

# Houses, Apartments and Condos: The Governance of Multifamily Housing

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## Abstract

Residential condominium ownership is positively and non-linearly correlated with building size. The usual explanation of high rentership in multifamily buildings – a coordination problem among unit owners – is inconsistent with the greater prevalence of condominium ownership in larger buildings. Drawing on the theory of the private provision of public goods and contracting, our model explains the ownership-building size correlation as a trade-off between the fixed costs of hiring third-party management and free-riding among building owners. Numerical solutions of the model closely match observed patterns of ownership in the data. Additional empirical evidence from the American Housing Survey is also consistent - condominium ownership is more desirable the larger the structure. In contrast to the extant literature that asks what sort of households choose owner-occupation, we identify a role for the building itself in explaining tenure outcomes.

## 1 Introduction

Single family houses are typically owned by the households who occupy them. While most households in multifamily structures are renters, there are patterns of ownership in this category of housing that are relatively unrecognized. As depicted in Figure 1 (which we discuss in greater detail below), the fraction of duplex units held as condominiums in the

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2011 American Housing Survey (AHS) is especially low at 9 percent. Nonetheless, the rate of condo ownership nearly doubles in buildings of around twelve units and continues to increase in even larger buildings. In Figure 2, we illustrate that this pattern is replicated both within cities and across regions in the U.S. A related fact suggesting that small multifamily buildings are somehow problematic for condominium ownership is that the relative share of the U.S. housing stock in two to four family buildings has declined continuously since 1973 (Eggers and Thackeray, 2007). The usual explanation of high rentership in multifamily buildings – a coordination problem among unit owners – is inconsistent with the greater prevalence of condominium ownership in larger buildings.

Drawing on the theory of the private provision of public goods, firm governance and contracting we show that the ownership-building size relationship is explained by a trade-off between the fixed costs of hiring third-party management and free-riding among building owners. Because owner management is relatively more important when third party management is expensive, single family and small multifamily buildings are optimally owned by sole investors who operate the properties. For larger buildings in which third party management becomes more affordable and free-riding problems worsen, joint ownership arrangements are more desirable. We find that data from the American Housing Survey confirms the implications of the model for ownership outcomes and shows that wealthy condo owners outbid the less wealthy for more optimally-sized multifamily buildings, as predicted by the model. Our paper contributes to a long line of housing literature on the decision to owner-occupy versus rent, starting with Henderson and Ioannides (1983). In contrast to the extant literature that asks what sort of *households* rent versus own, we identify a role for the building itself (and by extension the structure of cities) in tenure outcomes.

As noted above, in the U.S. aggregate homeownership rates are strongly correlated with the type of building that households occupy. Data from the AHS indicate that 83 percent of occupied single-family, detached housing in the U.S. is owner-occupied while only 14 percent of units in multifamily buildings are owner-occupied. According to Glaeser and Shapiro (2003), “[t]here are few facts in urban economics as reliable as the fact that people in multifamily units overwhelmingly rent and people in single-family units overwhelmingly own.” Because condominium ownership allows households to own and occupy individual units, the form of multifamily ownership is of obvious importance for the realization of homeownership, especially in more urbanized places. It is remarkable, therefore, that multifamily ownership patterns have received so little attention.<sup>1</sup>

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<sup>1</sup>The legal structure and governance of condominiums and cooperatives have been periodically investigated. Ellickson (1982), Barzel and Sass (1990), and Ben-Shahar and Sulganik (2005) have all examined voting rules in co-ownership arrangements. Hansmann (1991) and Schill, Voicu and Miller (2007) investigate trade-offs between condominium and cooperative governance structures. van der Merwe and Muniz-Arguelles

The problem we address also has overlap with the literature on club and local public goods as the number of club or community members increases. The standard model of free-riding with a variable number of participants (Cornes and Sandler, 1996) suggests that free-riding increases as that number rises. The experimental literature does not always produce this result, however (Ledyard, 1995). More to the point, Ostrom (1990) describes a number of mechanisms through which common property resource problems are solved, but somewhat contrary to the institutions discussed here, the emphasis in Ostrom (1990) is the ability of smaller groups to resolve the tragedy of the commons. Our paper contributes to this debate by identifying a particular mechanism – an economy of scale in the cost of management – that may allow groups to ameliorate free-riding.

In particular, we develop a portfolio model of investment choice in risky housing assets that can be owned through various alternative forms. A building may be solely owned by a single investor who is the full residual claimant to building value, or by multiple investors who are joint owners of the building. We lump a variety of mechanisms used to coordinate and govern investor relations under this latter heading.<sup>2</sup> Our model emphasizes contributions by both investors and third-party managers to building management, which in turn affects building value. Into this setting, we introduce two sources of frictions related to the number of building co-investors. First, management effort exerted by one investor is assumed to benefit all co-investors in a joint ownership arrangement. The result is that investors bear a personal cost of management effort but only receive a fraction of the benefit produced. Therefore, each investor in a jointly-owned multifamily building under-produces management effort. A sole owner of the same building produces greater total management effort than the sum of effort produced by two or more co-investors.

The second mechanism in the model related to building size is a fixed cost of effort supplied by third-party managers ("managers"). Thus, in small multifamily buildings, managers are relatively expensive. Adding co-investors increases the building size accessible to a given investor, providing access to lower average costs of third-party management (up to the cost-minimizing building size). Our model trades-off the free-rider problem among investors against economies of scale associated with hiring third-party management.

A main result is that single family housing is optimally owner-occupied. The smallest multifamily building sizes are particularly disadvantaged for purposes of condo ownership and are also solely owned by an investor. Nonetheless, as the average cost of hiring management

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(2006) survey legal solutions around the world with respect to securing co-owner pledges to share in costs. None of these studies directly compare trade-offs between the sole ownership and co-ownership of housing.

<sup>2</sup>For example, joint ownership is provided by condominium and cooperative arrangements as well as through partnerships and real estate investment trusts (REITs). In any of these arrangements, free-rider problems emerge just as in diffuse stock ownership arrangement for firms.

falls, bid prices for joint ownership arrangements begin sloping upwards with building size. Under certain parameter assumptions, numerical solutions show that investor participation in direct building management falls to zero as the number of co-investors increases, such that changes in bid prices with respect to building size are dominated by changes in third-party management costs and the slope of bid functions become nearly identical for investors in the largest buildings. Therefore, the identity of the marginal investor in larger buildings is heavily influenced by the distribution of wealth. The maintained assumption linking our analysis back to homeownership rates in multifamily buildings is that sole owners of buildings rent the residences therein, while (at least some) joint owners preserve an option to owner-occupy.

The empirical portion of the paper uses the AHS to test the model's prediction that joint investors with greater wealth will outbid others for larger buildings. To do this, we estimate the bids of condominium owners with respect to marginal quality, as measured by interior floor space. The bids are modelled as functions of both demographic and unit characteristics, particularly the number of co-investors as measured by the number of units in the building. In order to do this we first account for the selection of buildings into condominium status, as opposed to sole ownership, and then for the selection into owner-occupied condos in order to overcome observability problems discussed below. We then use two-step procedures for recovering the bid functions that were first proposed by Epple (1987) and Bartik (1987). Identification is possible due to the observed variation in the hedonic price function across U.S. regions. By splitting the sample into high and low wealth sub-samples, we find that low wealth investors are sensitive to building size and their bid gradient with respect to unit count is negative and quite steep. They, therefore, are the winning bidders for joint ownership of smaller buildings, and are then outbid as condominium buildings become larger.

The paper proceeds as follows. In the next section we present some of the stylized facts and quandaries to which we have alluded, and describe the literature in this area. Then we present the model, analyze it, and present numerical solutions. The succeeding section estimates the data analogs of the bid functions derived from the theory. This involves several econometric issues, which are addressed, and we present empirical evidence consistent with model predictions. We then conclude.

## 2 Homeownership and Structures

By way of context, about three-fourths of year-round housing in the U.S. is in single family structures. The vertical bars in Figure 1 show that most of the multifamily housing in the U.S. is quite small as well. Over 64% of the multifamily residences surveyed in the 2011

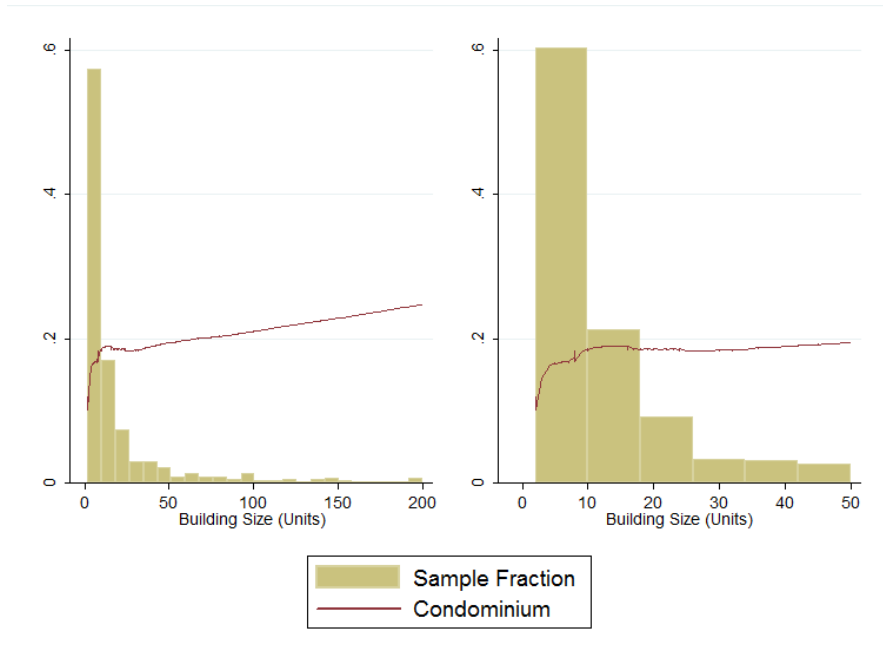


Figure 1: Condo Ownership, All Multifamily (AHS 2011)

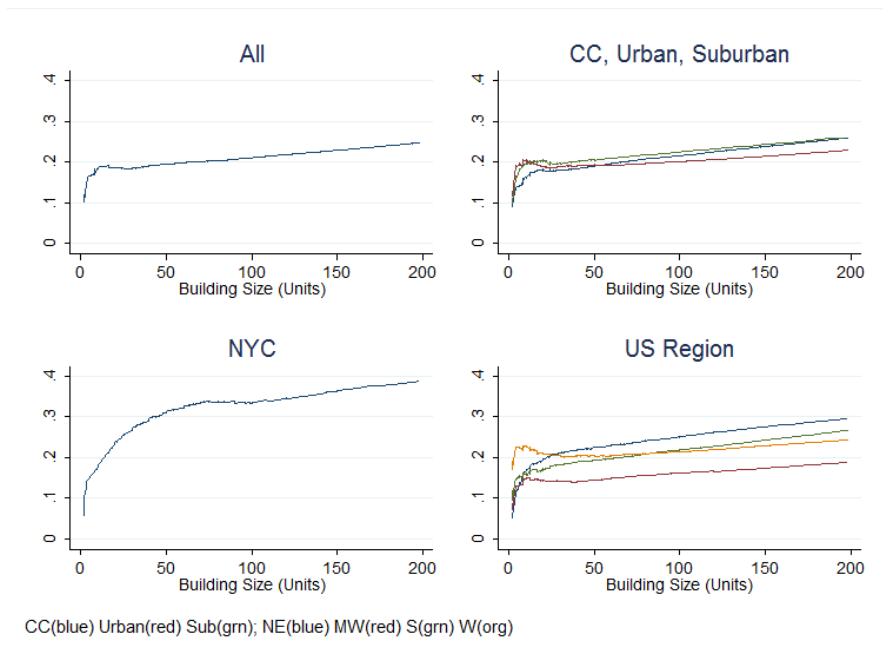


Figure 2: Condo Ownership, Sub-samples (AHS 2011)

AHS were in buildings of 12 or fewer residences. Confounding simple explanations of the homeownership-structure correlation, condo ownership steeply increases in small buildings (with 12 or fewer residences – see the right panel of Figure 1). Interestingly, the rate of condominium ownership for duplexes in the 2009 AHS data is just over 9%, whereas more than 25% of buildings with more than 50 units are organized as condominiums. This rise in condominium ownership is not sensitive to the location of housing within cities or across locations. Figure 2 reproduces Figure 1 for stratifications of the data by different geographic characteristics. We first categorize the data according to where the housing unit is located: within a central city, greater urban area or suburban area of an metropolitan region. We also display this graph for New York City, where there are a substantial number of large buildings. Finally, we graph the rate of condo ownership of housing units within multifamily buildings by region of the U.S. – Northeast, Midwest, South and West. The fact that condominium ownership increases with building size is an important feature of the U.S. housing landscape.

In order to examine the structure-tenure correlation more carefully than has been done previously, we ask a fundamental question: what is the relationship between building size and its ownership and governance? This is somewhat different than the traditional examination of homeownership (e.g. Coulson and Dalton, 2010; Hilber, 2004; Carrillo and Yezer, 2009) which asks: what characteristics of *households* make them more likely to become owner-occupiers? To introduce this conceptual shift, it is first useful to pin down our vocabulary. Rather obviously, by single-family housing we mean a stand-alone building intended as a residence for a single household. By multifamily, we refer to any arrangement of multiple residences within a building or structure.<sup>3</sup>

Because one of our main goals is to explain homeownership rates as a function of building size, it is important to recognize that homeownership in multifamily buildings requires a particular brand of governance like condominium or cooperative ownership, which provides investors with a proprietary claim to use and occupy a residence. In contrast, a limited partner in a partnership that owns a multifamily building has no *a priori* right to occupy a residence within the building. We would not consider the investor in this case to be a homeowner. (Nor for that matter would the Census or the IRS.) We do not directly model homeownership. Nonetheless, the ultimate connection between ownership and tenure should be evident. If joint ownership of multifamily buildings is disadvantaged relative to sole ownership for certain types of buildings, we would expect to observe greater rentership in multifamily buildings, all else equal. Thus, our analysis represents a necessary step in establishing the structure-tenure connection.

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<sup>3</sup>Multifamily ownership arrangements may also encompass multiple buildings, including multiple single-family houses, which we do not directly address in this paper.

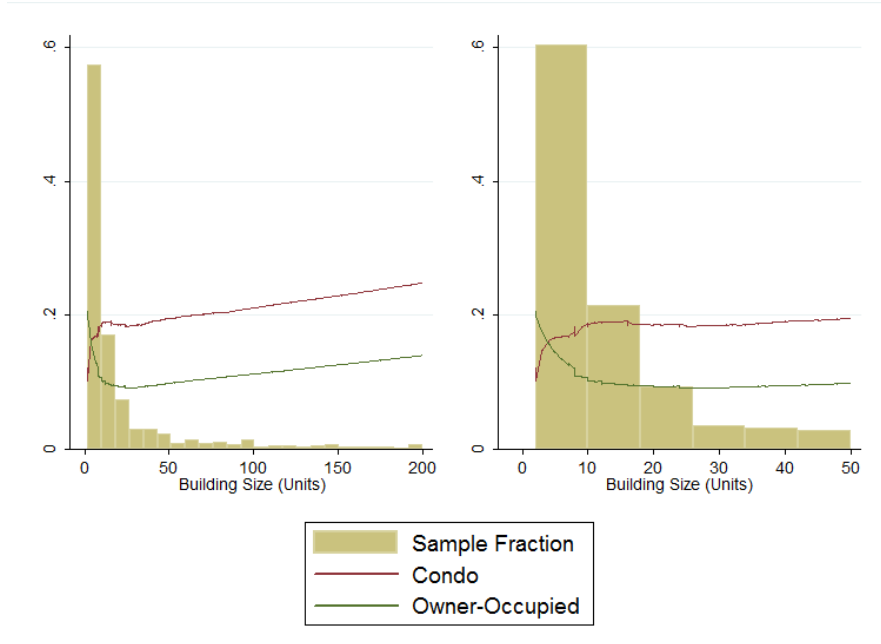


Figure 3: Condo Ownership and Homeownership (AHS 2011)

In contrast to our claim that condo ownership is a prerequisite for multifamily owner-occupation, the homeownership rate among small multifamily buildings is initially *higher* than the rate of condo ownership (Figure 3). Further, the homeownership rate of housing in the AHS survey moves inversely with the increase in condo ownership among very small buildings. A cross-tabulation of condo ownership and owner-occupation reveals that 26% of owner-occupied units in multifamily buildings are *not* in condominiums. We interpret these non-condo owner-occupiers as landlord-occupants, who occupy one unit within a small multifamily building and rent the remainder. This view is corroborated by the fact that 96% of these non-condo owner-occupiers are found in buildings with 12 or fewer units. If this interpretation is correct – it is difficult to imagine alternatives – then the owner-occupation of these units is entirely consistent with the model developed below. These owner-occupiers, like single family homeowners, avoid costly third party management by directly supplying management services to the building.

Literature on housing structures is sparse. Glaeser (2011*b,a*) and Glaeser and Shapiro (2003) somewhat informally argue that maintenance and upkeep are building-specific (as opposed to unit-specific) issues and so the problems of decentralized ownership of buildings makes condominiums and cooperatives difficult to manage. These difficulties make it more efficient to put those decisions in the hands of a single owner—a landlord—and make multifamily buildings rentals. However, this casual theorizing fails to explain the observed rise in the rate of condominium ownership among larger multifamily buildings. Linneman (1985)

discusses a trade-off between the costs of monitoring a landlord on one hand, and the value of having a landlord solve the free-riding problem, on the other. These trade-offs, in addition to economies of scale for landlords, are assumed to influence the relative productivity of landlord versus owner management in ways that are unmodeled in his investigation of tenure. Williams (1993) proposes a model of multifamily structure ownership which trades off the rental externality described in Henderson and Ioannides (1983) against economies of scale provided by landlords. This model again fails to explain the non-monotonic relationship between homeownership and building size, a fact that the author acknowledges. Ambrose and Goetzmann (1998), Turner (2003) and Hilber (2004) in varying ways find that the locational characteristics of buildings matter; where and when ownership is risky, homeownership by individual households is less likely. None of these papers comes to grips with the basic ownership-building size correlation.

Unlike Linneman (1985) and Williams (1993), we argue that economies of scale are not confined to rental buildings based on suggestive evidence from condominiums. Using data from the state of Massachusetts in 2009, Figure 4 plots the propensity of condo associations to employ professional management services as a function of the number of units in the building.<sup>4</sup> The rapid rise of this probability suggests that smaller buildings do not have sufficient scale to overcome the fixed costs of bringing in outside management services. Importantly, and even though exact services provided may differ, we expect that both joint owners and sole owners of buildings face similar scale economies in management costs. In the model that follows, we separately incorporate economies of scale for third party management, the agency costs from hiring managers, and the free-rider problem among co-investors.

### 3 Model

Our model is a basic one period, two date portfolio model in which an investor with initial wealth  $w_1$  chooses between a risk free asset and a risky housing asset at time one in order to maximize expected wealth (realized at time two). In the base model, housing investment is in a single building, or a fraction thereof, so that we may study the implications of building governance when there are multiple investors. Figure 5 sets out the simple timeline for our model. In addition to choosing the level of housing investment, investors choose the terms of a contract with third-party management at time one. Investors also participate in building management. Management effort by both investors and third-party management is exerted

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<sup>4</sup>This figure displays the smoothed rate at which buildings of different sizes are professionally managed as revealed by condo listings on MLS PIN in the state of Massachusetts during 2009.  $N = 53,303$  listings with non-missing information about the number of units in the building and whether the building is professionally managed (either by on-site or off-site management).



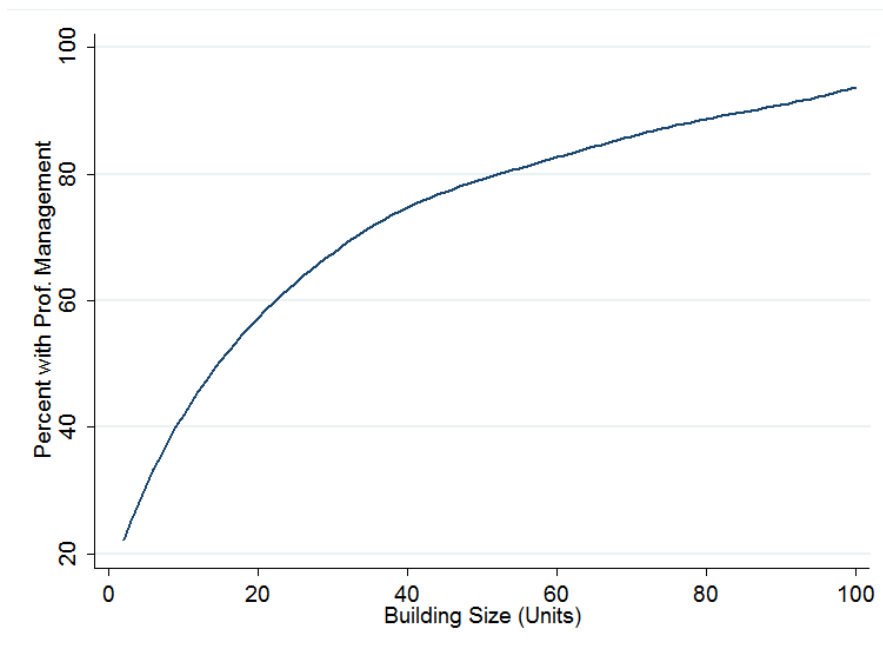


Figure 4: Professional Management in Condominiums

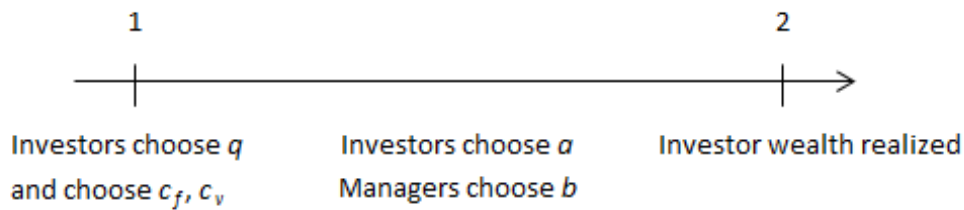


Figure 5: Model Timeline

after the investment is made, between time one and time two.

Taking the number of co-investors  $l$  in a building as given, the investor's indirect utility over wealth is given by

$$U(w_1, l) = \max_{q, c_f, c_v} E[v(\tilde{w})] \quad (1)$$

where  $v(\cdot)$  is the investor's utility function,  $\tilde{w}$  is uncertain time two wealth, and  $E$  is the expectations operator. Investors are price-takers who choose an amount of risky housing investment  $q$ , and must also choose a fixed and variable component of the manager's contract,  $c_f$  and  $c_v$ , respectively. The investor's optimization problem is subject to a third-party manager participation constraint, which is described below.

Investors place the balance of their initial wealth endowment  $w_1$  in risk free savings at the rate  $r$ .  $P$  is the price of a unit of housing services at time one and  $R$  is the periodic rent from a unit of housing services that is paid by a tenant at time one for the ensuing period. The net cost of investment in housing is therefore  $P - R$ . While we initially abstract from consideration of whether the housing investment is owner-occupied,  $R$  can be interpreted as imputed rent in the case of owner-occupation. Housing investment and building size are both measured in units of housing services, which comprises both a quantity and quality measure. Ownership shares of multiple investors within a single residential building are equal (although ownership shares can vary between buildings). This ensures that buildings have homogenous investors who will choose equal amounts of maintenance effort and third-party services.<sup>5</sup> We characterize building size as  $n = ql$  where  $l \geq 1$  represents the number of building investors.<sup>6</sup>

We assume that management and maintenance of a building is required in order to produce a flow of housing services to occupants (who are either the investors themselves or their tenants). Because we are interested in building-level issues created by different forms of building ownership, we set aside consideration of how a particular residence within a building is maintained. In their model of housing tenure, Henderson and Ioannides (1983) focus on maintenance externalities resulting from the utilization of a particular residence. In contrast, we focus on the administration of building services, and the maintenance of building systems and common areas, like lobbies, hallways and elevators. Knapp (1991) also examines demand for maintenance, but focuses on the transfer of control between developers and homeowners' associations. Our model highlights the sharing of management between investors who form the homeowners' association and third-party managers.

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<sup>5</sup>Likewise, Knapp (1991) assumes homogeneity of owner demand within associations due to the expectation that households stratify themselves into homogenous groups.

<sup>6</sup>In the optimization problem that follows, allowing building size to change with changes in investment ensures that  $q \leq n$ .

Management effort may come from both building investors and managers. Investors choose the terms of a contract with third party management ( $c_f, c_v$ ) for the building at time one. After time one (but before time two), each investor  $j$  in a building non-cooperatively chooses effort  $a_j$  that is contributed to building management. Managers also choose effort  $b$  for a particular building between time one and two. The two sources of effort are perfect substitutes and impact property value independently. Management effort exerted in one period is assumed to influence the flow of housing services in the next period. The benefits of effort in period one for housing services in a future period are capitalized into housing value at time two.

Building value at time two *per unit of housing services* is  $P + 2\phi_a \sqrt{\sum_i a_i} + (2\phi_b \sqrt{b} + \tilde{\varepsilon}) + \tilde{u}$ , where  $\phi_a$  and  $\phi_b$  are the marginal product of investor and manager effort, and  $\tilde{\varepsilon}$  and  $\tilde{u}$  are normally distributed noise with mean 0 and variance  $\sigma_\varepsilon^2$  and  $\sigma_u^2$ , respectively. Each investor's property value benefits from the collective effort of all building investors and from manager effort. The cost of a unit of investor effort is linear in both effort and building size, while building value is a concave function of effort.<sup>7</sup>

Manager effort is private information that is not observed by building investors. Rather, investors observe its impact on property value attributed to management, which incorporates the output of management effort along with a random component equal to  $2\phi_b \sqrt{b} + \tilde{\varepsilon}$ . Managers incur total cost of effort equal to  $b(d + \alpha n^2)$  which is linear in effort. This specification of costs allows for the average cost of effort per square foot of building to be u-shaped under certain parameter constraints. Therefore, there is an economy of scale in building size with respect to manager costs. Managers have constant absolute risk aversion (CARA) risk preferences represented by a negative exponential utility function with additively separable cost of effort,  $V(g, n) = -e^{-\rho(g - b(d + \alpha n^2))}$ , where  $\rho$  is the manager's coefficient of absolute risk aversion and  $g$  is her compensation. Investors and a third party manager write a linear contract of the form  $g = n \left( c_f + c_v \left( 2\phi_b \sqrt{b} + \tilde{\varepsilon} \right) \right)$ . Below, we derive the manager's certainty equivalent value of wealth  $\hat{z}$  and denote the manager's opportunity cost per unit of housing services as  $\bar{z}$ .

Second period wealth for investor  $j$  is

$$\tilde{w} = (w_1 - q(P - R))(1 + r) + q \left( P + 2\phi_a \sqrt{\sum_i a_i} + (1 - c_v) \left( 2\phi_b \sqrt{b} + \tilde{\varepsilon} \right) + \tilde{u} \right) - qc_f - a_j en \quad (2)$$

where the summation with respect to  $a_i$  is over  $l$  co-investors in a particular building. Notice that the variable payment to a manager reduces investor realization of value from third-party

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<sup>7</sup>We follow Edmans and Manso (2011) in choosing this functional form. In particular, this set-up ensures a constant technology between sole investors and joint investors in our model.

management by the fraction  $c_v$ . Investors bear a personal cost of management effort equal to  $a_j en$  but only benefit from their own effort to the extent of their ownership share. Therefore, investors under-invest in the management effort supplied to the building when  $l > 1$ .

Notice in (2) that each square foot of the building benefits from the total contributions of investor effort ( $\sum_i a_i$ ). In specifying the cost per unit of management effort contributed by each investor, we require the cost to increase linearly with building size ( $en$ ), and there is no economy of scale in investor effort. We do this in order to separately focus on free-riding among investor-owners and economies of scale in the context of third party management, with no loss of generality. In Appendix A, however, where we allow investors to own more than one building, we require  $a$  to be supplied separately to each building in a portfolio. Therefore, because investor effort must be duplicated across buildings, there is a cost advantage to owning a single large building as compared to an equal scale investment distributed across multiple buildings. In the absence of such an assumption, a single investor could supply effort to an infinite number of buildings. Within a building, the functional form of the production function transforming investor effort into value serves to put an upper bound on effort.

We assume that there is an open city with a fixed supply of residential buildings. Under the assumption that buildings are sold to the highest bidder, bidding by investors of different wealth, and under alternate ownership structures, determines market prices for multifamily buildings of different size as well as equilibrium ownership arrangements for these buildings. In the next sections we develop the analytic results of the model, and then numerically solve the model to provide additional insights.

### 3.1 Management Effort

In order to find investor bid prices, we first need to derive the optimal effort of investors, conditional on investment level and building size. We also derive manager effort as a function of expected compensation and building size.

#### 3.1.1 Investor Effort

Investor  $j$  takes the effort of all other investors within the same building as given and chooses effort to maximize (2). Due to the nature of joint production with other investors in the same building, investor  $j$ 's choice identifies the optimal level of effort for the entire building:

$$\frac{\phi_a}{el} = \sqrt{\sum_i a_i}. \quad (3)$$

If we assume a symmetric equilibrium (although asymmetric equilibria are feasible), the

optimal contribution of the individual investor is:

$$\frac{\phi_a^2}{e^2 l^3} = a_j. \quad (4)$$

Because investors bear the full cost of personal effort, but only obtain benefits for  $q < n$  units of housing services, they under-supply effort when  $l > 1$ . Notice that if an investor owns the whole building, the total effort supplied is  $\frac{\phi_a^2}{e^2}$  as compared to total effort of  $\frac{\phi_a^2}{e^2 l^2}$  among  $l$  co-investors. The difference between these levels widens as the number of building co-investors increases.

### 3.1.2 Manager Effort

The manager's expected utility is  $E \left[ -\exp \left( -\rho \left( e_f n + c_v n \left( 2\phi_b \sqrt{b} + \tilde{\varepsilon} \right) - b(d + \alpha n^2) \right) \right) \right]$ . Using the properties of the negative exponential utility function, we re-write the expected utility as

$$V(g, n) = -\exp \left( -\rho \left( c_f n + 2c_v n \phi_b \sqrt{b} - b(d + \alpha n^2) - \frac{\rho}{2} c_v^2 n^2 \sigma^2 \right) \right). \quad (5)$$

The manager's optimal choice of effort is given by

$$\frac{c_v^2 q^2 l^2 \phi_b^2}{(d + \alpha q^2 l^2)^2} = b. \quad (6)$$

Substituting, the utility of the manager can be expressed as  $V(g, n) = -\exp(-\rho \hat{z})$ , where

$$\hat{z} = c_f q l + q^2 l^2 \frac{(c_v \phi_b - \alpha q l)^2}{4d} - \frac{\rho}{2} (c_v q l \phi_b \sigma)^2 \quad (7)$$

is the certainty equivalent value of manager wealth.

## 3.2 Investment Choice

Substituting the definitions of  $a$  and  $b$  into (2), the investor's objective function in terms of three choice variables is

$$\max_{q, c_v, c_f} E[v(\tilde{w})] \quad (8)$$

subject to

$$\hat{z} = \bar{z}n. \quad (9)$$

We obtain four first order conditions from which we are able to simplify the system to

two equations in  $q$  and  $c_v$ . These are:

$$R(1+r) - rP - \bar{z} + \phi_a^2 \frac{2l-1}{el^2} + \frac{2ql\phi_b^2}{(d+l^2q^2\alpha)^2} (d + \alpha q^2 l^2 (1-c_v)^2) - \rho c_v q l \sigma^2 + \frac{E[v'\tilde{u}]}{E[v']} = 0, \quad (10)$$

and

$$(1-c_v) \frac{2ql\phi_b^2}{d + \alpha n^2} - \rho c_v q l \sigma^2 - \frac{E[v'\tilde{\varepsilon}]}{E[v']} = 0. \quad (11)$$

## 4 Investor Prices

To identify the slope of an investor's bid function with respect to the number of building investors, we invoke the envelope theorem and differentiate (1) with respect to  $l$ , holding all choice variables at their optimum. In order that investor utility remain unchanged, bid prices (per unit of housing services) are assumed to vary with  $l$ . This exercise yields:

$$\frac{\partial P(l)}{\partial l} = \frac{1}{r} \left( c_v \phi_b^2 (2-c_v) \frac{d - \alpha q^2 l^2}{(d + \alpha q^2 l^2)^2} - \frac{2}{el^3} \phi_a^2 (l-1) - \frac{\rho}{2} c_v^2 \sigma^2 \right). \quad (12)$$

It is difficult to sign this equation. The first term in parentheses on the right hand side of (12) represents a potential benefit of additional co-investors. Notice that by construction,  $d - \alpha n^2 \geq 0$  at building sizes no larger than the cost-minimizing size. For building sizes less than the cost-minimizing size, additional investors increase building size, thereby lowering the average costs of third party management. The next term in parentheses, which is equal to zero at  $l = 1$ , represents a price discount resulting from the free-rider problem for  $l > 1$ . The last term on the on the right hand side of (12) indicates that managers require greater compensation when building size increases (because they bear more total risk, holding  $c_v$  constant). Clearly, for large enough buildings the impact of an additional investor on bid prices is negative. Below the cost-minimizing size, utility, and therefore bid prices, may be increasing or decreasing.

In Appendix A, we extend the base model to the case of sole ownership. To uncover bid prices by wealthy investors for buildings that are smaller than their total investment demand, we must allow these investors to hold a portfolio of small buildings. While the cost of effort is linear in the size of the investor's portfolio, ownership of multiple buildings requires a duplication of effort for each building. Therefore, the costs of effort increases in the number of buildings owned. From this exercise we derive the slope of the sole investor's

bid function as building size increases,

$$\frac{\partial P}{\partial n} = \frac{1}{r} \left( c_v \phi_b^2 (2 - c_v) \frac{d - \alpha n^2}{(d + \alpha n^2)^2} - \frac{\rho}{2} c_v^2 \sigma^2 \right). \quad (13)$$

Compared to (12), recall that building size is  $n = ql$  in the joint ownership model. The remaining difference, once building size is accounted for, is the second term in parentheses in (12). This term quantifies the marginal discount for the free-rider problem. In the limit, however, this term goes to zero as the number of co-investors increases. Because the marginal impact of the free-rider problem dissipates at larger sizes, the wealth of the marginal investor will depend on the distribution of wealth in the market for large multifamily buildings that are jointly owned.

One of the main insights of this model is that among single family and small multifamily properties, sole ownership of buildings impacts value in two ways. Most obviously, sole ownership allows investors to avoid the free-rider problem. Second, however, because sole investors supply greater levels of investor management effort, they are able to substitute away from high third-party management costs in the context of small building. Empirically, the latter point is consistent with the extent of owner-occupation in single family and small multifamily housing – investors minimize management costs by living on site and directly supplying management effort when third-party management costs are high.

The other main result of the model is that the marginal impact of the free-rider problem associated with joint ownership arrangements, like condominiums, dissipates with building size. Larger buildings, for reasons of individual investor financial constraints and a desire for diversification, are more likely to be jointly owned regardless of free-riding issues. If larger buildings are more operationally efficient (for third-party managers), then wealthier joint investors will outbid less wealthy joint investors for large buildings.

## 5 Numerical Solutions

Analytically, we have derived a non-linear bid price curve for housing investors as a function of the number of building investors and initial investor wealth. In this section, we parameterize the model in order to arrive at numerical solutions for the model's choice variables. For an investor with initial wealth  $w_1$ , the expected utility from sole ownership of a building is  $\bar{U}(w_1, 1)$ . Requiring the investor to have utility of at least  $\bar{U}(w_1, 1)$ , we derive bids for partial ownership ( $q < n$ ) of increasing larger buildings. To compare bid prices under joint ownership arrangements to bids from investors who intend to be sole owners, we modify the base model to incorporate sole ownership of multiple buildings in Appendix A. This

formulation allows us to derive the bid prices for buildings of different size by letting a sole investor hold a portfolio of buildings. We consider a sequence of portfolios, starting with multiple single family houses, and each subsequent portfolio has fewer and larger buildings until the portfolio consists of just one building. Using these portfolios, we construct bid prices by holding the sole investor’s indirect utility equal to that obtained in the single family portfolio. We anchor housing prices with the average per square foot single family home price in the 2011 American Housing Survey, which is approximately \$94. For investor utility, we adopt a constant relative risk aversion form with  $E[v(\tilde{w})] = E\left[\frac{\tilde{w}^{1-\gamma}}{1-\gamma}\right]$ .

## 5.1 Initial Assumptions

We use with the following assumptions. First, we interpret units of housing services as square feet of space. Productivity parameters translating investor and manager effort into housing value are set to 0.5. The (real) risk free rate is equal to 2%. Manager opportunity cost ( $\bar{z}$ ) is set to \$0.15 per square foot of building size. The owner effort cost parameter is set to  $e = .3$  so that the optimal choice of  $a$  at  $l = 1$  for the average wealth investor produces a gross value of investor effort equal to approximately 3% of single family house value. Manager cost parameters  $\alpha$  and  $d$  are set so that the cost minimizing building size is 240,000 square feet (approximately, a 200 to 250 unit multifamily building). This size is a current industry norm for efficient operations of a multifamily rental building. The parameters are  $\alpha = 1.32e - 8$  and  $d = 750$ . Based on these assumptions, the total cost of third party management approximates evidence from internet searches about management fees per residence.

Investors’ utility shape parameter is  $\gamma = 3$ . Manager coefficient of absolute risk aversion is  $\rho = .5$ . Variances of the two noise parameters are initially set relative to the scale of manager output at the cost-minimizing building size and single family house prices, respectively. They are  $\sigma_\varepsilon^2 = 0.16$  and  $\sigma_u^2 = 12$ .

## 5.2 Results

Figure 6 depicts bid prices for an average investor with \$95,000 in initial wealth. This investor is assumed to be either a single family owner ( $l = 1$ ) or a condo owner ( $l > 1$ ) and is not allowed to own multiple buildings. Given our assumptions about parameters, this investor demands a single family home of approximately 1,200 square feet given a price of \$94 per square foot. Bid prices for condominium investments in buildings of increasing size are then solved for, holding the level of indirect utility derived from the single family exercise constant. Underlying these bid prices, investor effort declines dramatically with



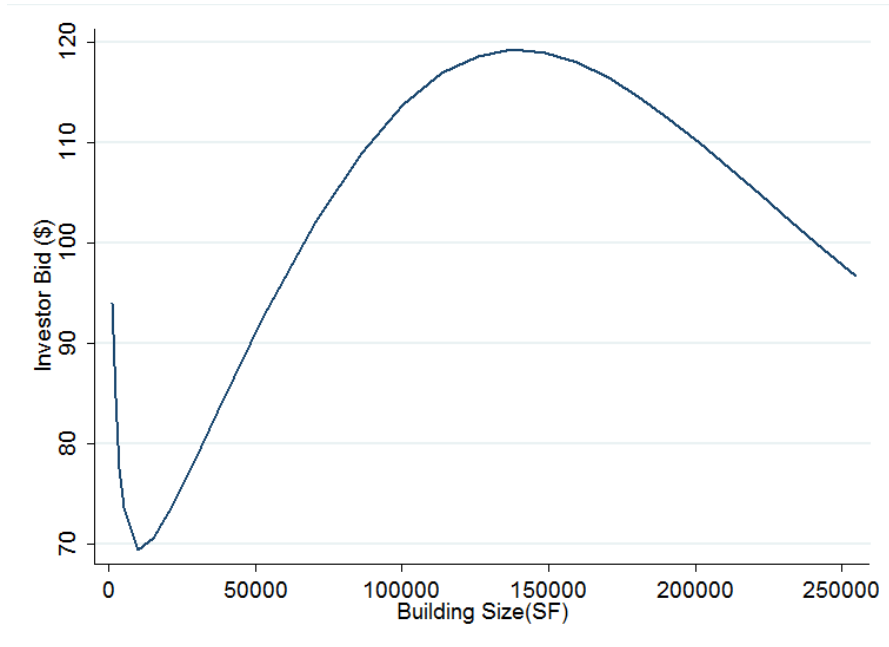


Figure 6: Investor Bids with Initial Wealth of \$95,000

the addition of co-investors, due to the free rider problem (left panel of Figure 7). Manager effort, on the other hand, increases non-linearly in the right-hand panel of Figure 7.

For households with initial wealth of more than \$95,000, we estimate the value of indirect utility from owning more than one single family house where each house is 1,200 square feet in size with an assumed price of \$94 per square foot. We then calculate the bid price for these same wealthy investors as they own fewer buildings of larger size, at that constant level of utility. Eventually, we produce a bid price for sole ownership of just one building. We then assume that the wealthy investor co-invests in a larger building by partnering with other investors. Therefore, ownership transitions to a joint ownership arrangement and the wealthy are subject to the same free-rider problems as less wealthy investors. The main difference is that the building size at which wealthier investors require partners is larger.

In Figure 8, we graph bid prices (per square foot) for initial investor wealth of \$95,000 and \$240,000. At the left of the graph, the investor with \$240,000 in initial wealth holds an optimized portfolio of single family houses that cost \$94 per square foot. The initial peak in this investor's bid prices comes at the point where the investor is a sole owner of a 2,900 square foot building. To the right of this peak, the investor becomes a joint owner of increasingly larger buildings, and the slope of the bid function eventually mirrors that of the investor with \$95,000 in initial wealth.

In Figure 9, we depict the bid prices for buildings of increasing size for investors with \$95,000, \$240,000, \$500,000, and \$1 million, respectively. Among investors with initial wealth

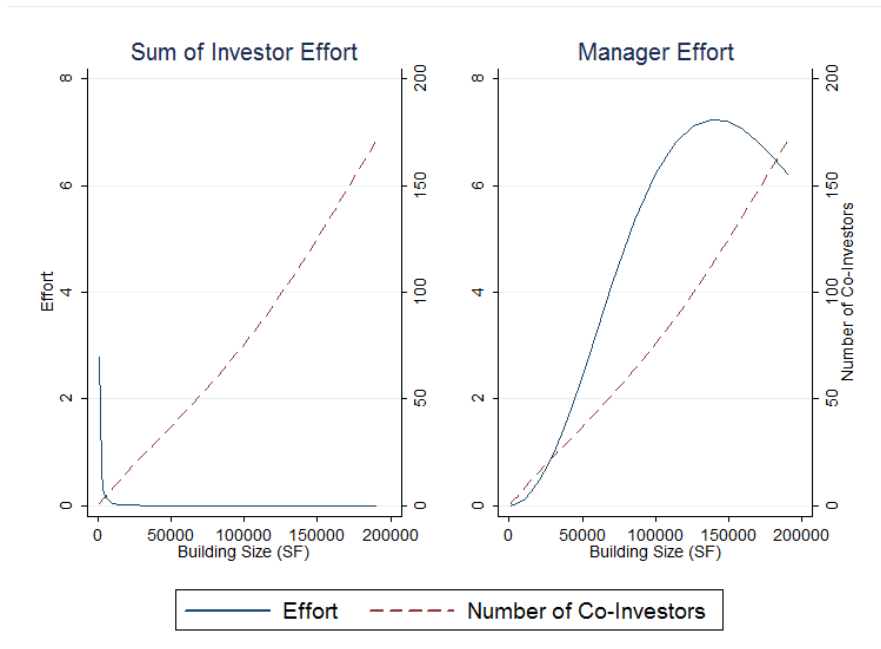


Figure 7: Investor and Manager Effort

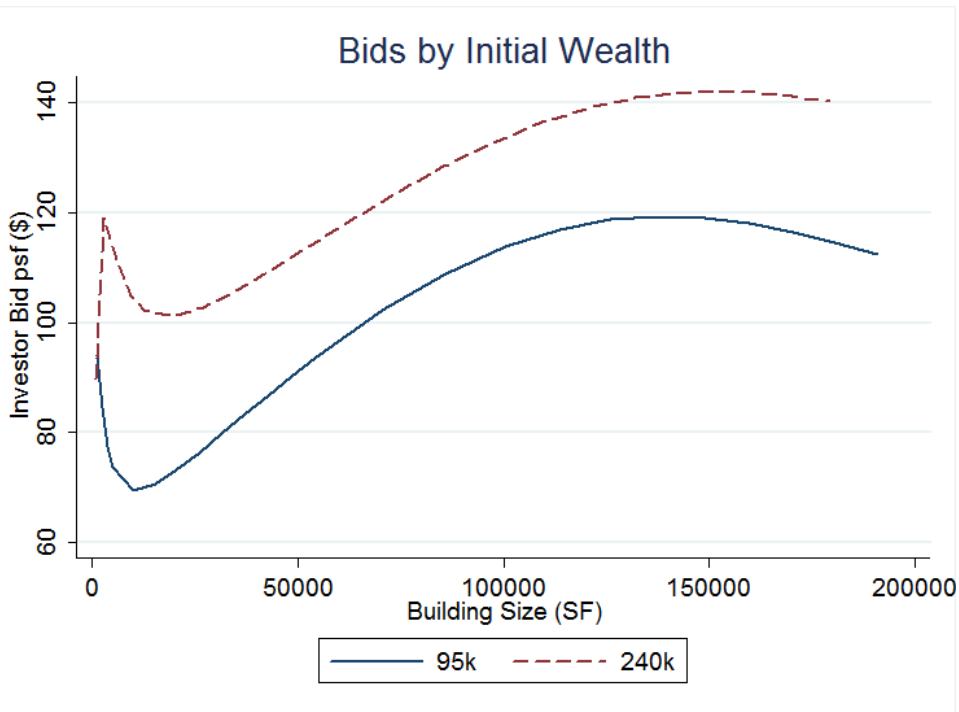


Figure 8: Investor Bids by Building Size

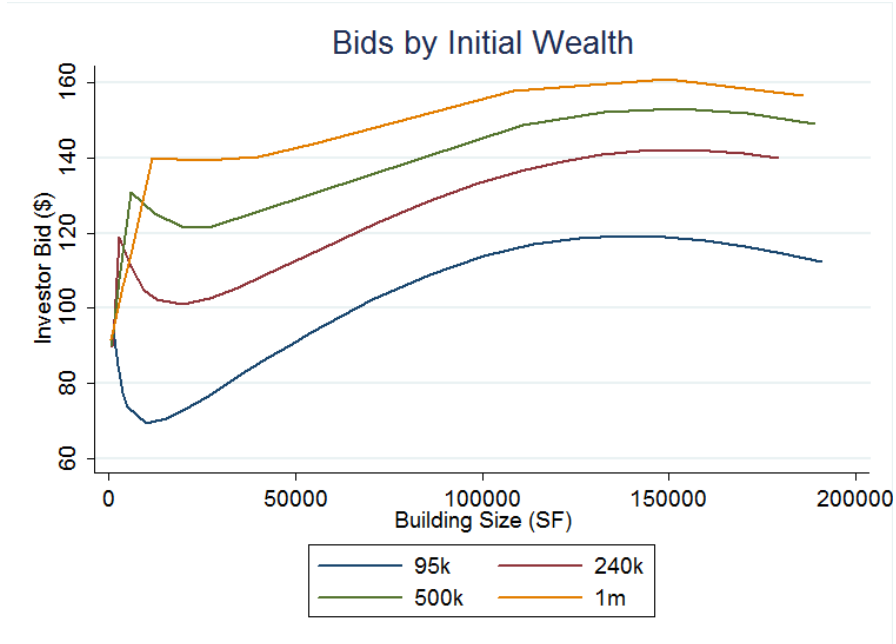


Figure 9: Investor Bids by Initial Wealth and Building Size

above \$95,000, the less wealthy have the steeper, upward sloping bid curves along the portion of the curve where they bid for sole ownership of buildings. This implies that bidding results in a domination of joint ownership arrangements by sole ownership, and sole owners sort by wealth into buildings of increasing size. Because sole owners outbid condo owners in particular, rental tenure should dominate owner-occupation in smaller multifamily buildings.

Overall, the analytic results and our numerical solutions fit the basic patterns of ownership that we observe in Figures 1 and 2. In the next section, we test the model’s finding that among joint ownership arrangements, the wealthy will outbid less wealthy investors for more optimally sized buildings. We do this by recovering the bid functions of condominium owners surveyed in the AHS.

## 6 Empirical Models

The theoretical model in the previous section suggests that investor demand for properties in which they perforce share ownership is likely to be affected by the number of co-investors. For small buildings, low-wealth investors are likely to lower bids (per square foot) as the number of co-investors rises because of the increasing level of free-riding that will take place. However, as the scale of the building increases, the ability of the partners to increasingly elicit effort from an outside manager overcomes the free rider problem, and bids begin to rise. The simulations suggested that these bids would flatten out and even decline at large

enough scale. The size of the bid is also, naturally enough, sensitive to investor wealth. The simulated bids suggested that for wealthier investors, the effect of the free rider problem is ameliorated by scale due the size of buildings in which the highest wealth investors become co-investors.

In order to do estimate bid functions, we need data on household wealth, household demographic variables, individual condominium prices, and structural attributes of the condominiums, particularly including the floorspace and the number of units in the condo building. All of these, save wealth, are available in the American Housing Survey, and this is our primary source of data. Other surveys calculate household wealth, but are short on the detailed data needed to estimate hedonic price functions for condo prices. Our resolution of this quandary is to use the Panel Survey of Income Dynamics (PSID) to estimate the determinants of wealth as a function of household characteristics, and use this regression function to estimate wealth for the households surveyed in the American Housing Survey. This is the first step in our empirical procedure.

The second step should have been to estimate the hedonic price function for condos – that is, a regression function that maps condo structural and locational attributes into condo prices. However, we are faced with two selectivity issues. The theoretical model implies that for a multifamily structure, the equilibrium ownership of the building is not random and stresses that the number of co-investors is influential in this decision. Other quality dimensions play a role as well. The empirical literature on homeownership (Hanson, 2012) speaks most directly to this point. It stresses that tax incentives such as the home mortgage interest deduction lead homebuyers to choose higher quality homes, in turn suggesting that buildings of higher quality are more likely to be condos. Quality is only partially observable, therefore the unobserved factors that influence the choice to become condo are correlated with the unobserved attributes that create the price. In other words it is necessary, in the hedonic model, to control for the selective nature of the condo sample. Secondly, as we outline below, the AHS surveys housing units, but of necessity, the interview subjects are the residents of the housing units. For about half of the condominium records in the sample, about half are owner-occupiers, and half are renters. The value of the condo investment (along with condo fees, if any) is reported only in the case that the interviewee is the owner-occupier. But the decision to be an owner-occupier is influenced by the same unobservable factors (albeit with different weights) as was the decision for the building to be organized as a condo in the first place.

Therefore, the estimation of the hedonic price model for condos consists of three stages. The first stage chooses between joint ownership through condominium governance and sole ownership by a landlord who rents the building’s units. The building developer sells to the

type of buyer that generates the highest bid. We do not observe the bids, but we assume they are summarized by the linear index

$$I_1^* = X_1\beta_1 + e_1 \quad (14)$$

where the index for observation number is suppressed.  $I_1^*$  is the net profit from condo organization relative to apartment organization,  $X_1$  is a vector of structural and locational characteristics, and  $e_1$  is the shock to relative profits encountered by the building developer. In the usual way, we do not observe  $I_1^*$  but only the decision, so we define  $I_1 = 1$  if the building is a condo, and  $I_1 = 0$  if not. Then

$$P(I_1 = 1) = P(e_1 > -X_1\beta_1) \quad (15)$$

and on the assumption that  $e_1$  is normally distributed,

$$P(I_1 = 1) = 1 - \Phi(-X_1\beta_1) = \Phi(X_1\beta_1) \quad (16)$$

where  $\Phi$  is the normal cumulative distribution function. Estimation of stage 1 can proceed as a normal probit.

Stage 2 asks, given condo organization, whether a unit with specified characteristics will be owner-occupied or rented by unit's owner. Again, there are unobserved benefits and costs accruing to each decision, summarized in a linear index:

$$I_2^* = X_2\beta_2 + e_2 \quad (17)$$

where we define the observable decision as  $I_2 = 1$  if the unit is rented and  $I_2 = 0$  if not. However, the owner-occupation decision is of necessity conditioned on the decision that the building be organized as a condo. Thus there is a selectivity issue if the unobservables that inform the second decision are correlated with those of the first. This suggests the joint estimation of stage 1 and 2 by maximizing the log likelihood function

$$\log L = \sum_{I_1=0} \Phi(X_1\beta_1) + \sum_{I_1=1, I_2=0} \Phi_2(X_1\beta_1, -X_2\beta_2, e_{12}) + \sum_{I_1=1, I_2=1} \Phi_2(X_1\beta_1, X_2\beta_2, \rho_{12}) \quad (18)$$

where  $\Phi_2$  is the bivariate normal cumulative distribution function and  $\rho_{ij}$  is generically the correlation coefficient of  $e_i$  and  $e_j$  (Poirier, 1980).

Turning now to the property values equation, we propose a standard hedonic equation

of the form

$$\log value = X_3\beta_3 + e_3 \quad (19)$$

we note that there is also a selection issue here, since we only observe values in the case where the building is condo, and the owner elects to owner-occupy. Using identical reasoning as above, we have

$$E(e_3|I_1 = 1, I_2 = 0) = \rho_{13}\xi_1 + \rho_{23}\xi_2 \quad (20)$$

with

$$\xi_1 = \frac{\phi(X_1\beta_1)(\Phi(-X_2\beta_2))}{\Phi_2(X_1\beta_1, -X_2\beta_2, \rho_{12})}, \quad (21)$$

and

$$\xi_2 = \frac{\phi(-X_2\beta_2)(\Phi(X_1\beta_1))}{\Phi_2(X_1\beta_1, -X_2\beta_2, \rho_{12})} \quad (22)$$

as in Lahiri and Song (2005) and Hotchkiss and Pitts (2005). The variables  $\xi_1$ , and  $\xi_2$  can be consistently estimated upon obtaining the parameter estimates from (18). Consistent estimates of value in the face of these two selection problems can be obtained through the least squares regression

$$\log value = X_3\beta_3 + \rho_{13}\xi_1 + \rho_{23}\xi_2 + e_3^* \quad (23)$$

on the owner-occupied condos only.

Having obtained consistent estimates of the value function for condominiums, our next step is to derive the bid functions that underlie it. As noted in the hedonic literature (e.g. Rosen (1974) and Epple (1987)) the hedonic function is the upper envelope of bids from different segments of the heterogeneous pool of demanders. If the heterogeneity is due to resource constraints – characterized here as wealth, but in Rosen as income – then normality and concavity of the utility function ensures a single crossing to any pair of bids, and a matching between quality – here characterized as square footage – and wealth. The bid function, the data analogue to the curves calculated in Figures 5 and 6 above, is a function that maps demographic and resource characteristics of the (successful) bidder, along with the structural characteristics, into the marginal price of the characteristic. That is, for some characteristic  $X^j$  we write the bid function as

$$\frac{\partial V}{\partial X^j} = Z\omega + X_3\tau + e_4 \quad (24)$$

where the dependent variable is the derivative of the hedonic function with respect to the characteristic – the marginal bid for a unit of that characteristic.  $Z$  is a vector of personal

characteristics. Rosen suggested that the estimation of what is in effect a Hicksian demand function is subject to the same kind of endogeneity bias that "ordinary" supply and demand estimation suffers from. In housing market applications, it is reasonable, however to assume (and we do so here) that housing supply is exogenous.

Nevertheless, Bartik (1987) and Epple (1987) note that another kind of simultaneity is present. The hedonic function is by design nonlinear in the characteristics. It must be in order for there to be variation in the dependent variable of (24). The marginal price and quantity of the attribute are simultaneously chosen. If shifts in the error term are caused by unobserved taste differences across consumers, then those shifts which (conditional on  $Z$ ) cause the choice of bid price, are correlated with the characteristic quantity on the right hand side of the equation. In short, because price and quantity are chosen jointly, quantity is endogenous. To consistently estimate the bid parameters, instruments are needed. Bartik (1987) notes that the instruments must be correlated with  $X$ , but uncorrelated with tastes, and variables that shift the budget constraint are therefore valid instruments. The particular implementation that is often used (Bartik, 1987; Coulson and Bond, 1990) is to allow the hedonic function to vary across (geographic) markets. The assumption is that hedonic variation is due to supply constraints and not differences in the distribution of unobservable tastes. Then market-specific variables – market binaries, for instance, but also these binaries interacted with  $Z$  – can serve as instruments. As Bartik (1987) notes, non-housing expenditure (or wealth) is an appropriate member of  $Z$ , which implies that total resources (i.e. total wealth) would be an appropriate instrument, when interacted with regional binary variables. We follow this procedure below, and estimate proxies for both total and non-housing wealth from the PSID data.

## 7 Empirical Results

In this section we present the three-stage estimation of the hedonic price function for condominium units. The first stage, recall, estimates the probability that the building in which the housing unit is located is jointly owned using condominium governance or solely owned. The second step estimates the probability that the specified unit, conditional on it being a condo, is owner-occupied (such that the value is observed). These two steps are estimated jointly in a maximum likelihood framework. The third step is estimating the hedonic function itself, conditional on the two selection criteria being fulfilled. While fully efficient estimates are realized only if the third step is estimated jointly with the first two, consistent estimates are possible in a two stage procedure, where the second stage merely adds the appropriate Mills ratios to the hedonic model.

<b>Housing Units</b>		
Total Records	186,448	
Mobile Homes, etc.	-5,586	
Public housing	-4,217	
Missing data	-66,513	
Sample Housing Units	110,132	

	<b>Single family</b>	<b>Multifamily</b>
Total Housing Units	83,077	27,055
Owner-Occupied	68,890 (83%)	3,324 (12%)
Units in Solely-Owned Bldg		22,154
Units in Condo Bldg		4,901
Owner-Occupied		2,427 (50%)

Table 1: 2011 AHS Sample

## 7.1 Data

Our data source is the 2011 American Housing Survey national sample. The AHS is a biennial survey of housing units and occupants conducted by the US Department of Housing and Urban Affairs. Table 1 outlines some initial facts about the survey. There are 186,448 housing units surveyed. We eliminate those for which some basic information is not available, particularly tenure status, structural status (single or multifamily) or key structural characteristics. We also eliminate mobile homes and public housing. About 75,000 units' records were set aside, primarily because the building was not a permanent structure (i.e. mobile home), not a "typical" housing unit (e.g. group quarters), the unit was vacant, or a household member was not available to interview. Of the remaining 110,132 observations, the table indicates that just over 27,000 (25%) are in multifamily buildings. Of these 4,900 (18%) are condominium units, of which half are occupied by their owners, and the others rented to other parties. Presumably, the solely-owned multifamily buildings are renter-occupied. The difference between owner-occupied units in multifamily buildings and owner-occupied condo units represents almost 900 owner-occupier-landlords, who solely-own a multifamily building, occupy one unit and rent the remainder. Our interpretation is based on the fact that 65% of these owner-occupied units in non-condo buildings are located in duplexes, and 96% are found in buildings with 12 or fewer units. It seems plausible that owners would also be managers in such small buildings. Overall, the descriptive statistics exhibited here are similar to what has been found in the US Census.

Table 2 lists, for units in multifamily structures, means and standard deviations, stratified by three main tenure-ownership groups. In the first panel, the data summary is presented



	<b>Rental Bldg Unit</b>		<b>Condo Rental</b>		<b>Condo Own-Occ.</b>	
	<b>(N = 22,154)</b>		<b>(N = 2467)</b>		<b>(N = 2434)</b>	
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
Number Units	29	64	39	84	43	89
Unit SF	917	841	1000	765	1349	1176
Baths	1.19	0.43	1.35	0.51	1.59	0.58
Half Baths	0.11	0.35	0.19	0.45	0.29	0.51
Fireplace	0.12	0.32	0.21	0.41	0.35	0.48
Porch	0.67	0.47	0.76	0.43	0.84	0.37
A/C	0.52	0.5	0.65	0.48	0.67	0.47
Garage	0.33	0.48	0.47	0.55	0.68	0.47
Yr. Built	1966	25	1974	21	1975	23
Central City	0.46	0.5	0.42	0.49	0.44	0.5
MSA, not CC	0.49	0.5	0.52	0.5	0.52	0.5

Table 2: Sample Statistics for Units in Multifamily Buildings, 2011 AHS

for units in solely owned buildings, in the second rentals in condo buildings and in the third, owner-occupied condos. The most obvious takeaway is that there are quality differences, sometimes substantial ones, across these various ownership arrangements. In particular, both types of condo units are larger, and embody more structural amenities, than rental units. Note also that condo units are, on average, newer, although this is partly due to the fact that in most states condominium and cooperative ownership arrangements were not permitted prior to the early 1960s. There are also notable differences between owner-occupied condos and rental condos, the latter being of lower quality than the former. These quality differences are expected, if only because of the greater tax advantages that higher quality units bring to owner-occupiers. These differences are important, since the observable quality differences may also herald unobservable quality differences which must be accounted for later.

A final element of data that we require is to impute wealth for our sample of AHS households. We estimate wealth functions using the Panel Survey of Income Dynamics that can be used to predict wealth for respondents to the AHS. The exact procedure is described in the Appendix C.

## 7.2 Hedonic Function Estimation

Turning now to the estimation of the three stage hedonic model, note that we first cull from the sample observations with unrealistic rents ( $< \$50$  per month) or values ( $< \$1000$ ). In both cases these are either properties with extremely low quality, not arms-length transactions, or transcription errors, so that it is appropriate to delete them. In estimating these models

it is useful to have "identifying variables" – i.e. variables that influence the choice of condo ownership that do not influence the decision to owner-occupy, and variables that influence the choice to owner-occupy but do not influence value (Lahiri and Song, 2005) . This can most easily be seen in the third stage estimation, where the bivariate Mills ratios are entered into the linear regression model of value. If the regressor set in each stage is similar, there can be collinearity issues between the Mills ratios and the determinants of value. There do not appear to be any valid exclusion restrictions that apply, however. Any characteristic that influences the choice to of the owner to occupy the unit (i.e. is of high quality) is likely to have influenced the decision to make the building condo in the first place, and is likely to have an influence on the asset value of the unit. We therefore take  $X_2$ ,  $X_3$ , and  $X_4$  to be identical, and rely on the nonlinear functional form to separately identify the coefficients of the characteristics and the Mills ratios.

The results of the first two (jointly estimated) stages are in Table 3. There are three notable results. The first result is that both condo and owner-occupied probability are strongly associated with unit quality. Almost every observable quality dimension has a coefficient that is both economically and statistically significant, although there are some deviations from this general rule. This confirms an earlier point, that the motives of housing consumers are influenced by the tax incentives to owner-occupation. Higher quality units are more likely to be condos, and owner-occupied condos. The second result is, corresponding to Figure 1, that the probability of a building being a condo is strongly, and nonlinearly, associated with the number of units. Figure 10 (which is a smoothed curve of the average predicted probability of condominium ownership across building sizes) shows a higher initial rate of condo ownership, followed by a significant drop in and then recovery of the smoothed, predicted probability of condo ownership according to building size. It must be noted, however, that the polynomial cannot imply a flattening of the probability profile over large unit counts, and that beyond 20 units or so, the implied probability rises with the unit count more strongly than that implied by Figure 1. Moving to the probability of owner-occupation, it is of substantial interest to observe that the probability of being owner-occupied is not a function of the number of units. This is highly suggestive: our theoretical model suggests that there are strong investment motives that govern the organization of building ownership. However, we have also noted in this section that the probability of condo ownership might also be influenced by tax incentives of housing consumers. If the unit count were a consumption motive for building organization (i.e. was indicative of higher quality) we would expect it to be a significant determinant of owner occupation, but it is not. Therefore the importance of unit count is strictly as it pertains to investment motives, as suggested by the theory. The third result is that, the test of correlation between the residuals of these two stages indicates

Dependent Variable:	Condo	Hown
No. Units	0.0043*** [0.000]	0.0005 [0.001]
Units Sq	-1.3E-05*** [0.000]	-2.04E-06 [0.000]
Units Cu	9.90E-09*** [0.000]	1.26E-09 [0.000]
Floor		0.0160** [0.007]
Baths	0.4059*** [0.021]	0.1702* [0.100]
Half Baths	0.3836*** [0.023]	0.0731 [0.078]
Fireplace	0.3827*** [0.026]	0.0725 [0.090]
Porch	0.1246*** [0.024]	0.1530** [0.065]
A/C	0.1355*** [0.023]	-0.016 [0.050]
Garage	0.4244*** [0.021]	0.3026** [0.125]
Unit SF	0.0001*** [0.000]	0.0001*** [0.000]
Yr Built	0.0027*** [0.000]	-0.0067*** [0.001]
Constant	-6.7298*** [0.952]	13.2685*** [1.857]
Observations	27,055	27,055

Coefficients reported; Standard errors in brackets

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

Table 3: Estimates of the Probability of Condo Ownership, Owner-Occupation

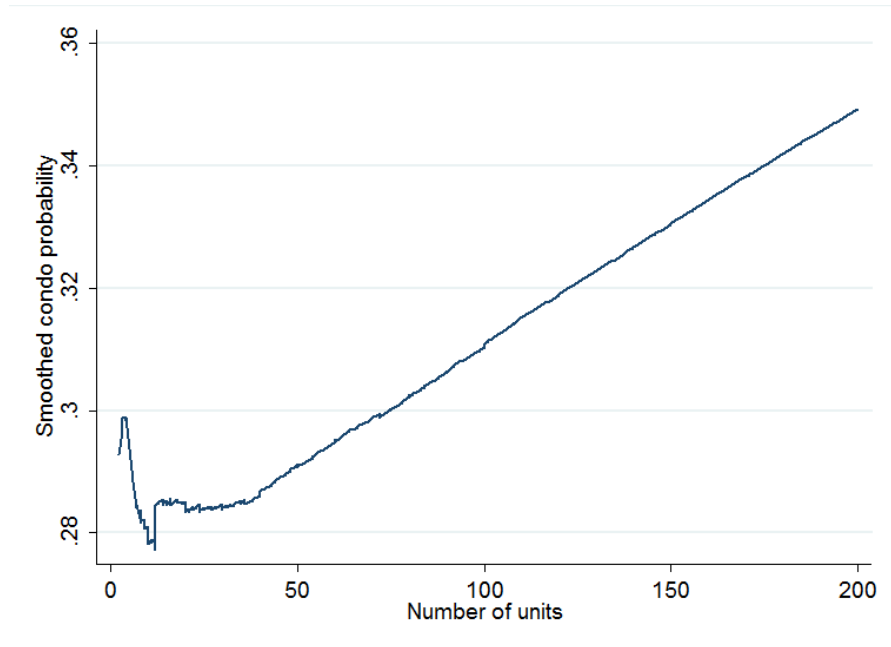


Figure 10: Predicted Probability of Condo Ownership

that the hypothesis of no correlation can be rejected; accounting for selectivity is important, presumably because omitted quality variables influence both decisions.

We turn now to the estimation of the value functions. As noted previously, we stratify the sample by geographic markets – in this case, Census regions. Note that we still include binaries for metropolitan areas, so that intra-regional variation in the hedonic functions is still allowed, but we restrict this variation to intercept terms. The marginal valuations of structural attributes is homogenous within regions, but heterogeneous across. The functional form uses value as the dependent variable (and not, say, its log) and the nonlinearity required to create sufficient variation is created by allowing floorspace to be entered as a cubic polynomial. The number of units is also entered in the regression as a cubic. Table 4 presents these estimates in which the dependent variable is value in thousands of dollars and the coefficients for the cubic polynomials as well geographic variables are suppressed. The parameter estimates vary substantially across regions, especially for the quality binaries such as fireplace, air conditioning, and the like. They also exhibit considerable heterogeneity in the unit count polynomial, but very few of these parameters are significant at standard levels of type I error. The number of bathrooms and the floor of the unit are all large and significant. Importantly, the polynomial factors of square footage are jointly significant, but even the linear term is not estimated particularly precisely. Of equal interest is the fact that the two Mills ratio terms do not have statistically significant coefficients, indicating that the unobserved quality factors that determined condo and ownership probabilities do not seem

	Northeast	Midwest	South	West
Number Units	1.006 [0.827]	-0.022 [0.289]	-0.048 [0.452]	0.811 [0.518]
Floor	24.811*** [6.098]	12.079*** [2.656]	12.981*** [3.018]	15.431*** [4.266]
Baths	455.695*** [44.042]	58.269*** [16.478]	71.991*** [18.329]	87.899*** [26.980]
Half Baths	305.869*** [51.615]	30.751** [15.124]	31.047** [13.078]	55.365** [24.061]
Fireplace	-24.698 [54.543]	-14.539 [15.737]	-11.614 [18.935]	107.540*** [22.503]
Porch	-31.375 [51.025]	-6.053 [20.752]	-1.261 [30.190]	12.451 [35.591]
A/C	-32.544 [47.100]	-16.92 [20.762]	-31.714 [46.067]	-96.282*** [21.261]
Garage	-118.801** [56.667]	-16.318 [22.742]	26.3 [23.411]	35.992 [51.250]
Unit SF	0.249* [0.138]	0.055 [0.042]	0.063* [0.033]	0.193* [0.100]
Yr. Built	-4.034*** [1.130]	1.359*** [0.491]	-0.29 [0.636]	-5.624*** [0.780]
ξ1	206.742 [270.595]	-207.594** [99.848]	-27.694 [91.006]	488.088*** [143.581]
ξ2	17.424** [8.619]	2.173 [1.881]	1.477 [1.144]	6.06 [5.337]
Constant	7,259*** [2,072]	-2,600*** [893]	490 [1,178]	10,392*** [1,414]
Observations	386	421	508	1,112
Adjusted R-squared	0.486	0.383	0.257	0.306

Standard errors in brackets

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

Table 4: Estimates of Hedonic Price Functions

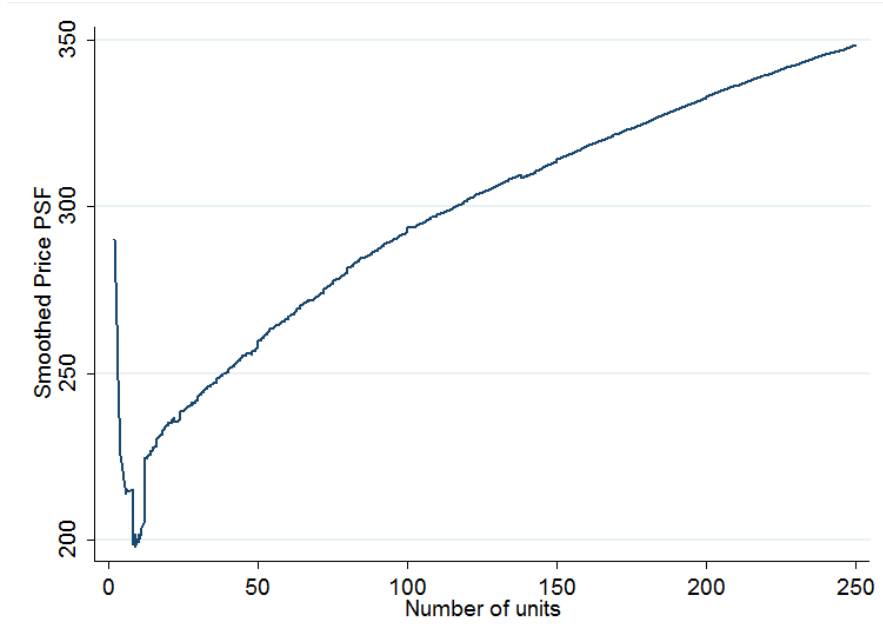


Figure 11: Smoothed Condo Price PSF and Building Size

to have a particularly definable impact on the price of the unit.

### 7.3 Bid Functions

The final stage in this exercise is to estimate bid functions for various wealth levels and unit counts. As a first look at the data from this point of view, we first estimate a bivariate nonparametric relationship between price per square foot and number of units. This is contained in Figure 11. Its resemblance to Figure 6, as derived from the theory model for low wealth investors, is striking. The purpose of the bid function estimation is to map portions of this curve to various wealth groups. In particular, what we observe from Figure 9, which presents theoretical bid curves for various wealth groups, is that if low wealth investors invest in real estate, they will only do so for buildings of small size, i.e. low unit counts. If that occurs, we should also observe a downward (in unit count) sloping bid function for those low wealth investors. At higher unit counts, however, higher wealth investors should be the winning bidders, but at this point, the slope of the bid functions for these wealthier people should be relatively flat. This would be congruent for both the theoretical and empirical graphs of Figures 9 and 11.

We use the value functions for the four regions in the previous sections to compute marginal prices-per square foot of interior floor space. These are displayed in Figure 12. Note that there are, as desired, differences across the four regions (although the South and

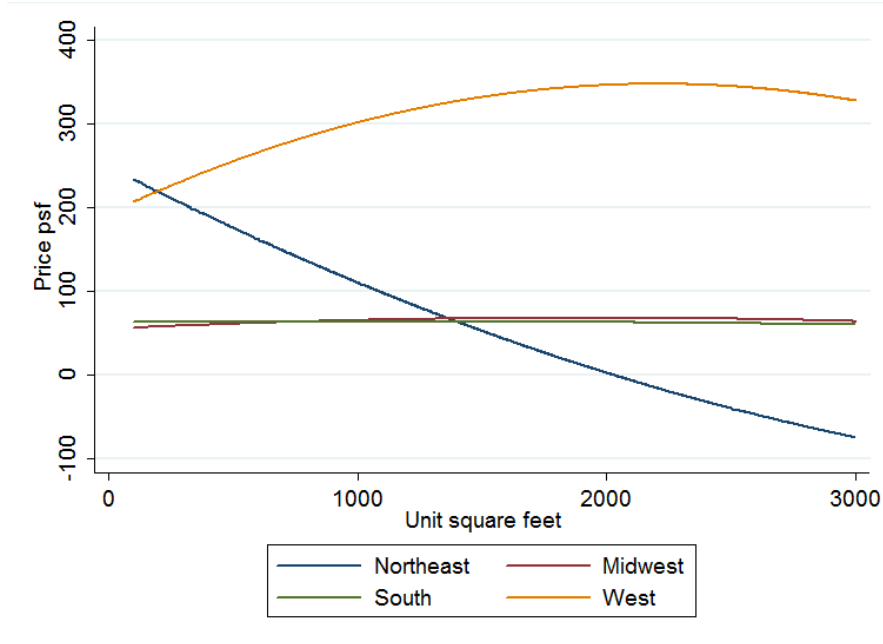


Figure 12: Predicted Bids for Marginal Quality by Low and High Wealth Households

Midwest are fairly similar) with the West having the highest marginal prices. As discussed above, these marginal floorspace prices are calculated for each owner-occupied condo. In order to most clearly see the difference in marginal bid functions for different wealth groups we split the sample into two parts, labeled “Low Wealth” and “High Wealth” with the dividing line at a non-housing wealth level of \$150,000. We separately estimate (24) for these two groups. We include the income of the household and the age of the household head as the demographic variables,  $Z$ , and include the housing characteristics from above as well. As noted, the unit square footage is included here, but is clearly endogenous, and so we instrument using regional binaries and these binaries interacted with total wealth. The cubic polynomial of unit count is importantly included as well. The results are displayed in Table 5 (geographic variables are suppressed). The coefficients of importance are first of all, that of unit square feet itself, which is negative for both high and low wealth households, as is quite appropriate for bid functions that display diminishing marginal rates of substitution. Note furthermore that the slope is greater for low wealth households, indicating that high wealth households outbid low wealth ones for larger units. More importantly for our purposes, note the coefficients for the polynomial in the number of units. From the linear terms, we can directly see that, for low unit counts at least, there is a pronounced negative slope for the bids from low wealth households, and that the bids from high wealth households are relatively flat. In Figure 13 we present bid functions for low and high wealth households that

	LowWealth	HighWealth
Unit SF	-0.514*** [0.114]	-0.489*** [0.099]
Number Units	-2.093*** [0.712]	0.029 [0.699]
Number Units, sq.	.0066*** [.002]	0.0007 [0.002]
Number Units, cu.	-4.91E-06** [1.96E-06]	-3.92E-07 [0.000]
Income	0.0002 [0.0005]	0.0002** [0.0001]
Head HH Age	1.11 [1.229]	2.096** [0.893]
Floor	-6.921 [4.609]	1.59 [7.939]
Baths	210.627*** [47.117]	323.487*** [62.681]
Half Bath	18.546 [32.642]	206.297*** [53.141]
Fireplace	-18.385 [27.013]	48.513* [26.203]
Porch	35.817 [32.213]	55.654 [40.774]
A/C	18.693 [29.211]	-109.991*** [25.071]
Garage	172.500*** [19.222]	84.095** [42.562]
Yr. Built	-1.529* [0.906]	-1.382** [0.689]
Constant	3,277* [1,792]	2,770** [1,312]
Observations	903	1,524
Wald Chi2	269.06	229.99
p-value	0.00	0.00

Robust standard errors in brackets

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

Table 5: Estimates of Bid Functions



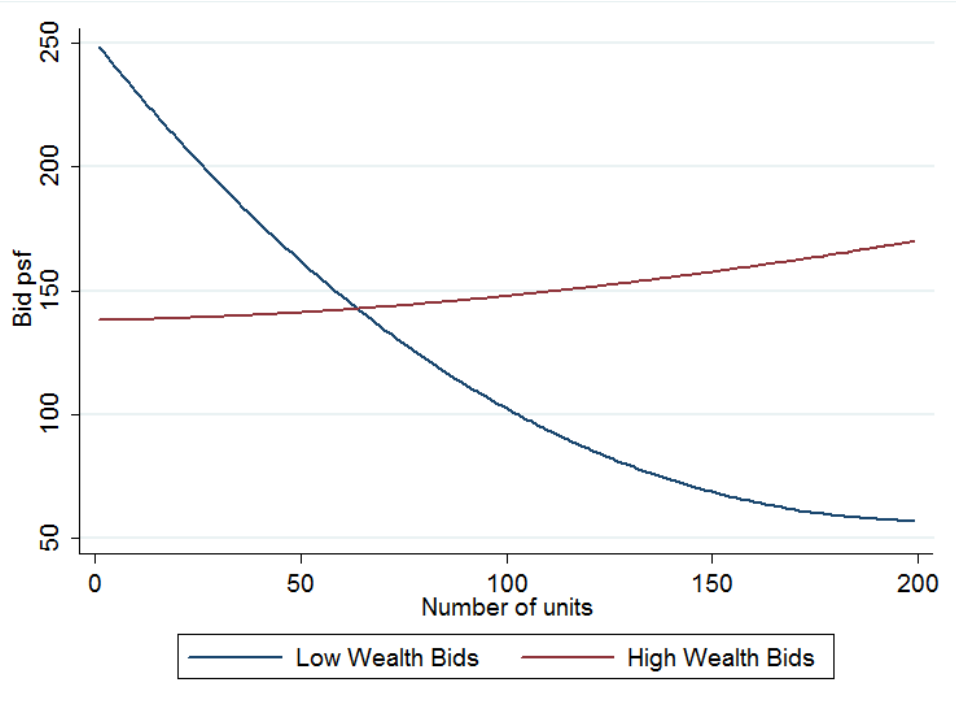


Figure 13: Predicted Bids of Low and High Wealth Households

are otherwise identical (in both  $X$  and  $Z$ ).<sup>8</sup> Low wealth households are observed to be very sensitive to unit count, and by extension to the free rider problem. High wealth households are not as sensitive to this, and so as the free rider problem worsens, and the expense from condo management rises, they begin to outbid low wealth households at around 60 units, in bigger condo buildings. This is exactly what our theory would predict.

## 8 Conclusion

The literature on housing tenure has largely failed to address the type of housing which households occupy, focusing instead on the households themselves. This paper confronts the trade-offs incurred by the need or desire to co-own buildings. We find that while small multifamily buildings are particularly disadvantaged for condo ownership – and this comprises much of the U.S. multifamily housing stock – economies of scale in the cost of third-party management may make joint ownership valuable in relatively larger buildings. In particular, the empirical estimation of bid functions confirms that wealthier households will tend to outbid the less wealthy for ownership of condominiums in larger buildings. These insights have implications for our understanding of homeownership in cities. For example, they may

<sup>8</sup> $X$  are set at approximate medians for a condo in the Seattle CMSA. The unit is 800 square feet, with 1 bath and a porch, and was built in 1975. The head's age is set at 49 years, and income is set at \$75,000.

help to explain phenomenon like the differential rate at which housing units in certain types of buildings filter down to lower income occupants (Rosenthal, forthcoming).

We also note that the theoretical and empirical results in this paper rely on an underlying legal system that effectively addresses governance issues with respect to owner contributions towards common building expenses, like water and sewer charges. This may not be an appropriate assumption in some legal environments and the failure of institutions to do so will further reduce the likelihood of observing joint ownership arrangements like condominiums. Conversely, the failure of laws to support landlord rights *vis a vi* tenants may result in a preference for joint ownership arrangements (Casas-Arce and Saiz, 2010).

Our results also shed light on the fundamental correlation between single family structures and ownership. The reason that single family homes are owner-occupied is that maintenance for smaller units does not scale up, at least under traditional business models. Unit size aside, it is surely more difficult to manage 500 (possibly dispersed) single family units than one building with 500 units. This issue is of vital importance due to the large amount of newly-vacated homes in the wake of the recent housing crisis. The conversion of these properties to rental units by large investors is underway, although it surely remains to be seen whether this is sustainable practice in the long run.<sup>9</sup>

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<sup>9</sup>See, for example, Olick (2013), although other reports (Hallman and Berman, 2013) reinforce our point that maintenance is indeed quite costly to scale up in single family portfolios.

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## Appendix A. Sole Investor Problem

In order to adapt the basic model to allow for ownership of multiple buildings, let  $\bar{q}$  represent the number of buildings, while  $n$  still denotes total building size. Total investment in risky housing assets is now  $\bar{q}n$ . The total cost of landlord effort is  $a\bar{q}(e\bar{q}n)$ . Notice that while the cost of effort is linear in effort within a building, ownership of multiple buildings requires a duplication of effort for each additional building. Using these adjustments, we initially take the number of buildings to be exogenous and define the landlord's indirect utility for a given number of buildings as

$$U^s(w_1, \bar{q}) = \max_{n, c_f, c_v} E[v(\tilde{w})].$$

Investor second period realization of wealth is:

$$\tilde{w}^s = (w_1 - \bar{q}n(P - R))(1 + r) + \bar{q}n \left( P + 2\phi_a\sqrt{a} + (1 - c_v) \left( 2\phi_b\sqrt{b} + \tilde{\varepsilon} \right) + \tilde{u} \right) - \bar{q}ne - ae\bar{q}^2n$$

As before, we solve for the interim choices of effort by the investor:

$$a^s = \frac{\phi_a^2}{e^2\bar{q}^2}.$$

Solving the investor's problem subject to the participation constraint of third-party managers, which remains unchanged relative to the base model, the equation that we use in numeric solutions to identify  $n$  is

$$R(1 + r) - rP - \bar{z} + (c_v^2 - 2c_v + 2) \frac{n\phi_b^2}{2(d + n^2\alpha)} - (2 - c_v) \frac{\rho}{2} c_v n \sigma^2 + \frac{E[v'\tilde{u}]}{E[v']} = 0.$$

The first order condition for  $c_v$  is identical to the base model.

We use the envelope theorem to derive a slope of the bid function for sole ownership of buildings:

$$\frac{\partial P}{\partial n} = \frac{1}{r} \left( c_v \phi_b^2 (2 - c_v) \frac{d - \alpha n^2}{(d + \alpha n^2)^2} - \frac{\rho}{2} c_v^2 \sigma^2 \right).$$

The first term on the right hand side is positive for buildings smaller than the cost-minimizing size. Whether or not the slope is positive or negative depends on the particular choice of parameters.

In the numerical solutions for sole investors in multifamily buildings, we initially invert the investor's maximization problem and taking  $n = 1200$  as given, solve for the size of the

investment  $q$ , as well as  $c_v$  and  $c_f$  at the single family price of \$94 per square foot. Once we obtain the investor's level of utility from owning a portfolio of single family houses, we then solve for  $n$ ,  $c_v$ ,  $c_f$ , and bid price at which utility is held constant conditional on an integer number of buildings being held in portfolio.

## Appendix B. Symbols

$U$	indirect investor utility
$v$	investor utility over wealth
$\tilde{w}$	investor (uncertain) wealth at time two
$P$	price per unit of housing services
$R$	periodic rent per unit of housing services
$w_1$	endowment at time 1
$\tilde{\varepsilon}, \tilde{u}$	normally distributed noise with mean 0 and variance $\sigma_{\varepsilon}^2$ and $\sigma_u^2$
$q$	housing investment in units of housing services
$r$	risk-free rate
$c_v$	variable component of third party manager compensation
$g$	total third-party manager compensation
$V(g)$	third party management's utility as a function of compensation
$c_f$	fixed component of third party manager compensation
$\rho$	coefficient of manager absolute risk aversion
$\phi_a, \phi_b$	investor and manager production parameter
$n$	building size in units of housing services
$\hat{z}$	third party manager's certainty equivalent wealth
$\bar{z}$	third party manager's opportunity cost per unit of housing services
$e, d, \alpha$	cost of effort parameters



## Appendix C. Imputing Wealth

We use the 2011 wave of the PSID; data on persons defined as household heads were downloaded, although total household wealth is the variable of interest. The response coding in the PSID is different from that used in the AHS, which required adjustments to the PSID responses. For instance, in coding education levels, the PSID uses responses 1 through 16 to code actual grad levels completed, while the AHS uses response 31 to code completion of first to fourth grades. For added predictive power, polynomials of age and income are also included in the specification. To additionally aid in the predictability of the sample, we eliminated observations with very large (over \$4,000,000) or very negative (less than -1,000,000) wealth. It is literally impossible to predict wealth that great or that far underwater using demographic variables. There were 47 observations all together in those two categories.

The R-squared of the wealth regression is 31% which, while not large, is respectable for this sort of exercise. The coefficients are sensible; age maps into wealth in a highly nonlinear manner, as would be expected, however wealth seems to be a linear function of income. The schooling and ethnic coefficients coincide with prior expectations, however it is of interest to note that those with between 1 and 10 years of schooling seem to do worse than those with no schooling at all (the omitted category). Figure 14 displays the density of both wealth (solid bars) and predicted wealth (clear). The regression model accurately reproduces the skewed nature of the wealth distribution displayed in the PSID, however it under-predicts the fraction of participants with wealth near the mode of the actual distribution and under-predicts the number with slightly higher amounts of assets.

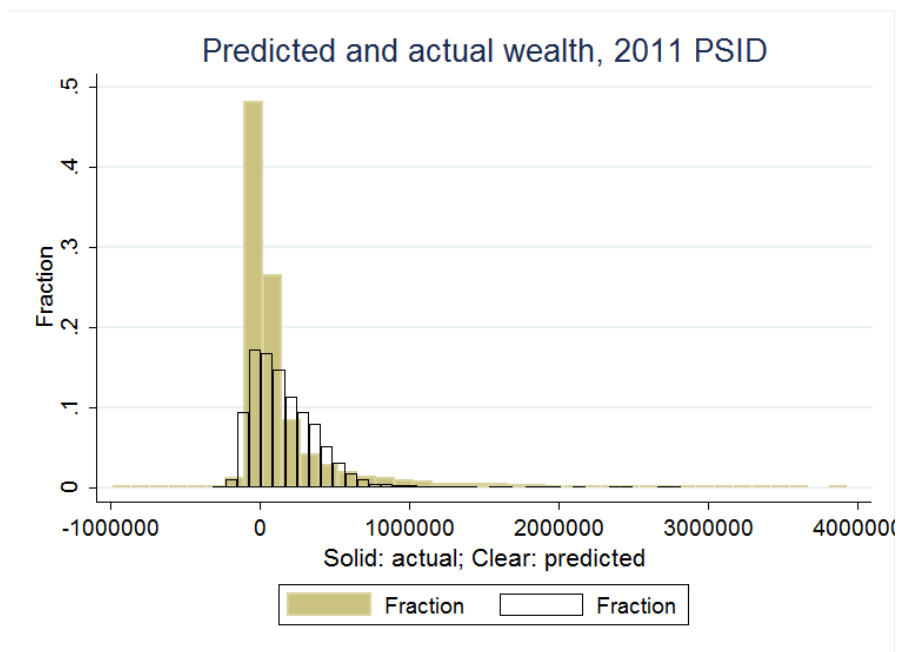


Figure 14: Actual and Predicted Wealth