Joint Dynamics of House Prices and Foreclosures*

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Abstract

In this paper we study the joint transitional dynamics of the foreclosures and house prices in a standard life-cycle incomplete markets model with housing and realistic mortgage structure. Mortgages are modeled as individual-specific long-term amortizing contracts, and the terms of the mortgage contracts (e.g. mortgage premium) are determined endogenously by the risk-neutral lenders in a market with perfect competition. We calibrate our model to match several long-run features of the US housing market. The model produces realistic housing and mortgage holding behavior along the age, income and wealth dimensions. We use the quantitative model to study the effects of several shocks generating the bust of the housing market. Specifically, we analyze the effects of a shock to the expectations about the long-term risk-free interest rate and the minimum downpayment requirement. We study the dynamics in the housing and the mortgage markets both at the steady-state and through the transition. We show that a simultaneous increase in the risk-free interest rate from 2% to 3% and in the minimum downpayment fraction from 0% to 20% can generate a 30% decline in the house price and almost five times increase (from 1.7% to 9.8%) in the foreclosure rate along the transition. Lastly, we analyze the effects of several policies in mitigating the consequences of adverse shocks in the housing and the mortgage market.

Keywords: Housing,house price, interest rate, mortgage contract, mortgage default, home equity

JEL Classifications: D91, E21, G01, R21

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1 Introduction

In this paper we study the joint transitional dynamics of the foreclosures and house prices in a standard life-cycle incomplete markets model with housing. In particular, we analyze how house prices and foreclosures in our model responds to several shocks such as interest rates and minimum downpayment requirement. In the nut shell, we show that, when calibrated to match the long term features of the US data, if the long term interest rate increases from 2 to 3 percent house prices decline around 15 percent and foreclosure rates increase from 1.7 percent to more than 4 percent. Similarly, if the minimum downpayment requirement increases from 0 to 20 percent house prices decline around 25 percent and foreclosure increases to around 6 percent. In addition, we study the effects of several policies in mitigating the consequences of the adverse shocks to the economy.

The model that we study is a standard life-cycle model with incomplete markets and idiosyncratic labor income uncertainty. We explicitly incorporate housing tenure choice into the model. Households are born as renters. Every period, renters decide whether or not to purchase a house. There is a continuum of risk-free lenders who offer mortgage contracts to prospective home buyers. A mortgage contract consists of a mortgage interest rate, loan amount, mortgage repayment schedule and maturity. Mortgages are fully amortizing, that is, homeowners have to pay the mortgage back in full until the end of the mortgage contract, as specified by the maturity. However, homeowners also have the option to sell their houses or default on the mortgage and return to the rental market. Selling a house is different from defaulting, because a seller has to pay back the outstanding mortgage balance to the lender whereas a defaulter has no obligation. Therefore, default occurs in equilibrium as long as the selling price is lower than the outstanding mortgage debt. Upon default, the household becomes a renter again and is excluded from the mortgage market for a certain number of years on average as a punishment.

There is free entry into the credit market and lenders are risk-neutral. So in equilibrium lenders make zero profit on each contract. Since mortgages are long-term contracts, it is essential for the lenders to infer the default probability of each household at every date and state, which depends on the income risk as well as the other characteristics of each household.

We calibrate our model to match several long term features of the US housing market. As our first exercise, we feed the economy with an unexpected and permanent interest rate shock where the risk-free interest rate increases from 2 to 3 percent. We show that, if we compare the first and the second steady state house prices are around 13 percent lower in the second steady state. On the other hand, foreclosure rate declines to 0.6 percent. Two reasons behind the lower foreclosure rate are that both wealth and the average down payment are larger in the second steady state.

The transitional dynamics differ considerably from the steady state comparisons. We show that, when a higher interest rate shock (from 2 percent to 3 percent) hits the economy, house prices decline rapidly and then increase gradually to its new steady state value. As the interest rate rises from 2% to 3%, house prices decline around 15% initially. During the transition, the mortgage default rate increases from 1.7% to around 4% initially, then declines to its new steady state value (0.6%). Hence, studying the transitional dynamics shows that a very important determinant of the foreclosure activity is the movement in the house prices, not the levels. Another point that emerges from the exercise is that declining prices induces more foreclosure due to negative home equity and more foreclosures puts downside pressure on housing prices. About 20 percent of the decline in housing prices can be attributed to this feedback mechanism.

Next we study the implications of an alternative shock: an increase in the minimum downpayment requirement. In the baseline model there is no minimum downpayment requirement. We assume that at the beginning of the period unexpectedly and permanently the minimum downpayment increases to 20 percent. This number is not far from the requirement during the 80's. The effects of this shock are higher compared to the interest rate shock that we mentioned. In response to this shock house prices decline more than 25 percent and foreclosures increase to around 6 percent. As the foreclosure rates decline overtime, the effect of foreclosures on house prices decline and house prices appreciate around 10 percent afterwards. In the long run the model converges to the new steady state where house prices are around 15 percent lower than the initial steady state. In the second steady state foreclosure rate is almost zero due to higher downpayment requirement. Lastly, we also analyze the case where these two shocks are observed simultaneously. As expected, the responses are larger. House prices decline around 30 percent and foreclosures increase to the levels around 10 percent.

We go further and study several policies, and question how the transition would look like in the presence of these policies. The first policy we analyze is the Federal Reserves (FED) policies. We assume that, after the initial decline in house prices real interest rates becomes 0.5 percent for several periods.[1] We find that temporary low interest rates have temporary effects. When there is only the interest rates shock, as a response to low interest rates house prices first appreciate and then depreciate. On the other hand foreclosures, due to higher volatility in house prices, remain high. When there is only an increase in minimum downpayment requirement low interest rate policy becomes much less effective.

We also study how the model would react to the same interest rate shock if the minimum downpayment requirement of 20 percent had been always in effect. In the benchmark calibration we use 0 percent minimum downpayment requirement. We find that with higher minimum downpayment requirement the response of house prices and foreclosures to shocks declines dramatically.

[1] We do not model how FED achieves low interest rates.

2 Related Literature

The run up of the housing prices prior to 2006 and the collapse afterwards induced an increased interest in the housing markets. The earlier studies are generally motivated to understand the increase in housing prices. After the collapse, both understanding the decline in the house prices and the increase in foreclosures have become the main motivation.

The model of this paper is based on the model developed in Guler (2012) where he studies the implications of the improvement in information technologies. One innovation of his paper is the modeling of the mortgage contract in detail for each individual. This extension enables us to study the default decision realistically.

Our paper is most closely related to work by Chatterjee and Eyigungor (2011). Like us, they also study the collapse of the house prices and rise in defaults in a standard incomplete market model. Different from our model, they assume that mortgage holders pay only the interest on their loans. Consequently, all homeowners have the same equity which is the downpayment amount. In our model, consumers pay out their loans before they retire, hence consumers have heterogenous levels of home equity. We believe this is a crucial difference between our model and Chatterjee and Eyigungor (2011) since home equity levels are found to be a primary determinant of the foreclosure decision. In their model the aggregate shock that they study is an unexpected increase in the stock of owner-occupied housing units. In addition they consider the lengthening of the foreclosure process and the disruption in the flow of credit from the financial sector as the sources of the movements in the housing market. While the framework and the questions that we analyze are similar, the reasons generating the process are different. As we mentioned earlier (and show in the next section), the exogenous processes that we think as the underlying sources of the shocks in the housing market are the higher real interest rates, higher unemployment rates and the disruption in the financial markets which we model as an increase in the minimum downpayment requirement. In addition, we analyze the implications of some policies such as monetary and financial policies in mitigating the effects of these adverse shocks.

Our paper differs from the related studies by Corbae and Quintin (2011) and Campbell and Cocco (2011). These papers also study the defaults in the housing market. The main difference is that they consider the house prices as an exogenous process. With this modeling assumption they cannot study the feedback between the house prices and foreclosures which has the potential to be significant. However, we model the house prices and foreclosures endogenously, hence are able to study the joint dynamics.

Another related study to ours is a recent study by Hatchondo, Martinez and Sanchez (2012). Similar to our study, they also study a life-cycle standard incomplete markets model with housing. They show that a calibrated version of the model matches the non targeted moments such as the distribution of the downpayments, hence a plausible borrowing behavior. They also study the implications of minimum downpayment requirement and income garnishment on the mortgage defaults. One key point that we depart from their study is that, in their model they assume house prices follow a stochastic process. In our model we assume that house prices are constant in the steady state and change only when an aggregate interest rate shock hits the economy. We find this difference important since to study the feedback mechanism between house prices and default, both of them should be endogenous. Another point we differ is the policies we analyze.

The earlier quantitative macro literature which studies the housing market finds that in a business cycle model it is difficult to generate significant volatility in the house prices. However, Arslan (2008) finds that the rigidities in the housing market together with mortgage contracts implies both large house price volatility and comovement of house prices and transaction volume. Sommers et al (2011) build a stochastic life cycle Aiyagari-Bewley-Huggett economy with exogenous down payment requirements and interest rates, and endogenous house prices and rent prices. They find that, fundamentals (ie downpayments and interest rates) can explain about half of the run up in the house price-rent ratio. In a related article Garriga et al (2012) show that interest rates and downpayment requirements generate volatilities in the house prices comparable to the data. Hence, it is no surprise that in our model we also have significant response of the house prices to the fundamentals which is necessary to have significant movements in foreclosures as well.

3 Model

We begin by describing the environment agents face in the economy. We then specify the decision problems of households and lenders. We finally define the equilibrium.

3.1 Environment

The economy is populated by overlapping generations of J period lived households and a continuum of lenders. Each generation has a continuum of households. Time is discrete and households live for a finite horizon. There is no aggregate uncertainty but households face idiosyncratic shocks to labor income, and markets are incomplete so that these shocks are not fully insurable. There is mandatory retirement at the age J_r. Retirement income is constant and depends on the income of the household at age J_r and the average income in the economy. Households can save at an exogenously given interest rate r but they are not allowed to make unsecured borrowing. Ex-ante, households are different in two dimensions: initial asset and initial income. Initial asset-income ratio is assumed to be log-normally distributed.

Households live in houses, which they can either rent or own. At the beginning of each period, a household is in one of the four housing statuses: *inactive renter*, *active renter*, *homeowner*, or *mover*. Inactive renters are the renters with default flag in their credit history whereas active renters are the renters without any default flag in their credit history[2]. In any period, active renters are always allowed to purchase a house, whereas inactive renters are not allowed to buy, and forced to stay as renters for that period. Homeowners are the households who start the period by owning a house. Lastly, movers are the owners who are forced to move from their current houses due to exogenous reasons, like a job-offer or family size change. The size of the house is fixed, i.e. there is no upgrading or downgrading the house size. However, since houses are big and expensive, their purchase is only through mortgages, which is also the only source of borrowing in the economy. A mortgage contract is a combination of interest rate and loan amount, specified by the downpayment fraction and house value. Maturity of the mortgages is assumed to be the remaining life time of the household until retirement[3]. Lenders only offer fixed-payment mortgages, so the payment is constant throughout the life of the mortgage[4]. There is no mortgage refinancing or home-equity line of credit.

Homeowners may leave their houses for three reasons: 1) due to the income shocks, they may be liquidity constrained, and cannot make the mortgage payments, 2) since all homeowners are forced to sell their houses upon retirement, some homeowners may choose to leave their houses before retirement due to consumption smoothing arguments, and 3) for exogenous reasons which are not explicitly modeled here [5]. Homeowners can leave their houses by two means. They can either sell their houses, or they can default on the mortgage. Selling a house is subject to capital gains/losses, and these gains/losses are assumed to be i.i.d and only realized after the household sells the house, i.e. upon observing the shock, the household cannot withdraw the house from the market. In the case of default, households become inactive renters, and they can become an active renter in the next period with probability δ[6]. The supply of rental and owner-occupied units is constant and targeted to match the average ownership rate in the US. Rental price is normalized to 0 and purchase

[2] This is to mimic the fact that default stays in the credit history of the individuals for a certain time, and makes it hard for her to find credit.

[3] Maturity of the mortgage, in reality, is a choice variable. However, in the current context to save from an extra state variable, we avoid this choice for now. Moreover, we assume that all homeowners are forced to sell their houses by retirement and spend their remaining life as renters. Since after retirement there is no uncertainty, housing tenure choice becomes uninteresting. So, to simplify the problem of the retirees, we ignore their housing tenure choice and force them to live as renters. This formulation will greatly simplify the computation of the value function at the time of retirement.

[4] Since we assume constant interest rate, traditional fixed rate mortgages and adjustable rate mortgages would have fixed payments throughout the life of the mortgage and they both fall into this category. These mortgages are not necessarily optimal contracts. A more convenient formulation should also include the mortgage payment as part of the contract and be determined in equilibrium. However, for simplicity we abstract from that and focus on the fixed payment mortgage contracts which are the dominant type of mortgages in the U.S. history.

[5] Like relocation due to job or family related reasons

[6] I have this assumption to mimic the fact that in reality default stays in the credit history of the households for a certain time period. Moreover it makes default costly for the individuals. This is also the reason I have the inactive renters as a different housing status.

price is solved endogenously given the fixed supply of owner-occupied units. The details of the model are explained below.

3.1.1 Households

Households derive utility from consumption and housing services. Preferences are represented by

$$E_0\left[\sum_{j=1}^{J_r}\beta^{j-1}u_k\left(c_j\right)+\beta^{J_r+1}W\left(w_{J_r},y_{J_r}\right)\right]$$

where $\beta<1$ is the discount factor, c is the consumption and k is the housing status: renter or homeowner. W represents the value function of the household at retirement given wealth w_{J_r} and income y_{J_r}[7]. The house size is fixed and the utility from housing services is summarized as two different utility functions: one for the renter, u_r and one for the homeowner, u_h. A homeowner receives a higher utility than a renter from the same consumption: $u_h(c)>u_r(c)$[8].

The log of the income before retirement is a combination of a deterministic and a stochastic component whereas after retirement, it is λ fraction of the income at age J_r plus η fraction of the average income in the economy, $\bar{y}$:

$$\begin{aligned}y_j\left(j,z_j\right) &= \left\{\begin{matrix}\exp\left(f\left(j\right)+z_j\right) & \text{if } j\le J_r\\ \lambda y_{J_r}\left(J_r,z_{J_r}\right)+\eta\bar{y} & \text{if } j>J_r\end{matrix}\right\}\\ z_j &= \rho z_{j-1}+e_j\end{aligned}$$

where y_j is the income at age j, $f\left(j\right)$ is the age-dependent deterministic component of the log income, and finally z_j is the stochastic component of the log income. The stochastic component is modeled as an $AR(1)$ process with ρ as the persistency level. The innovation to the stochastic component, e_t, is assumed to be i.i.d and normally distributed with mean 0 and variance σ_e^2. Households can save to smooth their consumption at the constant risk-free interest rate r, but there is no unsecured borrowing.

Households start the economy as active renters, and can purchase a house and become an owner at any period in time. However, an inactive renter is only allowed to purchase a house with probability δ. With $(1-\delta)$ probability, she is forced to live as a renter. Since houses are expensive items, their purchases can be done through securitized borrowing: mortgages. A purchaser chooses among a menu of feasible mortgage contracts, each specified with a loan amount and interest rate[9]. Since the mortgages are fixed-payment mortgages, the contract

[7] Since there is no housing tenure choice and uncertainty after retirement, household's problem is trivial and can be calculated analytically.

[8] Note that the size of the houses is fixed, that's why having separate utility functions for renter and homeowners imply that they get different housing services from the same house.

[9] Not every combination of mortgage interest rate and loan amount is feasible for the household. Lenders'

together with the maturity, remaining time to retirement, determine the periodic mortgage payments. As long as the household stays in the house, she has to make at least these payments. However, a mortgage holder is always allowed to make additional payments to reduce the principal amount. The homeowner has also the option to sell the house at any time period. However, selling the house is costly. Firstly, upon deciding to sell, the individual receives an i.i.d house price shock, ϵ_h, drawn from a distribution $F_h(\epsilon_h)$. This idiosyncratic shock changes the value of the house up or down[10]. Additionally, there are some other costs (transaction costs including real-estate costs and maintenance costs) associated with selling the house. So, a seller incurs a proportional cost, φ_h, of the house price. Moreover, a seller has to pay the outstanding mortgage debt back to the lender.

There is another option for the household to quit ownership. She can default on the mortgage. A defaulter has no obligation to the lender. Upon default, the lender seizes the house, sells it and pays back, if any, to the defaulter the amount net of outstanding mortgage debt and costs associated to selling the house. The lender's cost of selling the house is $\varphi_l \geq \varphi_h$ fraction of the house price. What makes default appealing for the household is the fact that a defaulter has no obligation to the lender whereas a seller has to pay back the debt in full. The same fact puts a risk of loss on the lender. The lender incurs a loss if the net value of the house is smaller than the outstanding debt upon default.

Default is not without any cost to the household. A defaulter becomes an inactive renter and can only enter to the housing market with probability δ. Since default is costly and the selling price of the house to the homeowner is at least the selling price to the lender, the homeowner who decides to leave the house only defaults if the outstanding mortgage balance is strictly higher than the selling price. Otherwise it is always optimal to sell the house rather than defaulting.

In each period homeowners receive a moving shock with probability ψ, and they are forced to quit to the rental market. To do that, they again have two choices, either sell the house or default on the current mortgage.

3.1.2 Lenders

There is a continuum of lenders and financial markets are perfectly competitive, and there is no cost to entry. Lenders are risk-neutral[11]. The economy is assumed to be an open economy and the risk-free interest rate, r, is set exogenously. Mortgage contracts are long-term

inference about the type of the household and competition among lenders restrict the contracts offered to the household in the equilibrium.

[10] We can think of this shock as a local shock. We need this feature of the model to create realistic mortgage default rates. Since house prices are fixed in the economy, without these local shocks, the incentive to default becomes very small.

[11] Securitization of mortgages helped lenders to diversify the risk they face and liquidate their asset holding. Risk-neutrality assumption corresponds to perfect securitization.

contracts and the maturity of the contract is directly determined by the time to retirement, which is assumed to be certain and observable. Lenders have full commitment to the contract and renegotiation is not allowed.

Each contract is characterized by a loan amount, d, and interest rate, r_m. Since households can default on the mortgage at any time, and transaction and further costs make the loan not fully securitized, lenders face a risk of loss on mortgage loans. Moreover, there is an additional per period servicing cost for mortgage loans, τ, which is assumed to be proportional to the loan amount.

3.1.3 Timing

The timing of the events is the following: Households are born as active renters. For any other period, the household starts the period either as a homeowner, a mover, an active renter or an inactive renter. At the beginning of each period, households realize their income shock, and decide about their housing statuses for the current period.

An active renter has two choices: continue to rent or purchase a house. If she decides to continue to rent, she pays the rental price, makes her consumption and saving choices, and reaches to the next period as an active renter. If she decides to buy a house, she goes to a lender. The lender offers a menu of mortgage contracts depending on the characteristics of the household. The household chooses the mortgage contract that maximizes her utility. However, the buyer is also allowed to pay the whole house price upon purchase in which case she will have no mortgage to pay. Upon deciding about the mortgage choice, she pays the downpayment and periodic mortgage payment implied by the mortgage contract, makes her consumption choice, and reaches to the next period as a homeowner with $(1-\psi)$ probability and mover with ψ probability. We do not allow owners to save once they do have a mortgage obligation. We also do not allow for refinancing, but owners with mortgage can always prepay the mortgage. This prepayment will reduce the outstanding mortgage balance, but it will have no effect on the mortgage interest rate.

A homeowner has three choices: stay, sell or default; whereas a mover has two choices: sell or default. If she decides to stay in the current house, she makes her consumption and mortgage payment choice, which has to be at least the amount dictated by an amortizing fixed rate mortgage, and starts the next period again as a homeowner or a mover depending on the moving shock. If she decides to sell the house, she receives the idiosyncratic house price shock, pays the outstanding mortgage debt back to the lender, makes her consumption and saving choices and begins the next period as an active renter. If she decides to default, she receives any positive remaining balance - the selling price of the house to the lender minus the outstanding mortgage debt - from the lender, makes her consumption and saving choices, and starts the next period as an active renter with δ probability and inactive renter with

$(1-\delta)$ probability.

An inactive renter has no housing tenure choice. She is forced to live as a renter. So, she pays the rental price, and only makes her consumption and saving choices and starts the next period as an active renter with δ probability and inactive renter with $(1-\delta)$ probability.

3.2 Decision Problems

We now turn to the recursive formulation of the household's and lender's problem. Note that since the mortgages are long-term contracts, the lender's problem also has dynamic structure. The lender has to calculate the default risk of the household through the life of the mortgage. Here, we first start with the recursive formulation of the household's problem, then we set up the lender's dynamic programming problem which is also closely related to the household's problem.

3.2.1 Household's Problem

We only focus on household's problem before retirement. The value function at the time of retirement can be calculated analytically given the utility specification. At the beginning of each period, the household is in one of the four housing positions: inactive renter, active renter, homeowner, or mover. After the realization of the income shock, the active renter and the homeowner make their housing tenure choices for the current period and start the next period with their new housing statuses. Let's denote V^r as the value function for an active renter after the realization of the income shock and just before the housing choice. Similarly, let V^h be the value function for a homeowner, V^m be the value function for a mover, and let V^e be the value function for an inactive renter. Note that in the current period inactive renters and movers have no housing tenure choice.

Inactive Renter. We start with the problem of an inactive renter. An inactive renter's problem is simple. She does not have any housing tenure choice, she is forced to be a renter in the current period. The only decisions she has to make are the consumption and saving allocations. She starts the next period as an active renter with probability δ and an inactive renter with probability $(1-\delta)$. Denoting the value function of an inactive renter with age j, period beginning saving a and income z as $V^e(a,z,j)$, the inactive renter's problem is given by:

$$V^e(a,z,j) = \max_{c,a'\geq 0} \left\{ u_r(c) + \beta E\left[\delta V^r\left(a',z',j+1\right) + (1-\delta)V^e\left(a',z',j+1\right)\right]\right\} \qquad (1)$$

subject to

$$c + a' + p_r = y(j,z) + a(1+r)$$

where c is the consumption, a' is the next period saving, and p_r is the exogenous rental price . Note that the inactive renter derives utility from consumption and renting a house.

Active Renter. Different from an inactive renter, an active renter has to make a housing tenure choice. After the realization of the income shock, an active renter has to decide whether to continue to stay as a renter or purchase a house in the current period. This means we need to define two additional value functions for the active renter. Define V^{rr} as the value function for an active renter who decides to stay as a renter and name such a household as *renter*. Her problem is very similar to the inactive renter's problem apart from the fact that she starts the next period as an active renter for sure. Given all these facts, we can write down the problem of the renter as:

$$V^{rr}(a,z,j) = \max_{c,a' \geq 0} \left\{ u_r(c) + \beta E V^r \left(a', z', j+1\right) \right\} \tag{2}$$

subject to

$$c + a' + p_r = y(j,z) + a(1+r)$$

The second possible choice of an active renter is to purchase a house. Define the value function for an active renter who decides to purchase a house as V^{rh} and name such a household as *purchaser*. Housing purchase is done through a mortgage contract. The purchaser, additional to the usual consumption choice, has to choose a mortgage contract. Lenders design the mortgage contracts depending on the observable of the household. Due to the perfect competition in the financial market, lenders have to make zero-profit on these mortgage contracts. So, only the contracts which make zero-profit are feasible and offered to the household. We denote the set of feasible contracts for a household with observable θ as $\Upsilon(\theta)$ where $\theta \equiv (a,z,j)$. A mortgage contract is specified with a loan amount d and interest rate, r_m. So, a typical element of the feasible contract set is $(d, r_m) \equiv \ell \in \Upsilon(\theta)$. We leave the construction of $\Upsilon(\theta)$ to the section we define the lender's problem. Since mortgages are due by retirement, which is deterministic, household's age captures the maturity of the mortgage contract. Moreover, since we only focus on fixed payment mortgages, the choice of the loan amount and interest rate, together with the age of the household, determine the amount of mortgage payments, m. The calculation of these payments is shown in the lender's problem. Out of the total financial wealth, net of the mortgage payment and the downpayment fraction, the household consumes the rest, and starts the next period as a homeowner. So, we can formulate the problem of the purchaser in the following way:

$$V^{rh}(a,z,j) = \max_{c,(d,r_m)\in\Upsilon(\theta)} \left\{ u_h(c) + \beta \left[(1-\psi) E V^h \left(z', j+1; d', r_m\right) + \psi E V^m \left(d', z', j+1\right) \right] \right\} \tag{3}$$

subject to

$$
\begin{aligned}
c+m\left(d,r_m,j\right)+p_h-d &= y\left(j,z\right)+a\left(1+r\right) \\
d' &= \left(d-m\left(d,r_m,j\right)\right)\left(1+r_m\right) \\
d &\leq \left(1-\lambda\right)p_h
\end{aligned}
\tag{4}
$$

where p_h is the house price and λ is the minimum downpayment requirement. Notice that although we impose a minimum downpayment requirement, the household can choose any downpayment higher than this amount. The household makes the downpayment immediately upon the purchase of the house, and mortgage payments are due by the beginning of each period. Outstanding mortgage debt decumulates according to equation (4). It says that next period outstanding mortgage debt, d', is the current period outstanding mortgage debt reduced by the mortgage payment, net of interest payment. Note that since the purchaser becomes a homeowner in the current period, she derives utility from both consumption and being a homeowner. Lastly, notice that purchaser starts the next period as a mover with ψ probability and as a homeowner with $(1-\psi)$ probability.

The value function for the renter together with the value function for the purchaser characterize the value function for the active renter:

$$
V^r = \max\left\{V^{rr}, V^{rh}\right\} \tag{5}
$$

Homeowner. A homeowner has three housing choices: stay in the current house, sell the house, or default on the mortgage. This requires us to define three additional value functions. Let V_i^{hh} be the value of a homeowner who decides to stay in the current house and name such a household as *stayer*. The stayer has to make at least the periodic mortgage payment, but also can prefer to make additional payments to reduce the outstanding mortgage debt. However, this will not effect the interest rate on the mortgage, Apart from the usual state variables (z, j), a stayer is also defined by her outstanding mortgage debt, d, and interest rate on the mortgage, r_m. The outstanding mortgage debt decumulates according to the same equation we defined in the purchaser's problem. In recursive formulation, the problem of the stayer becomes the following:

$$
V^{hh}\left(d,z,j;r_m\right) = \max_{c,d'}\left\{u_h\left(c\right)+\beta\left[\left(1-\psi\right)EV^h\left(d',z',j+1;r_m\right)+\psi EV^m\left(d',z',j+1\right)\right]\right\} \tag{6}
$$

subject to

$$
\begin{aligned}
c+d &= y\left(j,z\right)+d'/(1+r^m) \\
d' &\leq \left(d-m\left(d,r_m,j\right)\right)\left(1+r_m\right)
\end{aligned}
$$

Again, notice that a stayer starts the next period as a mover with ψ probability and as a homeowner with $(1-\psi)$ probability.

The second possible choice for a homeowner is to sell the house and become a renter, and name such a household as *seller*. The selling price of the house is exogenously set to $(1-\varphi_h)$ fraction of the purchase price p_h. This feature tries to capture the possible transaction costs, maintenance costs etc. Additional to this transaction cost, sellers also incur an i.i.d capital loss/gain, ϵ_h. Moreover, a seller has to pay the outstanding mortgage debt, d, in full to the lender. Denoting V_i^{hr} as the value function for a seller, the recursive formulation of her problem is the following:

$$V^{hr}(z,j;d,r_m) = \max_{c,a'\geq 0}\left\{u_r(c) + \beta E V_{j+1}^{r}\left(a',z',j+1\right)\right\} \tag{7}$$

subject to

$$c + a' + p_r = y(j,z) + p_h(1-\varphi_h)(1+\epsilon_h) - d$$

Again, since the seller becomes a renter in the current period, she pays the rental price and enjoys the utility of a renter.

The third and the last possible choice for a homeowner is to default on the mortgage. Name such a household as *defaulter*. A defaulter has no obligation to the lender. The lender seizes the house, sells it in the market and pays any positive amount net of the outstanding mortgage debt and selling costs back to the defaulter. For the lender, selling price of the house is assumed to be $(1-\varphi_l)\,p_h$. So, the defaulter receives $\max\{(1-\varphi_l)\,p_h, 0\}$ from the lender. Defaulter starts the next period as an active renter with probability δ. With $(1-\delta)$ probability she becomes an inactive renter. Denoting V^d as the value function for a defaulter, her problem becomes the following:

$$V^{d}(z,j) = \max_{c,a'\geq 0}\left\{u_r(c) + \beta E\left[\delta V^{r}\left(a',z',j+1\right) + (1-\delta)\,V^{e}\left(a',z',j+1\right)\right]\right\} \tag{8}$$

subject to

$$c + a' + p_r = y(j,z) + \max\{(1-\varphi_l)\,p_h - d, 0\}$$

Since the defaulter is a renter in the current period, she pays the rental price and enjoys the utility of a renter.

So, we can characterize homeowner's value function, which is the maximum of the above three value functions as:

$$V^{h} = \max\left\{V^{hh}, V^{hr}, V^{d}\right\} \tag{9}$$

Mover. A mover's problem is very similar to the homeowner's problem apart from the staying option. A mover can either choose to sell the house or default on the mortgage. So, we can characterize the value function of the mover as:

$$V^m = \max\left\{V^{hr}, V^d\right\}$$

3.2.2 Lender's Problem

Since the mortgages are long-term contracts, the lender's problem is also a dynamic problem. The lender has to design a menu of contracts, $\Upsilon(\theta)$, depending on the observable, θ of the purchaser. As we mentioned above, a mortgage contract is a combination of a loan amount and an interest rate: $(d, r_m) \in \Upsilon(\theta)$. Note that we do not include mortgage payment, m and maturity as parts of the mortgage contract, because maturity is directly determined through the age of the household, which is observable, and mortgage payment is assumed to be fixed and becomes a function of the loan amount, interest rate and household's age.

Present Value Condition. We first show how the mortgage payments are computed. Since the mortgages are fixed-payment mortgages, the payments are constant through the life of the mortgage. They are directly computed from the *present value condition* for the contract. This condition says that given the loan amount and the mortgage interest rate, the present discounted value of the mortgage payments should be equal to the loan amount. Since the lender has full commitment on the contract, he calculates the payments as if the contract ends by the maturity. Assuming the interest rate on the mortgage is r_m and current age of the household is j, this gives me the following formulation for the per-period payments of a mortgage loan with outstanding debt d:

$$\begin{aligned} d &= m + \frac{m}{1+r_m} + \frac{m}{(1+r_m)^2} + ... + \frac{m}{(1+r_m)^{J_r-j}} \\ m(d, r_m, j) &= \frac{1-\alpha}{1-\alpha^{J_r-j+1}} d, \text{ where } \alpha = \frac{1}{1+r_m} \end{aligned} \tag{10}$$

No-Arbitrage Condition. Next, given the mortgage payments and loan amount, the lender has to determine the mortgage interest rate. This rate is pinned down by the *no-arbitrage condition*. It says that given the expected mortgage payments, the lender should be indifferent between investing in the risk-free market, which is the only outside investment option for the lender, and creating the mortgage loan. Note that the expected payments are not necessarily the above calculated mortgage payments. If the household defaults when the outstanding mortgage debt is d, the lender receives $\min\{(1-\varphi_l) p_h, d\}$[12].

[12] Since default is costly, as long as $p_h(1-\varphi_h) \geq d$, the household sells the house rather than defaulting. This means, in equilibrium, when the household defaults, the lender receives $p_h(1-\varphi_l) \leq p_h(1-\varphi_h) < d$ and incurs some loss.

Before formulating the no-arbitrage condition, let us denote the value of a mortgage contract with outstanding debt d and interest rate r_m, offered to a household with current period characteristics (z,j) as $V^{\ell}(z,j;d,r^m)$[13]. Note that this function does not only represent the value of the contract at the origination, but also represents the continuation value of the contract at any time period through the mortgage life. Depending on the homeowner's tenure choices, the realized payments may change. If the household stays in the current house, the lender receives the calculated mortgage payment and the continuation value from the contract with the updated characteristics of the household and the loan amount. If the household defaults, then the lender receives $\min\{(1-\varphi_l)\,p_h, d\}$. If the household sells the house, the lender receives the outstanding loan amount, d.

Given that the opportunity cost of the contract is the risk-free interest rate, r, plus the per period transaction cost, τ, and the lender is risk-neutral, the value function for the lender becomes the following:

$$V_i^{\ell}(z,j;d,r_m) = \left\{\begin{array}{ll} d - \frac{d'}{1+r_m} + \frac{1}{1+r+\tau} EV_i^{\ell}(z',j+1;d',r_m) & \text{if hh stays} \\ \min\{p_h(1-\varphi_l),d\} & \text{if hh defaults} \\ d & \text{if hh sells} \end{array}\right\} \tag{11}$$

where d' is the policy function to problems (3) and (6) and finally $m(d,r_m,j)$ is defined by equation (10).

Now, we are ready to formulate the no-arbitrage condition. At the time of the origination of the contract, the lender observes all the characteristics of the household. This actually means that mortgage contracts are individualized and independent from all the other households in the economy. The lender can solve the household's problem and obtain the necessary policy functions (mortgage payment choice and housing choice) to evaluate the value of the contract at the origination. So, the no-arbitrage condition for a mortgage contract offered to a household with characteristics (z,j) becomes:

$$V^{\ell}(z,j;d,r_m) = d \tag{12}$$

Note that initial loan amount d is determined by the downpayment fraction: $d = (1-\phi)\,p_h$.

[13] Notice that asset level, a, is not part of the state variable for the lender. This is because we do not allow mortgage holders to save.

Table 1: calibration parameters

Parameter	Explanation	Value
σ	risk aversion	2
ρ	persistence of income	0.84
σ_ε	std of innovation to AR(1)	0.34
φ_h	selling cost for a household	10%
r	risk-free interest rate - initial	2%
λ	minimum downpayment	0%
φ_l	selling cost for a lender	10%
β	discount factor	0.94
γ_h/γ_r	utility advantage of ownership	1.23
ψ	moving probability	4%
δ	prob. of being an active renter	0.14

3.3 Equilibrium

Define the set of state variables for the household as Ω with a typical element (a, z, j) [14],and let $\theta \in \Theta \subseteq \Omega$ be the observable characteristics of the household by the lender.

Definition 1 ***Equilibrium:*** *An equilibrium to the economy is a set of policy functions* $\{c_s^*, a_s^*, d_s'^*, \ell_s^*, i_s^*\}$ *and a contract set* Υ_s *such that*

(i) *given the feasible contract set* Υ_s, $c_s^* : \Omega \times \Upsilon_s \to \Re$, $d_s'^* : \Omega \times \Upsilon_s \to \Re$, *and* $\ell_s^* : \Omega \times \Upsilon_s \to \Re^2$ *solve equations* $(1) - (3)$ *and* $(6) - (8)$, i_s^* *is a policy indicator function which solves equations* (5) *and* (9),

(ii) *given the policy functions each contract* $\ell \in \Omega \times \Upsilon_s$ *solves equation* (12) *and*

(iii) *no lender finds it profitable to offer another contract, which is not in the contract set,* $\Omega \times \Upsilon_s$, *i.e.* $\nexists (d, r_m)$ *such that* $V^\ell(\theta; d, r_m) > d$ *for* $\forall \theta \in \Theta$, *with* V^ℓ *defined as in equation* (11).

4 Calibration

A set of the parameters is directly taken from the literature. For the rest of the parameters, we calibrate the economy to match some relevant data moments for the 2002-2006 period. In particular, we calibrate the utility advantage of homeownership, γ_h, discount factor β and exogenous moving probability, ψ, to match the homeownership rate, mortgage default rate, and moving rate of households in the pre-2004 period. Table 1 presents the results of the calibration.

[14] The only relevant household for the lender is the purchaser, since contracts are only offered to them. And the state variable for a purchaser is, as mentioned earlier, (a, z, j)

External Calibration A model period is 1 year and households live for 70 periods. So, we assume households start the economy at the age 20, and live till the age of 90. The mandatory retirement period is 25, corresponding to the age of 65. Utility function for the households is the standard CRRA utility function with a slight modification to account for the benefit of homeownership: $u_k(c) = \frac{(\gamma_k c)^{1-\sigma}}{1-\sigma}$, $k \in \{r, h\}$ and γ_k is the utility advantage of being a renter $(k = r)$ or homeowner $(k = h)$[15]. We normalize $\gamma_r = 1$, and calibrate γ_h internally. We set the risk-aversion parameter, σ, to 2.

For the income process before retirement, we take the parameters to be consistent with the findings of Hubbard, Skinner and Zeldes (1994), Carroll and Samwick (1997) and Storesletten, Telmer and Yaron (2004). Using their income process, we simulate an economy for a sufficiently long time and estimate the resulting income profile as an AR(1) process. This gives us the income persistency, ρ, as 0.84 and standard deviation of the innovation to the AR(1) process, σ_ε, as 0.34. We approximate this income process with a 7-states first-order Markov process using the discretization method outlined in Tauchen (1986). For retirement income, we assume $\lambda = 0.35$ and $\eta = 0.2$, meaning the retiree receives 35% of the income at the time of retirement plus 20% of the mean income in the economy. The probability of becoming an active renter, while the household is an inactive renter, is set to 0.14, to capture the fact that the bad credit flag stays, on average, 7 years in the credit history of the household. The loss in the selling price of the house for the household and the lender is set to $\varphi_h = 10\%$[16]. The initial distribution of the income is assumed to be the stationary distribution, and initial wealth of individuals is set to 0.

The annual risk-free interest rate in the first steady-state is set to $r = 2\%$. The same rate is 3% in the second steady-state. The annual transaction cost of mortgages to the lender is set to 0%. Rental price is normalized to 0 in both steady states, and we solve for the house price which equalizes the fixed supply of owner-occupied units in both steady-states. Lastly, we set the i.i.d shock to the house price to take seven values following the estimates of Garriga and Schlagenhauf (2009) which uses data from American Housing Survey. We set $\varepsilon_h \in \{0.80, 0.903, 0.987, 1.059, 1.122, 1.179, 1.230\}$ with corresponding probabilities $\pi_\varepsilon = [0.0388, 0.2046, 0.4917, 0.1437, 0.0670, 0.0347, 0.0195]$, respectively.

[15] Given this utility specification, since there is no housing tenure choice and uncertainty after retirement, we can solve the value function at the time of retirement analytically: $W(w_r, y_r) = u_r(\bar{c}) \frac{1-\varkappa^{J-J_r+1}}{1-\varkappa}$, where w_r is the total wealth, including real estate, at the of retirement and y_r is the retirement income level, $\bar{c} = \frac{\alpha_1 y_r}{\alpha_2} + \frac{w_r}{\alpha_2}$, $\alpha_1 = \frac{1-\omega_1^{J-J_r+1}}{1-\omega_1}$, $\alpha_2 = \frac{1-\omega_2^{J-J_r+1}}{1-\omega_2}$, $\omega_1 = \frac{(\beta(1+r))^{1/\sigma}}{1+r}$, $\omega_2 = \frac{1}{1+r}$, and $\varkappa = \beta(\beta(1+r))^{\frac{1-\sigma}{\sigma}}$.

[16] Gruber and Martin (2003) estimates this cost for the homeowner as 7% using CEX data. Note that we abstract from various other sources of selling the house like house price change, unemployment shock, medical expense shock and we also exclude the depreciation on the houses. So, we think 10% is a reasonable estimate of the transaction cost for selling the house.

Table 2: Initial Steady State Results

	Data	Model: r=2%
Statistic		
Homeownership rate	68.8%	68%
Wealth-income ratio	4	3.9
Moving rate-owners	8.5%	8.6%
Foreclosure rate	1.7%	1.7%
Prices to income ratio	3.4	3.1
Average downpayment ratio	21.1	19.8%

Internal Calibration The remaining parameters of the model are calibrated internally to jointly match some of the key moments of the data. We use the initial steady-state in which the risk-free interest rate set to 2% to calibrate the parameters. These parameters are the utility advantage of owning a house, γ_h, discount factor, β, and moving probability, ψ. We internally calibrate these parameters to jointly match the following data moments: homeownership rate of 68%, average wealth-to-income ratio of 4, and homeowners' moving rate of 8.5%.

5 Initial Steady State Analysis

Table 2 shows the success of the model in matching some aspects of the data. Clearly, the model matches the homeownership rate, wealth-income ratio and moving rate of the homeowners very well since the parameters are chosen to deliver these results. However, the model also matches the data quite well along several other untargeted dimensions. The foreclosure rate in the model is 1.7%, which is the average foreclosure rate in the period 2001−2005. House price to income ratio that the model delivers is 3.1, whereas the corresponding number in the period 2001-2005 is 3.4. Lastly, average downpayment in the model is 19.8%, which is similar to the average downpayment of 21.1% in the period 2001-2005.

Given the success of the model in matching the data, we now turn into the workings of the model. The individuals in the model make three important choices regarding the housing market: when to buy the house, how much loan to obtain to buy the house, and when and how to exit from homeownership. Figure 1 displays renting vs owning choice as a function of wealth and income. Keeping household income constant, as wealth increases the household becomes more likely to choose owning over renting. Similarly, keeping the wealth constant as the income increases the household becomes more likely to own a house instead of renting. The presence of exogenously set positive utility premium attached to owning a house makes ownership more appealing compared to renting. However, since houses are expensive, not every household can buy a house whenever they want. Households need to accumulate some wealth to partially cover the cost of buying a house. We set the minimum downpayment requirement to 0%, meaning that households can buy a house by borrowing the whole purchase price from a lender. However, since the terms of the mortgage loans are determined endogenously depending on the characteristics of the households, higher leverage will imply higher risk of default. This, in turn, will be reflected as a higher premium on the loan. Since mortgage loans are long-term contracts, the benefit of waiting for several periods and accumulating enough wealth to reduce the mortgage premium might outweigh the cost of waiting, which is the loss of the utility advantage of ownership.

Whenever a homeowner with a mortgage chooses to rent in the next period, she has two options: sell or default on the mortgage. In case of selling, the household receives the selling price net of capital gain/loss and transaction costs. However she has to pay the outstanding mortgage debt back to the lender. Given that the transaction cost is positive and there is a chance of capital loss, it is likely that the household has negative equity in the house. As a result she might choose defaulting over selling. This is exactly what we see in Figure 1 which displays selling vs defaulting choice as a function of income and mortgage debt. First of all, for sufficiently small amount of debt, the household never defaults. This is because in this region the household has always positive equity in her house. As debt increases it is more likely that the household has negative equity which triggers default. Notice that even the household is in negative equity she might not choose to default, because default is costly due to exclusion from the housing market. As a result there are some states of the world in which the household sells the house although she is under water, and pays the difference through her wealth. However as income decreases this option becomes less likely. Thus as income decreases a household becomes more likely to default.

Figure 2 shows the homeownership rate over the life cycle. It is clear from this picture that early in the life-cycle a few households have enough asset to purchase a house. Over time, as households accumulate enough wealth to buy their houses. The figure also plots the fraction of households who buy their houses with a mortgage. Again early in the life-cycle most of the households buy their houses with a mortgage. But as they accumulate wealth, some households buy their houses by paying the whole purchase price. Notice that it is always better for the households to buy a house without mortgage, because we assume that households holding a mortgage cannot simultaneously save. As a result to avoid large volatilities in the consumption households always choose not to have a mortgage if their wealth is sufficiently large. Through the end of working period, homeownership rate slightly declines since we force retired individuals to live in rental units.

Figure 3 shows how the mortgage premium responds to the downpayment fraction for different income levels. As the downpayment fraction increases, loan-to-value ratio increases, meaning the amount of mortgage debt the household receives increases. Since higher mort-

Figure 1: Household Tenure Choices

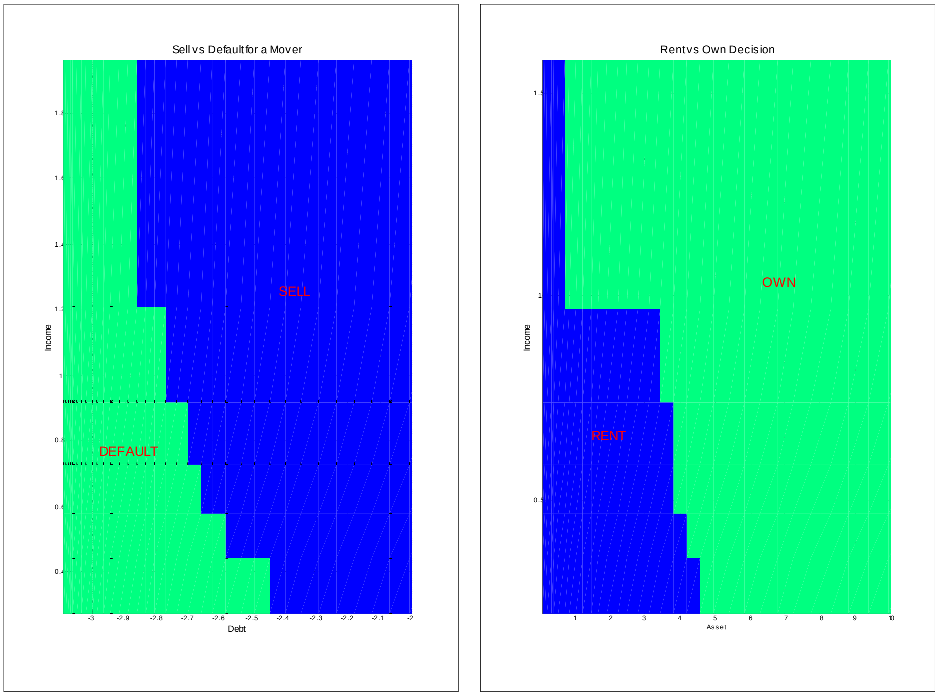


Figure 2: Homeownership over the Life Cycle

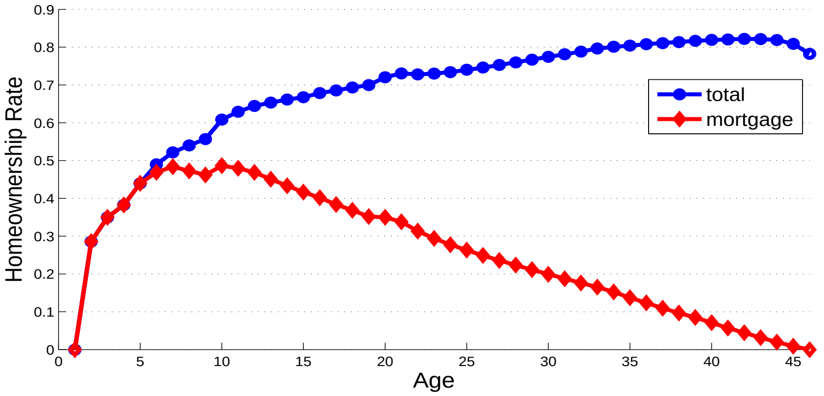
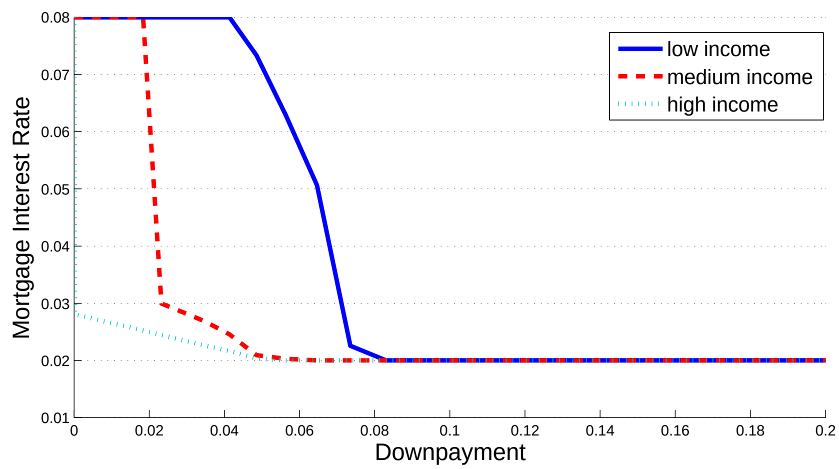


Figure 3: Mortgage Interest Rate as a Function of Downpayment for Different Income Levels

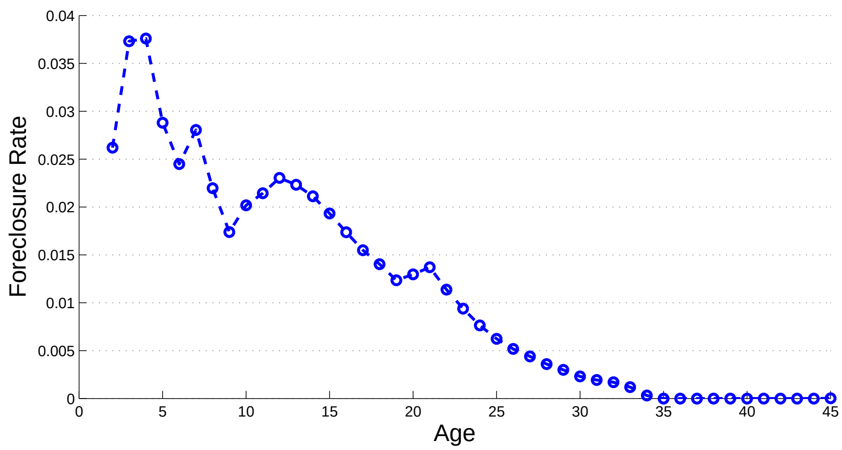


gage debt results a higher likelihood of default on the mortgage, lenders incorporate this increase in the default probability into the mortgage premium. Notice that households only default when house value net of transaction costs is smaller than the mortgage debt. In this case, household defaults, lender seizes the house, and sells the house in the market. Since the lender faces at the least the same costs the household faces in the housing transaction, the lender will incur some loss at the time of the house selling. This loss will be reflected as a default premium into the mortgage interest rate. In Figure 3, we see that for every income level, as downpayment fraction increases mortgage interest rate decreases. Moreover as the downpayment fraction exceeds a certain threshold, which is around 9% given current parametrization, the loan effectively becomes riskless, and the mortgage interest rate equalizes to the risk-free interest rate. Another observation we can derive from Figure 3 is the effect of income on the premium. As we know from Figure 1, higher income results less likelihood of default for the same amount of mortgage debt. Thus, we expect the premium to decrease as the income increases. This is what we see from Figure 3.

6 Steady State Comparisons

We assume that the economy has been in the steady-state with risk-free interest rate set to $r = 2\%$ and minimum downpayment set to 0%. We consider three scenarios. In the first scenario the economy is hit by an unexpected permanent interest rate shock. In the second scenario the interest rate stays constant but minimum downpayment requirement increases from 0% to 20% unexpectedly and permanently. In the last scenario both shocks are realized

Figure 4: Foreclosure Rate over the Life Cycle



together.

In the first scenario we assume risk-free interest rate jumps from 2% to $r = 3\%$ and stays there forever. The important nature of this exercise is that this shock is unexpected and permanent. The spirit of this exercise is to see the effect of expectations about the interest rate on the housing market. We assume that the economy is hit with a shock, and risk-free interest rate expectation is revised upwards permanently. The second shock that we feed the economy is an unexpected and permanent increase in the minimum downpayment requirement from 0% to 20%. We assume that in every steady-state, the homeownership rate is fixed at 68%, and we solve for the house prices endogenously. We preserve the same assumption when we solve the transition.

Table 3 presents the comparison of these steady-states along several important dimensions. SS1 corresponds to the steady-state of the benchmark economy. SS2 corresponds to the steady state of the economy with only interest rate shock: the risk-free interest rate is set to $r = 3\%$. SS3 corresponds to the steady state of the economy with only downpayment shock: the minimum downpayment is set to $\lambda = 20\%$. And finally SS3 corresponds to the steady state of the economy with both interest rate shock and downpayment shock: $r = 3\%$ and $\lambda = 20\%$. First of all, the house price declines by around 13% from the first steady state to the second one. As the risk-free interest rate increases, the cost of housing decreases. Since we fixed the homeownership rate to 68%, meaning housing supply is fixed, this requires a decrease in the house price since the cost of financing a house increases. The rate of the response of house prices to interest rates is significant and it is larger than most of the models studied in the literature. The existence of long term mortgage contracts and the transaction costs in the housing market are key ingredients in the model that generate large house price movements.

Table 3: Initial Steady State Results

	SS 1	SS 2	SS 3	SS 4
	r=2% **λ=0%**	**r=3%** **λ=0%**	**r=2%** **λ=20%**	**r=3%** **λ=20%**
Statistic				
Homeownership rate	68%	68%	68%	68%
Foreclosure rate	1.7%	0.6%	0%	0%
Price to income ratio	3.1	2.7	2.65	2.55
Average downpayment ratio	19.8%	26.4%	30.5%	32.1%

As housing is rigid, and the mortgages are long-term contracts with fixed rates, once an agent buys a house the effect of interest rates goes beyond one period.

In the second steady state the mortgage default rate is 0.6%. As the economy moves from low interest regime to high interest rate regime, mortgage payments become larger for a given loan amount (due to higher risk-free interest rates). This pushes the foreclosure rate up, however there is also an opposing effect. In the high interest rate regime, households put larger down payments (average downpayment increases from 19.8% to 26.4%) to avoid the higher cost of borrowing (due to higher interest rates), and they are also more wealthier (again thanks to higher risk-free interest rates, wealth-income ratio increases from 3.9 to 4.3). This, in turn, means households have lower leverage levels, which decreases the likelihood of default. The current calibration of the model suggests the first effect dominates and we observe a lower foreclosure rate in the second steady-state.

In the third and fourth steady states foreclosures are zero due to larger minimum downpayment requirements. House prices are lower and average downpayment is higher in the third and fourth steady states.

7 Transition

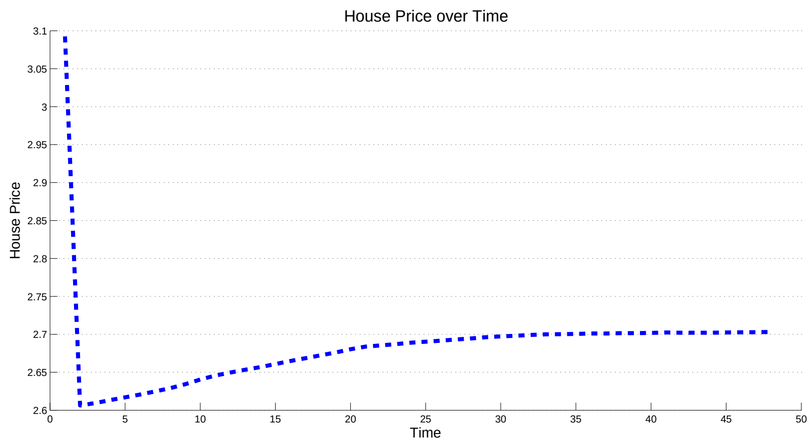
7.1 Transition from low interest rates to high interest rates

Comparing just the steady states masks important dynamics during the transition. To better understand the dynamic effects of a change in the interest rate, we also analyze the transition between the steady-states. However, since we do not model the rental supply explicitly, we need to take a stand on the response of homeownership and house prices to the change in the demand for housing during the transition. We either have to fix the homeownership rate or the house price during the transition. So, we conduct two separate transition exercises. First, we assume that homeownership rate is fixed to 68%, and we allow the house prices to respond to the change in the demand during the transition. In the second exercise, we assume that during the transition the house price is fixed to its second steady-state level, and analyze the transition by allowing the homeownership rate responding to the change in the demand for housing (will be presented in an Appendix later). It should be noted that in a world with flexible house prices and flexible homeownership rate, the effect of an increase in the risk-free interest rate on the mortgage and the housing market should be in between these two exercises.

Next, we analyze the transition by assuming the homeownership rate is fixed along the transition, and solve for the response of the house prices. In response to the rise in the risk-free interest rate, the demand for homeownership decreases as the cost of funding increases. However house prices need to decline sharply to the levels below the second steady state to achieve the homeownership rate of 68% as some households default due to the negative home equity. After the first fall, house prices increase gradually to the second steady state level (Figure 5). At this point we should note that there is a feedback mechanism from the house prices to the foreclosure rate and from foreclosure rate to the house prices. A fall in the house price initiates an increase in the foreclosure rate since some homeowners who end up having negative equity in their houses choose foreclosing. This increases the effective house supply. That extra supply pushes the house price even lower. Since households who default are stochastically excluded from purchasing a house, the increase in the foreclosure also decreases the effective demand of houses. As a consequence of this feedback mechanism the house price falls sharply after the interest rate shock. Moreover, as the transition path of the prices is perfectly seen by the consumers, the ones who want to sell have the incentive to sell before the house price falls further, the ones who want to buy has the incentive to wait until the house prices dip. After the house price dips, it starts gradually to appreciate to its new steady state value. As the house price starts to appreciate the foreclosure rate starts to decline.

An important difference between the steady states and the dynamics during the transition arises when we analyze the foreclosure rates. As we mentioned earlier the foreclosure rates are very similar at the first and second steady states. But during the transition, at its peak, the foreclosure rate increases sharply compared to the initial steady state level (Figure 6). The results from the steady state and the transition analysis reveal that what matters for the foreclosure rates is the change in the house prices, not the levels. House price is around 15% lower in the second state compared to the first steady state. Despite the significant difference in house price levels in two steady states the average foreclosure rate is only slightly lower in the second steady state. The reason is that at the steady states the percentage of homeowners who have negative equity and low income shocks are very similar. Whereas during the transition, households with low levels of home equity go to negative equity with lower house prices. With more homeowners who have negative housing equity foreclosure

Figure 5: House Prices During the Transition: Interest rates increases from 2 percent to 3 percent, permanently and unexpectedly.



increases.

One merit of our modeling strategy is that we optimally solve for downpayment ratios and mortgage interest rates jointly in the equilibrium. Consequently, we are able to study the dynamics of the downpayment ratio as well. In Figure 7 we plot the average downpayment requirement during the transition. The average downpayment in the model increases from 16 percent to 19 percent initially. After the first jump, the downpayment declines around 0.5 percent. Then, gradually downpayment ratio increases gradually to its new steady state value. Both downpayment ratio and the mortgage premia are affected by the default risk, and have very similar dynamics during the transition.

It is also informative to analyze how different wealth groups behave in this environment. For that purpose, in the following figure we plot the downpayment ratios for different wealth groups. For this exercise we fix the income to average income and the age to 40 and analyze the effect of the wealth. We should note that, for very low and very high levels of wealth the downpayment amounts do not change. While consumers with very low levels of wealth always pay the lowest possible amount, consumers with very high levels of wealth pay the full mortgage without borrowing. Since there is not any interesting dynamics for these two groups we exclude them from the current analysis. For the remainder we divide the consumers into three wealth groups (high, middle and low.) The common future for all wealth groups we analyze is that the downpayment ratio first sharply increases and then gradually declines (Figure 8). The difference is that, the amount of the increase is larger for the higher wealth group. This is because higher wealth groups have more opportunities to increase their downpayment amounts. Following the peak after the shock the downpayment ratio declines

Figure 6: Average Foreclosure Rate During the Transition: Interest rates increase from 2 percent to 3 percent, permanently and unexpectedly.

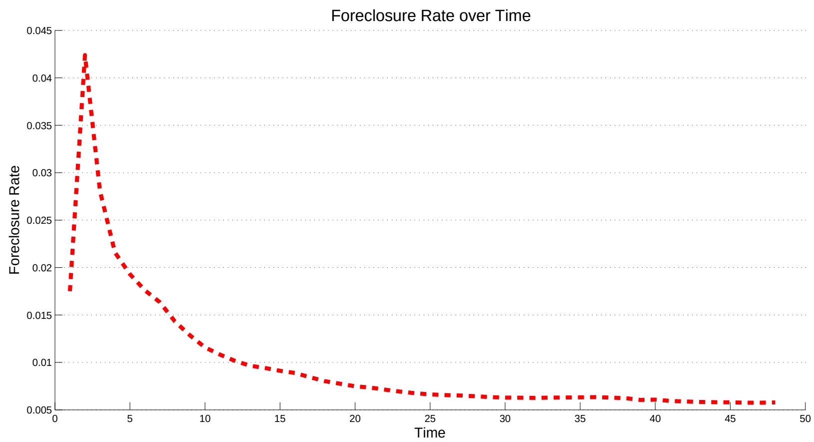


Figure 7: Average Downpayment Ratio During Transition: Interest rates increase from 2 percent to 3 percent, permanently and unexpectedly.

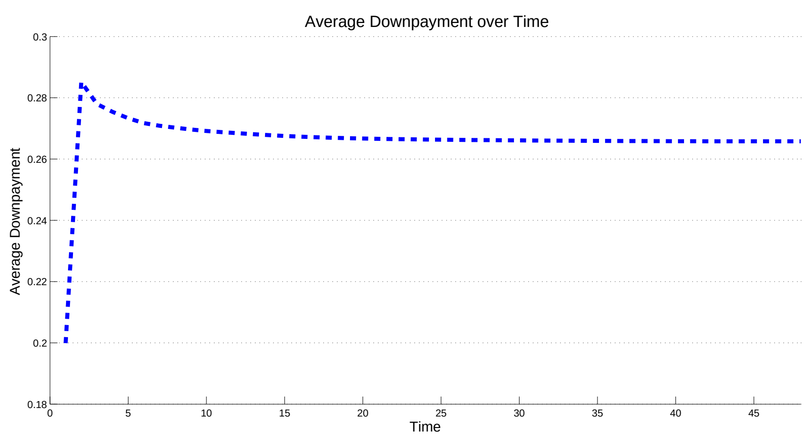
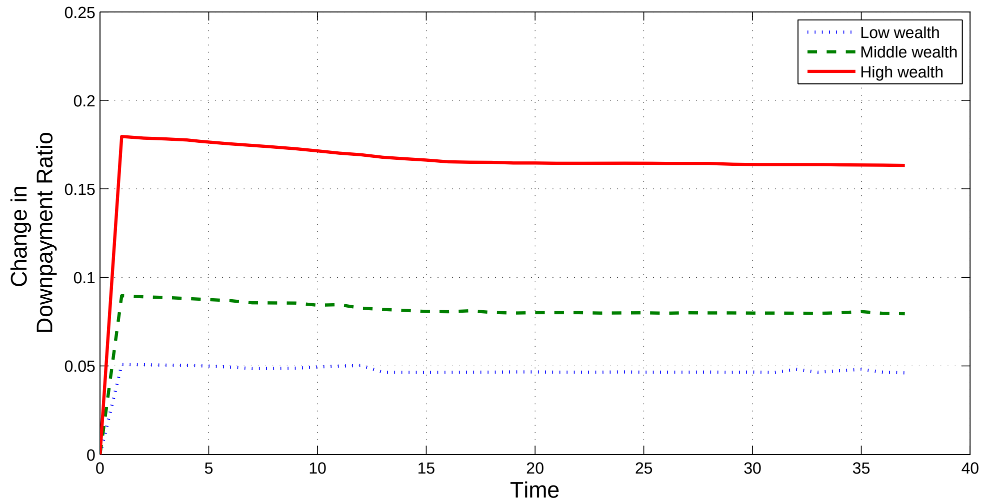


Figure 8: Changes in Downpayment Ratios for Different Wealth Groups



around 1.5 percentage points for the highest wealth group over the transition. For the middle and the low wealth groups the declines are around 1 and 0.5 percentage points.

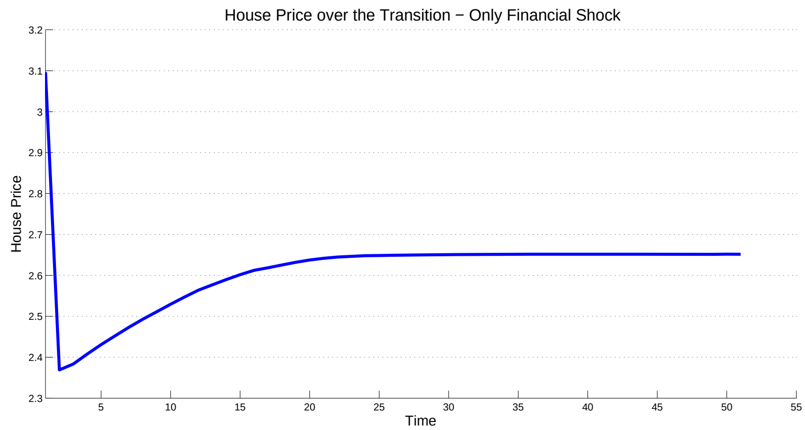
7.2 Transition From No Downpayment Requirement to 20% Downpayment Requirement

In our model, the lender is a risk neutral agent and has unlimited financing opportunities at the given interest rate. However, with the crisis, lenders had difficult times in financing themselves. Moreover they increased their lending standards which are hard to capture in our framework. To capture the implications of the stress in banking sector on the housing market we assume that along with higher interest rates the minimum downpayment requirement increases.

With the beginning of the crisis the financial industry came to the brink of a collapse. There are several mechanisms, such as the decline in collateral values, increase in haircuts which made the borrowing difficult for financial firms to raise funds to give loans. Besides, there was an urgency to deleverage which required selling assets on their book without issuing sizable loans. In addition, the uncertainty about the financial health of the banks made things even worse. These mechanisms do not exist in our model. To simulate the effects of financial turmoil we exogenously force banks in our model to require a 20 percent minimum downpayment unexpectedly and permanently.

Figures 9 and 10 show how house prices and foreclosures are affected from the shock. A 20 percent permanent and an unexpected increase in downpayment requirement causes around 25 percent decline in house prices initially. Afterwards, house prices increases gradually to

Figure 9: House Prices with Only Financial Shock: Minimum downpayment increases to 20 percent

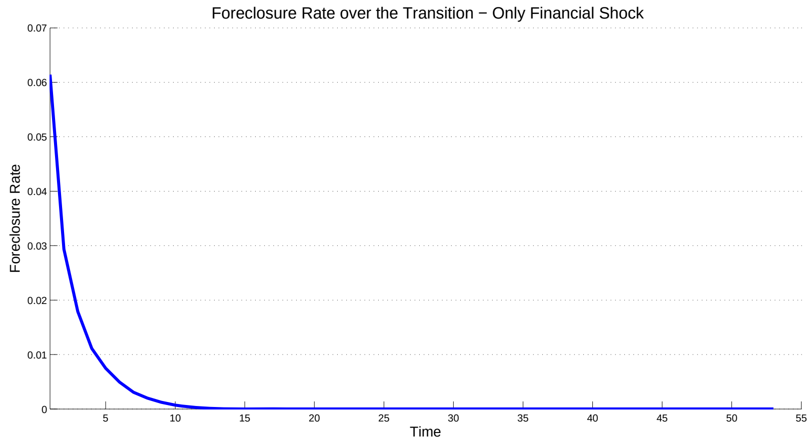


its new steady state level. Foreclosures sharply rise to around 6 percent on impact and then declines to the low levels around zero. Very low levels of foreclosures in the second steady state is due to larger downpayment requirement.

7.3 Transition to Larger Downpayment Requirement and Higher Interest Rates State

In the previous simulations we studied the effects of an interest rate shock and an increase in the minimum downpayment separately. However, in reality both of them happened during the crisis. In particular we study the transition where the interest rate increases from 2% to 3%, the minimum downpayment households are required to put during the purchase of the house increases from 0% to 20%. Figure 11 plots the path of the house prices with the tighter constraints. With tighter credit constraints, the house price, on impact, decreases by 30% (from 3.1 to 2.2) compared to a drop of 16% without the tighter credit constraints. The presence of tighter constraints together with the increase in the risk-free interest rate decreases the demand for homeownership further, and it needs a larger drop in the house price to set the homeownership rate at 68%. Moreover, the impact of the tighter constraints on the foreclosure rate is shown in Figure 12. Since the house prices decrease more compared to the economy without any minimum downpayment requirement, the foreclosure rate increases more sharply with tighter credit constraints. On impact, the foreclosure rate increases from 1.7% to 9.8% (an increase of 477%) compared to an increase from 1.7% to 4.3% (an increase of 153%) in the economy without any minimum downpayment requirement. Since the downpayment requirement is larger in the latter economy, the average downpayment is significantly larger

Figure 10: Foreclosures with Only Financial Shock: Minimum downpayment increases to 20 percent



in the economy with 20% minimum downpayment requirement.

8 Policy Experiments

As a response to the crises the FED rapidly decreased the funding rate. In addition to that policy, the FED also stated that the policy rates will stay low for a prolonged period of time. As of now, in the policy statements it is mentioned that the policy rates will be low until mid 2015. In this part of the paper we analyze the potential implications of this policy. In particular, we study the transition of our model. Particularly we assume that when the shock is realized and it is thought that it is permanent, unexpectedly the FED states that for a number of periods interest rates will stay low, but after that period interest rates will return to high levels. Consequently, agents in our economy know that low interest rate policy of the FED will be transitory.

In our policy analysis we assume that the FED communicates that interest rates will be 0.5% for a specified period of time. We also study how the length of the FED's low interest rate policy affects the transition. In particular, we choose 3 and 6 periods of low interest rate policies. We analyze the effectiveness of this policy in two different shock scenarios: interest rate and financial (higher downpayment requirement) shock.

In Figures 13 and 14 we plot house prices and foreclosures during the transition when there is FED policy in response to a higher interest shock (an increase from 2% to 3 %). We assume that economy stays in the high interest rate state for 2 periods. At the third period FED lowers interest rates to 0.5 percent and tells how long this policy will be effective.

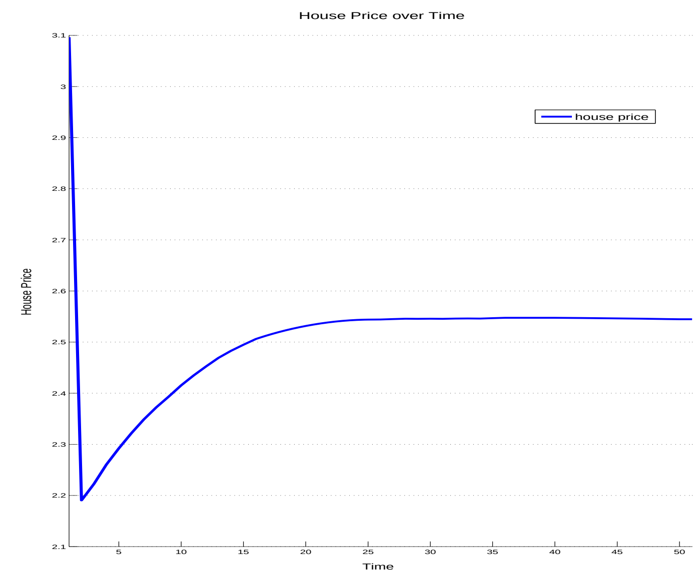


Figure 11: House Prices During the Transition: Interest rates increases from 2 percent to 3 percent and minimum downpayment requirement increases from 0 to 20 percent permanently and unexpectedly.

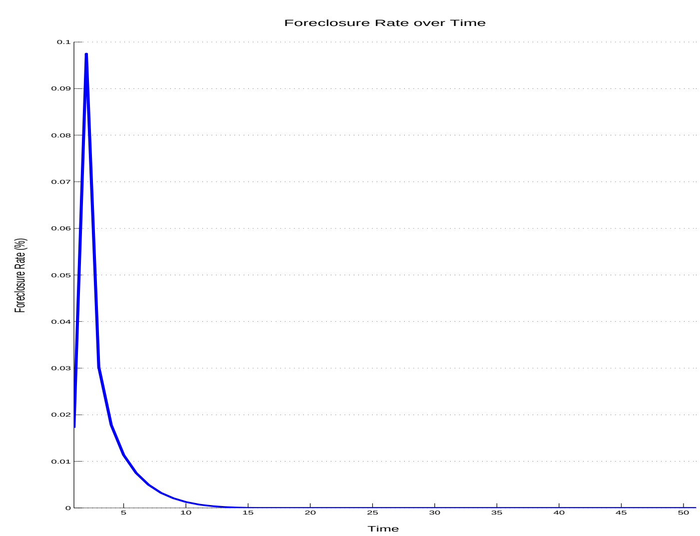
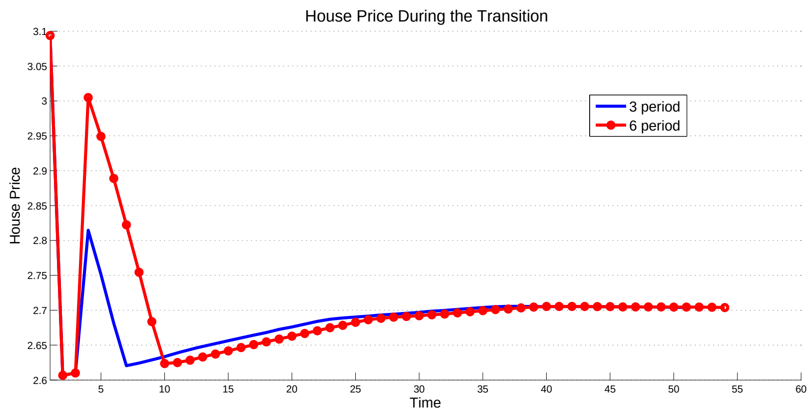


Figure 12: Foreclosure During the Transition: Interest rates increases from 2 percent to 3 percent and minimum downpayment requirement increases from 0 to 20 percent permanently and unexpectedly.

Figure 13: The Effects of FED's Policy on House Prices (Interest Rate Shock): First, 3 percent interest rate shock (permanent and unexpected) is realized. After 2 periods FED communicates the policy.



As can be seen from the figures the effects of this policy is larger if it is applied for longer. As a response to the FED's announcement, house prices appreciate rapidly and significantly and then declines to the low levels as policy end date becomes closer. As shown previously, without policy foreclosures increases sharply with the fall in house prices but then stabilizes. In the case with FED policy, foreclosures do not stabilize as rapidly as the case without FED policy. Hence, it takes more time for foreclosures to converge to low levels.

Next we study the potential effects of the FED policy in case of a financial shock. Here, as a short cut, we model the financial shock as a permanent increase in minimum downpayment requirement. We assume that other than the FED policy periods the interest rate is fixed to 2%. In Figures 15 and 16 we plot house prices and foreclosures during the transition when there is FED policy in response to a financial shock (an increase from 0 to 20%). The interest rate policy of the FED seems much less effective in this case compared to the previous interest rate shock case. The policy causes only a small appreciation in the house prices even if it is kept for 6 periods. Similarly, the policies do not have significant effects on the foreclosure. The main reason behind these results is that even the interest rates are very low, since people are constrained by the minimum downpayment requirement they cannot borrow and then the monetary transmission channel is hampered and monetary policy becomes ineffective.

9 Future Work

We plan to extend and improve the paper in several dimensions.

In the paper the level of the shocks that we feed to the model are not too precise. As a

Figure 14: The Effects of FED's Policy on Foreclosures (Interest Rate Shock): First, 3 percent interest rate shock (permanent and unexpected) is realized. After 2 periods FED communicates the policy.

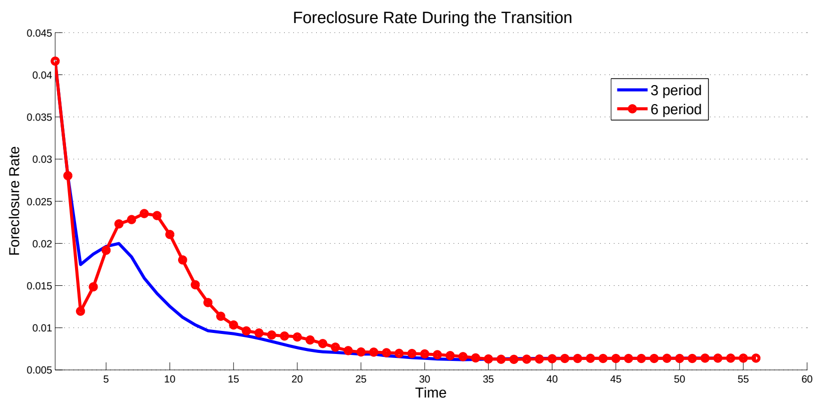


Figure 15: The Effects of FED's Policy on Foreclosures (Financial Shock): First, 3 percent interest rate shock (permanent and unexpected) is realized. After 2 periods FED communicates the policy.

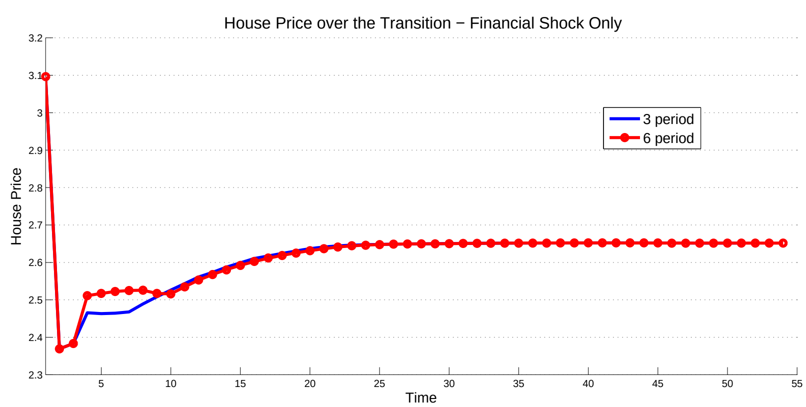
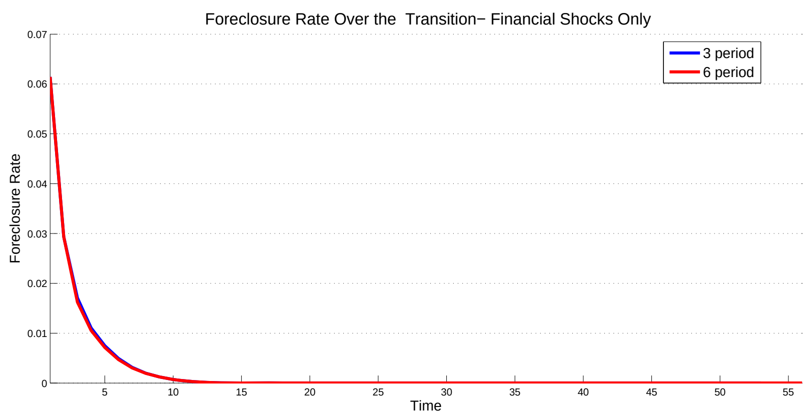
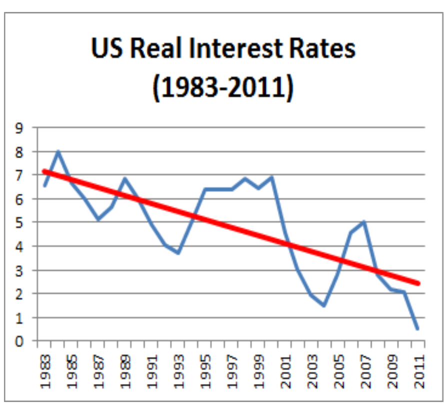
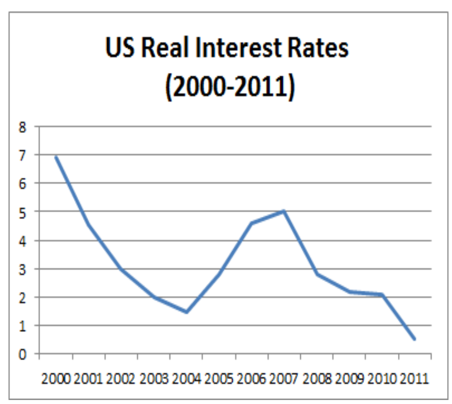


Figure 16: The Effects of FED's Policy on House Perices (Financial Shock): First, 3 percent interest rate shock (permanent and unexpected) is realized. After 2 periods FED communicates the policy.



step forward we plan to go to data and measure, as significant as possible, the sizes of the shocks. For instance we can measure the change in the returns on inflations indexed 10-year bonds. Or we can use expected real return on 10 year bond. For that purpose we use 10 year bond return and the long term inflations expectations to form expected real return. Prior to 2006 expected real return on 10-year bond declined to around 2 percent. After 2006 real rates increased around 1 percent. After the crisis with the unconventional monetary policy measures expected real rates turned to negative.





The movements in housing prices and foreclosure rates had dramatic effects also on the financial markets. Prior to the crisis credit was easily supplied to consumers. After the crisis financial institutions deleveraged considerably and decreased their credit supply. For our study this has potentially important effects on the housing market. Even though we have a financial sector which is more complex than many models, it lacks ingredients to endogenously

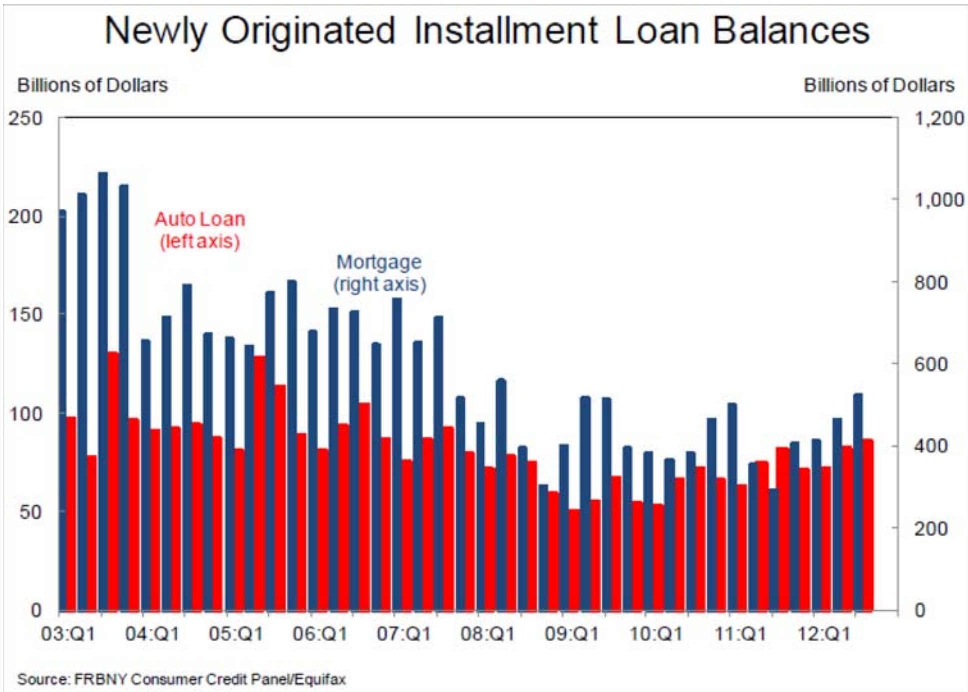


Figure 17:

generate deleveraging process. Hence, to mimic the behavior of the banking sector after the crisis we use the minimum downpayment requirement in our model to match the decline in the mortgage credits. However, whether the level of the shock that we used is realistic or not is a question. We plan to calibrate the level of this shock to match the decline in the mortgage credits we observe in the data.

The last factor we consider to add to the model is unemployment. We consider an increase in the unemployment probability to be an important risk in the housing market since unemployed are more likely to default on their loans. Prior to the crisis the unemployment rate was around 5 percent. With the crisis, unemployment rate rapidly increased to 10 percent. After topping at 10 percent it gradually declined to 7.8 percent and it is expected decline to the levels of 5 percent around 2018. As the crisis came up with higher unemployment and given that income shocks play an important role in the foreclosures, adding unemployment to the model may potentially add new insight to our understanding.

In addition to correctly identifying the levels of the shocks we plan to work on the assumption that the shocks are permanent. For that purpose we plan to study the implications of different transition paths on the equilibrium. For instance we have the expectations of interest rates and unemployment rate path over the next five years. In this way we plan to loosen the initial "permanent shock" assumption.

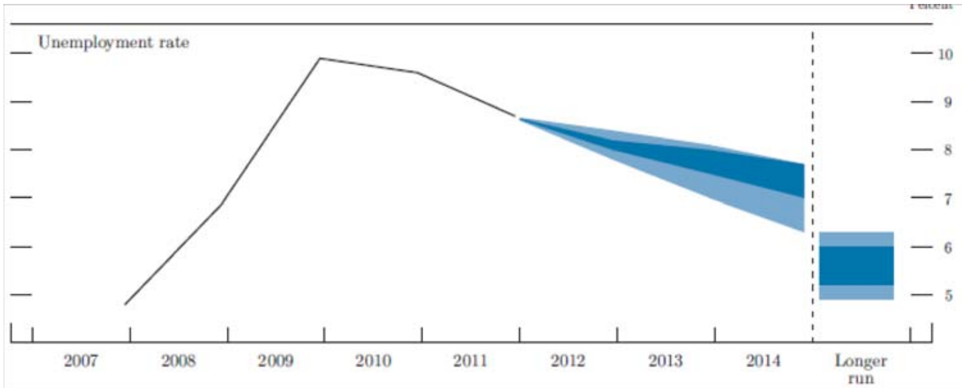


Figure 18:

10 Conclusion

To be completed...

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