given more weight due to the higher serial correlation of the shock.

From the model based results, the expected stochastic discount factor is **countercyclical** for the $\rho_A = 0.84$ case. This response is more countercyclical than in the empirical based results. At $\rho_A = 0.14$, the stochastic discount factor is also procyclical in the model based estimates.

These differences between data and model relate to earlier comments on the inability of the standard RBC model to match interest rate movements. In the data, interest rates are countercyclical, and thus the expected stochastic discount rate is procyclical. For leading models though, the interest rate is inversely correlated with output. The smoothing effects of procyclical interest rates, key to Thomas (2002), is not operative in the data.

Figure 1 summarizes these results. At $\rho_A = 0.14$, displayed in the top panel, both stochastic discount factors from the empirical specification based on the Euler equation and the model specification based on the Chat-Coop model are procyclical. The data based measure is more volatile than the model based one. For the case of $\rho_A = 0.84$, the stochastic discount factors are both countercyclical but in this case the model-based one is more volatile.

For the empirical specifications in Table 1 based on observed interest rates, the market returns on both T-bills and long-term bonds imply a strongly procyclical discount factor, regardless of the assumed serial correlation. This effect is stronger than in either the Euler-equation based empirical measures or model based measures.

These differences are important for our understanding of how much interest rate movements ultimately smooth the response of plant-level and aggregate investment to productivity shocks. If $\tilde{\beta}$ is positively correlated with the aggregate productivity shock, then the effect of $A_t$ on investment will be magnified. Alternatively, if $\tilde{\beta}$ is negatively correlated with $A_t$, then aggregate shocks will be smoothed. They also highlight the significance of the serial correlation of the aggregate shocks for the smoothing effects of interest rate movements.

the gap between the data and RBC model interest rate processes is well understood. here we are going beyond that to make the case that this difference matters for the behavior of aggregate investment...

# 4 Results

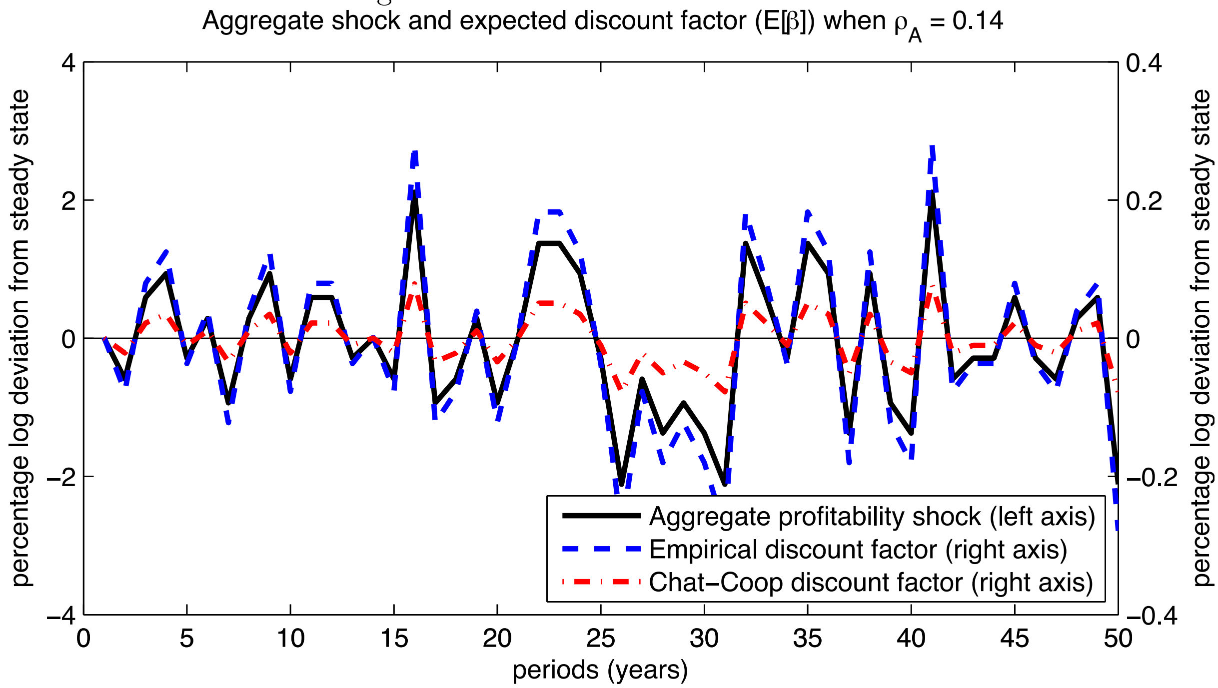
Using these processes for the stochastic discount factor, we study the response of investment to shocks. We do so first at the plant level and then in the aggregate.

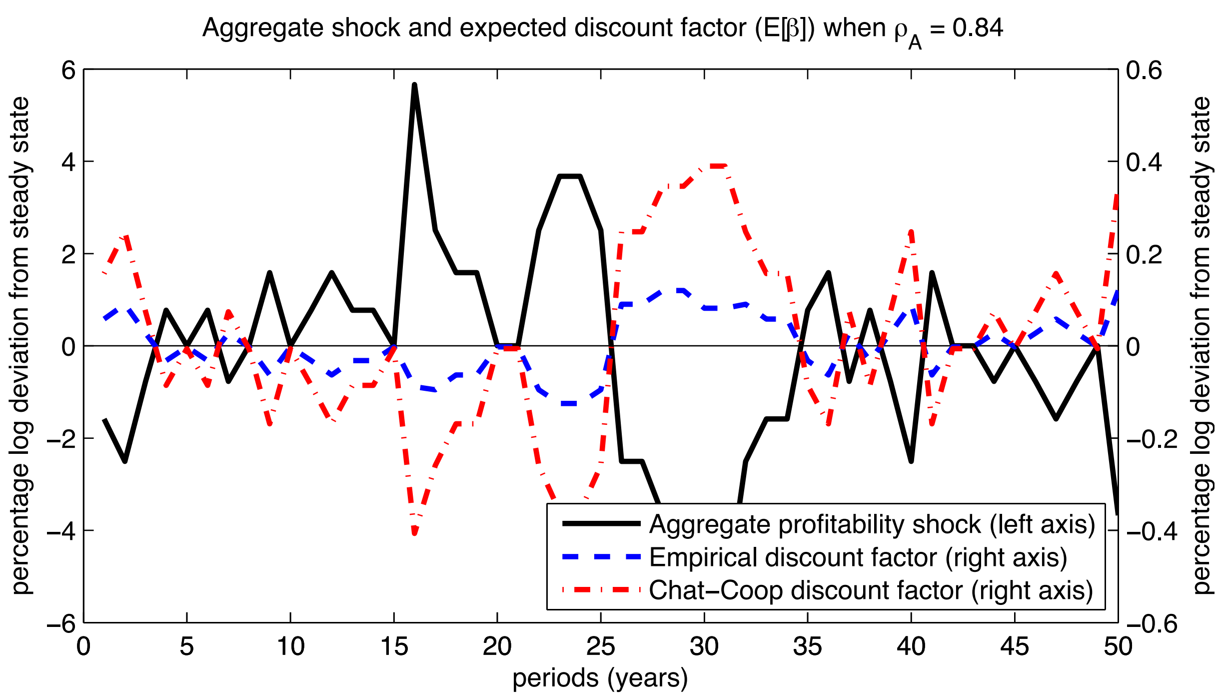
To obtain these results, we solve (1) for different specifications of the stochastic discount factor reported in Table 1. We then study investment choices at the plant-level and in aggregate. Thus the investment choices depend on empirically relevant representations of the stochastic discount factor.

For these simulations, we follow Cooper and Haltiwanger (2006) and assume $\alpha = 0.58$, $\rho_\varepsilon = 0.885$ $\sigma_\varepsilon = 0.1$ for the idiosyncratic shock process.[6] The adjustment are costs given

[6] In Cooper and Haltiwanger (2006) the estimates of the aggregate and idiosyncratic shock processes

Figure 1: Stochastic Discount Factors.




This figure shows the relationship between data and model based stochastic discount factors for two values of $\rho_A$.

by $\Lambda = 0.8, \nu = 0.15, qs = 0.98$. In the simulated data set, we follow 1000 plants for 500 periods.

## 4.1 Plant-level Implications

Table 3 reports moments at the plant-level for different interest rate processes.[7] These are the moments used in Cooper and Haltiwanger (2006) for the estimation of adjustment cost parameters.[8] There is an important point to gather from this table: these plant-level moments are essentially independent of the representation of the stochastic discount factor. Hence the parameter estimates from Cooper and Haltiwanger (2006), which assumes a constant discount factor, are robust to analysis allowing a stochastic discount factor.

This does not imply though that the investment decision is independent of the specification of the stochastic discount factor. Table 4 reports results of plant-level regressions on current state variables for different measures of the stochastic discount factor and different parameters for $\rho_A$.

There are three important results here. First, the response of investment to current productivity depends on the specification of the stochastic discount factor. Second, this response depends on $\rho_A$. Third, the response of investment to aggregate productivity is very different for the data based characterization of the stochastic discount factor compared to the model based version.

The response of investment to $A_t$ is generally positive and significant. There are two exceptions.

One exception is for the fixed $\beta$ case and the low value of $\rho_A$. At $\rho_A = 0.14$, the productivity process is closest to iid. This combined with the opportunity cost of investment, $\Lambda < 1$, implies that it is more costly to invest when $A_t$ is high without any apparent future benefit. This negative response of investment to a productivity shock is overturned for larger values of $\rho_A$ and for the other specifications of the stochastic discount factor.

A second exception comes from the model based versions of the stochastic discount factor. For the Chat-Coop case, investment at the plant-level is decreasing in $A_t$. This reflects the

correspond to profitability shocks, as technology, cost and demand shocks cannot be separately identified.

[7]In these tables, $r(A_t)$ is the 30-day T-bill.

[8]In that analysis, the correlation of productivity and investment was used rather than the correlation of plant-specific productivity and investment. In the data, these correlations are about the same.

fact that the stochastic discount factor is expected to fall when productivity increases.

Table 3: Plant-level moments from simulated data

| Model | $\rho_A$ | mean($\frac{I_i}{K_i}$) | Frac Inactive | Frac - | Spike + | Spike - | mean(abs($\frac{I_i}{K_i}$)) | Corr($I_i, I_{i,-1}$) | Corr($I_i, \varepsilon_i$) |
|---|---|---|---|---|---|---|---|---|---|
| | 0.14 | 0.090 | 0.87 | 0.000 | 0.13 | 0.000 | 0.72 | -0.13 | 0.19 |
| $\beta$ | 0.45 | 0.090 | 0.87 | 0.000 | 0.13 | 0.000 | 0.72 | -0.13 | 0.19 |
| | 0.84 | 0.090 | 0.88 | 0.000 | 0.12 | 0.000 | 0.72 | -0.13 | 0.20 |
| | 0.14 | 0.090 | 0.87 | 0.000 | 0.13 | 0.000 | 0.72 | -0.13 | 0.19 |
| $\tilde{\beta}(A_t, A_{t+1})$ | 0.45 | 0.090 | 0.87 | 0.000 | 0.13 | 0.000 | 0.72 | -0.13 | 0.19 |
| (Empirical) | 0.84 | 0.090 | 0.88 | 0.000 | 0.12 | 0.000 | 0.72 | -0.13 | 0.20 |
| | 0.14 | 0.090 | 0.87 | 0.000 | 0.13 | 0.000 | 0.72 | -0.13 | 0.19 |
| $\tilde{\beta}(A_t, A_{t+1})$ | 0.45 | 0.090 | 0.87 | 0.000 | 0.13 | 0.000 | 0.72 | -0.13 | 0.19 |
| (Chat-Coop) | 0.84 | 0.090 | 0.88 | 0.000 | 0.12 | 0.000 | 0.72 | -0.13 | 0.20 |
| | 0.14 | 0.090 | 0.87 | 0.000 | 0.13 | 0.000 | 0.72 | -0.13 | 0.18 |
| $r(A_t)$ | 0.45 | 0.090 | 0.87 | 0.000 | 0.13 | 0.000 | 0.72 | -0.12 | 0.18 |
| | 0.84 | 0.091 | 0.87 | 0.000 | 0.13 | 0.000 | 0.72 | -0.12 | 0.18 |

Note that for all the models, investment is increasing in the idiosyncratic productivity shock. This is partly due to the high serial correlation of that shock ($\rho_\varepsilon = 0.885$) and the fact that it does not influence the stochastic discount factor.

Tables 5 and 6 break the investment response into two components. The intensive margin indicating the investment response of the adjusters and the extensive margin regarding the choice to invest or not.

Comparing the intensive margin regressions in Table 5 with the results in Table 4, there are a couple of points to note. For the fixed $\beta$ case, the response to $A$ is positive for the adjusters. Once the selection effect from the extensive margin is removed, investment is increasing in productivity.

For the intensive margin the response to $K$ is almost zero. The explanation for the inverse relationship between investment rates and the stock of capital, shown in Table 4 must come from the extensive margin.

Comparing the extensive margin regressions in Table 6 with the linear probability model results in Table 4, the negative effects of high capital on the adjustment choice is very strong. The adjustment probability is increasing in $A_t$ for all specifications except for the fixed $\beta$ case with low $\rho_A$ **and** for all simulations using the Chat-Coop specification of the stochastic discount factor. The opposing signs in the response of the extensive margin to changes in $A_t$ for the empirical and Chat-Coop specifications of $\tilde{\beta}(A_t, A_{t+1})$ illustrates a key difference in the interest rate channel for plant-level investment decisions between these two models of the stochastic discount factor.

important to get into this extensive margin stuff. note from CHP that spikes are pro cyclical. seems to be a feature of Thomas as well. so what are GK saying is not consistent with Thomas and the data???

## 4.2 Aggregate Implications

so what can we link this all too in the aggregate data??? do more to compare Tables 8 and 9. what about the data on the fraction of adjustors correlated with A? so go back to that micro point... We aggregate our simulated data to study the response of aggregate investment to productivity shocks for different specifications of the stochastic discount factor. We also ask whether empirically relevant movements in the stochastic discount factor smooth out fluctuations in

Table 4: Plant-level investment regression on simulated data: $I_i(A, \varepsilon_i, K_i)$

| Model | $\rho_A$ | $A$ | $\varepsilon_i$ | $K_i$ | $R^2$ |
|---|---|---|---|---|---|
| $\beta$ | 0.14 | -12.78 | 28.47 | -0.46 | 0.36 |
| | | (3.22) | (0.26) | (0.00) | |
| | 0.45 | 1.36 | 29.18 | -0.46 | 0.36 |
| | | (2.32) | (0.26) | (0.00) | |
| | 0.84 | 23.59 | 29.82 | -0.43 | 0.36 |
| | | (1.37) | (0.27) | (0.00) | |
| $\tilde{\beta}(A_t, A_{t+1})$ (Empirical) | 0.14 | 24.58 | 28.40 | -0.46 | 0.36 |
| | | (3.22) | (0.26) | (0.00) | |
| | 0.45 | 7.08 | 29.10 | -0.46 | 0.37 |
| | | (2.32) | (0.26) | (0.00) | |
| | 0.84 | 13.76 | 29.74 | -0.43 | 0.36 |
| | | (1.37) | (0.27) | (0.00) | |
| $\tilde{\beta}(A_t, A_{t+1})$ (Chat-Coop) | 0.14 | -1.97 | 28.63 | -0.46 | 0.36 |
| | | (3.21) | (0.26) | (0.00) | |
| | 0.45 | -4.70 | 28.86 | -0.45 | 0.36 |
| | | (2.33) | (0.26) | (0.00) | |
| | 0.84 | -6.48 | 30.20 | -0.44 | 0.36 |
| | | (1.36) | (0.27) | (0.00) | |
| $r(A_t)$ | 0.14 | 80.89 | 27.44 | -0.44 | 0.36 |
| | | (3.23) | (0.26) | (0.00) | |
| | 0.45 | 91.40 | 27.14 | -0.43 | 0.36 |
| | | (2.34) | (0.26) | (0.00) | |
| | 0.84 | 91.15 | 26.39 | -0.41 | 0.36 |
| | | (1.45) | (0.26) | (0.00) | |

Table 5: Plant-level investment regression (Adjusters only): $I_i(A, \varepsilon_i, K_i)$

| Model | $\rho_A$ | $A$ | $\varepsilon_i$ | $K_i$ | $R^2$ |
|---|---|---|---|---|---|
| | 0.14 | 4.54 | 25.54 | 0.02 | 0.90 |
| | | (2.57) | (0.45) | (0.01) | |
| $\beta$ | 0.45 | 6.70 | 26.47 | 0.03 | 0.93 |
| | | (1.59) | (0.39) | (0.01) | |
| | 0.84 | 26.11 | 29.58 | 0.02 | 0.98 |
| | | (0.56) | (0.23) | (0.01) | |
| | 0.14 | 17.11 | 24.60 | 0.04 | 0.90 |
| | | (2.59) | (0.45) | (0.01) | |
| $\tilde{\beta}(A_t, A_{t+1})$ | 0.45 | 11.57 | 27.03 | 0.01 | 0.93 |
| (Empirical) | | (1.58) | (0.39) | (0.01) | |
| | 0.84 | 18.42 | 29.61 | 0.03 | 0.98 |
| | | (0.50) | (0.21) | (0.00) | |
| | 0.14 | 5.28 | 25.25 | 0.03 | 0.90 |
| | | (2.51) | (0.44) | (0.01) | |
| $\tilde{\beta}(A_t, A_{t+1})$ | 0.45 | 2.88 | 26.63 | 0.03 | 0.93 |
| (Chat-Coop) | | (1.58) | (0.39) | (0.01) | |
| | 0.84 | 2.80 | 29.67 | 0.03 | 0.99 |
| | | (0.45) | (0.19) | (0.00) | |
| | 0.14 | 32.77 | 24.14 | 0.05 | 0.89 |
| | | (2.91) | (0.44) | (0.01) | |
| $r(A_t)$ | 0.45 | 57.48 | 26.46 | 0.02 | 0.90 |
| | | (2.40) | (0.42) | (0.01) | |
| | 0.84 | 83.49 | 27.72 | 0.03 | 0.93 |
| | | (1.71) | (0.37) | (0.01) | |

Table 6: Linear probability regression using plant-level simulated data (extensive margin)

| Model | $\rho_A$ | $A$ | $\varepsilon_i$ | $K_i$ | $R^2$ |
|---|---|---|---|---|---|
| $\beta$ | 0.14 | -0.67 | 1.25 | -0.84 | 0.37 |
| | | (0.15) | (0.01) | (0.01) | |
| | 0.45 | 0.02 | 1.27 | -0.84 | 0.37 |
| | | (0.10) | (0.01) | (0.01) | |
| | 0.84 | 1.00 | 1.26 | -0.81 | 0.37 |
| | | (0.06) | (0.01) | (0.01) | |
| $\tilde{\beta}(A_t, A_{t+1})$ (Empirical) | 0.14 | 1.17 | 1.25 | -0.84 | 0.37 |
| | | (0.15) | (0.01) | (0.01) | |
| | 0.45 | 0.29 | 1.26 | -0.84 | 0.37 |
| | | (0.10) | (0.01) | (0.01) | |
| | 0.84 | 0.58 | 1.24 | -0.79 | 0.37 |
| | | (0.06) | (0.01) | (0.01) | |
| $\tilde{\beta}(A_t, A_{t+1})$ (Chat-Coop) | 0.14 | -0.11 | 1.26 | -0.85 | 0.37 |
| | | (0.15) | (0.01) | (0.01) | |
| | 0.45 | -0.29 | 1.26 | -0.83 | 0.37 |
| | | (0.10) | (0.01) | (0.01) | |
| | 0.84 | -0.33 | 1.27 | -0.81 | 0.37 |
| | | (0.06) | (0.01) | (0.01) | |
| $r(A_t)$ | 0.14 | 3.86 | 1.20 | -0.82 | 0.37 |
| | | (0.15) | (0.01) | (0.01) | |
| | 0.45 | 4.26 | 1.17 | -0.79 | 0.38 |
| | | (0.10) | (0.01) | (0.01) | |
| | 0.84 | 3.97 | 1.11 | -0.76 | 0.37 |
| | | (0.06) | (0.01) | (0.01) | |

aggregate investment due to the non-convexities at the plant level. We also study the cyclical patterns of investment on the extensive (fraction of plants investing) and intensive margins (investment rate conditional on investment).

#### 4.2.1 Basic Moments

Table 7 presents some basic correlations from the data as well as aggregated simulated data from the model. For the data, productivity is measured as describe in the appendix. The other measures are constructed from the Census Bureau's Annual Survey of Manufacturers (ASM) as reported and described in Gourio and Kashyap (2007). This data source is used because it provides important evidence for this analysis regarding the extensive margin investment decisions of manufacturing establishments. The reported statistics are based on annual data from 1974 to 1998.

The correlations based on the data reveal positive, but small, correlations between productivity and most of the aggregate series. The correlation between productivity and both aggregate investment and the aggregate investment rate are modestly positive across all filtering specifications. The correlation between productivity and the fraction of establishments with investment rates greater than 20 percent is approximately 0.2 across all filtering specifications. The correlation between productivity and the capital stock is near zero when $\lambda$ is 7 and 100, and negative when $\lambda$ is 100,000.

The correlation between productivity and investment in the simulated results depends critically on the specification of the stochastic discount factor: it is procyclical under the empirical models of the stochastic discount factor and countercyclical with the model-based specifications.

For the data-based version of the stochastic discount factor, either from $\tilde{\beta}(A_t, A_{t+1})$ or directly from observed interest rates, investment is positively correlated with aggregate productivity. As noted earlier in the discussion of Table 1, increases in productivity increase $\tilde{\beta}$ when $\rho_A = 0.14$ and $\rho_A = 0.45$. Therefore, the investment response to changes in $A_t$ is magnified by the changes in the stochastic discount factor. For $\rho_A = 0.84$, an increase in $A_t$ does lead to a fall in $\tilde{\beta}(A_t, A_{t+1})$, but the correlation between productivity and investment remains positive.

The results are quite different in the case of the model-based stochastic discount factor. Here investment is negatively correlated with the productivity shock. This comes from the interaction of the opportunity cost of adjustment and the fact that for $\rho_A = 0.45$ and $\rho_A = 0.84$, $\tilde{\beta}(A_t, A_{t+1})$ falls as $A_t$ increases.

These patterns are clearly sensitive to both the specification of the stochastic discount factor and the choice of $\rho_A$. For the fixed discount factor and a low value of $\rho_A$, investment is negatively correlated with the aggregate shock, both in terms of the investment rate and in the fraction of adjusters. This, as explained earlier, comes from the opportunity cost of adjusting.

These differences in the determination of the stochastic discount factor carry over to other moments. For the empirically based measured of $\tilde{\beta}(A_t, A_{t+1})$, the fraction of adjusters is positively correlated with the aggregate productivity shock. This correlation is negative for the model based versions. While there is a difference in the sign on the extensive margin, the mean investment rate for those who adjust is positively correlated with $A$ in both specifications of $\tilde{\beta}(A_t, A_{t+1})$.

The cyclical dynamics of the extensive margin may be a critical factor in determining whether interest rates smooth or amplify investment. The evidence from the data suggests that the extensive margin is procyclical with respect to productivity. Comparing results from the models, the extensive margin is procylical with respect to productivity for the empirical-based stochastic discount factor and countercylical for the model-based stochastic discount factor across all filtering specifications.

Table 8 reports regressions of the future aggregate capital stock on the current state for different values of $\rho_A$ for different values of the stochastic discount factor. Here we see that the coefficient on current productivity is positive for the empirically based stochastic discount factor but negative for the model based stochastic discount factor. Table 9 is the same exercise on actual data. The coefficient on $A_t$ is positive throughout, consistent with the data based version of the stochastic discount factor.

Figure 2 highlights the importance of the stochastic discount factor for aggregate investment. This is for the baseline non-convex adjustment cost case. As in Table 7, the two investment series respond very differently to variations in aggregate productivity. In many cases, the model and empirically based versions of the stochastic discount factor leads to opposite movements of aggregate investment in response to productivity shocks. This is, again, consistent with the differences reported at the plant-level in Table 4.

Figure 3 illustrates the role of the stochastic discount factor for the cyclical dynamics of

Table 7: Correlation of key variables with aggregate productivity ($A$)

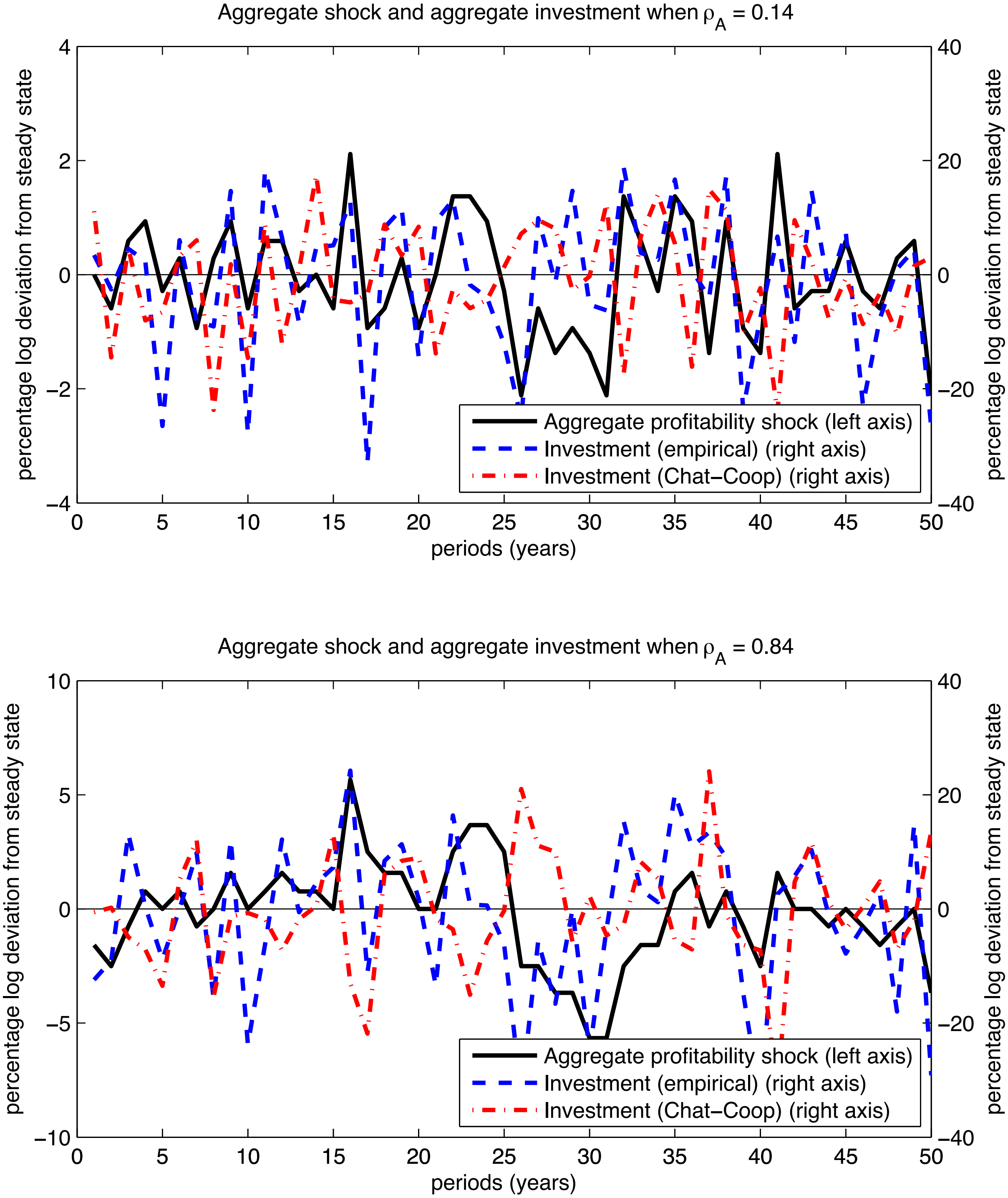
| Data or Model | $\lambda$ | $\rho_A$ | Aggregate variables $I$ | $K$ | $\frac{I}{K}$ | Extensive margin Fract. of adjusters | Intensive margin $\text{mean}(\frac{I_i}{K_i} \mid \frac{I_i}{K_i} > 0.2)$ |
|---|---|---|---|---|---|---|---|
| Data | 7 | 0.14 | 0.08 | -0.02 | 0.12 | 0.14 | NA |
| | 100 | 0.45 | 0.12 | 0.03 | 0.14 | 0.22 | NA |
| | 100000 | 0.84 | 0.05 | -0.36 | 0.23 | 0.19 | NA |
| $\beta$ | 7 | 0.14 | -0.51 | 0.03 | -0.50 | -0.55 | 0.61 |
| | 100 | 0.45 | -0.05 | 0.15 | -0.06 | -0.09 | 0.54 |
| | 100000 | 0.84 | 0.55 | 0.73 | 0.43 | 0.46 | 0.09 |
| $\tilde{\beta}(A_t, A_{t+1})$ (Empirical) | 7 | 0.14 | 0.61 | 0.15 | 0.58 | 0.62 | 0.01 |
| | 100 | 0.45 | 0.31 | 0.26 | 0.27 | 0.29 | 0.41 |
| | 100000 | 0.84 | 0.47 | 0.69 | 0.36 | 0.36 | 0.47 |
| $\tilde{\beta}(A_t, A_{t+1})$ (Chat-Coop) | 7 | 0.14 | -0.19 | 0.10 | -0.20 | -0.23 | 0.49 |
| | 100 | 0.45 | -0.31 | 0.04 | -0.30 | -0.36 | 0.65 |
| | 100000 | 0.84 | -0.27 | -0.22 | -0.24 | -0.33 | 0.83 |
| $r(A_t)$ | 7 | 0.14 | 0.79 | 0.10 | 0.76 | 0.78 | -0.74 |
| | 100 | 0.45 | 0.73 | 0.36 | 0.69 | 0.70 | -0.80 |
| | 100000 | 0.84 | 0.62 | 0.71 | 0.52 | 0.53 | -0.78 |

For the data results, the method used to construct the productivity data is described in the appendix. The other measures are constructed from the Census Bureau's Annual Survey of Manufacturers (ASM) as reported and described in Gourio and Kashyap (2007). The reported statistics are based on annual data from 1974 to 1998. The other results are based on simulated data from the model. The threshold used for the extensive and intensive margins is an investment rate (I/K) greater than 20 percent in absolute value.

Table 8: Aggregate capital regression on simulated data: Non-Convex Adjustment Costs

| Model | $\rho_A$ | $A_t$ | $K_t$ | $R^2$ |
|---|---|---|---|---|
| $\beta$ | 0.14 | -0.33 | 0.71 | 0.64 |
| | | (0.02) | (0.03) | |
| | 0.45 | -0.00 | 0.77 | 0.60 |
| | | (0.02) | (0.03) | |
| | 0.84 | 0.63 | 0.50 | 0.96 |
| | | (0.02) | (0.01) | |
| $\tilde{\beta}(A_t, A_{t+1})$ (Empirical) | 0.14 | 0.60 | 0.58 | 0.72 |
| | | (0.03) | (0.02) | |
| | 0.45 | 0.18 | 0.72 | 0.68 |
| | | (0.02) | (0.03) | |
| | 0.84 | 0.35 | 0.59 | 0.90 |
| | | (0.02) | (0.02) | |
| $\tilde{\beta}(A_t, A_{t+1})$ (Chat-Coop) | 0.14 | -0.10 | 0.78 | 0.60 |
| | | (0.02) | (0.03) | |
| | 0.45 | -0.12 | 0.77 | 0.63 |
| | | (0.02) | (0.03) | |
| | 0.84 | -0.09 | 0.74 | 0.66 |
| | | (0.01) | (0.03) | |
| $r(A_t)$ | 0.14 | 2.22 | 0.37 | 0.92 |
| | | (0.03) | (0.01) | |
| | 0.45 | 2.59 | 0.40 | 0.95 |
| | | (0.04) | (0.01) | |
| | 0.84 | 2.49 | 0.48 | 0.98 |
| | | (0.04) | (0.01) | |

Figure 2: Aggregate Investment with Non-convex Adjustment Costs.




This figure shows aggregate investment and the aggregate shock for the baseline model of non-convex adjustment costs. The top panel is for the case of $\rho_A = 0.14$ and the bottom panel is for the case of $\rho_A = 0.84$.

Table 9: Aggregate Capital Regression: Actual Data

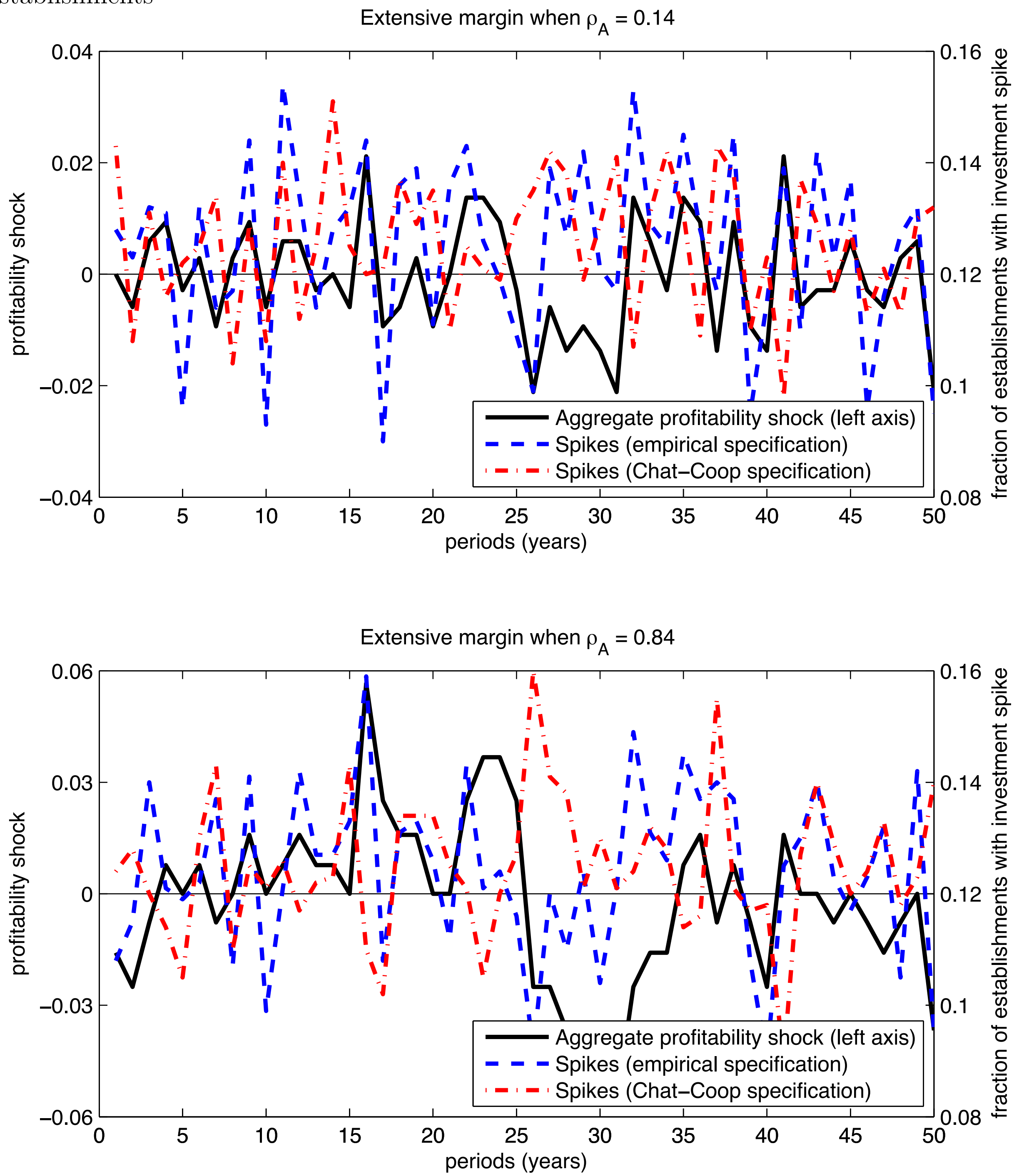
| $\rho_A$ | Constant | $A_t$ | $K_t$ | $R^2$ |
|---|---|---|---|---|
| 0.14 | -0.00 | 0.23 | 0.23 | 0.16 |
| | (0.00) | (0.08) | (0.12) | |
| 0.45 | -0.00 | 0.19 | 0.74 | 0.54 |
| | (0.00) | (0.09) | (0.09) | |
| 0.84 | -0.00 | 0.13 | 0.98 | 0.90 |
| | (0.00) | (0.06) | (0.05) | |

the extensive margin. Under the baseline non-convex adjustment cost case with $\rho_A = 0.14$, the fraction of establishment with investment spikes ($\frac{I}{K} > 0.20$) is positively correlated with productivity in the specification using the data-based stochastic discount factor. In other words, most of the upward movements in the fraction of establishments with spikes are associated with upward movements in productivity. When using the model-based stochastic discount factor, there is a slightly negative correlation. Most of the downward movements in the extensive margin for this case are associated with high levels of productivity. A similar pattern is observed in the lower panel, which displays the same models under a high value of the serial correlation of the productivity shock ($\rho_A = 0.84$). The low points for the model with the data-based stochastic discount factor occur with low levels of productivity, while the low points for the model with the model-based stochastic discount factor occur with high levels of productivity. These figures illustrate the correlations shown in Table 7, where the correlations observed in the data are of the same direction as the model with the data-based stochastic discount factor.

#### 4.2.2 Role of Adjustment Costs

not sure this belongs here. have we already made our point that the stochastic discount factor matters? are we vulnerable as we do not reproduce T or KT in any of our cases? or fine as the AC are different enough. maybe talk about the differences in that specification... Returning to the point emphasized in Thomas (2002), Khan and Thomas (2003) and Khan and Thomas (2008), we can use this

Figure 3: Extensive margin: relationship between productivity and the fraction of adjusting establishments




This figure shows aggregate productivity and the fraction of adjusting firms for the baseline model of non-convex adjustment costs under two different specifications of the stochastic discount factor. The top panel is for the case of $\rho_A = 0.14$ and the bottom panel is for the case of $\rho_A = 0.84$.

model to study the smoothing effects of the data based and model based stochastic discount factors. To do so, we compare the aggregate properties of investment in our baseline model with non-convex adjustment costs with the aggregate behavior of investment in a model with no adjustment costs. From the results of this literature, one would predict that the non-convexity does not matter in the aggregate.

We base this comparison in part on the evolution of the aggregate capital stock. If the non-convexity at the plant-level has no aggregate implications, then the state dependent evolution of the capital stock should be the same as in the model without any adjustment costs. This is consistent with the finding in Thomas (2002), Khan and Thomas (2003) and Khan and Thomas (2008).

For this exercise, we use the model based stochastic discount factor from the model of Chatterjee and Cooper (1993). That model has no adjustment costs. Motivated by the findings in Thomas (2002), Khan and Thomas (2003) and Khan and Thomas (2008), this is the appropriate discount factor to use under the null hypothesis that the non-convex adjustment costs have no aggregate implications.

Our results for the case of no adjustment costs are summarized in Table 10. This is comparable to Table 8, which is based on simulated data from a model with non-convex costs at the plant-level.

Comparing these tables, it is clear that the results are very different for all of the specifications of the stochastic discount factor. This is perhaps not as surprising for the fixed $\beta$ treatment since smoothing by aggregation is not feasible.

Even for the model based specification of the stochastic discount factor, the results with the non-convex adjustment costs are far from those without any adjustment costs. The response of the future capital stock to a variation in current productivity is negative for all values of $\rho_A$ in the case of non-convex adjustment costs and the model based stochastic discount factor. These responses are all positive in the case of no-adjustment costs.

Moreover, the results are quite different for the empirically based stochastic discount factor. There is no evidence here that interest rate movements smooth out the non-convexities at the plant-level so that aggregate capital movements are essentially identical to those in a model without adjustment costs.

The differences in aggregate investment under the different combinations of adjustment costs and stochastic discount factors are further illustrated in Figures 4 and 5. Not surpris-

Table 10: Aggregate capital regression on simulated data: no adjustment costs

| Model | $\rho_A$ | $A_t$ | $K_t$ | $R^2$ |
|---|---|---|---|---|
| $\beta$ | 0.14 | 0.77 | 0.24 | 0.68 |
| | | (0.03) | (0.03) | |
| | 0.45 | 0.35 | 0.84 | 0.73 |
| | | (0.02) | (0.03) | |
| | 0.84 | 0.10 | 1.81 | 0.95 |
| | | (0.01) | (0.03) | |
| $\tilde{\beta}(A_t, A_{t+1})$ (Empirical) | 0.14 | 0.08 | 3.11 | 0.90 |
| | | (0.01) | (0.05) | |
| | 0.45 | 0.19 | 1.38 | 0.83 |
| | | (0.02) | (0.03) | |
| | 0.84 | 0.18 | 1.20 | 0.91 |
| | | (0.02) | (0.03) | |
| $\tilde{\beta}(A_t, A_{t+1})$ (Chat-Coop) | 0.14 | 0.40 | 1.03 | 0.68 |
| | | (0.03) | (0.04) | |
| | 0.45 | 0.60 | 0.37 | 0.66 |
| | | (0.03) | (0.03) | |
| | 0.84 | 0.73 | 0.03 | 0.54 |
| | | (0.03) | (0.02) | |
| $r(A_t)$ | 0.14 | 0.01 | 7.85 | 0.98 |
| | | (0.01) | (0.05) | |
| | 0.45 | 0.01 | 8.42 | 0.99 |
| | | (0.00) | (0.04) | |
| | 0.84 | 0.01 | 6.81 | 1.00 |
| | | (0.00) | (0.03) | |

ingly, aggregate investment is considerably more volatile in the absence of adjustment costs. The role of adjustment costs is to dampen the response of investment to the aggregate shock. These differences are apparent for both values of $\rho_A$ in Figure 4.

Even for the model based version of the stochastic discount factor in Figure 5, these differences remain. There is no basis for the conclusion that these two models are observationally equivalent at the aggregate level.

It is clear that the findings of Thomas (2002), Khan and Thomas (2003) and Khan and Thomas (2008) do not hold in our environment. This is partly due to the use of data based stochastic discount factor and partly due to differences in the specification of adjustment costs.[9]

# 5 Conclusion

This paper studies the implications of interest rate movements on investment. In particular, we highlight how changes in interest rates influences the effects of productivity shocks on investment. We see that the smoothing effects of interest rates depends on the determination of that process. If, as we have emphasized here, the state dependent discount factor is determined from the data, then there is little smoothing of investment due to interest rate movements.

As this work proceeds, we will turn to an analysis of monetary policy which presumably underlies the interest rate process uncovered in the data. We can use our model to see how alternative monetary policies can influence investment behavior. Our results indicate an important channel for monetary policy: influencing the amount of plant-level lumpy investment that is smoothed through interest rate movements. When the lumpiness is not smoothed, the impact of monetary policy can itself be state-dependent.

[9]Relative to our model, Thomas (2002) has a lump-sum non-convex adjustment cost rather than the opportunity cost of investment. Further, that model has no quadratic adjustment costs and no irreversibility. There are no iid shocks to productivity but instead there are plant-specific stochastic adjustment costs. The numerical analysis assumes $\rho_A = 0.9225$. The plant's stochastic discount factor comes from the solution of the stochastic general equilibrium model. Understanding which of these model components is the source of the differences in results remains an open question.

# 6 Appendix: TFP process

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need to say how we got $A_t$ in the first place.. The measure of total factor productivity used in this analysis is constructed as a Solow residual following Stock and Watson (1999). The calculation includes nonfarm real GDP (source: BEA), nonfarm payroll employment (source: BLS), real nonresidential private fixed capital stock (source: BEA and authors' calculations), and a labor share of 0.65. The data sample is annual frequency from 1948 to 2008.

Our analysis is focused on business-cycle dynamics, so here we abstract from long-term growth in TFP by detrending the data. Various approaches have been used in the literature for detrending, so we employ three different detrending specifications to examine the sensitivity of the results. The first approach focuses specifically on business cycle frequencies (between 3 and 8 years). For this case, we detrend using the HP filter with the $\lambda$ parameter set to 7, which closely approximates a band-pass filter on annual data. The second approach is to remove a linear trend from the data, which we approximate by setting the HP filter parameter for $\lambda$ to 100,000. The third approach uses an intermediate value of $\lambda$ that is commonly used to filter annual data, $\lambda = 100$.

The parameters of the TFP process are estimated based on a log-normal AR(1) specification.

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_{A,t}, \quad \epsilon_A \sim N(0, \sigma^2_{\epsilon_A}) \tag{8}$$

Estimates of the shock process parameters are displayed in Table 11 for the three different detrending specifications. The estimate of serial correlation in TFP, $\rho_A$, is very sensitive to the detrending specification. If focusing on business cycle frequencies, $\lambda = 7$, there is little serial correlation in detrended TFP. On the other hand, detrended TFP has a much higher serial correlation when approximating the removal of a simple time trend ($\lambda = 100,000$). We consider the process parameter estimates from all three detrending specifications in our analysis to examine the role of the detrending assumption in modeling the relationship between interest rates and investment decisions.

Table 11: Parameter estimates for Solow-residual technology process

| $\lambda$ | $\rho_A$ | $\sigma_{\epsilon_A}$ |
|---|---|---|
| 7 | 0.14 | 0.012 |
| 100 | 0.45 | 0.015 |
| 100000 | 0.84 | 0.018 |

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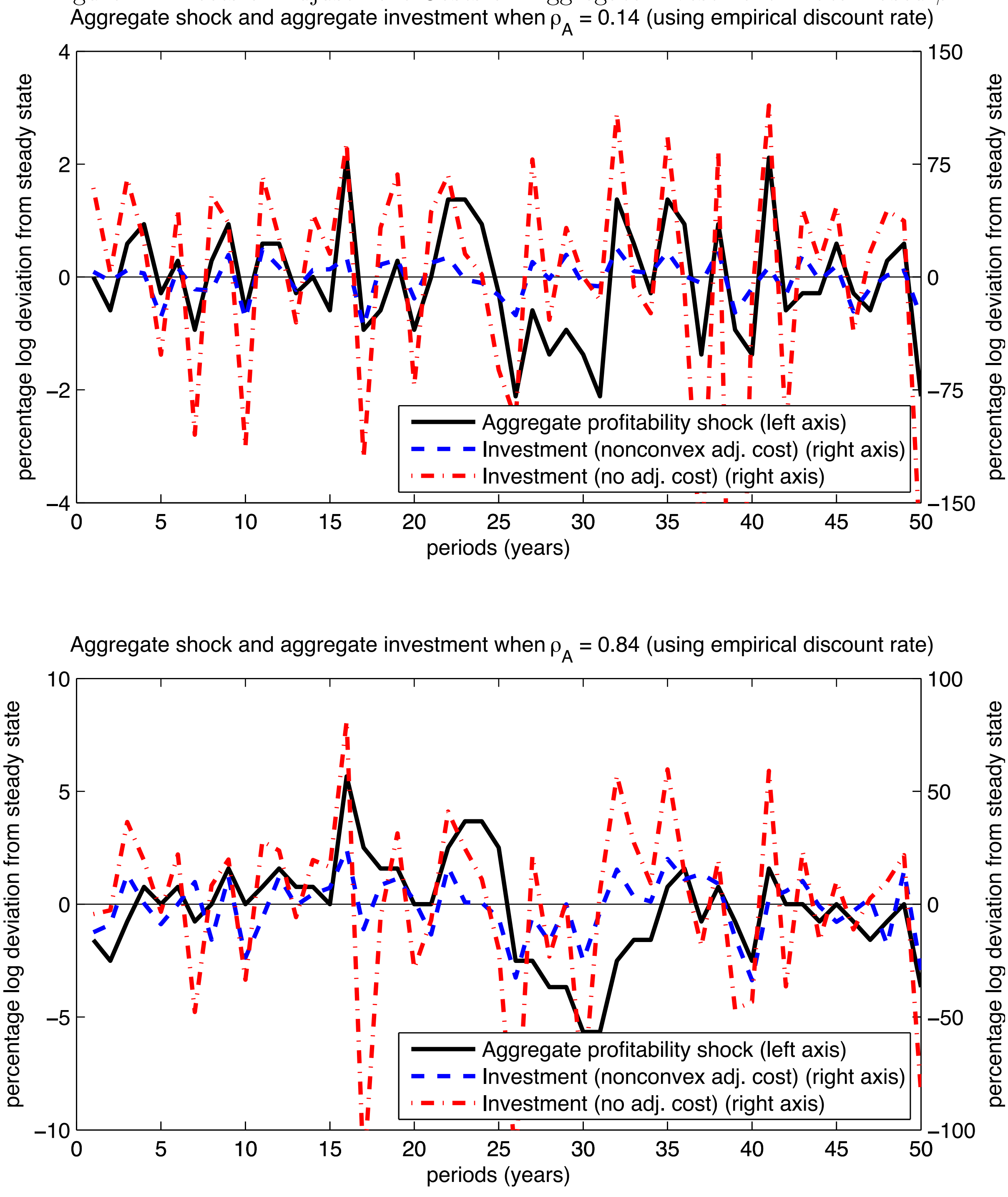
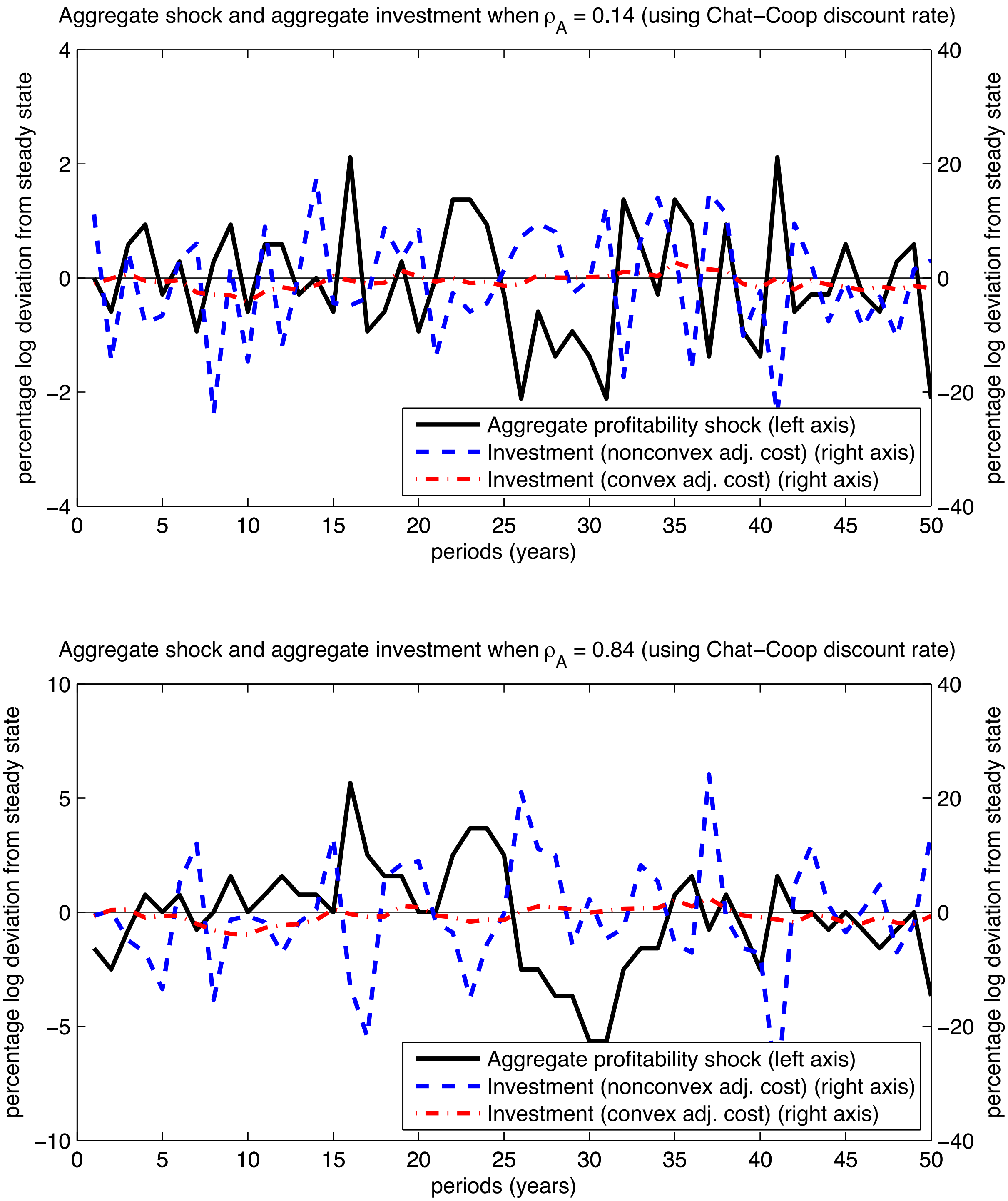
Figure 4: Effects of Adjustment Costs on Aggregate Investment: Data Based $\tilde{\beta}$




This figure shows aggregate investment and the aggregate shock for the baseline model of non-convex adjustment costs and for the case of no adjustment costs. The top panel is for the case of $\rho_A = 0.14$ and the bottom panel is for the case of $\rho_A = 0.84$. In both cases, the stochastic discount factor is based on the data.

Figure 5: Effects of Adjustment Costs on Aggregate Investment: Model Based $\tilde{\beta}$




This figure shows aggregate investment and the aggregate shock for the baseline model of non-convex adjustment costs and for the case of no adjustment costs. The top panel is for the case of $\rho_A = 0.14$ and the bottom panel is for the case of $\rho_A = 0.84$. In both cases, the stochastic discount factor is based on the model.