Housing Dynamics over the Business Cycle*

Finn E. Kydland[†], Peter Rupert[‡§] and Roman Šustek[¶]

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Abstract

Over the U.S. business cycle, fluctuations in residential investment are well known to systematically lead GDP. These dynamics are documented to be specific to the U.S. and Canada. In other economies, residential investment is broadly coincident with GDP. These observations are in sharp contrast with the properties of nearly all business cycle models with disaggregated investment. Including mortgages and interest rate dynamics aligns the theory with U.S. observations on residential investment and with international observations on housing starts. Longer time to build in housing construction then accounts for the coincident behavior of residential investment in other countries.

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[†]University of California–Santa Barbara; kydland@econ.ucsb.edu.

[‡]University of California–Santa Barbara

[§]Corresponding author: Peter Rupert, address: Department of Economics, 2127 North Hall, Santa Barbara, CA 93106, U.S.A., tel: +1 805 893-2258, fax: +1 805 893-8830, e-mail: rupert@econ.ucsb.edu.

[¶]University of Southampton; sustek19@gmail.com.

1 Introduction

Over the U.S. business cycle, fluctuations in residential investment are well known to systematically lead real GDP (e.g., Leamer, 2007). These dynamics, however, are documented here to be specific to the U.S. and Canada—in other developed economies, residential investment is, more or less, coincident with GDP. Nonresidential investment, on the other hand, exhibits exactly the opposite dynamics—in our sample of countries it is either coincident with or lagging GDP, making total investment in all countries coincident with GDP.

Such international evidence is in sharp contrast with the properties of nearly all business cycle models that disaggregate investment into residential and nonresidential. The home production models of Benhabib, Rogerson, and Wright (1991), Greenwood and Hercowitz (1991), and McGrattan, Rogerson, and Wright (1997) predict exactly the opposite pattern: that home investment *lags* the cycle and business investment *leads* the cycle. A business cycle model of Gomme and Rupert (2007), featuring a more detailed disaggregation of investment and investment-specific shocks, also exhibits this anomaly. So does a multi-industry model of housing construction with industry-specific shocks of Davis and Heathcote (2005).[1] In the class of models with home production, Gomme, Kydland, and Rupert (2001) demonstrate that while longer time to build in nonresidential—than residential—construction can reduce the discrepancy between models and data, it is not strong enough to overturn the lead-lag pattern. Fisher (2007) explores the potential role of complementarities between home and business capital. He shows that a traditional home production model can be consistent with the data once home capital has a positive effect on labor productivity in the market sector.[2]

The first objective of this paper is to provide further empirical evidence on the dynamics of

[1]The reason why the models predict the opposite pattern to that in the data is that output produced by business capital has more uses than output produced by home capital: the former can be either consumed or invested in both business and home capital, whereas the latter can only be consumed (e.g, as housing services). Investment in business capital thus allows more future consumption of both types of goods, market and home. This provides a strong incentive to invest in business capital first, in response to shocks that increase market output.

[2]Edge (2000), Li and Chang (2004), and Dressler and Li (2009) construct monetary models with a focus on the responses of the two types of investment to monetary policy shocks, identified by Bernanke and Gertler (1995).

residential and nonresidential investment with the aim of investigating the robustness of the anomaly in U.S. data and uncovering empirical regularities that may guide us in advancing the theory. One important finding is obtained from international data. In a sample of developed economies, the strong lead of residential investment observed in the U.S. is shared only by Canada. Nonetheless, international evidence does not support the lead-lag patterns inherent in business cycle models; other countries in our sample have residential investment, more or less, coincident with GDP. And international data on nonresidential investment, while not having the same cyclical properties as U.S. data, do not support the models either. Nonresidential investment is either lagging GDP (U.S.) or tends to be lagging to coincident with GDP (other countries). These patterns in the data are confirmed by robustness checks based on bootstrapping.

The data are then scrutinized in more detail in order to narrow down the potential sources of the lead-lag patterns in the U.S. and of the deviation from these patterns in other countries. Further analysis of U.S. data reveals that the cyclical lead of residential investment cannot be entirely attributed to Regulation Q and that the lead in residential investment is driven by those structures that rely on mortgage finance, which in the U.S. takes, predominantly, the form of a 30-year fixed-rate mortgage. In addition, the observed dynamics of the nominal mortgage interest rate *suggest* that mortgages are relatively cheap ahead of an economic upturn. Specifically, the mortgage rate is strongly negatively correlated with future output and positively correlated with past output—a pattern observed also for mortgage rates in other countries in our sample, both fixed- and adjustable-rate mortgage countries.[3]

International data on housing starts provide information about the likely reason behind the difference in the dynamics of residential investment in the U.S. and other countries. They

[3]In all countries in our sample, mortgage rates inherit the dynamics of government bond yields of comparable maturities. The lead-lag pattern noted here for mortgage rates has been previously pointed out for U.S. government bond yields by King and Watson (1996) and, more recently, Backus, Routledge, and Zin (2010). As these authors note, generating such lead-lag patters of interest rates endogenously within structural models is an unresolved issue. Henriksen, Kydland, and Šustek (2009) document this pattern also for government bond yields of a number of industrialized economies.

reveal much more uniformity across countries in the dynamics of housing starts than in the dynamics of residential investment. Nearly all countries in our sample exhibit housing starts strongly leading GDP—the same pattern as in the U.S. This finding suggests existence of significant cross-country differences in residential time to build—a period over which expenditures on investment projects are incurred and recorded in national accounts (i.e., a project that takes, for instance, one year to complete, has value put in place in a given quarter recorded in that quarter's accounts). Such a possibility is confirmed by available data on housing completions and existing cross-country comparative studies of the homebuilding industry.

The second objective of the paper is to evaluate the hypothesis suggested by the data within a fairly standard business cycle model with disaggregated investment. Specifically, to investigate (i) if the cyclical variation in the costs of mortgage finance, described below, provides a strong enough mechanism to overturn the standard predictions of the model; and (ii) if time to build in residential investment can quantitatively account for the cross-country differences in the cyclical dynamics of residential investment. To this end, mortgages and residential time to build are introduced into a calibrated home production model studied by Gomme et al. (2001).[4] The exogenous input into the model is an estimated VAR process for total factor productivity, the nominal mortgage interest rate, and the inflation rate. This guarantees that the lead-lag pattern of the mortgage rate (and of the inflation rate) is as in the data. A government closes the model. In a baseline case with one-period residential time to build, the model exhibits lead-lag patterns of residential and nonresidential investment similar to those in U.S. data, while also being in line with standard business cycle moments as much as other models in the literature.

The equilibrium effects of mortgage finance on investment dynamics can be summarized in the form of an endogenous time-varying wedge in the Euler equation for residential capital.

[4]Debt finance is considered only for residential investment. This feature of the model is justified by a well-known finding in the finance literature that in major developed economies, on average, only 16-25% of fixed assets in the nonfinancial corporate sector are financed through debt (Rajan and Zingales, 1995); nonresidential investment is primarily financed through retained earnings. In contrast, residential investment is heavily debt (mortgage) dependent.

The wedge, resembling an ad-valorem tax on residential investment, depends on expected future real mortgage payments over the entire life of a mortgage, discounted by the household's pricing kernel. As such, unlike the observed nominal mortgage rate, it captures the true cost of the mortgage to the household in the model. Its cyclical behavior, nonetheless, confirms the conjecture drawn from the observed mortgage rate dynamics. That is, that mortgages are relatively cheap ahead of a peak in GDP. Mortgage finance, however, does not have only a direct effect on residential investment. Indirectly it also affects nonresidential investment—as households want to keep consumption relatively smooth, large movements in residential investment ahead of an expected future increase in GDP lead to a delay in nonresidential investment. This keeps total investment, and consumption, coincident with GDP, as in the data and standard business cycle models.

While mortgage finance is crucial for producing residential investment leading output, longer time to build in housing construction pushes residential investment towards being coincident with output. This is because total expenditures on a residential project get spread out over a longer period of time. Housing starts, however, still lead output as in the data.

Following a seminal contribution by Iacoviello (2005), a number of authors have studied housing and housing finance in DSGE/business cycle models (a more complete discussion of the literature is provided in Section 3). The models in this class usually abstract from nonresidential capital.[5] However, as is apparent from the properties of the home production models, nonresidential capital has important implications for the cyclical behavior of residential investment. In addition, housing finance in the DSGE literature involves a sequence of one-period loans. Although it makes the models tractable, this form of finance misses important features of mortgage contracts. In particular, their very long repayment periods (up to 30 years), during which the principal is gradually amortized, and constant nominal

[5]The absence of nonresidential capital in these models—one of the few exceptions being, for instance, Iacoviello and Pavan (forthcoming)—is perhaps motivated by a different focus of that literature, being predominantly concerned with the interaction between borrowing constraints, home equity loans, consumption, and monetary policy.

periodic payments (certainly in the case of a fixed-rate mortgage and, in the absence of interest rate shocks, also in the case of an adjustable-rate mortgage). These features turn out to be important for our question; one-period loans fail to generate the sort of dynamics in the costs of housing finance (the wedge) necessary to produce the observed lead-lag pattern of residential investment. We propose a fairly accurate approximation of mortgage contracts capturing the two aforementioned features of mortgages. The approximation has only three state variables and two, easy to calibrate, parameters, replacing a pool of 120 vintages of mortgage contracts. Its parsimonious nature thus provides a simple way of introducing mortgages into DSGE models that other researchers may find useful in addressing a variety of questions. For instance, a fairly brief account of the role of mortgages in aggregate fluctuations and the transmission mechanism of monetary policy, provided by Campbell (2012), reflects the fact that these areas remain underexplored.[6]

The paper proceeds as follows. The next section presents the empirical findings. Section 3 describes the model and the mortgage approximation. Section 4 defines the equilibrium and characterizes the wedge due to mortgage finance. Section 5 calibrates the model and presents quantitative findings for one-period residential time to build. Section 6 then investigates the quantitative importance of the various model features and introduces multi-period residential time to build. This section also discusses the role of risk and refinancing. Section 7 concludes with a summary and suggestions for future research. The paper has three appendixes. Appendix A provides a description of the international data used in Section 2. Appendix B contains some additional derivations related to Section 4 and describes the computation of the equilibrium. Finally, Appendix C contains estimates of exogenous stochastic processes used for computational experiments in Sections 5 and 6.

[6]There is a literature, to a large extent isolated from the business cycle/DSGE literature, which does study mortgages in the macroeconomy. Its focus, however, is primarily on optimal mortgage and housing tenure choice in steady state (e.g., Chambers, Garriga, and Schlagenhauf, 2009). It models mortgages in a lot more detail than we do, incorporating various option-like features, such as refinancing and default. The focus on steady state makes the presence of such features analytically and computationally feasible.

2 Leads and lags in investment data

Our empirical analysis is based on quarterly data for the following countries and periods: Australia (1959.Q3-2006.Q4), Belgium (1980.Q1-2006.Q4), Canada (1961.Q1-2006.Q4), France (1971.Q1-2006.Q4), the U.K. (1965.Q1-2006.Q4), and the U.S. (1958.Q1-2006.Q4). Although the sample is somewhat limited, these are the only countries for which the breakdown of total investment into residential and nonresidential components is available from at least 1980 (we regard a period of about 25 years as the shortest that allows us to talk sensibly about business cycles).[7]

All investment data are measured as chained-type quantity indexes. The reported statistics are for logged data filtered with the Hodrick-Prescott filter; i.e., the statistics are for percentage deviations from 'trend'.[8] The cyclical behavior of a variable x is then conveniently summarized by its correlations with real GDP at various leads and lags; i.e., by corr(x_{t+j}, GDP_t) for $j = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$, where x_{t+j} and GDP_t are percentage deviations from trend. We adopt the following terminology, common in modern business cycle literature: we say that a variable is *leading* the cycle (meaning leading real GDP) if the highest correlation is at $j < 0$, as *lagging* the cycle if the highest correlation is at $j > 0$, and as *coincident* with the cycle if the highest correlation is at $j = 0$.

2.1 Total, residential, and nonresidential investment

To set the stage, we start with correlations for total investment, usually referred to in national accounts as gross fixed capital formation (GFCF), one of the five main expenditure

[7] Concerning other developed economies, the data are available as follows: Austria from 1988.Q1, Denmark from 1990.Q1, Finland from 1990.Q1, Germany from 1991.Q1 (annually from 1970), Ireland from 1997.Q1 (annually from 1970), Italy from 1990.Q1, the Netherlands from 1987.Q1, New Zealand from 1987.Q2, (annually from 1972), Portugal from 1995.Q1, and Spain from 1995.Q1. The data sources are the OECD Main Economic Indicators database, the OECD National Accounts database, and national statistical agencies. The data are also available for Japan from 1980.Q1, Norway from 1978.Q1, and Sweden from 1980.Q1. However, for the available time periods residential investment in these countries does not exhibit 'cyclical' fluctuations. Instead, in each of these countries the data are dominated by one episode: the financial and housing market crises in Norway (1987-1992) and Sweden (1990s) and the late 1980s housing boom and early 1990s bust in Japan.

[8] Similar results are obtained if, instead, the Christiano and Fitzgerald (2003) band-pass filter is used.

components of GDP. The correlations are presented graphically in Figure 1 (the figure caption contains the volatilities of the data). As the figure shows, in all six countries total investment is coincident with GDP. In addition, the volatility of total investment is between 2.5 times to 4 times the volatility of GDP; that is, in the ballpark of the much-cited volatility of U.S. investment, which is about 3 times as volatile as GDP. Such volatilities are also broadly in line with the prediction of a prototypical business cycle model with typical calibration.

Figure 2 displays the cross-correlations for residential and nonresidential structures (volatilities are reported in the figure caption). Residential structures include houses, apartment buildings, and other dwellings, whereas nonresidential structures include office buildings, retails complexes, production plants, etc. Together with equipment and software, residential and nonresidential structures make up GFCF. We will often refer to residential structures as 'residential investment' and to nonresidential structures as 'nonresidential investment'.[9] The well-known empirical regularity that over the U.S. business cycle residential structures lead GDP clearly jumps out of the chart for the U.S. This chart also shows that nonresidential structures have the opposite dynamics, lagging GDP over the business cycle. Such a stark difference in the dynamic properties of residential and nonresidential investment is to a lesser extent observed also in Canada, but in the remaining countries the two types of investment tend to be, more or less, coincident with GDP.

Even though the cross-correlations in Figure 2 are useful descriptive statistics summarizing the dynamic properties of the historical data, it would be useful to have a handle on how robust these empirical regularities are. For example, in the case of Belgium, although not clearly leading (based on our definition), residential structures tend to be more strongly correlated with GDP at leads than at lags and nonresidential structures are in fact lagging GDP a little. In order to assess the significance of the leads and lags in the data, we carry out the following robustness check. Using a block bootstrap method (e.g., Hardle, Horowitz, and Kreiss, 2001), 10,000 artificial data series of the same length as the

[9] In the case of Belgium and France the cross-correlations are for the sum of nonresidential structures and equipment and software as the two series are not available individually.

historical data are drawn for each country. Like the historical data, each artificial series is logged and filtered with the Hodrick-Prescott filter, the cross-correlations are computed, and the $j \in \{-4, ..., 0, ..., 4\}$ at which the highest correlation occurs is recorded. Figure 3 plots the histograms of these occurrences at the different j's.[10] For residential structures, the U.S. and Canada are the only countries for which the highest correlation is at a lead (i.e., at $j < 0$) in at least 95% of the draws, while for nonresidential structures only the U.S. has the highest correlation at a lag (i.e., at $j > 0$) in at least 95% of the draws. Nevertheless, with the exception of Belgium, all countries exhibit residential investment either leading or coincident with GDP; i.e., the highest correlation occurring at $j \leq 0$ in more than 95% of the draws. And, with the exception of the U.K., they exhibit nonresidential investment either lagging or coincident with GDP; i.e., the highest correlation occurring at $j \geq 0$ in more than 95% of the draws. The predictions of business cycle models with disaggregated investment, as reviewed in the Introduction, are thus not supported by the available international data. (Note that even in Belgium residential investment is not lagging, based on the 95% confidence level, and in the U.K. nonresidential is not leading, based on the same confidence level.)

2.2 Housing starts

While the U.S. and Canada look clearly different from the other countries in terms of the cyclical lead of residential structures, there is much more uniformity across the six countries in terms of the dynamics of housing starts.[11] The start of construction is defined across countries consistently as the beginning of excavation for the foundation of a residential

[10] The length of each block in the bootstrap is set equal to 20 quarters, which is sufficient to address the serial correlation in the historical data.

[11] The time periods used for housing starts differ slightly from the time periods used for residential structures due to different data availability. Housing starts are for the following periods: Australia (1965.Q3-2006.Q4), Belgium (1968.Q1-2006.Q4), Canada (1960.Q1-2006.Q4), France (1974.Q1-2006.Q4), and the U.S. (1959.Q1-2006.Q4). For the U.K., residential building permits are used instead of starts as the data on starts are available only from 1990.Q1. Based on a strong comovement between the two data series during the period 1990.Q1-2006.Q4, we take permits as a proxy for starts. For all countries the data come from the OECD MEI database.

building (single family or multifamily) and every month detailed surveys of home builders record the number of such activities.

The top half of Figure 4 plots the cross-correlations with GDP for the historical data (volatilities are in the figure caption). As is immediately apparent, housing starts lead GDP in all countries, possibly with the exception of Belgium. Using a similar robustness check as in the case of structures, the lead occurs in at least 95% of the draws in the cases of Canada, the U.K., and the U.S. And if the significance level is lowered to 90%, then also in the case of Australia and France, as the bottom half of Figure 4 shows.[12] Together with the data on residential investment, the data on housing starts suggest cross-country differences in completion times (time to build) in residential construction. Longer time to build means that investment expenditures on a housing project are recorded, as value put in place, in national accounts over a longer period of time. Residential investment thus may not exhibit a cyclical lead in countries with longer time to build even when housing starts do. Empirical evidence on residential time to build is discussed below.

2.3 Further details on the dynamics of residential structures

Available details on the different types of residential construction in the U.S., and a comparison of the data across time periods, provide an insight into the potential sources of the cyclical lead of U.S. residential investment. A comparison of some of the details with available evidence from other countries also provides information about the sources of the cross-country differences documented above. We first discuss the relevant characteristics of the different types of residential structures and time periods and then present the findings.

2.3.1 Single family vs multifamily structures

Most of residential construction in the U.S. consists of single family structures (houses). Their share in residential investment is five times as large as the share of multifamily struc-

[12] In the case of Belgium, even though starts do not lead, residential building permits lead by three quarters. In the other countries, building permits and starts exhibit essentially the same lead.

tures (mainly apartment buildings). Whereas new houses are primarily built for owner occupancy, most apartment buildings are built to rent (historical data from Census Bureau's Survey of Construction).[13] For our purposes, the main differences between the two types of structures are two-fold. First, time to build is longer for multifamily than for single family structures. Based on historical data from the Survey of Construction, the average period from start to completion for a typical single family structure is 6.2 months (5.6 months if only built-for-sale houses, as opposed to custom-built houses, are counted). For multifamily structures the average construction time is 10 months for all structure types and 13 months for 20+ unit structures, which make up the majority of multifamily construction.

Second, ownership of a house is financed differently from ownership of a multifamily structure.[14] House purchase finance is relatively simple and standardized. Based on historical data from the Survey of Construction, on average 76% of new houses are financed through a 30-year conventional mortgage (this includes also subprime and Alt-A mortgages not reported separately), 18% through FHA/VA insured mortgages, and 6% are paid for with cash. And the average loan-to-value ratio of conventional mortgages for newly-built homes has been relatively stable at 76% (Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10).[15] Debt thus plays a major role in financing newly-built house purchases and its importance has been relatively stable over time. In contrast, financing acquisitions of new multifamily structures is more involved, heterogenous, and, as discussed below, has changed dramatically over time.

2.3.2 Structural changes in housing finance in the 1980s

There are two reasons for splitting the U.S. sample period 1959.Q1-2006.Q4 into two subperiods in 1984. First, it is often argued that Regulation Q was responsible for residential

[13]Most of the historical data from the Survey of Construction used in this section are from either early 1960s or early 1970s to 2006.

[14]Construction, as opposed to the ultimate ownership, is in both cases typically financed by a short-term construction loan obtained by a home builder or a developer from a bank.

[15]The data on loan-to-value ratios exclude subprime and Alt-A mortgages. Their importance in the aggregate has been, however, isolated only to the last three years of our sample.

construction booms and busts in the U.S. before the 1980s, causing boom and bust cycles in the wider economy (e.g., Bernanke, 2007). This regulation set ceilings on interest rates that savings banks and savings and loans—the main mortgage lenders at the time—were allowed to pay on deposits. Regulation Q was eventually abolished in 1980 and largely phased out during the following four years. Second, the method of financing multifamily housing changed dramatically. As discussed by Bradley, Nothaft, and Freund (1998) and Colton and Collignon (2001), up until mid- to late 1980s limited partnerships, financing apartment housing through mortgages, have been the dominant form of apartment ownership in the U.S. Since then, however, they have been replaced by equity real estate investment trusts (REITs). As a result there has been substantial substitution of equity for debt as a means of financing apartment housing.[16]

2.3.3 Findings

The first two panels of Table 1 report the cross-correlations with GDP, as well as volatilities, for key data related to single family and multifamily housing investment in the U.S. The first panel is for the period 1958.Q1-1983.Q4, while the second panel is for the period 1984.Q1-2006.Q4. The first two rows in each panel are for the single family and multifamily components of residential investment in national accounts, followed by starts and completions. These 'construction data' are then complemented with 'financing data'. Namely, the net change in real mortgage debt outstanding obtained from the Flow of Funds Accounts, Table F.217.[17]

From the first panel of Table 1 we see that single family structures clearly lead GDP in the first period (1958.Q1-1983.Q4), with the highest correlation coefficient of 0.73 at $j = -2$.

[16]Significant changes occurred also in the market for single family housing finance. These changes, however, occurred on the side of mortgage lenders—deregulation of the primary mortgage market and development of a liquid secondary mortgage market through securitization (see, e.g., Green and Wachter, 2005). Mortgage debt, nevertheless, remained the main source of finance from the ultimate owner's perspective.

[17]Flow of Funds tables report *home* mortgages, defined as mortgages for 1-4 family properties, and *multifamily* mortgages, defined as mortgages for 5+ family properties. The fraction of new construction accounted for by 2-4 family properties is, however, negligible (completions data from the Survey of Construction). Home mortgages are thus a good proxy for single family property mortgages.

Multifamily structures are, in contrast, coincident with GDP, with the highest correlation coefficient of 0.51 at $j = 0$. In terms of starts, however, both types of structures lead GDP, with both having the highest correlation coefficient at $j = -2$ (0.70 and 0.61, respectively). The reason why multifamily structure investment from the national accounts is coincident with the cycle is a longer time to build. As noted above, it takes about four quarters to complete most multifamily housing construction, compared with just two quarters (at the most) for single family houses. This is reflected in the dynamics of completions: while completions of single family structures peak at $j = -1$, one quarter after the peak of starts, completions of multifamily structures peak at $j = 2$, four quarters after the peak of starts. Notice also that both single family and multifamily mortgages lead GDP, with the highest correlation coefficients of 0.69 and 0.46, respectively, at $j = -2$, the same as that for starts.

There are two key observations concerning the second period (1984.Q1-2006.Q4). First, investment in single family structures still leads GDP, even though the cross-correlations at all leads and lags are weaker than in the first period. Starts, completions, and single family mortgages have also similar dynamics to those in the first period, even though again the correlations are weaker.[18] Thus, although Regulation Q likely played a role in the cyclical dynamics of residential investment in the first period, perhaps accounting for the stronger correlations with GDP, it cannot be the only reason for why movements in residential investment precede movements in GDP. An additional argument against Regulation Q being the main source of such dynamics is that a clear lead in residential investment is observed also in Canadian data, especially for single family structures (the third panel of Table 1). Unlike U.S. mortgage lenders, Canadian banks did not face constraints such as those imposed by Regulation Q (Lessard, 1975).

Second, multifamily residential investment in the second period behaves like nonresidential investment in the sense that it lags GDP; starts are coincident with GDP and completions

[18]The mortgage data are especially substantially less correlated with GDP at all leads and lags than in the first period. In addition, they are much more volatile. This is even after home equity loans (broadly available from 1991) have been stripped out of the data. A likely explanation for the low correlations and the high volatility is refinancing, which became much more accessible during the 1980s.

lag GDP by three quarters.[19] Interestingly, this is despite the fact that mortgages for multifamily housing still lead GDP, even though, like in the case of single family housing, they are much more volatile and the correlations are weaker than in the first period. Such decoupling between mortgage finance and construction in the multifamily sector is consistent with the increased role of equity finance in multifamily housing noted above.

We close this subsection by following up on our earlier discussion regarding cross-country differences in completion times. The bottom panel of Table 1 reports the dynamics of starts and completions in the U.K (the only other country for which completions data are available; unlike in the U.S., direct measurement of completion times is not available). As we can see, U.K. completions tend to peak three to four quarters after starts, an indication of possibly twice as long time to build in the U.K. than in the U.S. (single family homes).[20]

To sum up, we draw the following lessons from the details of the data: (i) the cyclical lead of U.S. residential investment cannot be entirely attributed to Regulation Q; (ii) the lead is driven by those structures that rely on mortgage finance; and (iii) there are significant differences in residential time to build across countries, perhaps due to technological, supply chain, or regulatory constraints, or a different composition of residential investment in terms of single- and multifamily structures. Ball (2003) provides an overview of the structure and practices of housebuilding industries in different countries, pointing out large variations across countries in all these respects.

2.4 Dynamics of mortgage rates

The last piece of empirical observation we report concerns the cyclical dynamics of the mortgage rate—the nominal interest rate on mortgage loans. Even though by itself it does

[19]The generally weaker cross-correlations of multifamily structures with GDP in the second period are likely due to shocks specific to that market segment that occurred in the early and mid- 1980s. As discussed by Colton and Collignon (2001), changes in the U.S. tax code in 1981 (the Economic Recovery Tax Act) provided strong incentives for apartment construction. Most of these incentives were, however, eliminated by the 1986 Tax Reform Act.

[20]Completions data for the U.K. come from the Department of Communities and Local Government, Housing Statistics, Table 222.

not reflect the true costs of mortgage finance to consumers—which, as demonstrated in the next section, depend on the present value of real mortgage payments (interest and amortization) over the lifetime of the mortgage—the mortgage rate is a key factor affecting the costs and may therefore indicate how the costs behave over the business cycle. According to a number of studies (e.g., Scanlon and Whitehead, 2004; Calza, Monacelli, and Stracca, forthcoming), countries can be generally described as either FRM or ARM countries. For each country we therefore use the interest rate for the country's most common mortgage product, as documented in the above studies.

The cross-correlations of mortgage rates with GDP (and their volatilities) are reported in the first panel of Table 2, which reveals a common pattern across countries: mortgage rates are generally negatively correlated with future GDP and positively correlated with past GDP. Thus, on average, mortgage rates are relatively low before a GDP peak, tend to increase as GDP increases, and reach their peak a few quarters after a peak in GDP. The second panel, which reports the same statistics for government bond yields, shows that the cyclical dynamics of mortgage rates reflect the general behavior of nominal interest rates over the business cycle, rather than factors specific to the mortgage market (for FRM countries we take par yields on coupon government bonds of maturities close to the periods for which FRM mortgage rates are fixed; for ARM countries we take 3-month Treasury bill yields, as mortgage rates on ARMs are set, after some initial period, as a constant margin over a short-term government bond yield).[21] Because it is real—rather than nominal—mortgage payments that matter for the costs of mortgage finance, the last panel of Table 2 reports the dynamics of inflation rates. It shows that, with the exception of Belgium, the lead-lag pattern of inflation rates is similar to that of nominal interest rates: inflation rates are negatively correlated with future GDP and positively correlated with past GDP.

[21] For future reference we also include for the U.S. the yield on 3-month Treasury bills.

3 A business cycle model with mortgage finance

The findings of the previous section suggest that the cost of mortgage finance may be a key factor behind the observed lead of residential investment in the U.S. business cycle and of housing starts in the cycles of the other countries; longer time-to-build in residential construction in these other countries then determines the comovement between their GDP and residential investment, as recorded in national accounts. To evaluate this hypothesis within a theoretical framework, mortgages are introduced into a business cycle model with disaggregated investment. Specifically, into the model of Gomme et al. (2001), henceforth referred to as GKR, which shares with the other models in the literature the property that residential investment lags and nonresidential investment leads output.

Before getting into details, it is worth mentioning at the outset that we do not model the underlying frictions giving rise to mortgages and to their various features. A mortgage for our purposes is simply a long-term fully-amortizing loan with nominal payments, which is superimposed on purchases of new housing. Modeling demand for mortgages from first principles would make the model too large. And the focus here, in any case, is on the effects of the cyclical variation in the costs of mortgage finance, rather than on the deep reasons why mortgages exist. In this sense, we are taking the same shortcut as business cycle models with cash-in-advance constraints that do not model demand for money from first principles.[22]

3.1 Preferences and technology

A representative household has preferences over consumption of a market-produced good c_{Mt}, a home-produced good c_{Ht}, and leisure, which is given by $1 - h_{Mt} - h_{Ht}$, where h_{Mt} is time spent in market work and h_{Ht} is time spent in home work. The preferences are

[22]Gervais (2002), Rios-Rull and Sanchez-Marcos (2008), and Chambers et al. (2009) develop models with many of the micro-level details we abstract from. Their focus, however, is on steady-state analysis. Campbell and Cocco (2003) model a single household's mortgage choice that includes refinancing. Corbae and Quintin (2011) and Chatterjee and Eyigungor (2011) construct models that allow for foreclosures, focusing on steady-state equilibria. Iacoviello and Pavan (forthcoming) construct a model with some of the features in Gervais (2002) and with aggregate shocks. Housing finance in their model, however, takes the form of a one-period bond.

summarized by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, 1 - h_{Mt} - h_{Ht}\right), \quad \beta \in (0,1), \tag{1}$$

where $u(.,.)$ has all the standard properties and c_t is a composite good, given by a constant-returns-to-scale aggregator $c(c_{Mt}, c_{Ht})$. Time spent in home work is combined with home capital k_{Ht} to produce the home good according to a production function

$$c_{Ht} = A_H G(k_{Ht}, h_{Ht}), \tag{2}$$

where $G(.,.)$ has all the standard properties. In contrast to the home production literature, we abstract from durable goods and equate home capital with residential structures when mapping the model to data. We will therefore refer to home capital as 'residential capital'.[23]

Output of the market-produced good y_t is determined by an aggregate production function

$$y_t = A_{Mt} F(k_{Mt}, h_{Mt}), \tag{3}$$

operated by identical perfectly competitive firms. Here, A_{Mt} is total factor productivity (TFP) and k_{Mt} is market capital, which will be referred to as 'nonresidential capital'.[24] Firms rent labor and capital services from households at a wage rate w_t and a capital rental rate r_t, respectively. The market-produced good can be used for consumption, investment in residential capital, x_{Ht}, and investment in nonresidential capital, x_{Mt}.

We start with one-period residential time to build. Residential capital therefore evolves

[23] c_{Ht} is thus consumption of housing services and h_{Ht} is interpreted as time devoted to home maintenance and leisure enjoyed at home, rather than in a bar. Under enough separability in utility and production functions, which will be imposed under calibration, the period utility function can be rewritten such that it is a function of c_{Mt}, h_{Mt}, and k_{Ht} (Greenwood, Rogerson, and Wright, 1995). This makes it comparable with models that put housing directly in the utility function.

[24] Notice that in contrast to A_{Mt}, which is time varying (due to shocks), A_H is constant. GKR show that under enough separability in utility and production functions, which will be imposed under calibration, shocks to A_H do not affect market variables (i.e., time spent in market work, consumption of the market-produced good, and accumulation of the two types of capital). This is convenient as it allows us to abstract from home-production TFP shocks, which cannot be measured outside of the model.

as

$$k_{H,t+1} = (1 - \delta_H)k_{Ht} + x_{Ht}, \tag{4}$$

where $\delta_H \in (0,1)$. As in GKR, nonresidential capital, in contrast, has a J-period time to build, where J is an integer greater than one. Specifically, an investment project started in period t becomes a part of the capital stock only in period $t+J$. However, the project requires value to be put in place throughout the construction process from period t to $t+J-1$. In particular, a fraction $\phi_j \in [0,1]$ of the project must be invested in period $t+J-j$, $j \in \{1, ..., J\}$, where j denotes the number of periods from completion and $\sum_{j=1}^{J} \phi_j = 1$. Let s_{jt} be the size of projects that in period t are j periods from completion. Total nonresidential investment (i.e., investment across all on-going projects) in period t is thus

$$x_{Mt} = \sum_{j=1}^{J} \phi_j s_{jt} \tag{5}$$

and the projects evolve as

$$s_{j-1,t+1} = s_{jt}, \quad j = 2, \dots, J, \tag{6}$$

$$k_{M,t+1} = (1 - \delta_M)k_{Mt} + s_{1t}, \tag{7}$$

where $\delta_M \in (0,1)$.

3.2 Mortgages

So far the setup is exactly the same as in GKR. What makes the current model different is that residential investment is subject to a financing constraint

$$l_t = \theta p_t x_{Ht}, \tag{8}$$

where l_t is the nominal value of mortgage loans extended in period t, $\theta \in [0,1)$ is a loan-to-value ratio, and p_t is the aggregate price level (the price of the market-produced good in dollars). An empirical support for treating θ as a parameter, noted in the previous section, comes from the observation that, over time, there is very little variation in the fraction of *new* single family housing that is financed by mortgages and in the associated average loan-to-value ratio. Note that the constraint (8) serves a different purpose than the housing finance constraint in Iacoviello (2005) and similar models. In our case, housing finance is only used to purchase newly constructed housing. In contrast, in Iacoviello (2005) a fraction of the value of the entire housing stock is used as a collateral for general-purpose borrowing to satisfy preference for early consumption.

Mortgage debt requires that the household makes regular payments throughout the life of the mortgage. The household's budget constraint is thus

$$c_{Mt} + x_{Mt} + x_{Ht} = (1-\tau_r)r_t k_{Mt} + (1-\tau_w)w_t h_{Mt} + \delta_M \tau_r k_{Mt} + \frac{l_t}{p_t} - \frac{m_t}{p_t} + \tau_t, \qquad (9)$$

where τ_r is a tax rate on income from nonresidential capital, τ_w is a tax rate on labor income, m_t are nominal mortgage payments on debt acquired in the past, and τ_t is a lump-sum transfer.[25] Mortgage payments are given as

$$m_t = (R_t + \delta_{Dt})d_t, \qquad (10)$$

where d_t is nominal mortgage debt outstanding, R_t is an *effective* net interest rate on the outstanding mortgage debt, and $\delta_{Dt} \in (0,1)$ is an *effective* amortization rate of the outstanding mortgage debt. Notice that $\delta_{Dt} \in (0,1)$ implies that $m_t > R_t d_t$; i.e., a part of the outstanding debt is amortized each period. The variables d_t, R_t, and δ_{Dt} are state variables evolving recursively according to the laws of motion

$$d_{t+1} = (1-\delta_{Dt})d_t + l_t, \qquad (11)$$

[25] τ_r and τ_w are constant and, as in the rest of the home production literature, are introduced into the model purely for calibration purposes; τ_t is time-varying and its role is to ensure that the economy's resource constraint holds.

$$\delta_{D,t+1} = (1-\nu_t)\delta_{Dt}^{\alpha} + \nu_t\kappa, \qquad \alpha,\kappa \in (0,1), \tag{12}$$

$$R_{t+1} = \begin{cases} (1-\nu_t)R_t + \nu_t i_t & \text{if FRM,} \\ i_t & \text{if ARM.} \end{cases} \tag{13}$$

Here, $\nu_t \equiv l_t/d_{t+1}$ is the share of current loans in the new stock of debt and $(1-\nu_t) \equiv (1-\delta_{Dt})d_t/d_{t+1}$ is the share of outstanding unamortized debt in the new stock of debt. In addition, i_t is the net interest rate (either fixed or adjustable) on current loans and α and κ are parameters controlling the evolution of the amortization rate, which is described in further detail below. Notice that the assumption $\alpha,\kappa \in (0,1)$ implies that $\delta_{Dt} \in (0,1)$ for all t, as assumed above. Notice also that combining equations (10) and (11) gives the evolution of mortgage debt in a more familiar form: $d_{t+1} = (1+R_t)d_t - m_t + l_t$. Given that, as a first approximation, most countries can be characterized as either FRM countries or ARM countries, the household in the model operates only under either FRM or ARM environment.

3.2.1 An example and assessment of the mortgage

It is worth pausing here to explain in a little more detail the laws of motion (11)-(13) and their implications for the time path of mortgage payments, given by equation (10). For this purpose, let us suppose that the representative household has no outstanding mortgage debt and takes a fixed-rate mortgage in period $t=0$ in the amount $l_0 > 0$. Let us further assume that the household does not take any new mortgage loans in subsequent periods (i.e., $l_1 = l_2 = ... = 0$). Equations (10)-(13) then yield the following path of mortgage payments: In period $t=1$, the household's outstanding debt is $d_1 = l_0$, the initial amortization rate at which this debt will be reduced going into the next period is $\delta_{D1} = \kappa$, and the effective interest rate is $R_1 = i_0$. Mortgage payments in $t=1$ are thus $m_1 = (R_1+\delta_{D1})d_1 = (i_0+\kappa)l_0$. In period $t=2$ the outstanding debt is $d_2 = (1-\kappa)l_0$ and is reduced at a rate $\delta_{D2} = \kappa^{\alpha} > \kappa$ going into the next period. The interest rate R_2 is again equal to i_0. Mortgage payments in $t=2$ are thus $m_2 = (R_2+\delta_{D2})d_2 = (i_0+\kappa^{\alpha})(1-\kappa)l_0$ and so on. Notice that whereas the interest part of mortgage payments, $R_t d_t$, declines as debt gets amortized, the amortization part, $\delta_{Dt}d_t$, may increase if, for a given κ, α is sufficiently small. The parameter α thus allows calibration of the model such that m_t is approximately constant for a 'sufficiently long' period, thus approximating the constant mortgage payments during the lifetime of a typical mortgage contract.

Figure 5 provides a numerical example to illustrate these points further and to assess how well the mortgage in the model approximates a real-world contract. Here, one period corresponds to one quarter, $l_0 = \$250,000$, $i_0 = 9.28\%/4$ (the long-run average interest rate for a U.S. 30-year conventional FRM), $\alpha = 0.9946$, and $\kappa = 0.00162$. Panels A and B plot mortgage payments, m_t, and outstanding debt, d_t, respectively, for 120 quarters. Panel C then plots the shares of interest payments, $R_t d_t$, and amortization payments, $\delta_{Dt}d_t$, in mortgage payments, m_t. For comparison, the panels also plot the same variables obtained from a Yahoo mortgage calculator for a U.S. 30-year conventional FRM in the same amount and with the same interest rate. We see that the model captures two key features of the conventional mortgage. First, mortgage payments based on the calculator are constant; in the model they are approximately constant for the first 70 or so periods (17.5 years). Second, interest payments are front-loaded: they make up most of mortgage payments at the beginning of the life of the mortgage and their share gradually declines; the opposite is true for amortization payments.[26] How good is this approximation? By comparing the time paths in panel A, one may conclude that the approximation is poor, as after the 70th period the payments in the model significantly deviate from the payments in the real-world contract. Such conclusion would, however, be misguided. This is because mortgage payments far out are heavily discounted and thus matter little for decisions in period 0. A more suitable metric is therefore to measure the deviations in present value terms (we use $1/i$ as the annual discount factor), normalized by the size of the loan (i.e., $\$250,000$). This metric is plotted in panel D of the figure, which shows that throughout the 120 periods the approximation error is of the order of magnitude of $1e^{-4}$. The sum of these present-value errors is equal to

[26]If α was equal to one, the share of interest payments in m_t would be constant and m_t would decline linearly throughout the lifetime of the mortgage.

about 1% of the size of the loan. For comparison, when all monetary transaction costs of obtaining a real-world mortgage are counted (costs that we abstract from), they usually add up to at least 3% of the amount borrowed.[27]

3.2.2 The general case

So far we have only considered once-and-for-all house purchase. Of course, in response to shocks, the representative household adjusts x_{Ht}, and thus l_t, every period.[28] In this case, δ_{Dt} evolves as a weighted average of the amortization rate on the outstanding stock of debt and the initial amortization rate on new loans, with the weights being the relative sizes of the current stock and flow in the new stock, respectively. Similarly, in the FRM case, R_t evolves as the weighted average of the interest rate on the outstanding stock of debt and the current interest rate charged for new loans. An advantage of the approximation lies in its parsimonious nature. It effectively replaces 120 vintages of mortgage debt, each with a different amortization and (in the FRM case) interest rate, with just three state variables and two parameters. This should make it easy to introduce mortgages into a variety of DSGE models, if the question at hand makes such a model feature desirable.[29]

3.3 Exogenous process and closing the model

The inflation rate $\pi_t \equiv \log p_t - \log p_{t-1}$ and the current mortgage rate i_t follow a joint VAR(n) process with market TFP: $z_{t+1} b(L) = \varepsilon_{t+1}$, where $\varepsilon_{t+1} \sim N(0, \Sigma)$, $z_t = [\log A_{Mt}, i_t, \pi_t]^\top$, $b(L) = I - b_1 L - ... - b_n L^n$ (L being the lag operator), and $\Sigma = BB'$. As mentioned in the Introduction, this is to ensure that the lead-lag pattern of the two nominal variables is

[27] If we were to plot the time paths of m_t and d_t in the model beyond period 120, the picture would show that both indeed converge to zero, making also the approximation error in panel D to converge to zero.

[28] Calibration ensures that the probability of $x_{Ht} < 0$ occurring approaches zero.

[29] Even though many DSGE models include housing and housing finance, they do not have debt contracts resembling mortgages. Instead, housing is financed by a sequence of one-period loans. The interest rate applied to a loan is either the current short-term interest rate (e.g., Iacoviello, 2005, and many others), a weighted average of the current and past interest rates (Rubio, 2011), or evolving in a sticky Calvo-style fashion (Graham and Wright, 2007). Calza et al. (forthcoming) model FRM as a two-period loan, in which a half of the principal and a half of the total interest is paid each period, and ARM as a one-period loan.

as in the data. The model is closed by a government budget constraint. The government collects revenues from capital and labor taxes and operates the mortgage market by providing mortgage loans and collecting mortgage payments. Each period the government balances out its budget by lump-sum transfers to the household: $\tau_t = \tau_r r_t k_{Mt} + \tau_w w_t h_{Mt} - \tau_r \delta_M k_{Mt} + m_t/p_t - l_t/p_t$.

4 Equilibrium effects of mortgage finance

This section defines the equilibrium and shows how the equilibrium effects of mortgage finance can be conveniently summarized by a wedge in an Euler equation for housing investment. Due to space constraints, equilibrium conditions that are not essential for the current discussion are relegated to Appendix B. This appendix also describes the computational method, based on a linear-quadratic approximation of the economy (Hansen and Prescott, 1995), used to compute the equilibrium.

4.1 Equilibrium

The equilibrium is defined as follows: (i) the representative household solves its utility maximization problem, described below, taking all prices and transfers as given; (ii) r_t and w_t are equal to their marginal products; (iii) the government budget constraint is satisfied; and (iv) the exogenous variables follow the VAR(n) process. The aggregate resource constraint, $c_{Mt} + x_{Mt} + x_{Ht} = y_t$, then holds by Walras' Law. To characterize the equilibrium, it is convenient to work with a recursive formulation of the household's problem

$$V\left(s_{1t}, ..., s_{J-1,t}, k_{Mt}, k_{Ht}, d_t, \delta_{Dt}, R_t\right) = \max\{u\left(c_t, 1 - h_{Mt} - h_{Ht}\right)$$

$$+\beta E_t V\left(s_{1,t+1}, ..., s_{J-1,t+1}, k_{M,t+1}, k_{H,t+1}, d_{t+1}, \delta_{D,t+1}, R_{t+1}\right)\},$$

subject to (2) and (4)-(13). After substituting the constraints into the Bellman equation, the maximization is only with respect to (h_{Mt}, h_{Ht}, s_{Jt}, x_{Ht}). Here, x_{Ht} affects the period utility function, through its effect on l_t in the budget constraint, and the value function, through its effect on the laws of motion for $k_{H,t+1}$, d_{t+1}, $\delta_{D,t+1}$, and R_{t+1}.

There is enough separability in this problem that the variables related to mortgage finance (d_t, δ_{Dt}, R_t; i_t, π_t) show up only in the first-order condition for x_{Ht}

$$u_{1t}c_{1t}(1-\theta) - \theta\beta E_t\left[\widetilde{V}_{d,t+1} + \zeta_{Dt}(\kappa - \delta_{Dt}^{\alpha})V_{\delta_D,t+1} + \zeta_{Dt}(i_t - R_t)V_{R,t+1}\right] = \beta E_t V_{kH,t+1}. \quad (14)$$

Here, $\zeta_{Dt} \equiv \left(\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t\right) / \left(\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t + \theta x_{Ht}\right)^2$, $\widetilde{V}_{d,t+1} \equiv p_t V_{d,t+1}$, $\widetilde{d}_t \equiv d_t/p_{t-1}$ and $V_{kH,t}$, V_{dt}, $V_{\delta_D,t}$, and V_{Rt} are the derivatives of the value function with respect to the state variables specified in the subscript.[30] The variables $V_{d,t+1}$ and d_t are transformed in order to ensure their stationarity in the presence of a nonzero steady-state inflation rate. It is convenient to rearrange the first-order condition (14) as

$$u_{1t}c_{1t}(1+\tau_{Ht}) = \beta E_t V_{kH,t+1}, \quad (15)$$

where

$$\tau_{Ht} = -\theta\left\{1 + \frac{\beta E_t \widetilde{V}_{d,t+1}}{u_{1t}c_{1t}} + \frac{\zeta_{Dt}(\kappa - \delta_{Dt}^{\alpha})\beta E_t V_{\delta_D,t+1}}{u_{1t}c_{1t}} + \frac{\zeta_{Dt}(i_t - R_t)\beta E_t V_{R,t+1}}{u_{1t}c_{1t}}\right\} \quad (16)$$

is an endogenous time-varying wedge, discussed below. For $\tau_{Ht} = 0$, equation (15) has a simple interpretation: it equates marginal utility of market consumption today with discounted expected marginal lifetime utility of housing. The wedge acts like an ad-valorem tax, making an additional unit of housing more or less expensive in terms of current market consumption (the wedge can be positive or negative, depending on parameter values and exogenous shocks). In GKR, the mortgage finance constraint (8) is not present. Indeed, if $\theta = 0$, the

[30] We also adopt the convention of denoting, for example, by u_{2t} the first derivative of the u function with respect to its second argument.

wedge is equal to zero and the equilibrium is the same as in GKR. Thus, under $\theta = 0$, the model exhibits the same dynamics as the GKR model: x_{Ht} lags and x_{Mt} leads output. The question is if for $\theta \in (0,1)$ calibrated to the data, and given an estimated VAR(n) process for z_t, the wedge moves in such a way as to overturn this results and reproduce the lead-lag pattern in the data.

4.2 The wedge

The derivatives of the value function appearing in equations (15) and (16) are given by Benveniste-Scheinkman conditions. Here we focus only on $V_{kH,t}$ and $\widetilde{V}_{dt}$. The other derivatives of the value function are not of first-order importance for the current discussion and, due to space constraints, are included in Appendix B. $V_{kH,t}$ satisfies

$$V_{kH,t} = u_{1t}c_{2t}A_H G_{1t} + \beta(1-\delta_H)E_t V_{kH,t+1},$$

which, after recursive substitution, states that marginal lifetime utility of housing is given as the expected discounted sum of per-period marginal utilities of housing over its lifetime. For $\widetilde{V}_{dt}$, the Benveniste-Scheinkman condition states that

$$\widetilde{V}_{dt} = -u_{1t}c_{1t}\left(\frac{R_t+\delta_{Dt}}{1+\pi_t}\right)+\beta\left(\frac{1-\delta_{Dt}}{1+\pi_t}\right)E_t\left[\widetilde{V}_{d,t+1}+\zeta_{xt}(\delta^{\alpha}_{Dt}-\kappa)V_{\delta_D,t+1}+\zeta_{xt}(R_t-i_t)V_{R,t+1}\right], \tag{17}$$

where $\zeta_{xt} \equiv \theta x_{Ht}/\left(\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t+\theta x_{Ht}\right)^2$.

Equation (17) simplifies when either i) new loans are the same as old loans (i.e., $\delta^{\alpha}_t = \kappa$ and $R_t = i_t$) or ii) we consider again a once-and-for-all house purchase, implying that $\zeta_{Dt} = 0$ and $\zeta_{x,t+j} = 0$ for $j = 1, 2, ...$ In these special cases equation (17) becomes

$$\widetilde{V}_{dt} = -u_{1t}c_{1t}\left(\frac{R_t+\delta_{Dt}}{1+\pi_t}\right)+\beta\left(\frac{1-\delta_{Dt}}{1+\pi_t}\right)E_t\widetilde{V}_{d,t+1}, \tag{18}$$

which has a simple interpretation. After recursive substitution, it states that the marginal value of mortgage debt is given as the expected discounted sum of marginal per-period real mortgage payments, weighted by the marginal utility of market consumption, over the lifetime of the mortgage. Notice that if the mortgage was modeled as a one-period loan (i.e., $\delta_{Dt} = 1$), this condition would simplify further to a familiar $\widetilde{V}_{dt} = -u_{1t}c_{1t}(1 + R_t)/(1 + \pi_t)$, where $R_t = i_{t-1}$.

In the special cases (i) and (ii), the expression for the wedge (16) also simplifies

$$\tau_{Ht} = -\theta \left[1 + \beta E_t \widetilde{V}_{d,t+1}/(u_{1t}c_{1t})\right]. \tag{19}$$

Combining this equation with equation (18) provides a clear interpretation of the wedge: the wedge is equal to $-\theta$ times the difference (as $\widetilde{V}_{d,t+1}$ is negative) between the 'out-of-pocket' cost of financing an additional unit of housing, which is one unit of foregone market consumption today, and the debt cost of doing so, which is the present value of foregone market consumption in the future. Other things being equal, when the debt cost declines (i.e., $\widetilde{V}_{d,t+1}$ declines in absolute value), the wedge declines, encouraging more residential investment. In this sense, the wedge reflects the cost of mortgage finance to the household in the model.[31]

Of course, the household in the model chooses x_{Ht} every period, in response to shocks, and $\delta_{D,t+1}$ and R_{t+1} are the effective amortization and interest rates, respectively, on the entire stock of debt d_{t+1}, not just on the new (marginal) debt l_t. In this case, therefore, the simplified expressions (18) and (19) are insufficient to characterize the impact of x_{Ht} on the marginal mortgage payments because these conditions incorrectly state that the marginal effect on the periodic mortgage payments of the new mortgage debt l_t is $R_{t+1} + \delta_{D,t+1}$, the

[31]What makes the effects of the cost of mortgage finance nontrivial is an implicit assumption that the household is not pricing mortgages, and bonds generally. Any model that is to have nontrivial effects of mortgage finance on housing decisions has to rely on such an assumption. For instance, Koijen, Van Hemert, and Van Nieuwerburgh (2009) model the mortgage rate as following a process determined by an exogenous, reduced-form, term structure model; Chambers et al. (2009) assume market incompleteness resulting in the steady-state mortgage rate deviating from the pricing kernel of the households taking mortgages. In our model, the supply side of mortgage finance is subsumed in the exogenous VAR process and the government.

sum of the effective interest and amortization rates on the entire stock of outstanding debt next period. The terms $\zeta_{Dt}(\kappa - \delta^{\alpha}_{Dt})V_{\delta_D,t+1}$ and $\zeta_{Dt}(i_t - R_t)V_{R,t+1}$ in the general expression for the wedge (16), and the terms $\zeta_{xt}(\delta^{\alpha}_{Dt} - \kappa)V_{\delta_D,t+1}$ and $\zeta_{xt}(R_t - i_t)V_{R,t+1}$ in the general Benveniste-Scheinkman condition (17) take into account the effect of x_{Ht} on $\delta_{D,t+1}$ and R_{t+1}. The term $\zeta_{Dt}(\kappa-\delta^{\alpha}_{Dt})V_{\delta_D,t+1}$ in equation (16), for instance, reflects the fact that the new debt will be amortized next period at a lower rate than the current outstanding debt ($\kappa < \delta^{\alpha}_{Dt}$).

5 Results for one-period residential time to build

This section presents quantitative findings for the baseline experiment with one-period residential time to build. After describing the calibration, the results are reported, with much of the explanation of how the various model features affect the results left for the next section.

5.1 Calibration

The calibration is based on U.S. data and the parameter values are summarized in Table 3. One period in the model corresponds to one quarter and the functional forms are as in GKR: $u(.,.) = \omega \log c + (1-\omega)\log(1 - h_M - h_H)$; $c(.,.) = c_M^{\psi} c_H^{1-\psi}$; $G(.,.) = k_H^{\eta} h_H^{1-\eta}$; and $F(.,.) = k_M^{\varrho} h_M^{1-\varrho}$. The parameter A_H is normalized to be equal to one and the value of A_{Mt} in a nonstochastic steady state is chosen so that y_t in the nonstochastic steady state is equal to one.

As mentioned above, we abstract from consumer durable goods. In addition, housing services are modeled explicitly in the home sector. The data equivalent to y_t is thus GDP less expenditures on consumer durable goods and gross value added of housing. Nonresidential capital in the model is mapped into the sum of nonresidential structures and equipment & software (equipment & software is, more or less, coincident with GDP, although it is more strongly positively correlated with GDP at lags than at leads). If only nonresidential structures were used as the data equivalent to k_{Mt}, the share of capital income in GDP, ϱ, would be too low, making the model's dynamic properties difficult to compare with the literature. Because k_{Mt} includes equipment & software, J is set equal to 4 and ϕ_j is set equal to 0.25 for all j. These are the same choices as those of GKR. The parameter ϱ is set equal to 0.283, based on a measurement from the National Income and Product Accounts (NIPA) obtained by Gomme, Ravikumar, and Rupert (2011). Their NIPA-based estimate of $\tau_w = 0.243$ is also used. The depreciation rates are given as the average ratios of investment to the corresponding capital stocks. This yields $\delta_H = 0.0115$ and $\delta_M = 0.0248$. These are a little higher than the average depreciation rates from BEA Fixed Assets Accounts because the model abstracts from long-run population and TFP growth.

The parameter θ is set equal to 0.76, the average loan-to-value ratio for conventional single family newly-built home mortgages (Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10, 1963-2006). The values of the steady-state mortgage rate i and of the parameters α and κ are the same as those in Section 3.2.1: $i = 9.28\%$ per annum, $\alpha = 0.9946$, and $\kappa = 0.00162$. The steady-state inflation rate is set equal to 4.54% per annum, the average inflation rate for 1971-2006, which is the period for which the mortgage rate data are available. Given these values, the law of motion (12) implies a (quarterly) steady-state amortization rate of 0.0144, which, as in the U.S. economy, is higher than the depreciation rate for residential structures. The law of motion for debt (11) then implies a steady-state debt-to-GDP ratio of 1.64, which is a little lower than the average ratio (1958-2006) of home mortgages to GDP, which is 1.71 (for GDP less consumer durable goods and gross value added of housing).

The discount factor β, the share of consumption in utility ω, the share of market good in consumption ψ, the share of capital in home production η, and the tax rate on income from nonresidential capital τ_r are calibrated jointly. Namely, by matching the average values of h_M, h_H, k_M/y, k_H/y, and the after-tax real rate of return on nonresidential capital, using the steady-state versions of the first-order conditions for h_M, h_H, s_J, and x_H (see Appendix B), and the model's after-tax real rate of return on nonresidential capital, $(1 - \tau_r)(A_M F_1 - \delta_M)$, evaluated at the steady state. According to the American Time-Use Survey (2003), individuals aged 16 and over spent on average 25.5% of their available time working in the market and 24% in home production. We assume that half of home hours correspond to our notion of h_H. The average capital-to-GDP ratios are 4.88 for nonresidential capital and 4.79 for residential capital (in both cases consumer durable goods and gross value added of housing are subtracted from GDP). The average (annual) after-tax real rate of return on nonresidential capital is measured by (Gomme et al., 2011) to be 5.16%. These five targets yield $\beta = 0.988$, $\omega = 0.47$, $\psi = 0.69$, $\eta = 0.30$, and $\tau_r = 0.61$. As is common in models with disaggregated capital, the tax rate is higher than the statutory tax rate or a tax rate obtained from NIPA.

The parameterization of the exogenous stochastic process is based on point estimates of the parameters of a VAR(3) process for TFP, the mortgage rate for a 30-year conventional FRM, and the inflation rate, obtained for the post-reform period 1984.Q1-2006.Q4 (see Appendix C for details). By construction, this process implies that the mortgage and inflation rates are negatively correlated with future TFP and positively correlated with past TFP, producing cross-correlations with GDP similar to those in Table 2.

The economy's resource constraint is $c_t + x_{Mt} + x_{Ht} = y_t$. This implies constant unitary rates of transformation between the three uses of output, which makes the two types of investment extremely sensitive to the VAR shocks. To address this issue, we adopt the notion of *intratemporal* adjustment costs of Huffman and Wynne (1999), which make the production possibilities frontier concave. Specifically, $c_t + x_{Mt} + q_t x_{Ht} = y_t$, where $q_t = \exp(\sigma(x_{tH} - x_H))$, with $\sigma > 0$ and x_H being the steady-state residential investment.[32] Increasing x_{Ht} above x_H makes housing investment increasingly costly in terms of foregone c_t or x_{Mt}. This reflects the costs of changing the composition of the economy's production (Huffman and Wynne, 1999), as well as constraints on available residential land in a given period, on which an increasing stock of housing is placed (Davis and Heathcote, 2007, document that available residential land grows at an approximately constant rate). The curvature parameter σ is then

[32]Of course, x_{Ht} is then multiplied by q_t through out the model. The household takes q_t as given; i.e., q_t depends on the aggregate x_{Ht}.

chosen by matching the ratio of the standard deviations (for HP-filtered data) of residential investment (single family structures) and GDP, which, for the period 1984.Q1-2006.Q4, is 8.4. This yields $\sigma = 6.4$.

5.2 Cyclical behavior of the model economy

Table 4 reports the cyclical behavior of the model economy for the above calibration. It reports the standard deviations (relative to that of y_t) of the key endogenous variables and their cross-correlations with y_t at various leads and lags. The first thing to notice is that the introduction of mortgage finance into the model does not significantly affect the behavior of the basic variables, y_t, c_{Mt}, x_t, and h_{Mt}. These variables behave pretty much like in other business cycle models: market consumption is roughly 50% as volatile as output, total investment is about four and a half times as volatile as output, and market hours are roughly 60% as volatile as output; in addition, all three variables are strongly positively correlated with output contemporaneously, without any leads or lags.

Second, unlike in other models, residential and nonresidential investment exhibit dynamics similar to those in U.S. data. As in the data, x_{Ht} is more volatile than x_{Mt} and leads output; x_{Mt}, in contrast, although not strictly speaking lagging, is substantially more strongly correlated with output at lags than at leads. The reason why residential investment leads output in the model can be understood from the behavior of the wedge. As discussed in the previous section, the wedge captures the relative cost of mortgage finance to the household in the model. As Table 2 shows, mortgage rates lead output negatively and lag positively. The wedge turns out to inherit this dynamics, but is an order of magnitude more volatile than the mortgage rates. This induces more residential investment ahead of an increase in GDP. Interestingly, while the wedge generates a lot of action outside of the steady state, our calibration implies that in steady state its value is close to zero ($\tau_H = -0.0117$), producing essentially the same steady state as that in GKR.

6 The role of mortgages and residential time to build

In order to gain further understanding of the results, this section disentangles the quantitative effects of the various model features on the lead-lag patterns of the investment variables. In addition, it introduces multiperiod residential time to build. The results are reported in Table 5, where, for the ease of comparison, the first panel repeats the results for the benchmark economy of the previous section. The section closes with a discussion of the likely role of risk and refinancing.

6.1 Mortgages

We start by removing mortgage finance from the model ($\theta = 0$). The exogenous VAR process, however, stays the same. This guarantees that the underlying probability space of the current economy is the same as that of the benchmark economy and, thus, that the two economies differ only in terms of the value of θ. In this case, even though there is no mortgage finance under $\theta = 0$, households care about the mortgage and inflation rates as these variables help forecast future TFP. In particular, a low mortgage rate forecasts high TFP. The second panel of Table 5 shows that for $\theta = 0$ the lead-lag patterns disappear: both x_{Ht} and x_{Mt} become coincident with output, with very strong contemporaneous correlations; in addition, x_{Ht} becomes substantially less volatile than x_{Mt} and then in the benchmark. Even though the behavior of its components changes, the behavior of total investment, x_t, stays, more or less, the same. In fact, the dynamics of x_t stay broadly unchanged across all our experiments. This is because consumption smoothing constrains the response of total investment to shocks. For this reason, x_{Ht} and x_{Mt} can both be coincident with output only if at least one of the two becomes substantially less volatile than in the benchmark. A corollary of this result is that x_{Mt} has to lag output in the benchmark, as x_{Ht} leads output with high volatility. The results of this experiment also mean that, by themselves, expectations of higher future TFP, and thus higher future output and income, are not sufficient to produce residential investment leading the business cycle.

Next, consider again the case of no mortgage finance ($\theta = 0$), but, in addition, assume a linear production possibilities frontier ($\sigma = 0$). This makes changes in the output mix less costly than in the previous case and the benchmark. This economy is essentially the GKR economy (subject to small differences in calibration and the VAR process). In this case, as the third panel shows, both x_{Ht} and x_{Mt} become more volatile than in the previous case and the 'inverted' lead-lag pattern present in most existing models re-appears. As GKR show, this inverted lead-lag pattern would be even stronger if there was no time to build in nonresidential capital.

Panel 4 considers the case of a one-period loan ($\delta_{Dt} = 1\ \forall t$), which results when $\alpha = 0$ and $\kappa = 1$. In this case the model behaves as if $\theta = 0$. This is because the wedge—driven by $E_t[(1+i_t)/(1+\pi_{t+1})]$, the one-period ex-ante real interest rate implied by the VAR process—is too smooth and too little correlated with output to significantly, and systematically, affect the dynamics of the two types of investment.

Next, consider FRM vs ARM. Under ARM, the mortgage rate is reset every period, as specified in equation (13). The two ARM rates in Table 2 (Australia and the U.K.) exhibit the typical lead-lag pattern of other nominal interest rates in the table, especially short-term interest rates, including the yield on U.S. 3-month Treasury bills. We therefore continue with U.S. calibration and take the yield on 3-month U.S. T-bills as the data equivalent to i_t in the model under ARM. It would, however, be incorrect to replace the estimated VAR for the 30-year mortgage rate with an estimated VAR that contains the 3-month T-bill yield instead. This would change the underlying probability space and would invalidate the exercise as ceteris-paribus. Therefore, in order to keep the probability space constant when comparing the model's behavior under FRM and ARM, a four-variable VAR for $z_t = [\log A_{Mt}, i_t^{FRM}, \pi_t, i_t^{ARM}]$ is estimated, with the point estimates reported in Appendix C. Under FRM, households care about i^{ARM} to the extent that it helps forecast the other three exogenous variables; i^{FRM} plays a similar role under ARM.

Panels 5 and 6 of Table 5 contain the results for the four-variable VAR for FRM and

ARM, respectively. Under FRM, the lead-lag pattern of x_{Ht} becomes even more pronounced than in the benchmark. This improvement comes from the fact that the four-variable VAR captures the joint dynamics of $\log A_{Mt}$, i_t^{FRM}, and π_t somewhat better than the three-variable VAR. Under ARM, x_{Ht} is less volatile than under FRM and leads output by a long way. In the table this shows up as positive correlations at $j = -4$ and $j = -3$, but the correlations peak at $j = -6$, not shown in the table for space constraints. The strong lead of x_{Ht} can be understood from the behavior of the wedge, in conjunction with the behavior of the 3-month T-bill rate in Table 2. As Table 2 shows, the T-bill rate has similar dynamics as the 30-year mortgage rate in the sense that it is negatively correlated with future output and positively correlated with past output. But because, under ARM, future mortgage payments depend on future short rates, expectations of sharp interest rate increases accompanying future GDP growth make the wedge start to increase even when the current T-bill rate is still relatively low. This is why the wedge becomes much less negatively correlated with output (or even positively correlated) at $j's$ at which the T-bill rate is still strongly negatively correlated.[33] Compared with the data, while not necessarily inconsistent with the long lead of U.K. housing starts (Figure 4), the lead of x_{Ht} in the model does appear rather extreme. A potential resolution of this issues is discussed below.

6.2 Multiperiod residential time to build

So far, residential construction was assumed to have the standard one-period time to build. When residential construction takes more than one period, a distinction needs to be made between finished and unfinished houses, and between the value of finished houses and ongoing residential construction. Unfinished houses are treated here in a similar way as unfinished

[33]The different behavior of the wedge under ARM and FRM has two sources. First, even under expectations hypothesis an expected future path of short-term interest rates affects real mortgage payments differently under ARM than under FRM (expectations hypothesis only implies indifference between a given path of the short rate and a yield on a *zero-coupon* bond). The second source are cyclical variations of term premia, which generate a deviation of the 30-year mortgage rate from the rate implied by the expectation hypothesis. Decomposing these effects requires an estimated affine term structure model, which is beyond the scope of this paper.

nonresidential investment projects in the sense that the household makes out-of-pocket investments in residential projects. Upon completion it sells finished houses at a market price, which is denoted by q_t^*. The household also buys finished houses for its own use (think of the household as a homebuilder who likes houses of other makes than its own). Let n_t^* denote the number of newly constructed houses, occupiable next period, the household wants to purchase for its own use and let n_{1t} denote the number of houses, built by the household, and occupiable next period. With these modifications, the budget constraint becomes

$$c_{Mt}+x_{Mt}+q_t x_{Ht}+q_t^* n_t^* = (1-\tau_r)r_t k_{Mt}+\tau_r \delta_M k_{Mt}+(1-\tau_w)w_t h_{Mt}+q_t^* n_{1t}+l_t/p_t-m_t/p_t+\tau_t,$$

where $l_t = \theta p_t q_t^* n_t^*$ and $x_{Ht} = \sum_{\iota=1}^{N} \mu_\iota n_{\iota t}$, with $n_{\iota t}$ denoting residential projects ι periods from completion and $\sum_{\iota=1}^{N} \mu_\iota = 1$. The stock of houses for the household's own use evolves as $k_{H,t+1} = (1-\delta_H)k_{Ht} + n_t^*$ and the on-going residential projects evolve as $n_{\iota-1,t+1} = n_{\iota t}$, for $\iota = 2, ..., N$. In equilibrium, $n_t^* = n_{1t}$. The economy's resource constraint is the same as before, $c_{Mt} + x_{Mt} + q_t x_{Ht} = y_t$, except that $x_{Ht} = \sum_{\iota=1}^{N} \mu_\iota n_{\iota t}$. The variables $n_{1t}, ..., n_{N-1,t}$ become a part of the vector of state variables.

The bottom panel of Table 5 reports the results. Based on our discussion in Section 2.3, N is set equal to 4. The μ's and ϕ's are treated symmetrically, setting $\mu_\iota = 0.25\ \forall \iota$. In addition to the usual variables, x_t, x_{Ht}, x_{Mt}, and τ_{Ht}, the table also reports results for housing starts n_{4t} (structures started in period t and four periods away from completion) and completions n_{0t} (structures that in period t became a part of the usable housing stock h_t). As the table shows, x_{Ht} now reaches the highest correlation at $j = 0$, while starts lead by two quarters and completions lag by two quarters. These patterns are similar to those in U.S. data for multifamily structures in the first subperiod, when multifamily housing still relied heavily on mortgage finance. This result also provides an explanation for why residential investment is coincident with output in many countries in our sample, despite the fact that housing starts lead output as in the U.S.

6.3 Discussion: risk and refinancing

As discussed above, under ARM the lead of residential investment in the model is rather extreme. In purely mechanical terms, aligning residential investment more closely with GDP requires that the coefficient on the mortgage rate in the optimal decision rule for residential investment is reduced (in absolute value) and the coefficient on TFP is increased. Financial advisors often make the point that ARM is more risky than FRM because of its interest rate variability. To the extent that this is true, risk-averse households should respond less to the mortgage rate under ARM, in order to keep future consumption smooth across the states of the world. This is what the so-called risk-sensitive preferences, a special case of Epstein and Zin (1989) preferences, imply; see, for instance, Backus, Routledge, and Zin (2004). In our framework with the representative household, however, their effect is limited.[34] Even for very high values of the risk aversion parameter, the dynamics of the investment variables are found to be essentially the same as in the benchmark, a result reminiscent of Tallarini (2000). This is because the representative household's pricing kernel, which now also includes the continuation of lifetime utility from period $t+1$ onwards, depends on aggregate consumption. Aggregate consumption, however, is not directly affected by interest payments—it is given by the aggregate resource constraint, not the household's budget constraint. In a setting with heterogenous consumers, however, the variability of interest rates under ARM may play quantitatively more important role. Especially, as Epstein-Zin preferences price in long-run risk, which is present in the case of ARM, as mortgages are long-term contracts and interest rate shocks are highly persistent.

A simplifying feature of the mortgage in the model is the absence of the option to refinance. Refinancing complicates modeling of mortgages as, being an option, it introduces a kink in the payoffs from the contract. An informal argument, however, can be made that, at least for our question, abstracting from refinancing should not make a big difference. First,

[34] The reason why we work with risk-sensitive preferences, rather than the more general Epstein-Zin preferences, is that risk-sensitive preferences are much easier to handle computationally. In particular, while preserving the role of risk, they fit in the Hansen and Prescott (1995) linear-quadratic approximation framework, which is used through out our computational experiments (see Appendix B).

consider the case of no refinancing. Suppose that the FRM rate temporarily declines and is expected to mean revert. This is the standard situation in the model and, according to the model, households take out mortgages when the rate is low. Now suppose that households can refinance. Of course, they will not exercise the option along the increasing path of the FRM rate. Thus, in this case, the presence of refinancing does not affect the timing of when to take a mortgage. Suppose, instead, that the FRM rate temporarily increases and is expected to mean revert. In the absence of refinancing, the households in the model reduce demand for mortgages until the mortgage rate has sufficiently declined (waiting means that they are trading off lower mortgage costs for foregone utility of housing). Now suppose that households can refinance. And, for simplicity, take the extreme case that they can do so at zero costs. Then the FRM mortgage is like an ARM mortgage because it is profitable to refinance every period along the declining path of the interest rate. According to the model, under ARM, households reduce demand for mortgages when the ARM rate temporarily increases. Thus, again, the presence of refinancing should not, at least qualitatively, affect the timing.

7 Concluding remarks

A well known feature of the U.S. business cycle is that residential investment leads GDP. Nonresidential investment, on the other hand, lags GDP. We document that in most other developed economies both types of investment are, more or less, coincident with GDP. In almost all countries, however, residential construction activity measured by housing starts leads GDP. In contrast, most existing business cycle models predict the opposite dynamics; that residential investment lags and nonresidential leads output. Our empirical analysis points to mortgage finance and residential time to build as potential reasons behind the dynamics of residential construction observed in the data.

In order to evaluate the effects of these factors within a quantitative-theoretical framework, mortgages are introduced into an otherwise standard business cycle model with disaggregated investment. The complexity of such a task is greatly reduced by devising an accurate approximation of mortgages. Feeding into the model the observed cyclical dynamics of nominal mortgage interest rates and inflation, which are very similar across countries, produces dynamics of residential and nonresidential investment similar to those in U.S. data. Increasing time to build in residential construction then makes residential investment more coincident with GDP as in most other countries. This is because value put in residential projects is added over a longer period. Housing starts, however, still lead output. The equilibrium effects of mortgage finance can be summarized by an endogenous time-varying wedge in an otherwise standard Euler equation for housing investment. The wedge depends on expected future real mortgage payments over the life of the mortgage, weighted by the household's pricing kernel. As such, it captures the costs of mortgage finance to the household in the model. The wedge is shown to be strongly negatively correlated with future GDP, inducing households to invest in residential capital before GDP peaks. Consumption smoothing then dictates that nonresidential investment has to be postponed.

While there are no doubt many idiosyncracies in the national housing, construction, and mortgage markets that play a role in the observed housing dynamics, the objective of this study was to see if it is possible to understand the broad features of the data within a common theoretical framework. The model provides a transparent mechanism consistent with the international data. Indeed, an open question remains (not just for us but more generally), what drives the observed lead-lag patters of nominal interest rates.

A broader lesson from the analysis is that interest rate dynamics, in conjunction with long-term mortgage contracts, have a quantitatively significant effect on the economy. In our framework this shows up only in the composition of total investment, not in other aggregate variables. It is, however, worth exploring if such effects can transmit also into the broader economy, especially aggregate output. This, of course, requires a richer framework than the one used here. Our way of modeling mortgages, however, should make it relatively easy to introduce mortgages into a variety of DSGE models more suitable for addressing such questions.

Another interesting avenue for future research is to study the macroeconomic effects of risk inherent in ARM due to highly persistent interest rate shocks and the long-term nature of mortgage contracts. In our case of a representative household, such effects are found to be small as the household receives aggregate consumption, which is not directly affected by interest payments. A framework with heterogenous agents, however, may deliver a different answer.

Finally, as our focus is on new residential construction, we have abstracted from pricing the existing housing stock. But our finding that interest rate movements have a large effect on residential investment under mortgages suggests that the transmission of interest rate dynamics to house prices may also be sizable. Accounting for the relatively high volatility of house prices in the data remains an outstanding issue. Incorporating mortgages in business cycle models aimed at studying this problem may be a fruitful way of proceeding.

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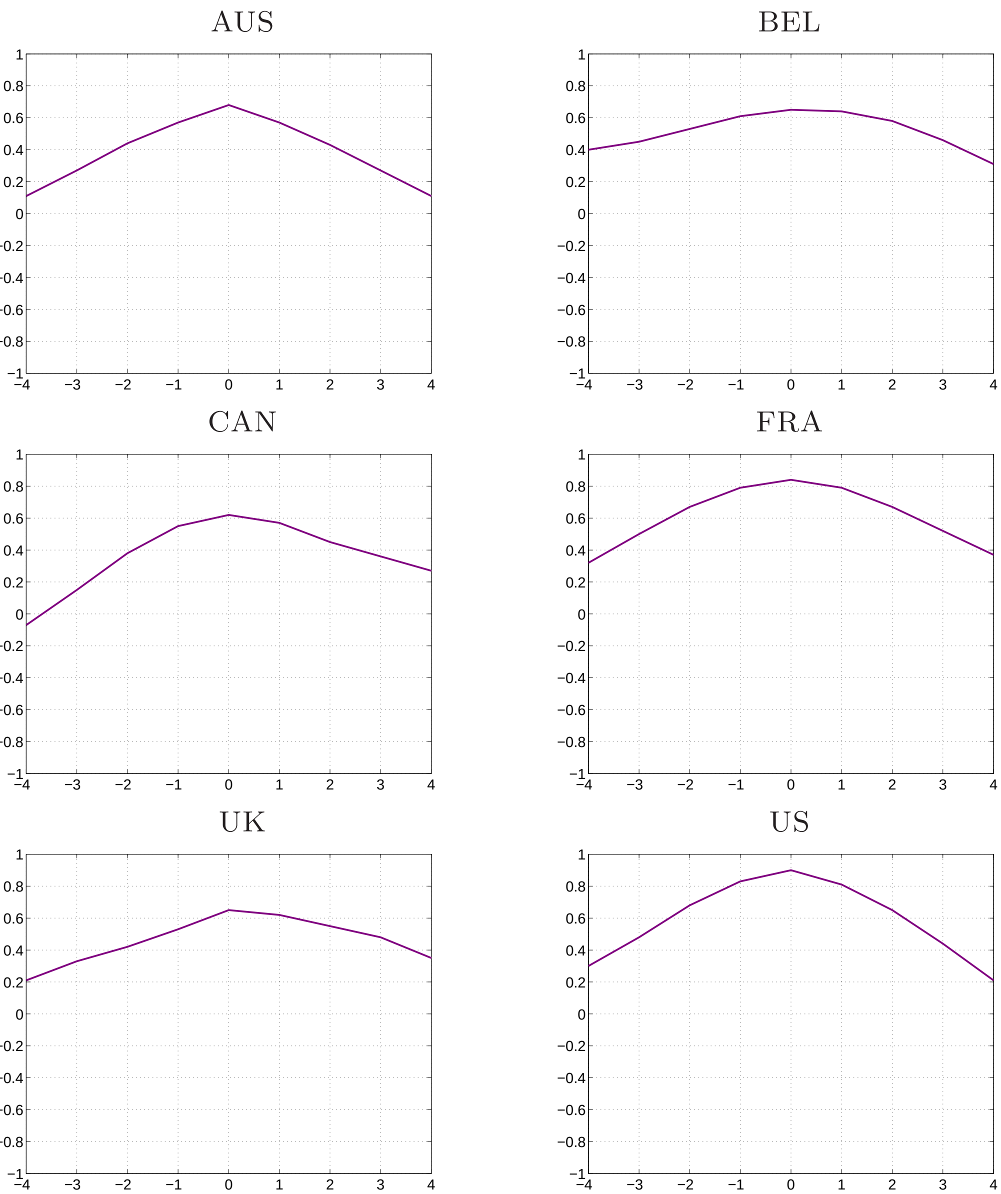

Figure 1: Cyclical dynamics of total fixed investment (gross fixed capital formation). The plots are correlations of real investment in $t+j$ with real GDP in t; the data are logged and filtered with Hodrick-Prescott filter. The volatility of total fixed investment (measured by its standard deviation relative to that of real GDP) is: AUS = 3.98, BEL = 3.93, CAN = 3.32, FRA = 2.65, UK = 2.55, US = 3.23.

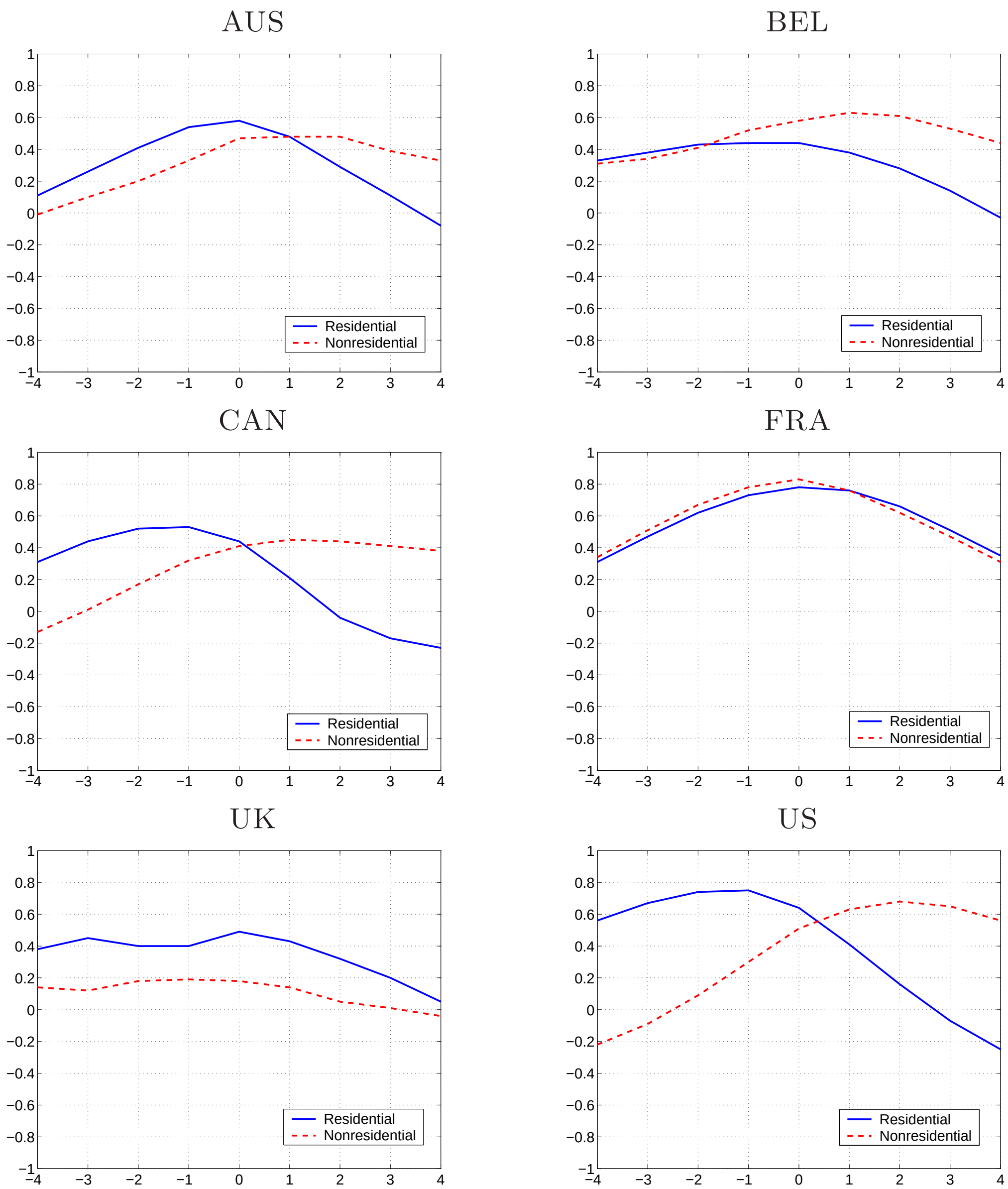

Figure 2: Cyclical dynamics of residential and nonresidential structures. The plots are correlations of real investment in $t+j$ with real GDP in t; the data are logged and filtered with Hodrick-Prescott filter (in the case of BEL and FRA nonresidential is the sum of structures and equipment). The volatility of residential (nonresidential), relative to that of real GDP, is: AUS = 5.95 (6.96), BEL = 7.97 (4.36), CAN = 4.39 (3.97), FRA = 3.05 (3.24), UK = 5.02 (3.24), US = 6.42 (3.40).

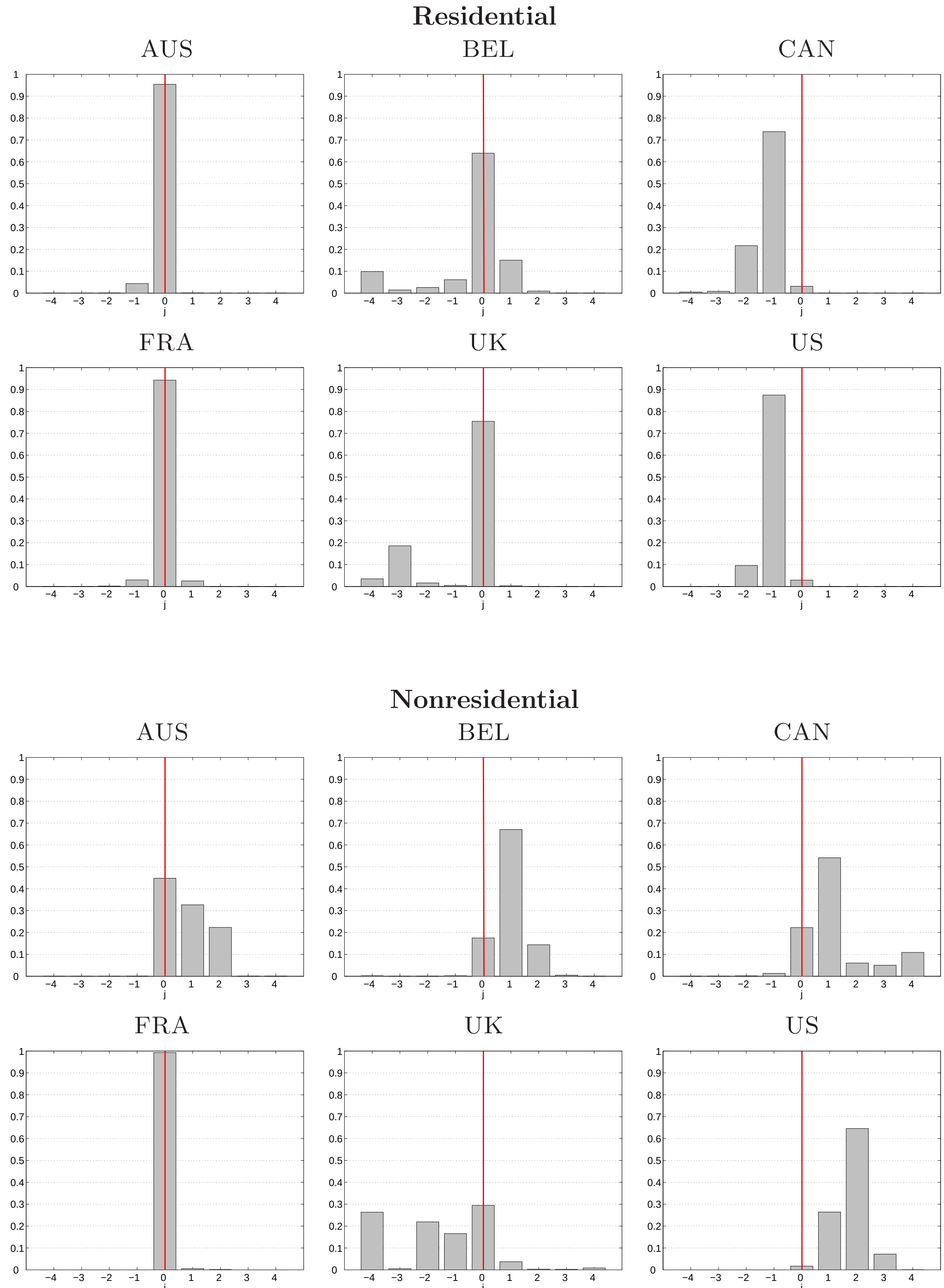

Figure 3: Statistical significance of leads and lags in structures dynamics. Histograms show the frequency with which a given j has the highest correlation coefficient in a sample of 10,000 cross-correlograms based on bootstrapped data (in each case a series is block-bootstrapped and then logged and HP filtered; a cross-correlogram is then computed for the HP-filtered series).

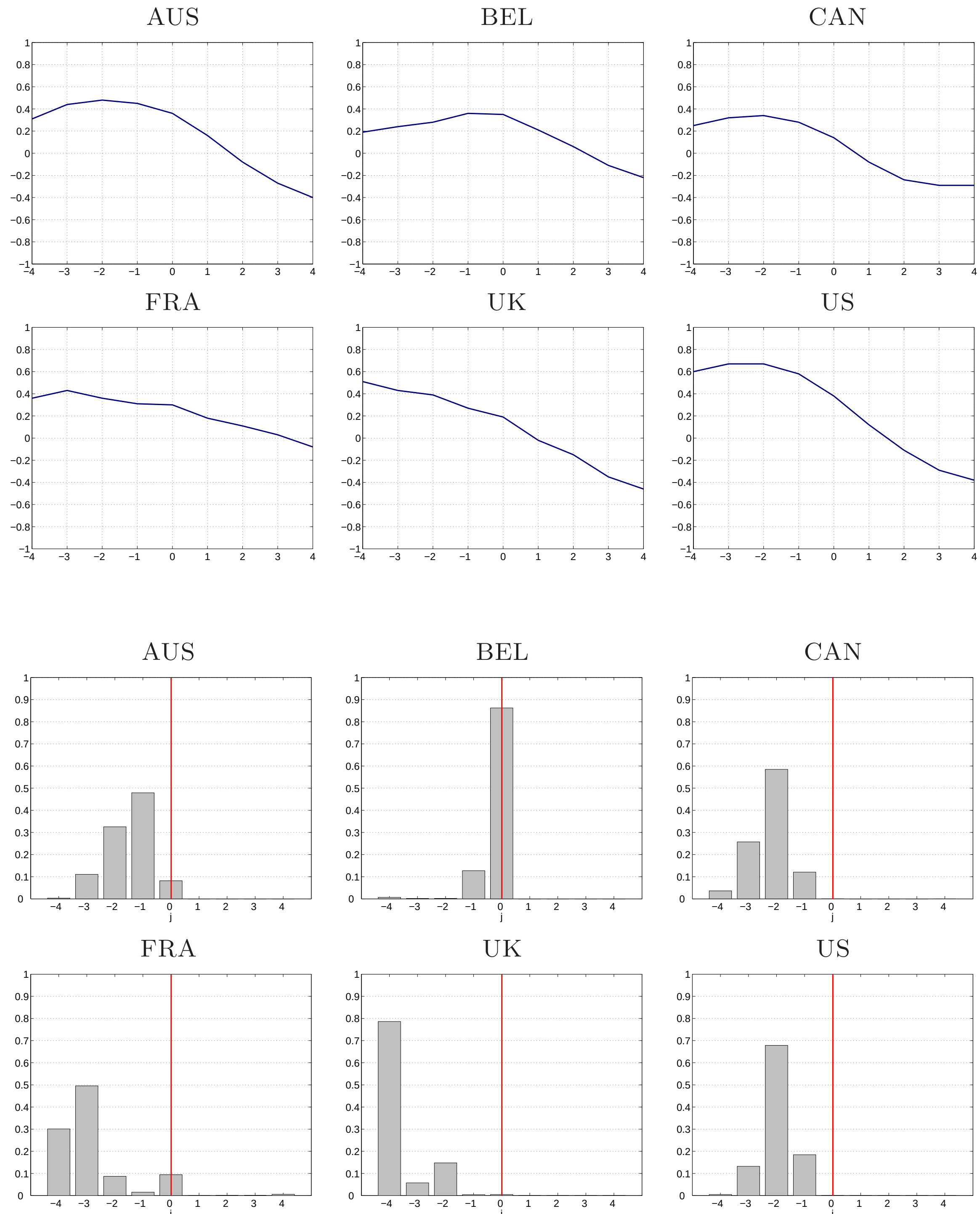

Figure 4: Housing starts. The top six charts plot cross-correlations in the historical data (logged and HP-filtered); the bottom six charts show the statistical significance of leads and lags in housing starts dynamics; i.e., the frequency with which a given j has the highest correlation coefficient in a sample of 10,000 cross-correlograms based on bootstrapped data. The volatility of housing starts in the actual data, relative to that of real GDP, is: AUS = 8.80, BEL = 11.67, CAN = 9.95, FRA = 6.24, UK = 7.86, US = 9.72. Note: due to a relatively short length of starts data for the U.K., residential building permits are used instead as a proxy (the two series co-move very closely during the period for which both are available).

Table 1: Residential investment—further details[a]

	Relative std. dev.[b]	Correlations of real GDP in t with a variable in $t+j$: j = -4	-3	-2	-1	0	1	2	3	4
United States: 59.Q1–83.Q4										
Residential structures										
Single family	8.84	0.58	0.65	0.73	0.72	**0.62**	0.39	0.14	-0.11	-0.30
Multifamily	11.40	0.13	0.25	0.38	0.48	**0.51**	0.46	0.34	0.21	0.07
Starts										
1 unit	8.85	0.61	0.68	0.70	0.61	**0.39**	0.12	-0.12	-0.33	-0.42
5+ units	14.16	0.39	0.52	0.61	0.60	**0.50**	0.30	0.10	-0.08	-0.22
Completions[c]										
1 unit	7.33	0.66	0.74	0.80	0.81	**0.78**	0.60	0.37	0.16	-0.05
5+ units	9.56	-0.02	0.12	0.27	0.42	**0.58**	0.75	0.77	0.73	0.65
Mortgages[d]										
Single family	14.22	0.45	0.56	0.69	0.63	**0.49**	0.30	0.15	-0.10	-0.23
Multifamily	17.41	0.43	0.42	0.46	0.45	**0.39**	0.31	0.16	0.03	-0.11
United States: 84.Q1–06.Q4										
Residential structures										
Single family	8.40	0.51	0.57	0.60	0.57	**0.48**	0.28	0.05	-0.13	-0.25
Multifamily	10.42	-0.02	-0.01	0.07	0.14	**0.22**	0.27	0.31	0.32	0.30
Starts										
1 unit	9.32	0.42	0.47	0.44	0.35	**0.23**	-0.01	-0.17	-0.29	-0.37
5+ units	16.43	0.05	0.16	0.27	0.40	**0.44**	0.40	0.33	0.21	0.13
Completions										
1 unit	6.51	0.36	0.43	0.52	0.50	**0.45**	0.33	0.15	-0.02	-0.16
5+ units	13.71	0.06	0.04	0.06	0.14	**0.23**	0.31	0.40	0.43	0.39
Mortgages[d]										
Single family	18.55	0.16	0.21	0.14	0.11	**0.10**	0.04	0.09	0.11	0.05
Excl. MEW[e]	20.83	0.18	0.21	0.11	0.06	**0.03**	-0.01	0.04	0.08	0.04
Multifamily	68.83	0.25	0.24	0.13	0.09	**0.03**	0.02	-0.03	-0.03	-0.01
Canada										
Residential structures[f]										
Single family	7.21	0.33	0.44	0.48	0.44	**0.27**	0.01	-0.29	-0.44	-0.42
Multifamily	6.60	-0.08	-0.13	-0.16	-0.10	**-0.08**	-0.05	0.03	0.10	0.05
United Kingdom[g]										
Starts	8.35	0.28	0.28	0.26	0.18	**0.10**	-0.08	-0.22	-0.38	-0.41
Completions	5.14	0.22	0.31	0.41	0.44	**0.48**	0.36	0.28	0.13	-0.01

[a] The series are logged (except for shares and multifamily mortgages) and filtered with Hodrick-Prescott filter; multifamily mortgages are expressed as a ratio to their mean due to negative values in the data.
[b] Standard deviations are expressed relative to that of a country's real GDP.
[c] 1968.Q1-1983.Q4.
[d] Net change in home and multifamily mortgages, deflated with GDP deflator (home = 1-4 family properties, multifamily = 5+ family properties). The fraction of new construction accounted for by 2-4 family structures is small, home mortgages are therefore a good proxy for single family housing mortgages, for which data are not available.
[e] MEW = mortgage equity withdrawal (home equity loans).
[f] 1981.Q1-2006.Q4.
[g] 1990.Q1-2006.Q4.

Table 2: Cyclical dynamics of mortgage rates[a]

		Relative std. dev.[b]	Correlations of real GDP in t with a variable in $t+j$: j = -4	-3	-2	-1	0	1	2	3	4
Mortgage rates[c]											
AUS	ARM	0.59	-0.29	-0.22	-0.16	-0.03	**0.12**	0.25	0.39	0.48	0.50
BEL	FRM 10 yrs	0.89	-0.17	0.01	0.19	0.38	**0.56**	0.63	0.60	0.53	0.41
CAN	FRM 5 yrs	0.77	-0.52	-0.41	-0.24	-0.04	**0.19**	0.38	0.45	0.45	0.43
FRA	FRM 15 yrs	0.87	-0.10	-0.02	0.10	0.20	**0.30**	0.36	0.35	0.31	0.27
UK[d]	ARM	1.29	-0.68	-0.52	-0.31	-0.06	**0.17**	0.36	0.49	0.55	0.56
US	FRM 30 yrs	0.55	-0.59	-0.55	-0.46	-0.29	**-0.07**	0.09	0.16	0.21	0.23
Government bond yields[e]											
AUS	3-m	1.07	-0.19	-0.06	0.10	0.24	**0.34**	0.44	0.52	0.45	0.34
BEL	10-yr	0.75	-0.01	0.20	0.33	0.49	**0.53**	0.50	0.43	0.31	0.19
CAN	3-5-yr	0.73	-0.42	-0.25	-0.06	0.17	**0.39**	0.52	0.54	0.50	0.41
FRA	10-yr	0.86	-0.12	-0.02	0.10	0.21	**0.29**	0.31	0.28	0.25	0.24
US	10-yr	0.53	-0.45	-0.39	-0.29	-0.11	**0.04**	0.09	0.10	0.12	0.09
	3-m	0.88	-0.45	-0.30	-0.10	0.17	**0.39**	0.48	0.51	0.49	0.46
Inflation rates[f]											
AUS		1.96	-0.19	-0.09	-0.02	0.12	**0.15**	0.21	0.31	0.28	0.17
BEL		1.80	0.15	0.19	0.13	0.15	**0.15**	0.17	0.17	0.16	0.15
CAN		1.44	-0.16	-0.03	0.05	0.16	**0.26**	0.35	0.32	0.35	0.35
FRA		1.72	-0.23	-0.13	-0.03	0.11	**0.20**	0.27	0.30	0.32	0.31
UK		2.80	-0.28	-0.22	-0.12	-0.01	**0.03**	0.18	0.23	0.29	0.27
US		1.28	-0.27	-0.13	-0.01	0.18	**0.37**	0.45	0.47	0.50	0.49

[a] GDP is in logs; all series are filtered with Hodrick-Prescott filter; time periods differ across countries due to different availability of mortgage rate data: AUS (59.Q3-06.Q4), BEL (80.Q1-06.Q4), CAN (61.Q1-06.Q4), FRA (78.Q1-06.Q4), UK (65.Q1-06.Q4), US (71.Q2-06.Q4).
[b] Standard deviations are expressed relative to that of a country's real GDP.
[c] Based on a typical mortgage for each country, as reported by Calza et al. (forthcoming) and Scanlon and Whitehead (2004). ARM = adjustable rate mortgage (interest rate can be reset within one year), FRM = fixed rate mortgage (interest rate can be at the earliest reset only after 5 years). The number of years accompanying FRMs in the table refers to the number of years for which the mortgage rate is typically fixed.
[d] U.K. mortgage rate data are available only from 1995.Q1. 3-m T-bill rate is used as a proxy for the adjustable mortgage rate for the period 1965.Q1-1994.Q4; the correlation between the two interest rates for the period 1995.Q1-2006.Q4 is 0.97. As the 3-m T-bill rate is used for this purpose, it is omitted from the next panel of the table.
[e] Constant maturity rates.
[f] Consumer price indexes, q-on-q percentage change at annual rate.

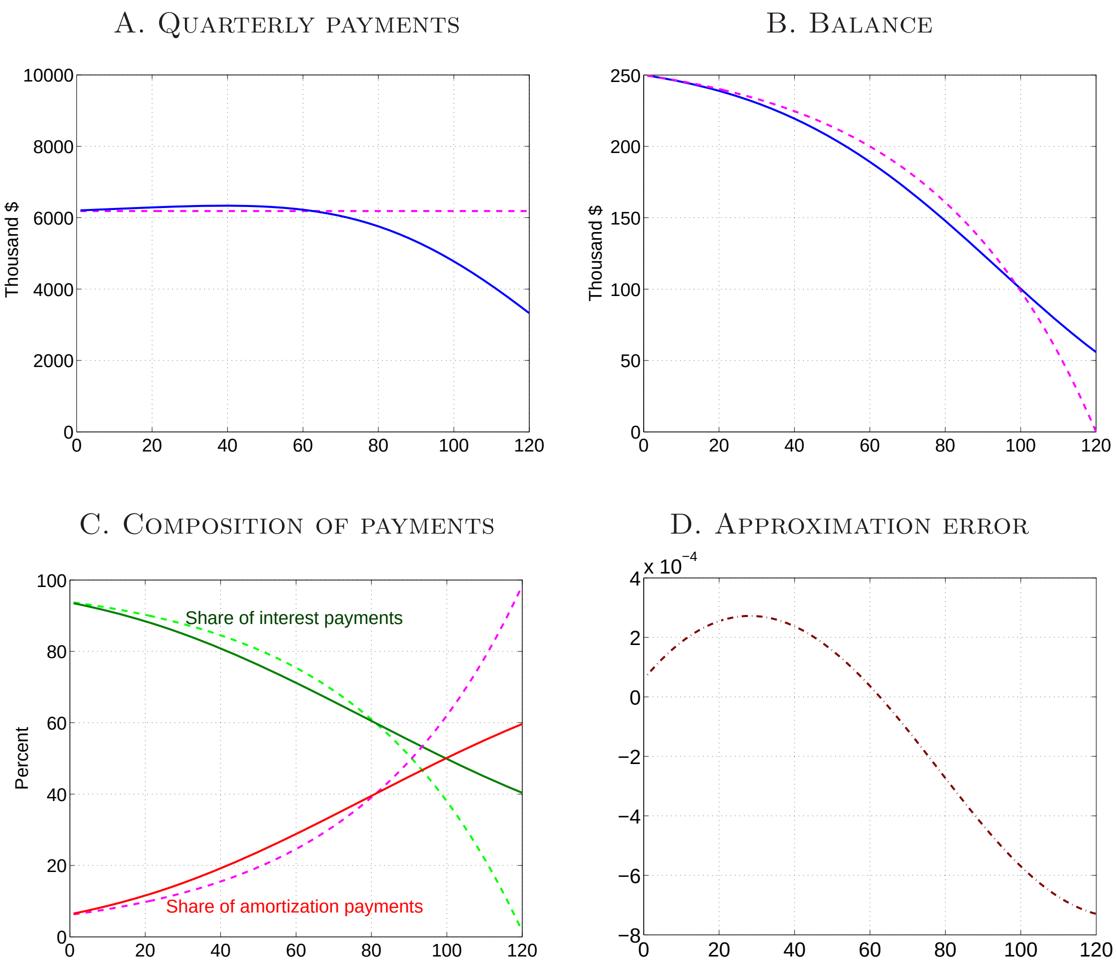

Figure 5: Mortgage: model vs real-world calculator. Solid line=model, dashed line=mortgage calculator. Here, $l_0 = \$250,000$, $4 \times i = 9.28\%$, $\alpha = 0.9946$, and $\kappa = 0.00162$. The approximation error is expressed as the present value (using $1/i$) of the difference between payments in the model and in the mortgage calculator (panel A), divided by the size of the loan.

Table 3: Calibration

Symbol	Value	Definition
Preferences		
β	0.988	Discount factor
ω	0.472	Consumption share in utility
ψ	0.692	Share of market good in consumption
Home technology		
δ_H	0.0115	Depreciation rate
η	0.305	Capital share in production
Nonresidential time to build		
J	4	Number of periods
ϕ_j	0.25	Fraction completed at stage j
Market technology		
δ_M	0.0248	Depreciation rate
ϱ	0.283	Capital share in production
σ	6.4	PPF curvature parameter
Tax rates		
τ_w	0.243	Tax rate on labor income
τ_r	0.612	Tax rate on capital income
Mortgages		
θ	0.76	Loan-to-value ratio
κ	0.00162	Initial amortization rate
α	0.9946	Adjustment factor
Other		
i	0.0232	Steady-state mortgage rate
π	0.0113	Steady-state inflation rate

Note: The parameters of the exogenous stochastic process are contained in Appendix C.

Table 4: Cyclical behavior of the model economy[a]

υ_{t+j}	Rel. st.dev.[b]	Correlations of y in period t with variable υ in period $t+j$: j = -4	-3	-2	-1	0	1	2	3	4
Model—main aggregates and hours										
y	1.01	-0.03	0.19	0.48	0.75	**1.00**	0.75	0.48	0.19	-0.03
h_M	0.56	0.10	0.31	0.57	0.76	**0.89**	0.68	0.41	0.07	-0.21
c_M	0.48	-0.21	-0.09	0.13	0.38	**0.70**	0.52	0.38	0.29	0.28
x	4.42	0.07	0.29	0.56	0.78	**0.93**	0.71	0.43	0.10	-0.18
Model—investment components and wedge										
x_H	8.45	0.19	0.34	0.50	0.55	**0.51**	0.31	0.11	-0.13	-0.32
x_M	4.33	-0.12	0.03	0.25	0.50	**0.78**	0.70	0.52	0.31	0.12
τ_H	3.26	-0.21	-0.33	-0.43	-0.43	**-0.32**	-0.17	-0.02	0.18	0.34

[a] Calibration is as in Table 3. The entries are averages for 200 runs of the length of 92 periods each, the same as the number of periods for 1984.Q1-2006.Q4. All variables are in percentage deviations from steady state, except the wedge, which is in percentage point deviations from steady state. Before computing the statistics for each run, the artificial series were filtered with the HP filter.
[b] Standard deviations are measured relative to that of y; the standard deviation of y is in absolute terms.

Table 5: Impact of various model features on investment dynamics

	Rel.	Correlations of y in period t with variable υ in period $t+j$:							
υ_{t+j}	st.dev.	j = -4	-3	-2	-1	0	1	2	3	4
Benchmark[a] (FRM, 30 years, $\theta = 0.76$)										
x	4.42	0.07	0.29	0.56	0.78	**0.93**	0.71	0.43	0.10	-0.18
x_H	8.45	0.19	0.34	0.50	0.55	**0.51**	0.31	0.11	-0.13	-0.32
x_M	4.33	-0.12	0.03	0.25	0.50	**0.78**	0.70	0.52	0.31	0.12
τ_H	3.26	-0.21	-0.33	-0.43	-0.43	**-0.32**	-0.17	-0.02	0.18	0.34
No mortgage finance ($\theta = 0$)										
x	4.21	0.08	0.27	0.52	0.76	**0.98**	0.75	0.46	0.15	-0.10
x_H	0.78	-0.07	0.06	0.30	0.55	**0.84**	0.55	0.37	0.28	0.34
x_M	5.79	0.09	0.28	0.52	0.76	**0.97**	0.74	0.46	0.14	-0.14
τ_H	–	–	–	–	–	–	–	–	–	–
No mortgage finance and linear PPF ($\theta = 0$, $\sigma = 0$)										
x	4.77	0.14	0.29	0.50	0.72	**0.99**	0.69	0.43	0.19	0.04
x_H	14.66	-0.19	-0.08	0.02	0.20	**0.54**	0.51	0.52	0.48	0.50
x_M	6.32	0.36	0.41	0.54	0.59	**0.52**	0.22	-0.07	-0.29	-0.48
τ_H	–	–	–	–	–	–	–	–	–	–
1-period loan ($\delta_{Dt} = 1\ \forall t$)										
x	4.30	0.07	0.27	0.52	0.76	**0.98**	0.75	0.47	0.17	-0.11
x_H	0.83	-0.08	0.10	0.35	0.63	**0.86**	0.66	0.45	0.43	0.17
x_M	5.85	0.08	0.28	0.52	0.76	**0.97**	0.74	0.46	0.14	-0.13
τ_H	0.21	-0.01	-0.04	-0.08	-0.13	**-0.04**	-0.15	-0.09	-0.22	0.23
FRM (4-variable VAR)										
x	4.95	0.18	0.33	0.56	0.77	**0.95**	0.74	0.48	0.20	0.01
x_H	8.46	0.35	0.49	0.56	0.55	**0.48**	0.31	0.16	-0.04	-0.23
x_M	5.55	-0.08	-0.01	0.23	0.50	**0.80**	0.69	0.50	0.31	0.23
τ_H	2.87	-0.34	-0.49	-0.51	-0.47	**-0.34**	-0.22	-0.10	0.08	0.26
ARM (4-variable VAR)										
x	4.68	0.17	0.27	0.50	0.73	**0.97**	0.74	0.46	0.18	0.02
x_H	2.59	0.32	0.22	0.05	-0.17	**-0.43**	-0.54	-0.60	-0.60	-0.54
x_M	7.56	0.09	0.20	0.44	0.69	**0.96**	0.77	0.53	0.29	0.13
τ_H	1.01	-0.26	-0.20	0.02	0.27	**0.61**	0.58	0.59	0.56	0.58
Residential time to build										
x	4.32	0.08	0.28	0.54	0.77	**0.95**	0.69	0.40	0.08	-0.17
x_H	6.51	0.18	0.32	0.47	0.57	**0.60**	0.42	0.14	-0.16	-0.40
n_4	8.89	0.33	0.40	0.50	0.48	**0.38**	-0.10	-0.33	-0.40	-0.34
n_0	8.88	-0.05	-0.02	0.06	0.18	**0.33**	0.40	0.50	0.48	0.38
x_M	4.11	-0.13	0.05	0.31	0.60	**0.90**	0.80	0.62	0.38	0.14
τ_H	3.17	-0.22	-0.34	-0.43	-0.42	**-0.29**	-0.16	-0.02	0.18	0.34

[a] Calibration as in Table 3.

Note: n_4 = housing starts (houses that in period t are four periods away from completion), n_0 = housing completions (houses that in period $t-1$ were one period away from completion and that in period t become a part of the housing stock).

Appendix

A. International data used in Section 2

Only those data for which details were not already provided in Section 2 are listed here. These are data on GDP, total investment, residential and nonresidential structures, and mortgage and interest rates. **Australia**. Real quantities: GDP, private GFCF, private GFCF nondwelling construction total, private GFCF dwellings total (all in chained dollars, SA, 1959.Q3-2006.Q4, *Australian Bureau of Statistics, National Accounts*); Mortgage rate: standard variable housing loans lending rate, banks (1959.Q3-2006.Q4, *Reserve Bank of Australia*); Interest rate: 3-month T-bill yield (1960.Q1-2006.Q4, *Global Financial Data*). **Belgium.** Real quantities: GDP at market prices, GFCF total, GFCF in dwellings, GFCF by enterprises, self-employed workers and non-profit institutions (all in chained 2006 euros, SA, 1980.Q1-2006.Q4, *BelgoStat Online, National Accounts*); Mortgage rate: fixed rate on loans for house purchasing (1980.Q1-2006.Q4, *Global Financial Data*); Interest rate: 3-month T-bill yield (1980.Q1-2006.Q4, *Global Financial Data*). **Canada.** Real quantities: GDP, residential structures, nonresidential structures, single dwellings, multiple dwellings (all in chained 2002 dollars, SA, *Statistics Canada, National Accounts*, 1961.Q1-2006.Q4, except for single and multiple dwellings, which are for 1981.Q1-2006.Q4); Mortgage rate: conventional mortgage lending rate, 5-year term (1961.Q1-2006.Q4, *Statistics Canada*); Interest rate: 3-month T-bill yield (1961.Q1-2006.Q4, *Global Financial Data*). **France.** Real quantities: GDP, total GFCF, GFCF of non financial enterprises—including uninc. entrep., GFCF of households—excluding uninc. entrep. (all in chained euros, SA, 1971.Q1-2006.Q4, *INSEE, National Accounts*); Mortgage rate: mortgage lending rate (1978.Q1-2006.Q4, *Global Financial Data*); Interest rate: money market rate (1971.Q1-2006.Q4, *International Financial Statistics* and *Datastream*). **United Kingdom.** Real quantities: GDP at market prices, GFCF total, GFCF dwellings, GFCF other new buildings and structures (all in chained 2002 pounds, SA, 1965.Q1-2006.Q4, *Office for National Statistics, United Kingdom Economic Accounts*); Mortgage rate: sterling standard variable mortgage rate to households (1995.Q1-2006.Q4, *Bank of England*); Interest rate: 3-month T-bill yield (1965.Q1-2006.Q4, *Office for National Statistics*). **United States.** Real quantities: GDP, private fixed investment, private residential fixed investment, private fixed investment single family, private fixed investment multifamily, private fixed investment structures (all in chained 2000 dollars, SA, 1958.Q1-2006.Q4, *FRED* and *Bureau of Economic Analysis, National Income and Product Accounts*); Mortgage rate: 30-year conventional mortgage rate (1971.Q1-2006.Q4, *FRED*); Interest rate: 3-month T-bill yield (1958.Q1-2006.Q4, *FRED*).

B. Equilibrium—details and computation

This appendix provides the full set of optimality conditions for the household's problem of Section 4 and describes the method used to compute the equilibrium of the model.

The household's optimal decisions are characterized by four first-order conditions for h_{Mt}, h_{Ht}, s_{Jt}, and x_{Ht}. These are, respectively,

$$u_{1t}c_{1t}(1-\tau_w)w_t = u_{2t},$$

$$u_{1t}c_{2t}A_H G_{2t} = u_{2t},$$

$$u_{1t}c_{1t}\phi_J = \beta E_t V_{s_{J-1},t+1},$$

$$u_{1t}c_{1t}(1-\theta) - \theta\beta E_t\left[\widetilde{V}_{d,t+1} + \zeta_{Dt}(\kappa - \delta_{Dt}^{\alpha})V_{\delta_D,t+1} + \zeta_{Dt}(i_t - R_t)V_{R,t+1}\right] = \beta E_t V_{kH,t+1}.$$

Here $\widetilde{V}_{d,t+1}$ and ζ_{Dt} are defined as in the main text; that is, $\widetilde{V}_{d,t+1} \equiv p_t V_{d,t+1}$ and $\zeta_{Dt} \equiv \left(\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t\right) / \left(\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t + \theta x_{Ht}\right)^2$, where $\widetilde{d}_t \equiv d_t/p_{t-1}$. The first-order condition for s_{Jt} is accompanied by Benveniste-Scheinkman conditions for s_{jt} $(j = J-1, ..., 2)$, s_{1t}, and k_{Mt}, respectively,

$$V_{s_jt} = -u_{1t}c_{1t}\phi_j + \beta E_t V_{s_{j-1},t+1}, \quad j = J-1, ..., 2,$$

$$V_{s_1t} = -u_{1t}c_{1t}\phi_1 + \beta E_t V_{k_M,t+1},$$

$$V_{k_M,t} = u_{1t}c_{1t}\left[(1-\tau_r)r_t + \tau_r\delta_M\right] + \beta(1-\delta_M)E_t V_{k_M,t+1}.$$

The first-order condition for x_{Ht} has four Benveniste-Scheinkman conditions, for d_t, δ_{Dt}, R_t, and k_{Ht}. These are, respectively,

$$\widetilde{V}_{dt} = -u_{1t}c_{1t}\frac{R_t + \delta_{Dt}}{1+\pi_t} + \beta\frac{1-\delta_{Dt}}{1+\pi_t}E_t\left[\widetilde{V}_{d,t+1} + \zeta_{xt}(\delta_{Dt}^{\alpha} - \kappa)V_{\delta_D,t+1} + \zeta_{xt}(R_t - i_t)V_{R,t+1}\right],$$

$$V_{\delta_D,t} = -u_{1t}c_{1t}\left(\frac{\widetilde{d}_t}{1+\pi_t}\right) + \left[\zeta_{xt}(\kappa - \delta_{Dt}^{\alpha}) + \frac{(1-\delta_{Dt})\alpha\delta_{Dt}^{\alpha-1}}{\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t + \theta x_{Ht}}\right]\left(\frac{\widetilde{d}_t}{1+\pi_t}\right)\beta E_t V_{\delta_D,t+1}$$

$$-\left(\frac{\widetilde{d}_t}{1+\pi_t}\right)\beta E_t\widetilde{V}_{d,t+1} + \zeta_{xt}(i_t - R_t)\left(\frac{\widetilde{d}_t}{1+\pi_t}\right)\beta E_t V_{R,t+1},$$

$$V_{Rt} = -u_{1t}c_{1t}\left(\frac{\widetilde{d}_t}{1+\pi_t}\right) + \frac{\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t}{\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t + \theta x_{Ht}}\beta E_t V_{R,t+1},$$

$$V_{k_Ht} = u_{1t}c_{2t}A_{Ht}G_{1t} + \beta E_t V_{k_H,t+1}(1-\delta_H),$$

where ζ_{xt} is defined as in the main text: $\zeta_{xt} \equiv \theta x_{Ht}/\left(\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t + \theta x_{Ht}\right)^2$. Notice that the terms involving $\widetilde{V}_{d,t+1}$, $V_{\delta_D,t+1}$, and $V_{R,t+1}$ appear only in the first-order condition for x_{Ht}, as claimed in the main text. These terms drop out if $\theta = 0$. In this case the optimal decisions are characterized by the same conditions as in GKR, implying the same allocations and prices.

The equilibrium is computed by combining the linear-quadratic approximation methods of Hansen and Prescott (1995) and Benigno and Woodford (2006). Specifically, after transforming the model so that it is specified in terms of stationary variables π_t and $\widetilde{d}_t \equiv d_t/p_{t-1}$ (instead of nonstationary variables p_t and d_t), the home production function (2) and the budget constraint (9), with l_t and m_t substituted out from equations (8) and (10), are substituted in the period utility function $u(.,.)$. The utility function is then used to form a Lagrangian that has the nonlinear laws of motion (11)-(13) as constraints.

This Lagrangian forms the return function in the Bellman equation to be approximated with a linear-quadratic form around a nonstochastic steady state, with the variables expressed as percentage deviations from steady state. The steps for computing equilibria of distorted linear-quadratic economies, described by Hansen and Prescott (1995), then follow; with a vector of exogenous state variables $\Omega_t = [z_t, ..., z_{t-n}]$, a vector of endogenous state variables $\Phi_t = [s_{1t}, ..., s_{J-1,t}, k_{Mt}, k_{Ht}, \widetilde{d}_t, \delta_{Dt}, R_t]$, and a vector of decision variables $\Upsilon_t = [h_{Mt}, h_{Ht}, x_{Ht}, s_{Jt}, \widetilde{d}_{t+1}, \delta_{D,t+1}, R_{t+1}, \lambda_{1t}, \lambda_{2t}, \lambda_{3t}]$, where $\lambda_{1t}, \lambda_{2t}, \lambda_{3t}$ are Lagrange multipliers for the non-linear constraints (11)-(13).[35] The use of the Lagrangian ensures that second-order cross-derivatives of the nonlinear laws of motion (11)-(13), evaluated at steady state, appear in equilibrium decision rules (Benigno and Woodford, 2006). The alternative procedure of substituting out $\widetilde{d}_{t+1}$, $\delta_{D,t+1}$, and R_{t+1} from these laws of motion into the period utility function is not feasible here as these three variables are interconnected in a way that does not allow such substitution. The Lagrangian is

$$L_t = u\left(c(c_{Mt}, c_{Ht}), 1 - h_{Mt} - h_{Ht}\right) + \lambda_{1t}\left[d_{t+1} - (1-\delta_{Dt})d_t - l_t\right]$$

$$+\lambda_{2t}\left[\delta_{D,t+1} - (1-\nu_t)\delta_{Dt}^{\alpha} - \nu_t\kappa\right] + \lambda_{3t}\left[R_{t+1} - (1-\nu_t)R_t - \nu_t i_t\right],$$

with the remaining constraints of the household's problem substituted in the consumption aggregator $c(.,.)$, as mentioned above. For our calibrations the steady-state values of the Lagrange multipliers (λ_{1t}, λ_{2t}, λ_{3t}) are positive, implying that the above specification of the Lagrangian is correct in the neighborhood of the steady state.

The Lagrange multipliers are instrumental for computing the wedge, τ_{Ht}. Notice from equation (16) that the wedge depends on conditional expectations of the derivatives of the value function. The multipliers, which are obtained as an outcome of the solution method, provide a straightforward way of computing these expectations. The mapping between the multipliers and the expectations is obtained from the first-order conditions for d_{t+1}, $\delta_{D,t+1}$, and R_{t+1} in the household's problem. Forming the Bellman equation

$$V\left(z_t, ..., z_{t-n}, s_{1t}, ..., s_{J-1,t}, k_{Mt}, k_{Ht}, d_t, \delta_{Dt}, R_t\right)$$

$$= \max\left\{L_t + \beta E_t V\left(z_{t+1}, ..., z_{t-n+1}, s_{1,t+1}, ..., s_{J-1,t+1}, k_{M,t+1}, k_{H,t+1}, d_{t+1}, \delta_{D,t+1}, R_{t+1}\right)\right\},$$

the respective first-order conditions are

$$\lambda_{1t} + \lambda_{2t}\left[\frac{(1-\delta_{Dt})\delta_{Dt}^{\alpha}d_t + p_t\theta\kappa x_{Ht}}{d_{t+1}^2}\right] + \lambda_{3t}\left[\frac{(1-\delta_{Dt})d_t R_t + p_t\theta i_t x_{Ht}}{d_{t+1}^2}\right] + \beta E_t V_{d,t+1} = 0,$$

$$\lambda_{2t} + \beta E_t V_{\delta_D,t+1} = 0,$$

$$\lambda_{3t} + \beta E_t V_{R,t+1} = 0.$$

When the model is transformed so that it is specified in terms of π_t and $\widetilde{d}_t$, rather than p_t

[35] In the version with residential time to build, the $n_{\iota t}$'s become a part of Φ_t and n_t^* becomes a part of Υ_t, but with q_t^* being its counterpart in the aggregate counterpart to Υ_t.

and d_t, the first of these conditions changes to

$$\widetilde{\lambda}_{1t} + \lambda_{2t}\left[\frac{(1-\delta_{Dt})\delta_{Dt}^{\alpha}\widetilde{d}_t}{(1+\pi_t)\widetilde{d}_{t+1}^2} + \frac{\theta\kappa x_{Ht}}{\widetilde{d}_{t+1}^2}\right] + \lambda_{3t}\left[\frac{(1-\delta_{Dt})\widetilde{d}_t R_t}{(1+\pi_t)\widetilde{d}_{t+1}^2} + \frac{\theta i_t x_{Ht}}{\widetilde{d}_{t+1}^2}\right] + \beta E_t \widetilde{V}_{d,t+1} = 0,$$

where $\widetilde{\lambda}_{1t} \equiv p_t \lambda_{1t}$.

C. VAR estimates

The exogenous VAR process used in Section 5 is estimated on U.S. data for logged and linearly detrended Solow residual, the interest rate on the conventional 30-year FRM, and the CPI inflation rate. The estimation period is 1984.Q1-2006.Q4. The series for the Solow residual is taken from data accompanying Gomme and Rupert (2007). The capital stock used for the construction of the residual is the sum of structures and equipment & software (current costs deflated with the consumption deflator), which is consistent with our mapping of k_{Mt} into the data in the rest of the calibration. The number of lags in the VAR is determined by the multivariate AIC. The point estimates (ignoring the constant term) are

$$z_{t+1} = \begin{pmatrix} 0.933 & -0.543 & -0.283 \\ 0.023 & 0.953 & 0.020 \\ 0.021 & 0.431 & 0.246 \end{pmatrix} z_t + \begin{pmatrix} 0.118 & -0.070 & 0.183 \\ -0.016 & -0.134 & 0.036 \\ 0.111 & -0.249 & 0.164 \end{pmatrix} z_{t-1}$$

$$+ \begin{pmatrix} -0.147 & 0.633 & 0.117 \\ 0.036 & -0.011 & 0.043 \\ -0.084 & -0.197 & 0.187 \end{pmatrix} z_{t-2} + \begin{pmatrix} 0.0049 & 0 & 0 \\ 0.0002 & 0.0009 & 0 \\ -0.0011 & 0.0009 & 0.0026 \end{pmatrix} \epsilon_{t+1},$$

where $z_t = [\log A_{Mt}, i_t, \pi_t]^\top$ and $\epsilon_{t+1} \sim N(0, I)$. These point estimates are used to solve the model and run the computational experiments in Sections 5 and 6. Note that as in our computational experiments we are interested only in unconditional moments, the ordering of the variables in the VAR is irrelevant.

In Section 6, a four-variable VAR is also used. Here, $z_t = [\log A_{Mt}, i_t^{FRM}, \pi_t, i_t^{ARM}]^\top$, where i_t^{FRM} is, as before, the interest rate on the conventional 30-year FRM and i_t^{ARM} is the yield on a 3-month Treasury bill. Here, AIC dictates four lags. The point estimates are

$$z_{t+1} = \begin{pmatrix} 0.858 & 0.014 & -0.157 & -1.232 \\ 0.044 & 0.849 & 0.042 & 0.008 \\ 0.085 & 0.172 & 0.241 & 0.554 \\ 0.049 & 0.127 & 0.021 & 1.362 \end{pmatrix} z_t + \begin{pmatrix} 0.070 & -0.221 & 0.192 & 2.122 \\ -0.020 & -0.070 & 0.048 & -0.006 \\ 0.103 & -0.023 & 0.162 & -0.721 \\ -0.041 & -0.107 & -0.010 & -0.346 \end{pmatrix} z_{t-1}$$

$$+ \begin{pmatrix} -0.302 & -0.168 & 0.036 & -2.277 \\ 0.005 & 0.051 & 0.045 & -0.036 \\ -0.140 & 0.097 & 0.204 & 0.406 \\ -0.004 & 0.123 & -0.032 & -0.090 \end{pmatrix} z_{t-2} + \begin{pmatrix} 0.231 & 1.124 & 0.053 & 0.615 \\ 0.027 & -0.189 & -0.062 & 0.178 \\ 0.012 & -0.458 & -0.153 & 0.060 \\ 0.032 & -0.219 & -0.010 & 0.063 \end{pmatrix} z_{t-3}$$

$$
+ \begin{pmatrix} 0.0042 & 0 & 0 & 0 \\ 0.0003 & 0.0008 & 0 & 0 \\ -0.0009 & 0.0008 & 0.0025 & 0 \\ 0.0001 & 0.0003 & 0.0001 & 0.0006 \end{pmatrix} \epsilon_{t+1},
$$

where $\epsilon_{t+1} \sim N(0, I)$.