Agglomeration Economies, Geography and the Value of Urban Land[1]

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Abstract

We examine how growth in wages and residential rents are affected by land supply constraints. Our city model incorporates production externalities and allows firms and workers to choose location as well as intensity of land use. When business structures cannot adjust (short run), cities with more severe constraints on the supply of urban land see higher growth in wages and rents in response to a positive shock to the demand for urban land. But, when business structures can adjust (long run), cities with more severe constraints experience lower growth in wages and rents as they are less able to exploit the positive external effects of greater employment density.

Keywords: Agglomeration economies, land supply constraints, productivity growth
JEL Codes: E10, R30

1 Introduction

What is the role of geography for the value of urban land? Urban economics suggests one answer. Imagine a location that is severely constrained by geography and cannot easily support a large population. Such a location cannot benefit as much from production externalities that are at the heart of urban agglomerations. All else the same, such a location will have lower incomes and land prices (or not develop into a city at all). Although it is challenging to tease out the effects of geography alone from the myriad of other factors that affect city incomes and land prices, a growing list of creative studies have unearthed evidence that seems consistent with this prediction. Bleakely and Lin (2012), Nunn and Puga (2010), Rosenthal and Strange (2008) and Combs, Duranton, Gobillon, and Roux (2010) show that geography that allows easier expansion (horizontal or vertical) tend to have higher wages (and, by implication, land rents).[1]

There is also an alternative view grounded in basic demand-supply analysis. When geography limits the physical expansion of a city, an increase in the demand for urban land results (all else the same) in a larger increase in the price of land in the city. This view also appears to be borne out in studies that utilize the variation in the growth rate of house prices in the recent boom episodes. Glaeser, Gyourko, and Saiz (2008) and Huang and Tang (2013) find evidence that houses prices responded more strongly to demand shocks in cities where supply of urban land is less elastic in terms of the Saiz (2010) measure of undevelopable land.

On the face of it, these two perspectives —equally compelling in their own way— suggest

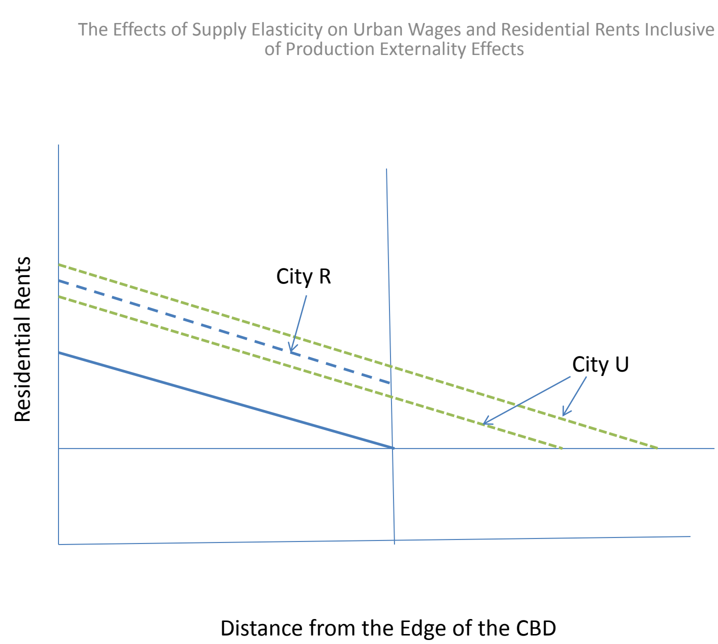
[1] Saiz (2010) finds that cities with less developable land have higher house prices in 2000. However, his sample only includes metro areas with population in excess of $500,000$ in 2000 and therefore does not account for smaller urban areas that might have failed to grow because of physical constraints and currently have low incomes and land prices. In Bleakely and Lin (2012), the size of the upstream watershed area of portage cities has a positive effect on population density and the hourly wage of workers in year 2000. A larger watershed area is a proxy for ease of expansion because it means that the headwaters of the river, which is typically in a hilly or a mountainous region, is more remote from the city. Nunn and Puga (2010) show that ruggedness of countries has adverse effects on country incomes in year 2000 other than in Africa (where ruggedness afforded protection from the slave raids). Rosenthal and Strange (2008) and Combs, Duranton, Gobillon, and Roux (2010) show that properties of the soil that lower construction costs are positively correlated with current incomes and use such soil properties as instruments to identify agglomeration effects.

opposite effects of geography on the value of urban land. In this paper, we reconcile these seemingly contradictory views in the context of a spatial city model with production externalities. Our findings can be summarized in Figure 1. The horizontal axis gives the location of land relative to the city center and the vertical axis has residential land rents. Consider two cities that have the same fundamentals and the same initial city size S. Residential rent gradient for the two cities are shown by the solid downward sloping line (rents are declining from the city center). Suppose a shock increases the demand for urban land in both cities. This could be the result of productivity growth in these cities or aggregate population growth. In either case, more people want live in these cities. One city can expand at a fixed cost (unrestricted city U) but the other cannot expand beyond point S (restricted city R). The dashed line is the position of the new rent gradient for the restricted city and shows that the increase in demand leads to an increase in residential rents in all locations.

The shift in the rent gradient of the unrestricted city may be either below or above the dashed line. The basic demand-supply view —which ignores production externalities— would say that the shift will be smaller (below the dashed line) because the expansion of the city boundary brings more land into the city and reduces demand pressure on existing city locations. With production externalities, whether the shift is smaller or bigger depends on the strength of the external effect. Specifically, suppose that output per unit of land in a given location is proportional to $z^{\gamma}n^{\theta}b^{\psi}$, where z is a measure of proximity to other workers in that location (described in more detail later in the paper) and n and b are employment and building densities in that location. Then, in the long run (when both employment and structures are variable), a sufficient condition for the rent gradient for city U to be above the gradient for city R is for the production externality parameter γ to be as large or larger than the share of land in production $(1 - \theta - \psi)$. In the short run (when only employment is variable and commercial structures are fixed), the sufficient condition for the rent gradient for city U to be above the gradient for city R is for γ to be as large or larger than $(1 - \theta)$, or equivalently, for the externality parameter to be as large or larger than sum of the shares of land and structures in production.

To understand the intuition behind these results, note, first, that the unrestricted city will absorb more workers than the restricted city. Then, for the rent gradient to rise more in the unrestricted city, wages need to rise more in the unrestricted city (so that utility is the same in both cities). For this to happen, the production externality needs to be strong enough to overcome the diminishing returns to labor that comes from the scarce factors of production. In a spatial model, land at different distances from the city center are not perfect substitutes and the amount of land at any given distance from the center is fixed. If the number of workers in the city doubles, the business sector as a whole cannot double its existing usage of land. What it can do is bring more land into the business sector but this land is not as productive and it can bid only a limited amount away from workers who use it for residential purposes. In the long run, land is the only scare resource; in the short run, land is scarce and structures are fixed. The more important these scarce or fixed factors are to production, the stronger the externality effect needs to be for wages to rise more as more workers move into the unrestricted city.

Figure 1



The quantitative evidence regarding these parameters suggests that γ is larger than $(1-\theta-\psi)$, or, equivalently, γ is larger than the share of land in business costs. Thus, when all non-land factors of production can be varied, the restricted city is likely to see lower growth in wages and residential land rents. The evidence on the share parameters also suggests that γ is less than $(1-\theta)$. Therefore, in the short run, when commercial structures cannot immediately respond to worker inflows, wage and residential land rent growth will be higher in the restricted city.

Thus, our model is able to reconcile the seemingly contradictory findings in the two strands of the empirical literature cited earlier. Studies that focus on recent episodes of rapid growth in house prices will find that supply restrictions contribute to higher wages and house prices as the time period in question is relatively short. In contrast, studies that focus on the long run will find that supply restrictions lower wages and house prices.

One point to highlight is the fact that the unrestricted city ends up with higher wages in the long run does not imply that cities are in an unstable equilibrium —where rising wages continually attract workers (from other locations) until all workers live in one giant city. Stability depends on more than just production parameters because workers need land for residential purposes as well and, empirically, the residential demand for urban land is strong enough to ensure stability (this point will be clear later in the paper).

The paper is organized as follows. Section 2 describes the environment. Sections 3 and 4 develop the equilibrium implications of this environment for the location decisions and intensity of land use by firms and workers. Section 5 analyzes how an urban growth boundary affects business and residential rents and other variables of interest when there is an increase in the demand for urban land, and discusses the empirical evidence on the magnitude of the externality and land-intensity effects. Section 6 concludes.

2 The Environment

A natural environment to explore the effects of land supply constraints on city incomes and land values is one in which businesses and workers compete for land and there is a positive production externality that makes firms want to locate close to one another other. We use the model developed in a companion paper (Chatterjee and Eyigungor (2014)). Our model, which is squarely in the tradition of Mills (1969), Muth (1969) and Alonso (1964), has important precursors (Lucas and Rossi-Hansberg (2002) and Fujita and Ogawa (1982)) but is novel in its modeling of the production externality. This results in a tractable structure suitable for comparative statics analysis.

Space is modeled as a flat plain extending in all directions with a point marked off as the city center. We focus on allocations that are symmetric relative to the center, so a location is described fully by its distance r from the center.

Utility function of a worker depends on the consumption of the single *numeraire* good avaliable in this economy and on the service flow from land. A worker who resides in location r has utility

$$U = c^{\beta}(r)l(r)^{1-\beta}, \quad \beta \in (0,1), \tag{1}$$

where $l(r)$ is the consumption of land in location r and $c(r)$ is consumption at location r.

A firm has a technology to produce the single consumption good. The production function of a firm that uses one unit of land at location s is

$$Y(s) = Az(s)^{\gamma}n^{\alpha}(s), \quad \alpha \in (0,1), \gamma > 0, \tag{2}$$

where $n(s)$ is the number of workers per unit of land at location s, A is a TFP term that is common to all firms in the city, and $z(s)$ is a variable—defined more precisely below—that captures how many other workers are in close proximity to the firm. We do not explicitly model factors of production other than land and labor. We show later in the paper that when structures are explicitly included as another factor of production, production possibilities effectively reduce to (2) if structures can be freely adjusted. In that section we examine how our results change when structures are explicitly included but cannot be varied in the short run.

A key assumption is that the proximity between any two firms is measured by the sum of the distance of the two firms from the city center. In other words, if one firm is located on a circle of radius r and the other firm is located on a circle of radius s, the distance of the firms to each other is simply $(r+s)$. The assumption that distance between two firms is measured by the sum of the lengths to the city center is reasonable if communication between workers in different firms requires travel to a central meeting place and the road system is radial. A second justification of this assumption is given below.

Let $N(s)$ denote the number of workers employed by firms at all locations s. Then, for a firm at location r, the level of access to workers at other firms is

$$z(r) = \int_0^\infty 2\pi s \exp\left(-\delta\left(r+s\right)\right) N(s) ds.$$

Here δ is a parameter that governs how quickly communication possibilities attenuate with distance. Since $z(0) = \int_0^\infty 2\pi s \exp\left(-\delta s\right) N(s) ds$, the above definition implies

$$z(r) = z(0) \exp\left(-\delta r\right). \tag{3}$$

Thus the measure of other workers that a business can communicate with declines exponentially at the rate δ with distance from the city center.

As will become evident, (3) is the reason our model predicts that all density and price gradients follow exponential functions and is the reason why the model is tractable. Given the importance of (3), we might ask, what other distance measures generate (3)? If we denote the general distance function as $\nu(r,s)$ and require that $\nu(r,s) = \nu(s,r)$ (symmetry), then it is straightforward to show that any symmetric distance function that generates (3) must be a linear transform of $r + s$.[2] Thus a second justification for our distance measure is that it is the only (symmetric) measure that is consistent with (3) and, therefore, with exponential density and price gradients.

There is a technology for commuting. This technology allows workers to commute to any firm that is located on the straight line that connects the worker's residential location to the city center. We follow Anas, Arnott, and Small (2000) and Lucas and Rossi-Hansberg (2002) and assume that a worker who resides in location s and commutes to a firm at location r has $\exp(-\kappa|s-r|)$ unit of time to devote to production, where $\kappa > 0$.[3]

There is also a technology for converting land from its natural state into land that can be used by workers and firms. The cost of converting a unit of natural land into developed land is d units of the consumption good.

Finally, following convention, it is assumed that all land in the economy is owned by entities outside of the model. These entities decide whether to convert any given unit of natural land into developed land and then rent the developed land to workers and firms.

3 Monocentric City with Endogenous Business District

In this paper, we focus on the case where the city is monocentric, meaning that there is central business district of positive radius in which all production is concentrated, with workers living

[2]For the general distance function $z(r) = \int_0^S \exp(-\nu(r,s))N(s)ds$. We require that $z(r) = z(0)\exp(-\delta r)$, where δ is some positive constant. Then $z(0) = \int_0^S \exp(-\nu(r,s) + \delta r)N(s)ds$. Since this relationship must hold for any r, it follows that $\nu(r,s)$ must be of the form $a + \delta r + f(s)$. From symmetry $a + \delta r + f(s) = a + f(r) + \delta s$, which in turn implies $f(s) - f(r) = \delta \cdot (s - r)$. Hence $\nu(r,s) = A + \delta \cdot (r + s)$.

[3]As noted in Anas, Arnott, and Small (2000), this assumption is key to obtaining an exponentially declining land rent and population density function without making counterfactual assumptions on the structure of preferences for land. Coupled with our assumption regarding how proximity between firms is calculated, we can extend the negative exponential form to commercial rents as well as employment density. Note also that, to a first-order approximation, the (net) income of a commuter is $w(r)[1 - \kappa|s-r|]$, which corresponds to the common assumption that the commuting cost is proportional to the hourly wage and linear in the distance traveled.

in the surrounding residential ring. Empirically, this is the pattern most relevant for US cities in that the fraction of land devoted to business use is generally declining from the city center, although it is not quite 1 in the business district and rarely does it become 0 away from it.

Monocentricity imposes a restriction on parameter values. To derive this restriction, let $w(r)$ be the wage paid by a firm at location r and let $q_F(r)$ be the maximum rent a firm would be willing to pay for a unit of land at location r. This quantity is simply $Az(r)^\gamma n^*(r)^\alpha - w(r)n^*(r)$, where $n^*(r)$ is the optimal choice of n conditional on locating at r. Then,

$$q_F(r) = [(1-\alpha)/\alpha]\left[\alpha Az(r)^\gamma w(r)^{-\alpha}\right]^{1/(1-\alpha)}. \tag{4}$$

The maximum rent a firm is willing to pay depends positively on the location's productivity and negatively on the location's wage.

Turning to workers, we let $q_H(r,s)$ be the maximum rent a worker would be willing to pay for a unit of land at location r, given that he will work at location s. Conditional on paying $q_H(r,s)$ in rent, a worker's optimal utility is $\beta^\beta(1-\beta)^{1-\beta}w(s)\exp(\kappa|s-r|)q_H(r,s)^{-(1-\beta)}$. If U is the maximum utility a worker can obtain from locating elsewhere,

$$q_H(r,s) = (1-\beta)\,\beta^{\beta/(1-\beta)}(w(s)\exp(\kappa|s-r|)/U)^{1/(1-\beta)}. \tag{5}$$

Thus, the maximum rent a worker is willing to pay for land at r depends positively on his wage and negatively on U.

For the city to be monocentric, there must exist a boundary $S_F < S$ such that all locations $r \in [0, S_F)$ are devoted to production and all locations $r \in (S_F, S]$ are devoted to residential use. Since workers must be indifferent between working within the business district,

$$w(r) = w(0)\exp(-\kappa r) \text{ for } r \in [0, S_F]. \tag{6}$$

Using this information in $n^*(r)$ and using the expression for $z(r)$ in (3) yields

$$q_F(r) = q_F(0) \exp\left(-\frac{\delta\gamma - \kappa\alpha}{1-\alpha} r\right) \text{ for } r \in [0, S_F), \tag{7}$$

which is declining in r provided $\delta\gamma - \kappa\alpha > 0$.

Given that workers earn the same regardless of where they work, the maximum rent a worker is willing to pay for land at location $r \in [0, S]$ and still get a utility of U is

$$q_H(r) = (1-\beta)\, \beta^{\frac{\beta}{1-\beta}} \left(\frac{w(0)\exp(-\kappa r)}{U}\right)^{\frac{1}{1-\beta}} = q_H(0) \exp\left(-\frac{\kappa}{1-\beta}\right) \text{ for } r \in [0, S]. \tag{8}$$

For the monocentric structure to be an equilibrium outcome, the two bid rents must be the same at the boundary of the CBD, and the slope of the firm's bid rent function must be steeper than the slope of the worker's bid rent function. The slope of the worker's bid rent function at S_F is $[-\kappa/(1-\beta)]q_H(S_F)$ and the slope of the firm's bid rent function at S_F is $[(\kappa\alpha - \delta\gamma)/(1-\alpha)]q_F(S_F)$. Since at the boundary of the CBD $q_H(S_F) = q_F(S_F)$, the necessary slope condition boils down to:

$$\kappa < \frac{(1-\beta)\gamma\delta}{(1-\beta\alpha)}. \tag{9}$$

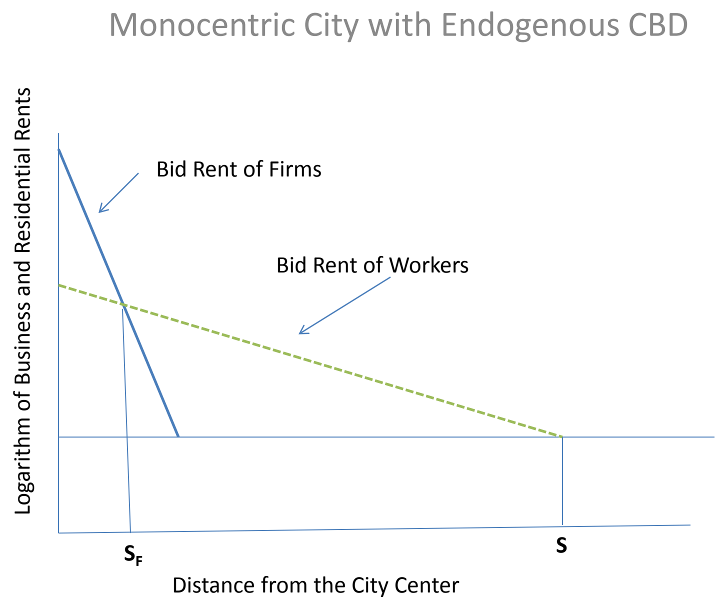
Note that since both α and β are less than unity, (9) implies that $\gamma\delta > \alpha\kappa$. Therefore, the firm's bid rent function is downward sloping, as assumed.[4] Figure 2 summarizes the situation.

4 Equilibrium

We imagine a system of cities, each of which delivers utility U to its residents. In this section, we find the equilibrium population (P) and equilibrium size (S) of a city that has

[4]As discussed in Chatterjee and Eyigungor (2014), this environment is consistent with two different urban forms, depending on parameter values. If condition (9) is violated, firms and workers will co-locate the employment will be dispersed over the entire city.

Figure 2



productivity A, can expand at cost d and delivers utility U.

The determination of equilibrium can be broken down into two parts. In the first part, P and S are taken as given and the equilibrium employment and residential density functions along with the equilibrium wage and rent functions are determined as functions of P and S (the equilibrium will also imply a utility level U in the city). In the second part, we will endogenize S also (which will be determined by the condition that says the rent at the boundary equals to the expansion cost d) and so U and S are determined as functions of P and d.

The task of determining the various equilibrium functions is made very simple by the fact that all these functions are negative exponentials, where the only unknowns are the values of these functions at $r = 0$ (the city center). Furthermore, these unknown values are all determined once $n(0)$ and $z(0)$ are determined. To see this, note that $w(0)$ is simply the marginal product of labor at the city center. Therefore

$$w(0) = \alpha A z(0)^{\gamma} n(0)^{\alpha - 1}. \tag{10}$$

And $q_F(0)$ is output at 0 minus the wage bill at 0 (since all "surplus" must go to the owners of land) and so,

$$q_F(0) = [(1 - \alpha)/\alpha] w(0) n(0) = (1 - \alpha) A z(0)^{\gamma} n(0)^{\alpha}. \tag{11}$$

To determine $q_H(0)$, we use the fact that the bid rents for businesses and workers are the same at S_F, which implies

$$q_F(0) \exp(-[\delta\gamma - \kappa\alpha]/[1 - \alpha] S_F) = q_H(0) \exp(-[\kappa/(1 - \beta)] S_F). \tag{12}$$

Therefore

$$q_H(0) = q_F(0) \exp\left(\frac{\kappa - \delta\gamma + \beta\delta\gamma - \beta\kappa\gamma}{(1 - \alpha)(1 - \beta)} S_F \right). \tag{13}$$

While this equation depends on S_F, we will show below that S_F is, in fact, pinned down by S alone (recall that we are taking both P and S as parametrically given in this part). Therefore, the first part of the equilibrium problem boils down to simply determining $n(0)$ and $z(0)$.

To proceed, we observe that the expression for $n^*(r)$, along with the expressions for $w(r)$ in (6) and $z(r)$ in (3) gives

$$n(r) = n(0) \exp\left(-\frac{\delta\gamma - \kappa}{1 - \alpha} r \right) \text{ for } r \in [0, S_F]. \tag{14}$$

The values of $n(0)$ and $z(0)$ are determined by invoking two market-clearing conditions. First, there is the labor-market-clearing condition. To develop this condition, we note that the total supply of labor time available at the border of the CBD, taking into account the time lost in commuting, is $\int_{S_F}^{S}[2\pi r/l(r)]e^{-\kappa(r-S_F)}dr$. If the employment density at a CBD location r is $n(r)$, the labor time needed at the border of the commercial district to fulfill this demand is $e^{\kappa(S_F-r)}n(r)$. Therefore, the total time needed at the border of the CBD to satisfy total labor demand inside the commercial district is $\int_0^{S_F} 2\pi rn(r)e^{\kappa(S_F-r)}dr$. Equality of labor demand and supply then requires

$$\int_0^{S_F} 2\pi rn(r)\exp\left(\kappa(S_F-r)\right)dr = \int_{S_F}^{S}\frac{2\pi r}{l(r)}\exp\left(-\kappa(r-S_F)\right)dr,$$

which, using the fact that $l(r) = (1-\beta)w(0)e^{-\kappa r}/q_H(r)$ and the expressions for $n(r)$ and $q_H(r)$ derived earlier, simplifies to

$$n(0)w(0)(1-\beta)\int_0^{S_F} r\exp\left(-\frac{\delta\gamma-\kappa\alpha}{1-\alpha}r\right)dr = q_H(0)\int_{S_F}^{S} r\exp\left(-\frac{\kappa}{(1-\beta)}r\right)dr.$$

Using (11) and (13) we can further simplify this equation to

$$\left[\int_{S_F}^{S} r\exp\left(-\frac{\kappa}{1-\beta}r\right)dr\right] =$$
$$\frac{(1-\beta)}{(1-\alpha)}\alpha\left[\int_0^{S_F} r\exp\left(-\frac{\gamma\delta-\alpha\kappa}{1-\alpha}r\right)dr\right]\exp\left(\frac{-\kappa+\delta\gamma+\beta\kappa\alpha-\beta\delta\gamma}{(1-\alpha)(1-\beta)}S_F\right). \quad (15)$$

Observe that this is an equation that implicitly defines S_F as a function of S, as promised. The following Lemma establishes that there is a unique S_F corresponding to each S that is strictly increasing in S and converging to a finite limit as S increases unboundedly.

Lemma 1 *For each $S>0$, (15) uniquely determines $S_F(S)\in(0,S)$. Furthermore, $S_F(S)$ is strictly increasing in S and* $\lim_{S\to\infty} S_F(S) = \bar{S}_F > 0$.

Proof. See Appendix.

The second market-clearing condition requires that the total number of residents in the city must equal the total population of the city, P. This requires that $P = \int_{S_F}^{S}[2\pi r/l(r)]dr$.

Since $l(r) = (1-\beta)w(0)\exp(-\kappa r)/q_H(0)\exp\left(-\frac{\kappa}{1-\beta}r\right)$, this implies

$$P = \frac{q_H(0)}{(1-\beta)w(0)}\int_{S_F(S)}^{S} 2\pi r\exp\left(-\frac{\beta\kappa}{(1-\beta)}r\right)dr.$$

Using (11), (13), and (15), we obtain

$$n(0) = \frac{P}{2\pi}\frac{1}{\left[\int\limits_{0}^{S_F(S)} r\exp\left(-\frac{\gamma\delta-\alpha\kappa}{1-\alpha}r\right)dr\right]}\frac{\left[\int\limits_{S_F(S)}^{S} r\exp\left(-\frac{\kappa}{1-\beta}r\right)dr\right]}{\left[\int\limits_{S_F(S)}^{S} r\exp\left(-\frac{\kappa\beta}{1-\beta}r\right)dr\right]}. \tag{16}$$

Since $z(0) = n(0)\int_0^{S_F(S)} 2\pi r\exp\left(-\left[\frac{\delta\gamma-\kappa}{1-\alpha}+\delta\right]\right)rdr$, knowing $n(0)$ allows us to pin down the level of the external effect at the city center. This completes the first part of the equilibrium determination problem.

Before proceeding to the second part, it is useful to report how allocations and prices within the city are affected by changes in the supply of urban land, holding A and P fixed. For $n(0)$, we see from (16) that the first fractional term is decreasing in S since $S_F(S)$ is increasing in S. The effect on the second fractional term seems unclear because an increase in S increases both the numerator and the denominator. Notice, however, that both integrals calculate a "mean distance" with weights that decline exponentially with distance and the weights decline *faster* for the numerator term (since $\beta > 0$). Thus an increase in S increases the numerator proportionately less than the denominator and $n(0)$ is declining in S. Since $z(0)$ is proportional to $n(0)$, an increase in S is also a force depressing $z(0)$. However, an increase in S increases the geographic reach of the external effect by increasing S_F and this is a force that elevates $z(0)$. But if $\delta > \kappa$, the first effect dominates and $z(0)$ is also decreasing in S.[5]. When $\gamma < 1$, which is the empirically relevant case (γ is generally estimated to be less than 0.10), (9) implies that $\delta > \kappa$. Therefore, we can without much loss of generality restrict out attention to this case. Then, it is easy to verify:

[5]These results are formally established in Chatterjee and Eyigungor (2014)

Proposition 1 *(The Effect of Change in City Size S): If A and P are held constant, (i) employment density (and employment), the level of the production externality, and rents at the city center are decreasing in S, (ii) if $\alpha+\gamma \leq 1$, wages at the city center are increasing in S, otherwise the effect is ambiguous and (iii) if $\beta(\alpha+\gamma) \leq 1$, U is increasing in S, otherwise the effect is ambiguous.*

We now turn to the second part of equilibrium determination, namely, the determination of S and U, given A and P. Since it costs d units of the consumption good to convert one unit of undeveloped land into urban land, developers (the entities that own all land in this economy) will continue to develop urban land until the rent at the city boundary S is equal to the cost of development. Therefore, S is determined by

$$q(S; A, P) = d, \tag{17}$$

where $q(S; A, P)$ is the rent at the city boundary when TFP is A and population is P. The following Lemma establishes how $q(S; A, P)$ varies with S.

Lemma 2 *$q(S; A, P)$ is strictly decreasing in S and strictly increasing in A and P. Furthermore, $\lim_{S\to 0} q(S; A, P) = \infty$ and $\lim_{S\to\infty} q(S; A, P) = 0$.*

Proof. See Appendix.

All else the same, rents fall with S because workers who live at the boundary earn the least. Complementing this effect is the fact that, recorded in Proposition 1, rents at the city *center* are also declining with S. The latter effect pushes down rents in all locations in the city, including the boundary. The "Inada-type" conditions of $q(S; A, P)$ are also intuitive: Rents in locations very far from the city center must be very low to compensate for the very large amount of time lost in commuting to a job. If the boundary is very close to the city center, employment density at the center must be very high, which would require very high rents there and, by extension, at the city boundary. Given Lemma 2, it follows that, for any A, P, and d, there is a unique S, denoted $S_d(A, P)$ that solves (17).

The following proposition describes how S is affected by changes in TFP, population, and costs of development. These properties follow directly from Lemma 2.

Proposition 2 *$S_d(A, P)$ is strictly increasing in A and P and strictly decreasing in d. Furthermore, $\lim_{P \to 0} S_d(A, P) = 0$ and $\lim_{P \to \infty} S_d(A, P) = \infty$.*

Finally, we come to the relationship between U, the utility deliverable by a city, and A and P when the city boundary adjusts so that rent at the boundary is d. We will denote this relationship by the function $U_d(A, P)$. We are primarily interested in understanding how this function behaves with respect to variations in P, since migration in or out of the city is the key adjustment mechanism for cities. It is a convenient feature of the model that this function can be expressed as a composition of two functions: An "outer" function, denoted $V_d(A, S)$, which gives the utility deliverable by a city given A and S and rent at the boundary of d, and an "inner" function, which is just $S_d(A, P)$. Thus, $U_d(A, P) = V_d(A, S_d(A, P))$. The benefit of this decomposition is that the $V_d(A, S)$ function has a closed-form expression that allows easy assessment of its shape with respect to variations in S. And, since $S_d(A, P)$ is strictly increasing in P (Proposition 2), the shape of $U_d(A, P)$ with respect to P is simply a shape-preserving rescaling of $V_d(A, S)$.

To develop the $V_d(A, S)$ function, we use two conditions. The first condition is that rent at the city boundary must be d, i.e., $d = q_H(0) \exp(-\kappa/(1 - \beta))S$. This condition implies that S and d pin down rents at the city center. We have already seen, however, that rents at the city center are determined by A, $n(0)$, and $z(0)$. Since $z(0)$ is itself pinned down by $n(0)$, it follows that the first condition fully determines $n(0)$ as a function of A, S, and d.

The second condition equates the utility obtained by a worker who resides at the city boundary when the city size is S and rent at the boundary is d to the utility delivered by the city to any worker, which is V. This equality gives $V = \beta^\beta (1-\beta)^{1-\beta} d^{-(1-\beta)} w(0) \exp(-\kappa/(1-\beta)S)$. Since $w(0)$ is ultimately determined by $n(0)$, and $n(0)$ is determined by A, S and d (from above), the expression for V yields $V_d(A, S)$.

To determine the shape of $V_d(A,S)$ with respect to S, it is convenient to examine $\ln(V_d(A,S))$. Collecting terms that do not depend on S into a "constant" D, we have

$$\begin{aligned}
&\ln(V_d(A,S)) = \\
&D + \frac{\gamma}{\alpha+\gamma}\ln\left[\int_0^{S_F(S)} r\exp\left(-\frac{\delta(\gamma+1-\alpha)-\kappa}{1-\alpha}r\right)dr\right] + \\
&-\kappa\frac{1-\beta(\alpha+\gamma)}{(1-\beta)(\gamma+\alpha)}S + \left(\frac{\gamma+\alpha-1}{\gamma+\alpha}\right)\frac{-\kappa+\delta\gamma+\beta\kappa\alpha-\beta\delta\gamma}{(1-\alpha)(1-\beta)}S_F(S).
\end{aligned} \tag{18}$$

From Lemma 1, however, we know that $\lim_{S\to 0} S_F(S) = 0$ and $\lim_{S\to\infty} S_F(S) = \bar{S}_F$. Therefore, $\lim_{S\to 0}\ln(V_d(A,S)) = \lim_{S\to\infty}\ln(V_d(A,S)) = -\infty$. Whether the function generally has a single peak is not easy to establish, although in our numerical simulations we have found it be single-peaked (i.e, inverted U in shape). When S is small the rate of change of $\ln V_d$ is likely to be dominated by the logarithmic term and, therefore, will be large and positive. Hence the function is likely to be initially increasing in S. More importantly, when S is large the behavior of $\ln V_d$ is dominated by the term involving S, since S_F converges to a constant. Hence the function is eventually declining in S. To summarize:

Lemma 3 *Assume* $1-\beta(\alpha+\gamma) > 0$. *Then,* $\lim_{S\to 0} V_d(A,S) = \lim_{S\to\infty} V_d(A,S) = 0$. *In addition,* $V_d(A,S)$ *is eventually declining in* S.

As mentioned earlier, because $S_d(A,P)$ is strictly increasing in P, $U_d(A,P)$ inherits all the properties of $V_d(A,S)$. Therefore, we have the following proposition:

Proposition 3 *Assume* $1-\beta(\alpha+\gamma) > 0$. *Then,* $\lim_{P\to 0} U_d(A,P) = \lim_{P\to\infty} U_d(A,P) = 0$. *In addition,* $U_d(A,P)$ *is eventually declining in* S.

The condition $1-\beta(\alpha+\gamma) > 0$ is our analog of what Fujita, Krugman, and Venables (1999) call the "no-black-hole condition." If this condition is violated, then, as is evident from the expression of $\ln(V_d(A,S))$, utility deliverable by the city would be increasing in S. Since $S_d(A,P)$ is strictly increasing in P, utility deliverable by the city would be strictly increasing in P. The model would then imply that the entire population of an economy would tend to gravitate to one giant city—the "black hole," so to speak. To rule this out, β must be low enough, or equivalently, the importance of land in the worker's utility must be high enough.[6]

5 City Growth and Land Supply Constraints

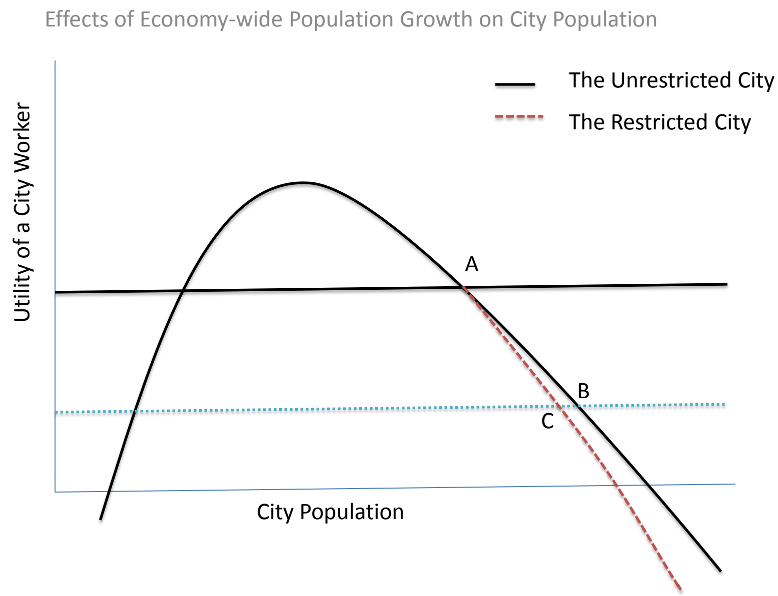
In this section, we use the model to explore the effects of an increase in the demand for urban land on city wages and rents when there are physical constraints on the supply of new urban land.[7] We will consider two cities that are in full spatial equilibrium with land rent at the boundary equal to d. The cities have identical primitives and are identical in terms of size and population. We will assume that each city is on the monotonically declining portion of the $U_d(A, P)$ function, so that comparative statics results are well-defined.

We consider two different ways in which the demand for urban land can increase. In one case there is an increase in the population of the economy due to natural growth or immigration and the new arrivals have to locate in one or the other city. The other case is an increase in TFP in the two cities which draws in workers from the rest of the economy. Following the increase in demand, we will assume that one of these two cities is free to convert undeveloped land into developed land at the cost d (i.e., it can expand the boundary of the city), but other city is physically restricted from doing so (i.e., geography prevents it

[6]Lucas and Rossi-Hansberg (2002) (and also Lucas (2001)) assume a condition that is stronger, namely, $\alpha + \gamma < 1$. Although this condition is also labeled a "no-black-hole condition," it is needed to rule out a different kind of black hole, one in which all firms pile up at 0 (the city center) with each firm using a vanishingly small amount of land but enjoying unboundedly high external effect, i.e., it is needed to rule out the case where $z(0)$ diverges to ∞. However, this case is not a concern for us because $z(r)$ is known to have the negative exponential form and, hence, productivity at the city center is naturally bounded above by city size and total population.

[7]There is small theoretical literature on the effects of urban land-use restrictions on city wages and rents. Brueckner (1990), Ding, Knaap, and Hopkins (1999), and Brueckner (2007) study the impact of urban growth boundaries in the context of the standard monocentric city model with a (negative) congestion externality, while Bertaud and Brueckner (2005) examine the impact of building-height restrictions, again in the standard monocentric city model. However, none of these studies allow for production externalities and therefore miss the production-side effects of such controls.

Figure 3

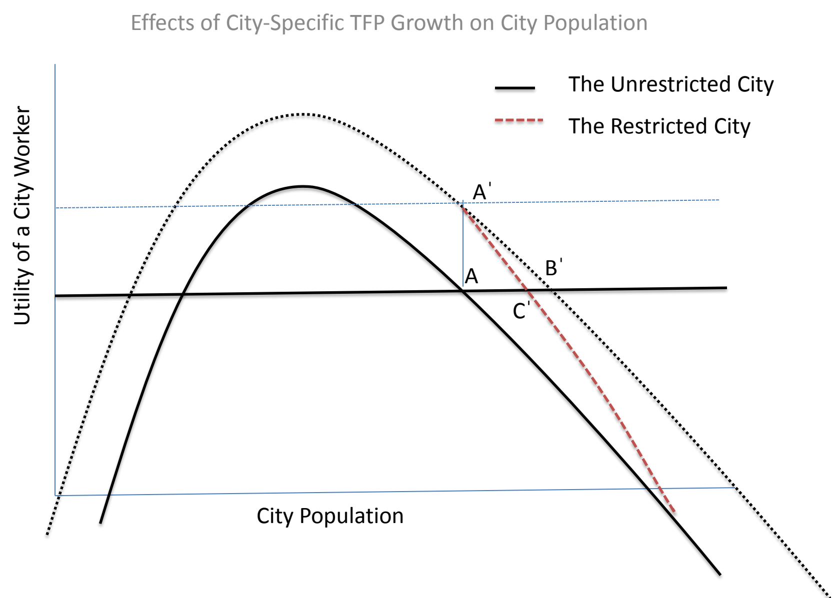


from pushing out the boundary of the city). We call the former the Unrestricted City and the latter the Restricted City.

The effects of the two types of shocks on the population of these two cities are illustrated in Figures 3 and 4. Figure 3 shows the impact of an increase in economy-wide population. The solid hump-shaped line plots $\ln(U_d(A, P))$ against P for the unrestricted city. The solid horizontal line is the utility available to a worker in any other city in the economy prior to the increase in population. We assume that the city is at the point labeled A (which corresponds to a stable equilibrium in the usual sense). The increase in population then results in a drop in the utility deliverable by cities in general, so the horizontal line shifts down to the dotted one. People move into the city until the city reaches the point labeled B. The dashed line in the figure is the utility curve for the restricted city. Since this city cannot expand its physical boundary, the decline in utility in response to increased in-migration is larger (at any level of P) relative to the unrestricted city. People move into the city until the restricted city reaches the point labeled C.

Figure 4 shows what happens if the two cities receive a positive TFP shock. The solid

Figure 4



lines have the same interpretation as in Figure 1, and the initial position of both cities is A. The shock leaves the general level of utility (the solid horizontal line) unchanged, but shifts the $\ln(U_d(A,P))$ for the two cities upward to the dotted line. The city draws in population from elsewhere in the economy until it reaches point B. The dashed line is the utility curve for the restricted city. Once again, it lies below the utility curve for the unrestricted city. The restricted city also draws in people from the rest of the economy until it reaches point C. We can, however, view this adjustment as happening from A' to B and C, which is exactly like a drop in the general level of utility from what is available at A' to the solid horizontal line. Thus the effects of the two types of demand shocks are fundamentally similar.

What we take from Figures 2 and 3 is that, following the increase in total population or TFP, both cities will experience increases in population. Proposition 2 implies that the unrestricted city will be physically larger than the restricted city following the shock. From the figures, it is also clear that population will increase more in the unrestricted city than in the restricted one. Of course, in the new equilibrium, both cities will deliver the same utility to workers residing there. In what follows, we analyze the impact of demand shocks on employment density, wages, and land rents in the two cities. We analyze land rents last because it is easier to understand why land rents behave as they do once we understand how employment density and wages are affected by the demand shocks.

5.1 Demand Shocks and Employment Density

We will focus on employment density at the city center since that will determine what employment density will be in any other location. Examining the expression for $n(0)$ in (16), we see that it is not immediately possible to tell how $n(0)$ compares across restricted and unrestricted cities: The unrestricted city has higher P and larger S relative to the restricted city. However, if we use the fact that, in equilibrium, both cities must deliver the same utility to workers, it becomes possible to compare employment densities.

Observe that, in both the restricted city and the unrestricted city, the firm's bid rent and the worker's bid rent for the city center coincide. This implies $(1-\alpha)\,Az(0)^{\gamma}n(0)^{\alpha} = (1-\beta)\,\beta^{\frac{\beta}{1-\beta}}\left(\frac{w(0)}{U}\right)^{\frac{1}{1-\beta}}$, where U is the common utility delivered by the two cities. Using the fact that both $w(0)$ and $z(0)$ can be expressed in terms of only $n(0)$, S, and other parameters, it is possible to express $n(0)$ in terms of U, S, and other parameters:

$$n(0) = KA^{\frac{\beta}{1-\beta(\alpha+\gamma)}}U^{\frac{-(1-\beta)}{1-\beta(\alpha+\gamma)}} \times \tag{19}$$
$$\exp\left(\frac{[\delta\gamma-\kappa]-\beta[\delta\gamma-\alpha\kappa]}{(1-\alpha)(1-\beta(\alpha+\gamma))}S_F\right)\left[\int_0^{S_F} 2\pi r\exp\left(-\left[\frac{\delta\gamma-\kappa}{1-\alpha}+\delta\right]r\right)dr\right]^{\frac{\gamma\beta}{1-\beta(\alpha+\gamma)}},$$

where K is a positive quantity that depends on parameters. By virtue of the "no-black-hole condition" $1-\beta(\alpha+\gamma)>0$ and the upper bound on κ in (9) it follows that $n(0)$ is increasing in S_F. Since S_F is strictly increasing in S (Lemma 1), it follows again that, in the new equilibrium, employment density in the center of the unrestricted city must exceed that in the center of the restricted city. We summarize this discussion in the following proposition:

Proposition 4 *If two cities have the same fundamentals and one of the cities has an urban growth boundary that is just binding, an increase in the demand for urban land will cause* *employment density in the unrestricted city to rise more than in the restricted city.*

5.2 Demand Shocks and (Product) Wages

As in the case of employment density, it is sufficient to consider what happens to wages offered by firms locating at the city center, namely, $w(0)$. In any city, $w(0) = \alpha A z(0)^{\gamma} n(0)^{\alpha-1}$. Using the relationship between $z(0)$ and $n(0)$, this implies

$$w(0) = \alpha A \left[2\pi \int_0^{S_F} r \exp\left(- \left[\frac{\delta\gamma - \kappa}{1-\alpha} + \delta \right] r \right) \right]^{\gamma} n(0)^{\gamma+\alpha-1}. \tag{20}$$

We already know that, in the new equilibrium, the unrestricted city will be larger in size and it will have a higher employment density. From the above expressions it follows that if $\alpha + \gamma \geq 1$ then wages at the city center will be higher in the unrestricted city relative to the restricted one. By continuity this ordering will also prevail when $\alpha + \gamma$ is slightly less than 1, but it may or may not prevail when $\alpha + \gamma$ is substantially less than 1. Since $(1 - \alpha)$ is simply the exponent to land in the production function and γ is the exponent to the level of agglomeration in the city, wages at the center of the unrestricted city will exceed those in the center of the restricted city, provided agglomeration is more important in production than land. Summarizing, we have the following proposition:

Proposition 5 *If two cities have the same fundamentals and one of the cities has an urban growth boundary that is just binding, and if $\gamma \geq 1 - \alpha$, an increase in the demand for urban land will cause the wage offered at the center of the unrestricted city to exceed the wage offered at the center of the restricted city.*

5.3 Demand Shocks and Urban Land Rents

It is helpful to break up the discussion in terms of how demand shocks affect business rents and how they affect residential rents. Once again, it is sufficient to focus on the rents at the city center.

The bid rent for a firm at the city center, $q_F(0)$, is $(1-\alpha)Az(0)^\gamma n(0)^\alpha$. Using the relationship between $z(0)$ and $n(0)$, this implies

$$q_F(0) = (1-\alpha)A\left[2\pi\int_0^{S_F} r\exp\left(-\left[\frac{\delta\gamma-\kappa}{1-\alpha}+\delta\right]r\right)\right]^\gamma n(0)^{\gamma+\alpha}. \qquad (21)$$

We already know that, in the new equilibrium, the unrestricted city will be larger in size and it will have a higher employment density. Therefore, business rents at the center of the restricted city will be higher than business rents at the center of the restricted city. Summarizing, we have the following proposition:

Proposition 6 *If two cities have the same fundamentals and one of the cities has an urban growth boundary that is just binding, an increase in the demand for urban land will cause rents paid by businesses at the center of the unrestricted city to exceed the rents paid by businesses at the center of the restricted city.*

Turning to residential rents, we can proceed by considering again what happens to the bid rent for residential space at the centers of the two cities. We have

$$q_H(0) = \beta^{\beta/(1-\beta)}(1-\beta)w(0)^{1/(1-\beta)}U^{-1/(1-\beta)}. \qquad (22)$$

Since U is the same for both cities, the ordering of workers' bid rent for space at the center of the city depends on the ordering of wages at the center of the city. Therefore, the conditions that govern the ranking of $w(0)$ also govern the ranking of $q_H(0)$: Specifically, if $\gamma \geq 1-\alpha$, then wages at the center will be higher in the unrestricted city in the new equilibrium and therefore workers will be willing to bid more for land at the center of the unrestricted city than in the restricted city. Summarizing, we have the following:

Proposition 7 *If two cities have the same fundamentals and one of the cities has an urban growth boundary that is just binding, an increase in the demand for urban land will cause* *the bid rent for residential space at the center of the unrestricted city to exceed the bid rent for residential space at the center of the restricted city if* $\gamma \geq 1 - \alpha$.

Since rent in any location is simply $\max\{q_F(r), q_H(r)\}$, the immediate implication of Propositions 6 and 7 is that, in the new equilibrium, land rents may be higher in every comparable location in the unrestricted city relative to the restricted city. Specifically, denoting the physical size of the restricted city by $\bar{S}$ and land rents in the restricted and unrestricted cities by $q_R(r)$ and $q_{UR}(r)$, respectively, we have the following corollary:

Corollary 1 *If* $\gamma \geq (1 - \alpha)$ *then, in the new equilibrium,* $q_{UR}(r) > q_R(r)$ *for all* $r \in [0, \bar{S}]$.

6 Business Structures and City Growth

In this section, we introduce structures as a separate factor of production. Let

$$Y(r) = Az(r)^{\gamma} N(r)^{\theta} B(r)^{\psi} L(r)^{1-\psi-\theta}, \tag{23}$$

where $B(r)$ denotes the level of structures (building size) in location r. The exponent to structures is ψ and the exponent to labor is correspondingly reduced to $\theta = \alpha - \psi$. The exponent to land, therefore, stays the same at $1 - \alpha = 1 - (\psi + \theta)$.

We will first examine the long run case where structures can be varied along with labor. Our goal is to first show the equivalence of this setup to previous sections. For the equivalence to hold, assume that the per unit flow cost at which building services can be obtained in location r is $\omega w(r)$. The optimal choice of $B(r)$ satisfies

$$B(r) = \frac{\psi}{\omega\theta} N(r).$$

Substituting this back into the production function and expressing it in intensive form yields:

$$y(r) = A\left(\frac{\psi}{\omega\theta}\right)^{\psi} z(r)^{\gamma} n(r)^{\alpha}.$$

This is identical to (2), except for the constant multiplying A. Also, with structures explicitly modeled,

$$w(r) = \theta A z(r)^{\gamma}\left(\frac{\psi}{\theta\omega}\right)^{\psi} n(r)^{\alpha-1}.$$

Thus share of labor in total output is now θ, the share of building services is ψ, and the share of land rent remains $(1-\alpha)$. It is evident that these alterations will not affect the comparative statics results reported in Section 5.

Next we analyze what happens if there is an increase in the demand for urban land but business structures cannot change. Since business structures are fixed, it makes sense to also assume that S_F is fixed as well. The goal is to understand how Propositions 4-7 are modified in the presence this short-run rigidity in structures.[8]

Let's again take two cities that are initially identical. The business structure density gradient $b(r)$, as well as S_F, are now exogenously given and identical in both cities.

$$\bar{b}(r) = \bar{b}(0)\exp\left(-\frac{\delta\gamma-\kappa}{1-\alpha}r\right) \text{ for } r \in [0, \bar{S}_F].$$

The exogeneity of $b(r)$ means, in effect, that TFP now decays exponentially from the city center. In general, the rate of decay of TFP will affect the equilibrium employment density gradient. However, since the gradient of $b(r)$ is the gradient for the initial equilibrium, the employment gradient for the case where b and S_F cannot adjust is the same as in the model

[8] Since we do not model residential structures explicitly, the assumption in this part of the analysis is that residential structures can be altered in the short run. This simplification is meant to capture the fact that business structures are more difficult to change owing to high building density in the business district compared to the residential district.

in which these are endogenous. In particular, we may verify that

$$n(r) = \left[\theta A\bar{b}(0)^{\psi}\frac{z(0)^{\gamma}}{w(0)}\right]^{\frac{1}{1-\theta}}\left[\exp\left(\left(\frac{(\kappa-\gamma\delta)}{1-\alpha}\right)r\right)\right].$$

Since the gradient for $n(r)$ remains unchanged, the gradients for all other endogenous variables remain identical to the model where $b(r)$ and S_F are endogenous.

For concreteness, we analyze the case where both cities receive a positive shock to A. As before, one city cannot expand its boundary while the other city can at cost d. In the unrestricted city, two equilibrium conditions need to be satisfied. The first condition is the labor market balance condition. This condition is the same as (15). Given $\bar{b}(0)$ and $\bar{S}_F$, this condition reduces to

$$n(0)^{\frac{1-\beta(\gamma+\theta)}{1-\beta}}\left[\theta A\left(\int_0^{\bar{S}_F}2\pi s\left[\exp\left(\left(\frac{(\kappa-\gamma\delta)}{1-\alpha}-\delta\right)s\right)\right]ds\right)^{\gamma}\bar{b}(0)^{\psi}\right]^{-\frac{\beta}{1-\beta}}\times$$
$$U^{\frac{1}{1-\beta}}\int_0^{\bar{S}_F}r\exp\left(-\frac{\delta\gamma-\kappa\alpha}{1-\alpha}r\right)dr = \beta^{\frac{\beta}{1-\beta}}\int_{\bar{S}_F}^{S}r\exp\left(-\frac{\kappa}{(1-\beta)}r\right)dr.$$

The other condition is that rent at the city boundary be d, which is the same as (17). This condition reduces to

$$(1-\beta)\,\beta^{\frac{\beta}{1-\beta}}\left(\frac{\theta A z(0)^{\gamma}\bar{b}(0)^{\psi}n(0)^{\theta-1}\exp(-\kappa S)}{U}\right)^{\frac{1}{1-\beta}} = d. \tag{24}$$

Eliminating $n(0)$ gives us the following equilibrium condition for S:

$$A\left(\frac{1}{d}\right)^{1-\beta(\gamma+\theta)}U^{-(\gamma+\theta)} = K\left[(\exp(+\kappa S))^{\frac{1}{1-\beta}}\right]^{1-\beta(\gamma+\theta)}\left(\int_{\bar{S}_F}^{S}r\exp\left(-\frac{\kappa}{(1-\beta)}r\right)dr\right)^{1-(\gamma+\theta)}.(25)$$

Since $\theta<\alpha$, the "no-black-hole" condition implies that $1-\beta(\gamma+\theta)>0$. Hence the first term on the r.h.s of (25) is increasing in S. The second term on the r.h.s. will be increasing in S if $\gamma+\theta<1$, which is the empirically relevant case (the empirical evidence is discussed in the next section). Therefore, the r.h.s of (25) is increasing S. If there is an increase in A or a decline in U (due to increase in aggregate population), the unrestricted city will expand in size. The labor market balance condition (24) (which applies to both cities) tells us that $n(0)$ will be strictly higher in the unrestricted city relative to the restricted city. Now, observe that

$$w(0) = \theta A z(0)^{\gamma} \bar{b}(0)^{\psi} n(0)^{\theta-1}.$$

Since S_F is the same (fixed) in both cities, $z(0)$ is the same multiple of $n(0)$ in both cities. Therefore, when $\gamma+\theta < 1$, $w(0)$ will be lower in the unrestricted city relative to the restricted city. Since utility offered by the two cities must be the same, it follows immediately that residential rents in the unrestricted city will rise *less* than in the restricted city.[9] Summarizing we have

Proposition 8 *Assume business structures are fixed and $\gamma < 1 - \theta$. If two cities have the same fundamentals and one of the cities has an urban growth boundary that is just binding, an increase in the demand for urban land will cause employment density in the unrestricted city to exceed that in the restricted city. Wages and bid rents for residential land at the center of the restricted city will exceed the wages and bid rents for residential land at the center of the unrestricted city.*

7 Evidence on Agglomeration and Share Parameters

As we have seen, the impact of urban land supply constraints on city wages and rents depend on the magnitudes of the production externality parameter γ and the share parameters α (long run) and θ (short run). In this section we discuss the empirical evidence on these parameter values.

[9]If $\gamma + \theta > 1$, the utility deliverable by the city will be an inverted-U function of S and there will be two values of S that will deliver the given utility level. As long as the city ends up on the downward sloping portion of the utility curve following the shock, the increase in A or decrease in U will cause the unrestricted city to expand in size. In this case, the unrestricted city will experience a larger increase in wages and residential rents than the restricted city.

Turning first to γ, Melo, Graham, and Noland (2009) (Table 2, p. 355) report that estimates of this parameter (across various types of datasets, methodology and measures of agglomeration) range between -0.366 and 0.319 for the US, with the mean (and median) estimate at 0.036. The median estimate for other developed countries ranges between 0.028 (Canada) and 0.083 (UK). Across industrial groupings, the median estimate of γ for manufacturing is 0.036 and for services it is 0.142. Studies that use some measure of market potential to measure agglomeration, the median estimate for γ is 0.076. Measures based on average density or size imply median estimates of 0.039 and 0.030, respectively.

Turning to $(1-\alpha)$, one estimate comes from Brinkman (2013) who uses data on commercial land prices and quantities for Columbus, OH, and estimates $(1-\alpha)$ to be 0.015. Ciccone (2002) also suggests 0.015 as a reasonable estimate for (non-farm) business land share. Finally, Rappaport (2008) uses 0.016, citing unpublished results by Jorgenson, Ho, and Stiroh (2005). These findings indicate that $(1-\alpha) = 0.015$ is a reasonable estimate.

Estimates for share of structures, ψ, seem sparse. Valentinyi and Herrendorf (2008) report the share of structures in US manufacturing to be 0.09. It is possible service industries use less structures, but we do not have information on service industries.[10]

For $(1-\beta)$, Davis and Heathcote (2007) estimate that land accounts for 36 percent of the value of aggregate housing stock. Given that households spend about 25 percent of their budget on housing (which includes the services from structures and land), a plausible value of $(1-\beta)$ is around $0.10 \approx 0.25 \times 0.36$.

What we take from this discussion is that γ is likely to exceed land share $(1-\alpha)$. Thus the evidence indicates that in the long run the externality effect dominates the land-intensity effect. Furthermore, with $\alpha + \gamma \approx 1$ and $\beta \approx 0.90$, the no-black-hole condition is satisfied. On the other hand, the estimate of ψ implies that $\theta = \alpha - \psi$ is 0.9. Thus, the estimate of $\theta + \gamma$ is well below 1. These estimates suggest that in the short run (when structures

[10] The authors definition of services includes housing services. Since housing services tend to be more intensive in structures than business services, their estimate of the share of structures (0.15) is probably too high for our purposes.

are fixed), an increase in the demand for urban land is likely to raise wages and residential land prices more in cities that are geographically constrained. But, as structures adjust, this relationship is likely to be reversed.

8 Conclusion

The goal of this paper is to assess, theoretically and quantitatively, the role of geography for the value of urban land. In the model, urban agglomerations arise because of positive production externalities that make firms want to locate close to each other. The model makes clear predictions about how constraints on the physical expansion of the city affects city wages and land values when there is an increase in the demand for urban land. In the long run, when all non-land factors of production can be varied, constraints on physical expansion is predicted to hurt city wages and residential rents. In the short run, when business structures are fixed, constraints on physical expansion is predicted to enhance city wages and residential rents. As noted in the introduction, these seemingly contradictory effects of geographical constraints appear to have support in empirical studies that separately focus on the long and short runs. The fact that the time horizon is predicted to matter for assessing the impact of geographical constraints on the growth of city wages and rents is an important finding that is not presaged in earlier work.

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APPENDIX

Proof of Lemma 1 in the text

Given any $S > 0$, (9) (the upper bound on κ) implies that the r.h.s. of (15) is increasing in S_F. The l.h.s. of (15) is clearly decreasing in S_F. Furthermore, the r.h.s. is 0 for $S_F = 0$ while the l.h.s. is strictly positive, and the r.h.s. is strictly positive for $S_F = S$ while the l.h.s. is 0. Therefore, for each $S > 0$ there is a unique $S_F \in (0, S)$ that ensures (15) is satisfied. Observe also that as S goes up and S_F does not change, the integral on the l.h.s. goes up. Since the r.h.s. is increasing in S_F, the equilibrium S_F must be strictly higher. Thus $S_F(S)$ is strictly increasing in S.

To prove the second part, we observe that since $S_F(S) < S$ for all S, it must be the case that $\lim_{S\to 0} S_F(S) = 0$. To prove the other limiting result, we will first establish that $\lim_{S\to\infty} S_F(S)$ is bounded above. Let S_n be an increasing sequence diverging to ∞. Let $S_F(S_n)$ be a corresponding sequence of S_F that satisfies (15). Then $S_F(S_n)$ is also a strictly increasing sequence. Next, observe that

$$\int_{S_F(S_n)}^{S_n} s \exp\left(-\frac{\kappa}{1-\beta} s\right) ds = -\left[\frac{(1-\beta)}{\kappa}\right]^2 \left[e^{-\frac{\kappa}{(1-\beta)} s}(ks+1)\right]_{S_F(S_n)}^{S_n}.$$

If $S_F(S_n)$ diverges to infinity along with S_n, the above integral will converge to 0. This will imply that the l.h.s. of (15) will converge to 0 while the r.h.s. will diverge to ∞, which is impossible. Hence, $S_F(S_n)$ must be bounded above. Since $S_F(S)$ is strictly increasing, it follows that $\lim S_F(S)$ must converge to some number $\bar{S}_F > 0$. ■

To prove Lemma 2 in the text, we need the following two lemmas.

Lemma 1 (A) *Let* $0 \leq s_L < s_U$. *Let* $\Lambda(s_L, s_U) = [\int_{s_L}^{s_U} se^{k_2 s} ds]/[\int_{s_L}^{s_U} se^{k_1 s} ds]$. *Then,* $\Lambda(s_L, s_U)$ *is increasing (decreasing) in both* S_U *and* S_L *if* $k_1 < (>) k_2$.

Proof. We will first establish the following two sets of inequalities. If $k_1 < k_2$, then

$$e^{(k_2-k_1)s_L} < \frac{\int\limits_{s_L}^{s_U} se^{k_2 s}ds}{\int\limits_{s_L}^{s_U} se^{k_1 s}ds} < e^{(k_2-k_1)s_U}, \tag{26}$$

and if $k_2 < k_1$, then

$$e^{(k_2-k_1)s_U} < \frac{\int\limits_{s_L}^{s_U} se^{k_2 s}ds}{\int\limits_{s_L}^{s_U} se^{k_1 s}ds} < e^{(k_2-k_1)s_L}. \tag{27}$$

Turning first to the l.h.s. inequality in 26, we observe that $se^{k_2 s} = se^{s_L k_2 + (s-s_L)k_2}$and $se^{k_1 s} = se^{s_L k_1 + (s-s_L)k_1}$. Multiplying both sides of the latter equation by $e^{(k_2-k_1)s_L}$ yields $e^{(k_2-k_1)s_L}se^{k_1 s} = se^{s_L k_2 + (s-s_L)k_1} \leq se^{s_L k_2 + (s-s_L)k_2} = se^{k_2 s}$, where the inequality follows because $k_2 > k_1$ and $s - s_L \geq 0$. Furthermore, the inequality is strict for all $s \in (s_L, s_U]$. Therefore, integrating the first and last expressions in the chain with respect to s, we have

$$e^{(k_2-k_1)s_L} \int\limits_{s_L}^{s_U} se^{k_1 s}ds < \int\limits_{s_L}^{s_U} se^{k_2 s}ds.$$

Turning to the r.h.s. of the inequality, we observe that $se^{k_2 s} = se^{s_U k_2 + (s-s_U)k_2}$and $se^{k_1 s} = se^{s_U k_1 + (s-s_U)k_1}$. Multiplying both sides of the latter equation by $e^{(k_2-k_1)s_U}$ yields

$$e^{(k_2-k_1)s_U}se^{k_1 s} = se^{k_2 s_U + (s-s_U)k_1} \geq se^{s_U k_2 + (s-s_U)k_2} = se^{k_2 s},$$

where the inequality follows since $k_2 > k_1$ and $s - s_U \leq 0$. Furthermore, the inequality is strict for all $s \in [s_L, s_U)$. Therefore, integrating the first and last terms in the chain with respect to s, we have

$$e^{(k_2-k_1)s_U}\int\limits_{s_L}^{s_U} se^{k_1 s}ds > \int\limits_{s_L}^{s_U} se^{k_2 s}ds. \quad \blacksquare$$

The proof of 27 is entirely analogous.

We now turn to the proof of the Lemma. We begin with the case in which $k_1 < k_2$. Observe that

$$\frac{\partial \ln(\Lambda(s_L, s_U))}{\partial s_U} = \frac{s_U \exp(k_2 s_U)}{\int\limits_{s_L}^{s_U} se^{k_2 s}ds} - \frac{s_U \exp(k_1 s_U)}{\int\limits_{s_L}^{s_U} se^{k_1 s}ds}.$$

Suppose, to get a contradiction, that $\partial\Lambda(s_L, s_U)/\partial s_U \leq 0$. Then, we must have

$$\frac{s_U \exp(k_2 s_U)}{\int\limits_{s_L}^{s_U} se^{k_2 s}ds} \leq \frac{s_U \exp(k_1 s_U)}{\int\limits_{s_L}^{s_U} se^{k_1 s}ds}.$$

Or, given that all elements are positive, we have

$$\exp([k_2 - k_1]\, s_U) = \frac{s_U \exp(k_2 s_U)}{s_U \exp(k_1 s_U)} \leq \frac{\int\limits_{s_L}^{s_U} se^{k_2 s}ds}{\int\limits_{s_L}^{s_U} se^{k_1 s}ds}.$$

But this contradicts the r.h.s. inequality in Lemma 1. Therefore, $\partial\Lambda(s_L, s_U)/\partial s_U > 0$. Analogous proof can be given for the case in which $k_2 < k_1$. $\blacksquare$

Remark: Let $I(s_U, s_L, k) = \int\limits_{s_L}^{s_U} s\exp(-ks)\,ds$. Then (i) $\lim_{s_U, s_L\to\infty} I(s_U, s_L, k) = 0$ and (ii) $\lim_{s_U\to\infty, s_L\to\underline{s}} I(s_U, s_L, k) = \bar{I} > 0$.

Observe that

$$\int\limits_{s_L}^{s_U} se^{-ks}ds = \frac{s_U e^{-ks_U} - s_L e^{-ks_L}}{-k} - \frac{e^{-ks_U} - e^{-ks_L}}{k^2}.$$

To prove (i), we notice that, as s_U and s_L go to infinity, the second term goes to 0, and the first term (on an application of L'Hospital's Rule to s/e^{ks}) also goes to 0. To prove (ii), we observe that if s_U goes to infinity and s_L converges to $\underline{s}$, then $I(s_U, s_L, k)$ converges to

$$\frac{-\underline{s}e^{-k\underline{s}}}{-k} + \frac{e^{-k\underline{s}}}{k^2} > 0. \quad \blacksquare$$

Lemma 2 (A) *(The Effects of a Change in Population): If A and S are held constant, (i) employment density and the level of the production externality change proportionately with P, (ii) the elasticity of rents in any location with respect to P is $\alpha + \gamma$, (iii) the elasticity of wage in any location with respect to P is $\alpha + \gamma - 1$, and (iv) elasticity of U with respect to P is $\beta(\alpha + \gamma) - 1$.*

Proof. If A and S are held constant, (i) follows because a change in P will change $n(0)$ (and therefore $z(0)$) proportionally. From this fact, we can infer (ii) using (11), we can infer (iii) using (6), and we can infer (iv) from the fact that $U = \beta^\beta (1-\beta)^{1-\beta} w(r) q(r)^{-(1-\beta)}$. $\blacksquare$

Proof of Lemma 2 in the text

To prove the first part, we note that $q_H(S; A, P) = q_H(0)e^{-\frac{\kappa}{(1-\beta)}S}$. Since $e^{-\frac{\kappa}{(1-\beta)}S}$ is decreasing in S, it is sufficient to show that, if we hold A and P constant, $q_H(0)$ is decreasing in S. To begin, note that $q_F(0)e^{-\frac{\delta\gamma - \kappa\alpha}{1-\alpha}S_F} = q_H(0)e^{-\frac{\kappa}{(1-\beta)}S_F}$, which implies that $q_F(0)/q_H(0) = e^{-\frac{-\kappa+\delta\gamma+\alpha\beta\kappa-\beta\delta\gamma}{(1-\alpha)(1-\beta)}S_F}$. By (9), the r.h.s. of the latter equation is increasing in S_F. Since $S_F(S)$ is increasing in S, it follows that $q_F(0)/q_H(0)$ is increasing in S. From Proposition 1 we know, holding A and P constant, that $q_F(0)$ is decreasing in S. Therefore $q_H(0)$ must be decreasing in S. And, if we hold fixed S and P, $q_H(0)$ is proportional to A and therefore $q(S; A, P)$ is increasing in A. And, holding fixed S and A, we see that $q_H(0)$ is increasing in P by Lemma 2(A). Therefore $q(S; A, P)$ is increasing in P.

We now turn to limiting behavior of $q_H(S; A, P)$.

Part (i): $\lim_{S\to\infty} q_H(S;A,P) = 0$. Consider

$$q_H(S;A,P) = (1-\beta)\,\beta^{\frac{\beta}{1-\beta}} \left(\frac{w(0)\exp(-\kappa S)}{U}\right)^{\frac{1}{1-\beta}}.$$

Using (10), (11), (13), (16), and the expression for $z(0)$, we can express the ratio of $w(0)$ to U as

$$\frac{w(0)}{U} = KP^{(1-\beta)(\gamma+\alpha)}A^{-1}\left(\int\limits_{S_F}^{S} s\left(\exp\frac{-\kappa\beta}{1-\beta}s\right)ds\right)^{-(1-\beta)(\gamma+\alpha)} \times$$
$$\left(\int_0^{S_F} s\exp\left(\frac{\kappa-\delta(\gamma+1-\alpha)}{1-\alpha}s\right)ds\right)^{\gamma(1-\beta)} \times$$
$$\exp\left(\frac{(-\kappa+\delta\gamma+\beta\kappa\alpha-\beta\delta\gamma)(\gamma+\alpha-1)}{(1-\alpha)}S_F\right),$$

where K is a positive constant. Given that $\lim_{S\to\infty} S_F(S) = \bar{S}_F$, the last two terms approach finite numbers. And, by Lemma 1, $\int\limits_{S_F}^{S} s\left(\exp\frac{-\kappa\beta}{1-\beta}s\right)ds$ appoaches a strictly positive finite number. Thus, we can conclude that, as $S\to\infty$, the ratio $w(0)/U$ approaches a finite number as well. Therefore, the limiting behavior of $q_H(S;A,P)$ is governed by the limiting behavior of $\exp(-\kappa S)$. Hence, $\lim_{S\to\infty} q_H(S;A,P) = 0$.

Part (ii): $\lim_{S\to 0} q_H(S;A,P) = \infty$

Since $S > S_F(S), S\to 0$ implies $S_F(S)\to 0$. Then, it is easiest to show that $q_F(0) = (1-\alpha)\,z(0)^{\gamma}n(0)^{\alpha}$ goes to infinity, which would imply that $q_H(S;A,P)$ goes to infinity also. Turning first to $n(0)$, we observe that

$$n(0) = \left[\frac{\int\limits_{S_F}^{S} s\exp\left(-\frac{\kappa}{1-\beta}s\right)ds}{\int\limits_{S_F}^{S} s\left(\exp\frac{-\kappa\beta}{1-\beta}s\right)ds}\right]\frac{P}{2\pi\left[\int\limits_0^{S_F} s\exp\left(\frac{\alpha\kappa-\gamma\delta}{1-\alpha}s\right)ds\right]}.$$

We know from Lemma 1 that

$$\exp\left(\kappa S_F\right) < \frac{\left[\int\limits_{S_F}^{S} s\left(\exp\frac{-\kappa\beta}{1-\beta}s\right)ds\right]}{\left[\int\limits_{S_F}^{S} s\exp\left(-\frac{\kappa}{1-\beta}s\right)ds\right]} < \exp\left(\kappa S\right).$$

This implies that as S and S_F converge to 0 (and so both $\exp\left(\kappa S_F\right)$ and $\exp\left(\kappa S\right)$ converge to 1) the term in square brackets converges to 1. We also know that $\left[\int\limits_{0}^{S_F} s\exp\left(\frac{\alpha\kappa-\gamma\delta}{1-\alpha}s\right)ds\right]$ goes to zero as S_F goes to zero, so $n(0)$ goes to infinity as S goes to zero.