

The Great Housing Boom in China*

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Abstract

China's decade-long housing boom looks nothing short of a gigantic bubble familiar to many countries: In big cities the price-to-income ratio reached 30 to 1 and the vacancy rate stood at 30% or above. This paper provides a theoretical framework to shed light on the causes and consequences of the great housing boom in China. We argue that the boom could be a rational bubble rooted in China's unprecedented economic transition—which features persistent high returns to capital. We argue that the very expectation that China's high capital returns driven by cheap labor and resource relocations are not sustainable in the long run can induce investors to seek alternative store of value for their growing wealth, thus triggering a self-fulfilling housing bubble in a financially underdeveloped economy with limited supply of financial assets. The bubble would exhibit a rapidly rising housing price-to-disposable income ratio during the transition path regardless of whether housing provides rents or utilities. This prediction is consistent with China's "ghost town" phenomenon and decade-long faster-than-income-growth housing bubble, which cannot be explained by standard neoclassical growth theory. We show the bubble could prolong China's economic transition and severely reduce social welfare. Our model also sheds considerable light on similar housing bubbles existed in other emerging economies during their rapid economic growth and transition periods.

Keywords: Housing Bubble, Capital Misallocation, Crowding-Out, Chinese Economy, Economic Transition.

JEL Codes: E22, E23, O16, O53, P23, P24, P31.

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1 Introduction

Standard neoclassical theory suggests that housing (land) prices can grow at most as fast as aggregate productivity under a fixed land supply. But this is not true in China. Housing prices in China experienced rapid and steady accelerations far above productivity growth since the housing reform in the late 1990s and, especially, in the recent decade. In many cities, the growth of housing prices significantly outpaced the growth of disposable income. For example, data based on 35 major Chinese cities show that in most of these cities the growth rate for real housing prices between 2006-2010 are between 15% to 30% per year, far exceeding the 10% growth rate for real disposable income in these cities.¹ As a consequence, the nation-wide average house price-to-income ratio increased from around 8 in 1999 to 13 in 2010, and this ratio reached about 30 in big cities such as Beijing and Shanghai, despite unprecedented income growth. The increase in housing prices is also accompanied by rapidly rising land values in China. Using data from the local land auction market in Beijing, Wu, Gyourko, and Deng (2010) show that real constant quality land values have increased by nearly 800% since the first quarter of 2003.

Because of the phenomenal rate of return in the housing market, real estate has become a vital investment opportunity for households, state-owned enterprises, private firms, as well as commercial banks. So housing investors in China include people from all walks of life, regardless of their income level and profession, and enterprises of all types, regardless of their sizes and ownerships. In fact, capital gains from housing investment have become an important (or even the only) source of profits for many state-owned and private firms. Given the sheer size of China's housing market (with a population of 1.3 billion and 230 millions new migrants into cities in recent decade and a similar-size migration in the next decade) and the long duration of the rapid housing-price growth, what we are witnessing can be truly called one of the greatest housing booms in human history.

The astonishingly high price-to-income ratio seems to suggest excess demand in the housing market; yet "ghost towns" and massive empty apartments across big cities in China appear to indicate excess supply. As if this mismatch in demand and supply is not puzzling enough, there exists even a bigger puzzle: housing prices keep growing rapidly in recent years despite the alarmingly high vacancy ratio.

¹See Wu, Deng, and Liu (2012, Fig. 5).

On the one hand, the great housing boom appears no different from a typical housing bubble experienced by many countries, such as Japan in the 1970s and 80s and the United States in the 2000s, except that the Chinese situation looks worse: The housing price-to-income ratio in big Chinese cities is twice as high as in Japan when Japan's housing bubble burst in 1990, and six times as high as in the United States at the peak of the U.S. housing bubble in 2005-2006. Vacancy rates in China are between 25% and 30%, well above the normal range of 5 to 10 percent in advanced countries. On the other hand, the great housing boom also looks unique to China: the ghost towns, ghost malls, and ghost apartments in China make even the unprecedented Japanese bubble thirty years ago look pale.

Not surprisingly, the great housing boom in China has generated global attentions. The fear is that the rapid housing price growth is not sustainable and a collapse of the Chinese housing market may intensify the current world slump and significantly prolong the world-wide recession amidst of the world financial and debt crisis, given China's increasing role as an engine of global economic growth. The great housing boom has also caused great concerns by the Chinese government, as excessive investment in housing can crowd out investment in fixed capital and other real economic activities, and a sudden collapse of the bubble may cause massive bank failures and jeopardize China's economic growth potentials.

What economic forces are at work to generate the great housing boom in China? Are the faster-than-income-growth housing boom sustainable? What are the economic costs of the great housing boom?

Many conventional views exist to explain the great housing boom in China. One theory views the boom as a natural consequence of China's rapid income growth and rising urban demand for housing. When total income grows rapidly but the supply of land is inelastic, a growing demand for housing can easily push up housing prices. This theory predicts that housing prices can grow as fast as aggregate income growth. Another view is that the housing boom is a pure bubble because in financially underdeveloped economies houses can serve as an attractive store of value even if they provide no utilities (as in the classical Samuelson model of fiat money). This view can explain the ghost-town phenomenon in China but requires the additional assumption that the rate of return to capital (or interest rate) in China is excessively low so that capital is not an idea asset to invest compared to housing. But the rate of return to capital in China is not low: Its real rate of return has been above 20% per year over the past decades despite decades-long excessive investment (Bai, Hsieh and Qian, 2006). Therefore, these conventional views cannot explain why the growth rate of housing prices in China have remained so high for so long, far exceeding the growth rate

of aggregate disposable income despite the excessively high rate of return to capital.²

This paper provides a theoretical framework to address these puzzles. We show that the great housing boom in China can be a rational bubble arising naturally from China's unprecedented economic transition—which features phenomenal rates of return to capital driven largely by newly emerging private firms and massive relocations of cheap labor from rural to urban areas, from poor to rich cities, and from SOEs (state-owned enterprises) to private enterprises during the transition.

As unprecedented as it is, however, no rational investors would expect China's growth miracle to continue forever. The cheap labor resource may one day be exhausted; the high return to capital may eventually come to an end; and the cheap credit supply due to financial repression cannot possibly last forever. In deed, a recent survey of the biggest private firms in China shows that the top concerns for their future profitability are the rising costs in (i) raw materials, (ii) labor, (iii) credit, and (iv) tax burdens. Thus, the rational anticipation of increasing costs and declining capital returns in the future would motivate rational investors to seek alternative stores of value beside capital.

Part of China's rapid growth came from the government's massive investment in infrastructures. As a result, China's public debt has increased rapidly over the past decade. The burden of repaying the debt will ultimately fall upon future generations. This anticipation of rising corporate taxes further reinforces the public's expectation of low future after-tax capital returns in the private sector, further encouraging people to seek alternative stores of value for their growing wealth.³

Capital controls and an underdeveloped financial market in China (as is the case for many developing countries) have limited the availability of stores of value for the rapidly increasing wealth held by households and entrepreneurs; thus, investing in housing becomes one of the best choices in China for capital gains.⁴ We show that this expectation-driven strong demand for housing as an alternative store of value, based on the foresight that China's low-cost and high-capital-return economy will eventually come to an end, can generate a large, fast-growing, and self-fulfilling housing bubble at the present. That is, even if housing provides no rents or utilities, rational agents would still hold it as a store of value if its rate of return

²There is another puzzle the standard neoclassical model cannot explain: consumption-to-income ratio in China has been declining over the past decades (see Wen, 2012). If housing provides utilities and its prices grow faster than income, then as a normal good the consumption-to-income ratio should also increase over time.

³In China government tax income comes mainly from corporate taxes, instead of household income, because of limited capacity in income-tax collection.

⁴Laws in China prohibit people purchasing land as a store of value. Otherwise it may be more efficient to use land rather than vacant houses as a store of value.

(capital gain) equals or exceeds that of capital—which is exceptionally high during transition but would surely be significantly reduced after the transition ends. Consequently, a bubbly equilibrium exists in which the growth rate of housing prices equals the rate of return to capital. Hence, along the transition path we will observe a S-shaped housing price-to-income ratio—with housing price growing much faster than disposable income in the initial stage of the transition but eventually converging to the growth rate of disposable income in the long run. This prediction appears consistent with the Chinese data.

We show that the housing bubble could greatly prolong China’s economic transition and reduce social welfare. Unlike some existing bubble models, in our model housing bubbles can exist without dynamic inefficiency, due to the disparity between social and private rate of returns to capital. Hence, by crowding out private capital formation and other productive activities, it creates a negative externality and reduces the permanent income of all agents. Accordingly, the occurrence of the housing bubble generates a substantial degree of resource misallocation and welfare losses, prolonging economic transition and slowing down aggregate economic growth.

Our model not only rationalizes the great housing boom in China, but also shed considerable light on housing bubbles experienced in many emerging economies, such as the Asian four tigers. These economies all experienced a transition period featuring low wage growth and high capital returns sustained by labor relocations, and also suffered from underdeveloped financial markets and the lack of store of value for their growing wealth. Hence, with currently high capital returns and anticipated low future capital returns, people opt to seek alternative stores of value for their rapidly accumulated wealth. Given the capital control policies widely adopted by governments to prevent capital outflows, land and housing stocks become the natural choice of asset investment.

Our paper is related to several strands of the literature. First, our theory relies on a similar mechanism as in Song, Storesletten and Zilibotti (2011, “SSZ” hereafter) to generate a persistently high rate of return to capital along transition. Specifically, SSZ provide an overlapping generations model with two sectors: one operates with inefficient technology and the other has superior technology but little access to bank capital, thus must rely on self finance. They show that labor relocation from the former to the latter sector can generate endogenous productivity growth at the aggregate level, and account for China’s high income growth, high capital returns, and large capital account surplus. In SSZ, however, investors’ only portfolio choice is between capital investment and bank deposit. As a result, their paper is silent on why housing bubbles may occur in China, and thus unable to explain why the allocative

efficiency in China can become worse as a result of the housing bubble—a phenomenon also shared by other emerging economies in Asia. By introducing housing as a bubble asset into the SSZ model, our paper not only sheds light on the formation of housing bubbles along the economic transition, but also their social costs in terms of resource misallocation and welfare. In particular, we explain why entrepreneurs in China have strong incentives to own empty apartments that generate zero rents or utilities. By showing housing bubble as a natural consequence of economic transition and studying what policies may correct the consequent distortions, our paper complements SSZ in understanding the typical growth pattern of emerging economies like China.

Our paper also contributes to the emerging literature on China’s housing price puzzle. Most works in this area focus on why the housing price level is so high in China. For example, Wei, Zhang and Liu (2011) provide a theory to link the high housing price *level* in major cities of China to these areas’ high household saving rates due to unbalanced sex-ratio. In sharp contrast, the focus of our paper is on why housing prices in China have *grown* faster than aggregate income over the last decade. To understand China’s growing housing bubble, models that only explain high housing price level from the demand side are not sufficient. More importantly, by shifting the analysis from housing price level to housing price growth, our paper sheds light on why the rapid growth of housing prices may create resource misallocation and prolong China’s economic transition, an issue silent in Wei *et. al* (2011).

Our paper fits into the fast-growing literature of economic development and resource misallocation, with a focus on financial under-development.⁵ We share the similar view that financial under-development, especially financial repression, is key to resource misallocation along transition. To our knowledge, we are the first to incorporate housing as a bubble asset in this literature and analyze in detail its social and welfare costs in terms of economic transition.

Finally, our theory is related to the existing theories on housing bubble. Tirole (1985) and Farhi and Tirole (2011) emphasize the crowding-out effects of bubble assets on capital accumulation and its negative welfare effects. On the other hand, Ventura (2012), Martin and Ventura (2012), Caballero and Krishnamurthy (2006), and Kocherlakota (2009) show that bubbles can crowd in capital accumulation when firms are borrowing constrained and can use the bubble as collateral or a store of value to facilitate intertemporal consumption smoothing. We show that even if the bubble asset can serve as collateral for borrowing, it

⁵See, for example, Buera and Shin (2011) and Moll (2011).

can still crowd out capital formation and hinder welfare.

The remaining part of the paper is organized as follows: In Section 2, we present the empirical facts about China’s growing housing bubble. Section 3 describes a simple 2-period benchmark model to illustrate our essential findings. Section 4 reports the model’s simulation results. In Section 5, we conduct a quantitative analysis based on a multi-period version of our model. Section 6 concludes.

2 Empirical Facts

2.1 Chronology

China economic reform started in 1978. In the era of planned economy (between 1950 and 1978), all housing (apartments) in the city were provided by the government at subsidized rents. All institutions, no matter whether they are hospitals, schools, or firms, are obligated to provide housing to their workers. This situation changed gradually since the reform. In particular, from 1982 to 1985, more than 1,600 cities in China launched pilot projects of housing reforms. Most of these projects focused on privatizing the existing public apartments and let residents to pay the market-determined rents. However, due to the delay of wage reforms and the lack of a financial system to provide loans, the first round housing reform failed.

In 1991, the city of Shanghai built a system of publicly pooled funds for housing finance. This experiment was later introduced to the entire country between 1994 and 1997. The pooled funds provided loans to enterprises and public institutions to build private housing units and to individuals to purchase housing units—which was also the only channel for individuals to obtain loans in those days. During this period, about 20% to 30% of the existing housing stock was traded in the market, so the bulk of housing units was still provided by the government at subsidized rates.

Things changed dramatically in 1998, in which year China’s State Council lunched a new round of housing reform and issued “Notice on the Further Deepening of Urban Housing Reform” (the 23rd Decree). After that, public housing provision essentially ended nationwide and bank mortgage loans became available to home builders and home buyers in addition to publicly pooled funds. Consequently, China entered an era of housing market boom. The share of private housing units in total housing units increases from 30% to more than 70% between 1998 and 2010.

2.2 Housing Prices and Disposable Income

The National Bureau of Statistics of China (NBSC) provides two major housing price indexes. Based on these housing price indexes, the average growth rate of housing prices in China is below the average growth rate of household disposable income. However, Wu, Deng, and Liu (WDL 2012) argue that these measures are severely biased downward because they include all existing houses in areas that are not yet included by market transactions or do not have realistic market values. These authors instead use independently constructed housing price index based on newly-built housing sales in 35 major Chinese cities. Their price index demonstrates that the government-published data are severely downward biased and fail to capture the dramatic increases in housing prices across the nation. Based on their data set on newly-built housing sales, the average housing prices in China's 35 major cities have increased from a level of 100 in early 2004 to a level of 250 in late 2009, implying an average annual growth rate of 17% per year. If we ignore the negative impact of the financial crisis, the average growth rate was 20% per year between 2004 and 2008 and this growth rate become 25% in the year 2009 (see Panel A of Figure 1). In big cities the growth rate is even higher. For example, in Shanghai and Beijing the average real growth rate of housing prices during the same period is 2-3 times higher than the average real growth rate of disposable income (see Figure 1B).

Yang, Chen, and Monarch (YCM 2010) show that during the post-reform period between 1978 and 2007, China's average nationwide real wage growth is only about 6.5% per year. Wage growth was particularly low (about 4%-5% per year) for the first sub-period of 1978-1998. But since the fully-fledged housing reform and the SOE reform in 1998, the growth rate of real wage accelerated to about 10% per year (8.5% per year in the manufacturing sector) in the second sub-period in 1998-2007, roughly caught up with the average national income growth rate in China (see, e.g., Figure 1 in YCM, 2010). So the gap between real housing price growth and real wage growth in the post housing reform period is at least 7 percentage points. This implies that an initial price-to-income ratio of 8 in 1999 would become 20 in 2013, consistent with the information reported in the opening paragraph in the Introduction and the facts presented in Panel B of Figure 1.⁶

⁶Economic growth in China is highly uneven, including wage income. YCM (2010) also documents that in some sectors the growth rate of real wage is nearly as high as or even above the average growth rate of housing prices. This inequality in income growth also suggests that high- and upper middle income classes in China are fully capable of opting to use housing as their preferred store of value despite the high average price-to-income ratio. Basically, with a rising average housing price-to-income ratio, although the average household in China becomes increasingly difficult to use housing as a store of value even if they want to, this is not true for rich and upper middle income households since their income levels are able to keep pace with

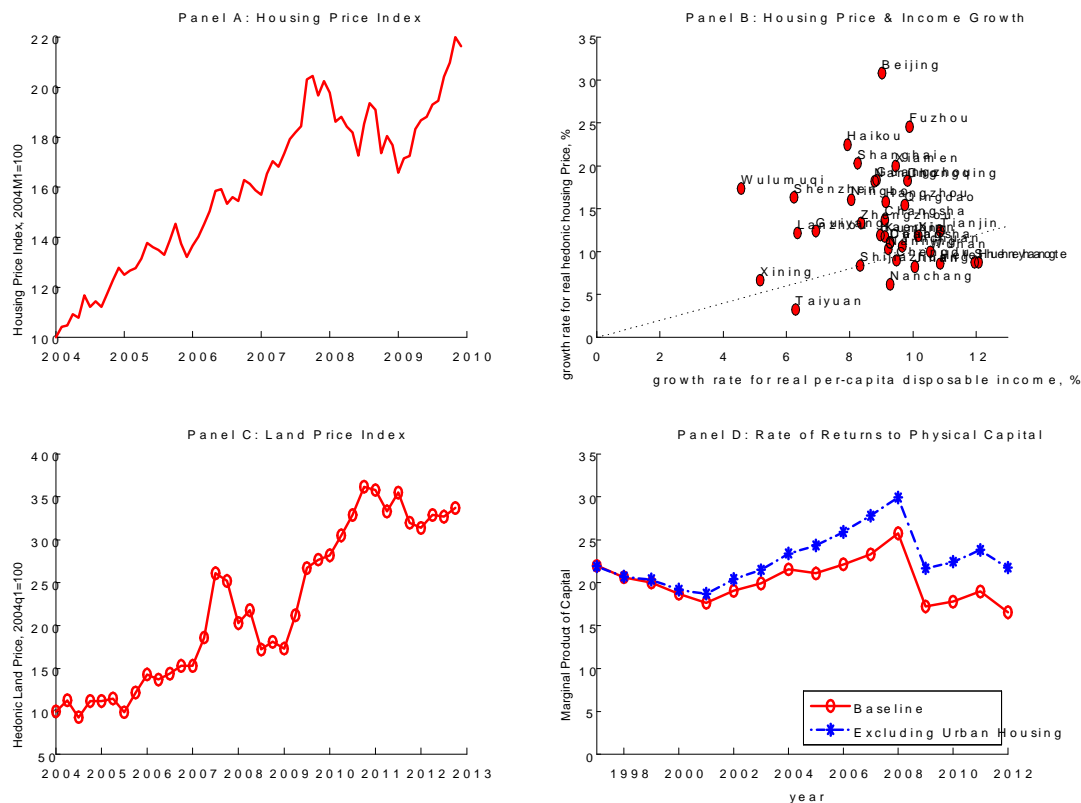


Figure 1: Housing Prices in China

The increase in housing prices is also accompanied by rapidly rising land values in China (Panel C of Figure 1). Using data from the local land auction market in Beijing, Wu, Gyourko, and Deng (2010) show that real constant quality land values have increased by nearly 800% since the first quarter of 2003. Among the market participants of land purchases, state-owned enterprises have played an important role and they in general paid 27% more than other bidders for an otherwise equivalent land parcel. A rapidly rising land value benefits the land owners (the government in China) but hurts firms, retailers, and consumers in terms of office building and rental costs.

Based on data from China Statistical Yearbook 2012, the real estate sector has experienced a spectacular boom since the full-fledged housing reform in 1998. The share of total real-estate investment in GDP increased by more than three folds, from 4.2% in 1999 to 13.2% in 2011. In particular, a booming residential investment accounts for about 70% of the real estate boom: its share in GDP rose from 2.4% in 1999 to 9.5% in 2011, a 4-fold the housing price growth.

expansion—the average nominal growth rate of residential investment is 25.5% per year and the average nominal growth rate of GDP is 14% per year, so residential investment growth is more than 11 percentage points above the growth rate of China’s nominal GDP. These statistics reinforce the previous housing price data on China’s great housing boom.

Accompanying the fast housing price growth in China is the persistently high rate of returns to capital. Panel D of Figure 1 shows that the rate of return to capital is on average 20% between 1998 and 2012. In particular, it increases steadily from 18 percent in 2001 to 26 percent before the global financial crisis hit in 2008.

2.3 The Crowding-Out Effects on Capital Investment

The rapidly growing housing bubble in China has been crowding out investment and capital formation of both SOEs and private firms. We measure the crowding-out effects by estimating the correlation coefficients between housing price growth and investment growth. To remove seasonal effects, all the growth rates are on year-to-year basis, which means growth rate comparing with the same month in last year. Table 1 presents the correlation between real housing price growth (deflated by Consumer Price Index) and real investment growth (deflated by PPI). Column 2 and 3 show the results based on housing prices at the national level. The nationwide housing price index may not fully represent the extent of the housing bubble in China, because of the highly unbalanced growth and inequality across regions. Therefore, column 4 and 5 show the results based on housing price index at major city level. Besides reporting the correlation between the investment and housing price growth in current period, we also lagged the housing price growth by 1 to 6 months to see how current housing price growth predicts future investment growth.

From Table 1, we can see that growth of investment on real estate sector is significantly positively correlated with housing price growth, while investment on other sector is significantly negatively correlated with housing price growth. More importantly, the results show that current housing price increases are a strong predictor of future drop in investment growth, with the peak correlation between housing price growth and investment growth reached at a lead of 5 month. Such a result is consistent with our model described in the next section. Column 4 and 5 suggest that if we use housing price index from cities where the housing prices have experienced sharper increases, we obtain even stronger negative correlation between housing price growth and investment growth, both contemporaneously and across periods.

Table 1. Correlation: Housing Price Growth & Fixed Investment Growth

	Nationwide		Major Cities	
	Real Estate Inv.	Other Inv.	Real Estate Inv.	Other Inv.
Current	0.5255*	-0.3212*	0.4532*	-0.5470*
$t - 1$	0.4765*	-0.4046*	0.3725*	-0.6567*
$t - 2$	0.4115*	-0.4499*	0.2394	-0.7331*
$t - 3$	0.3320*	-0.5025*	0.1363	-0.7875*
$t - 4$	0.2710*	-0.5467*	0.0666	-0.8011*
$t - 5$	0.2025	-0.5438*	-0.0830	-0.7750*
$t - 6$	0.1288	-0.5171*	-0.2195	-0.7009*

*Significant at 5%.

2.4 Other Facts Concerning the Key Assumptions in Our Model

Timing of Housing Reform and SOE Reform. Under China's planned economy, SOEs were the major employers in the cities and they played the pivotal role of maintaining low unemployment and ensuring social stability. SOEs are required to provide all social and pension benefits to employees, the SOE sector had not only low productivity and limited profits, but also high debt burdens. Naturally, SOEs suffered severe losses during the initial reform period, especially for the small and medium sized SOEs. By the mid-1990s, the Chinese Government realized that their gradualist reform policy could no longer manage the mounting losses of SOEs and decided to take more aggressive steps, first allowing the privatization of small and medium SOE and then, beginning in 1997, moving forward with more aggressive restructuring, accomplished through large scale housing privatization and shifting the federal responsibility of health insurance, unemployment insurance and pension provisions to local governments, employers and employees themselves (see YCM, 2010). Therefore, China's housing reform started roughly at the same time and moved in pace with its reform on the SOE sector. For this reason, we treat housing reform and SOE reform as simultaneous events in our model. Namely, before the housing reform, there were no market for houses and SOEs are the only enterprises. Workers deposit their savings into the state-owned banking system, which is channeled into SOEs for capital allocation. After the reform, house becomes a market commodity. Although it provides no utilities, it can be held as a store of value. At the same time, the private sector emerges, which relies on own savings to accumulate capital and compete with the SOEs for labor resources.

Financial Repression and Interest Rate Control. China has made significant progress since 1978 in opening its economy to the outside world, but financial reform significantly lags its economic reform in goods-producing sectors. China's financial repression is easy to

see in Figure 2 where interest rates are essentially flat with the deposit rate lying substantially below the lending rate. Funds are channeled through state-owned banks to the conventional sector mainly occupied by state-owned enterprises (SOEs). There are few investment alternatives for household savings, stock markets are poorly regulated and dominated by SOEs, interest rates are set by government, the capital account is closed, and the exchange rate is fixed or tightly managed. Through a system of strict capital controls where the state directly manages the banking sector and financial intermediation, the government has been able to maintain or suppress interest rates at below market-clearing levels. A fixed and low interest rate was initially imposed by the government as a development strategy to subsidize industrialization with cheap capital (see Lin, 2012). A below-market interest rate reflects the government’s goal of achieving a maximum rate of capital accumulation and a high level of employment in the SOE sector. Also, when the interest rate is fixed at a level below the market-determined rate, SOEs would be able to earn positive profits despite inefficiency. The profits are, however, not redistributed back to the households, they are instead re-invested to maximize the economy’s capital stock.

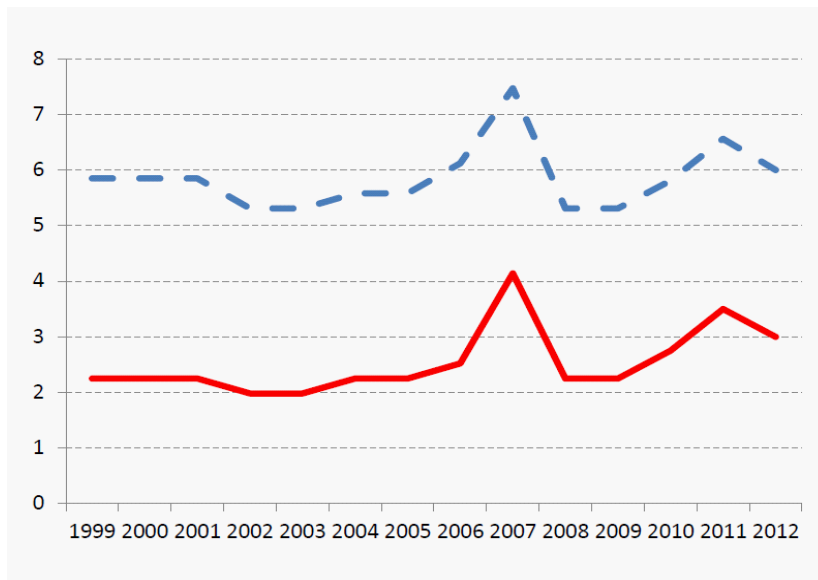


Figure 2. China’s One-Year Nominal Interest Rates (%): Deposit (solid) and Lending (dash).

3 The Benchmark Model

We extend the SSZ model to incorporate an intrinsically valueless asset—housing, and prove that a faster-than-income *growing bubble* in housing prices exists even if housing provides

no rents or utilities to investors. We emphasize the growing nature of the bubble because the existing bubble literature often focuses exclusively on static bubbles or bubbles that grow at the rate of technology. We focus instead on bubbles that can grow faster than the rate of productivity growth. In particular, we show how the very expectations—that the excessively high rate of return to capital during transition is not sustainable in the long run—can generate a self-fulfilling housing bubble that grows faster than aggregate income along the transition path.

3.1 The Environment

The economy is populated by overlapping generations of two-period lived agents.⁷ Agents work when young and consume the return from their savings when old. Agents have heterogeneous skills. Within each cohort, a measure $N_t/2$ of agents have no entrepreneurial skills. They choose to become workers and supply labor to firms. And the rest of the agents have entrepreneurial skills—so they choose to become entrepreneurs. Entrepreneur skills can be transmitted from parents to children. The population N_t grows at an exogenous rate ν .

Before the economy starts, the government owns one unit of housing (land), which is in fixed supply. At the beginning of the first period, the government sells it to the market and consumes all the proceeds.

3.2 Technology

There are two production sectors and thus two types of firms. Labor is perfectly mobile across the two sectors, but capital is not. The first sector is composed of conventional firms—F-firms, which for simplicity are owned by a national bank and operated as standard neoclassical firms. Workers can work in either sector but deposit their savings into the national bank. The bank lends capital to F-firms to produce output.

The second sector is an unconventional or emerging sector, which is composed of high-productivity firms—E-firms. The E-firms are operated by entrepreneurs with overlapping generations. More specifically, E-firms are owned by old entrepreneurs, who are residual claimants on profits and hire their own children as managers. E-firms have higher total factor productivity (TFP) than F-firms. However, E-firms cannot rent capital from the national bank.⁸ As a result, they must self-finance capital investment through own savings. By

⁷We first use a 2-period model to illustrate our main results and then extend it to a 50-period model later on to conduct calibrated quantitative analysis.

⁸We will relax this assumption in a later section.

contrast, F-firms can rent capital from the national bank at a fixed interest rate R . Accordingly, along transition, an F-firm can still survive despite with less productive technology. Over time, however, labor is gradually reallocated from F-firms to E-firms as E-firm sector's capital stock expands.

The technologies of the two types of firms follow constant returns to scale

$$\begin{aligned} y_{Ft} &= (k_{Ft})^\alpha (A_t n_{Ft})^{1-\alpha}, \\ y_{Et} &= (k_{Et})^\alpha (A_t \chi n_{Et})^{1-\alpha}, \end{aligned}$$

where y , k , and n denote output, capital stock and labor, respectively. $\chi > 1$ reflects the assumption that E-firms are more productive than F-firms. Technological growth is constant and exogenously given by $A_{t+1} = A_t(1+z)$.

3.2.1 The Worker's Problem

Workers can deposit their savings into the bank and earn a fixed interest rate R . But for simplicity and without loss of generality, we assume that workers cannot speculate in the housing market.⁹ The worker's consumption-saving problem is

$$\max_{c_{1t}^w, c_{2t+1}^w} \log c_{1t}^w + \beta \log c_{2t+1}^w$$

subject to

$$\begin{aligned} c_{1t}^w + s_t^w &= w_t \\ c_{2t+1}^w &= s_t^w R \end{aligned}$$

where w_t is the market wage rate, c_{1t}^w , c_{2t+1}^w and s_t^w denote respectively the consumption when young and old, and the worker's savings. τ_{t+1} is a lump-sum transfer from the bank (to be specified below). The first order conditions imply

$$s_t^w = \frac{1}{1 + \beta^{-1}} w_t$$

Namely, the optimal level of saving is proportional to the income received when young.

⁹In an appendix upon request, we show that allowing workers to invest in housing does not change our results—the dynamics of housing price is unaffected.

3.2.2 The F -Firm's Problem

In each period, F -firms maximize profits by solving the following problem

$$\max_{k_{Ft}, n_{Ft}} (k_{Ft})^\alpha (A_t n_{Ft})^{1-\alpha} - w_t n_{Ft} - R k_{Ft},$$

where the rental rate for capital is the same as the deposit rate, R . The first-order conditions are

$$\begin{aligned} w_t &= (1 - \alpha) k_{Ft}^\alpha A_t^{1-\alpha} n_{Ft}^{-\alpha} \\ R &= \alpha k_{Ft}^{\alpha-1} A_t^{1-\alpha} n_{Ft}^{1-\alpha} \end{aligned} \quad (1)$$

This gives

$$w_t = (1 - \alpha) A_t \left(\frac{\alpha}{R} \right)^{\frac{\alpha}{1-\alpha}} \equiv (1 - \alpha) A_t \kappa_F^\alpha \quad (2)$$

where $\kappa_F \equiv \frac{k_{Ft}}{A_t n_{Ft}} = \left(\frac{\alpha}{R} \right)^{\frac{1}{1-\alpha}}$. Note that along transition, the (detrended) wage rate, $\frac{w_t}{A_t}$, is constant, due to a constant rental rate for capital. When the transition ends, all F -firms disappear, so equation (2) no longer holds.

3.2.3 The E -Firm's Production

Following SSZ, we assume that young entrepreneurs get paid a management fee m_t —that is a fixed $\psi < 1$ fraction of the output produced.¹⁰ Therefore, the old entrepreneur's problem can be written as

$$\max_{n_{Et}} (1 - \psi) (k_{Et})^\alpha (A_t \chi n_{Et})^{1-\alpha} - w_t n_{Et}$$

The first-order conditions imply

$$(1 - \psi) (1 - \alpha) (k_{Et}/n_{Et})^\alpha (A_t \chi)^{1-\alpha} = w_t = (1 - \alpha) A_t \left(\frac{\alpha}{R} \right)^{\frac{\alpha}{1-\alpha}}, \quad (3)$$

where the second inequality comes from (2) based on the assumption of perfect labor mobility across firms. Equation (3) immediately implies a linear relationship between n_{Et} and k_{Et}

$$n_{Et} = [(1 - \psi) \chi]^{-\frac{1}{\alpha}} \left(\frac{R}{\alpha} \right)^{\frac{1}{1-\alpha}} \frac{k_{Et}}{\chi A_t}. \quad (4)$$

¹⁰SSZ provides a microfoundation for young entrepreneur's management fee as a fixed fraction of output: There exists an agency problem between the manager and owner of the business. The manager can divert a positive share of the firm's output for his own use. Such opportunistic behavior can only be deterred by paying managers a compensation that is at least as large as the funds they could steal, which is a share ψ of output.

Such a linear relationship is obtained because of the constant interest rate R , which implies a constant wage rate. Accordingly, labor is reallocated to E-firms at a speed equal to the growth of the private capital stock in the E-firm sector. The profit of the E-firm is

$$\begin{aligned}\pi(k_{Et}) &= (1 - \psi)(k_{Et})^\alpha (A_t \chi n_{Et})^{1-\alpha} - w_t n_{Et} \\ &= (1 - \psi)^{\frac{1}{\alpha}} \chi^{\frac{1-\alpha}{\alpha}} R k_{Et} \\ &\equiv \rho^E k_{Et},\end{aligned}$$

where the second equality is obtained by using (4). Whenever F-firms exist, the return to capital in E-firms, $\rho^E \equiv [(1 - \psi)]^{\frac{1}{\alpha}} \chi^{\frac{1-\alpha}{\alpha}} R$, is a constant. This is because n_{Et} increases linearly in k_{Et} . As a result, $\kappa_E = \frac{k_{Et}}{A_t n_{Et} \chi}$ is a constant during the transition.

3.2.4 The Consumption-Saving Problem of the Young Entrepreneur

The young entrepreneur obtain $m_t = \psi(k_{Et})^\alpha (A_t \chi n_{Et})^{1-\alpha}$ as income when young, and decides consumption and portfolio allocation in housing investment, bank deposits, and physical capital investment. The return for capital investment is simply ρ^E . By arbitrage, the return to capital must be equal to or larger than the capital gains from housing:

$$\frac{P_{t+1}^H}{P_t^H} \leq \rho^E. \quad (5)$$

Hence, a young entrepreneur's income when old is simply $\rho^E s_{Et}$, where s_{Et} denotes total savings. Therefore, a young entrepreneur's consumption-saving problem is

$$\max_{s_{Et}} \log(m_t - s_{Et}) + \beta \log \rho^E s_{Et}$$

The first-order conditions imply

$$s_{Et} = \frac{1}{1 + \beta^{-1}} m_t.$$

We assume that a fraction ϕ_{Et} of s_{Et} is invested in firms' capital, and the rest in housing, such that $K_{Et+1} = \phi_{Et} s_{Et} N_t$. Entrepreneurial housing demand is then $(1 - \phi_{Et}) s_{Et} N_t = p_t^H H_{Et}$. The optimal portfolio ϕ_{Et} is pinned down in equilibrium.

3.2.5 The Bank's Problem

For exposition, we assume that each period the bank simply absorbs deposits from young workers, rent them to F-firms at interest rate R , and invest the rest in foreign bonds with the same rate of return.

3.2.6 Time Line

To summarize, in each period the economic events in our model unfold as follows:

1. At the beginning of period t , production of E-firms and F-firms takes place. The capital stock used by E-firms is k_{Et} , which is from the savings of the entrepreneur when young. The capital stock used by F-firms is K_{Ft} , which is rented from the bank in the last period (pre-determined in period $t - 1$). Each young worker gets w_t regardless which sector they work in. Each young entrepreneur gets m_t .
2. The young entrepreneur chooses consumption and make saving decision s_{Et} . Young workers make consumption and deposit decisions.
3. Housing market opens. Old entrepreneurs sell housing stock held in the previous period, H_{Et-1} , in the housing market, consume, and die. Young entrepreneurs make portfolio decision ϕ_{Et} and invest a fraction of wealth $(1 - \phi_{Et}) s_{Et}$ in housing, $P_t^H H_{Et}$.
4. F-firms repay their rentals to the bank.
5. The currently old workers consume and die. So do the currently old entrepreneurs.

3.2.7 Law of Motion for K_{Et}

We now derive the law of motion for the capital stock held by E-firms. Since E-firm is self-financed, we have

$$\begin{aligned} K_{Et+1} &= \phi_{Et} s_{Et} N_t \\ &= \phi_{Et} \frac{1}{1 + \beta^{-1}} m_t N_t \end{aligned}$$

Note that $m_t = \psi (k_{Et})^\alpha (A_t \chi^{n_{Et}})^{1-\alpha} = \psi k_{Et} \left(\frac{1}{\kappa_E} \right)^{1-\alpha}$, where

$$\kappa_E \equiv \frac{k_{Et}}{A_t \chi^{n_{Et}}} = \kappa_F [(1 - \psi) \chi]^{-\frac{1}{\alpha}}.$$

Hence

$$\begin{aligned}
K_{Et+1} &= \phi_{Et} \psi \left(\frac{1}{\kappa_E} \right)^{1-\alpha} \frac{1}{1+\beta^{-1}} k_{Et} N_t \\
&= \phi_{Et} \psi \left\{ [(1-\psi)\chi]^\frac{1}{\alpha} \left(\frac{R}{\alpha} \right)^{\frac{1}{1-\alpha}} \right\}^{1-\alpha} \frac{1}{1+\beta^{-1}} K_{Et} \\
&= \phi_{Et} \frac{\rho^E \psi}{(1-\psi)\alpha} \frac{1}{1+\beta^{-1}} K_{Et} \\
&= \phi_{Et} \gamma_E K_{Et}
\end{aligned} \tag{6}$$

where $\gamma_E \equiv \frac{\rho^E \psi}{(1-\psi)\alpha} \frac{1}{1+\beta^{-1}}$. The second equality follows equation (4), whereas the third equation follows the definition of ρ^E . Equation (6) shows that the growth rate of private capital ($\frac{K_{Et+1}}{K_{Et}}$) increases with the share of entrepreneurial savings in physical capital, ϕ_{Et} .

With equations (4) and (6), we can derive the law of motion for labor in the E-firm sector as

$$\frac{N_{Et+1}}{N_{Et}} = \frac{K_{Et+1}}{K_{Et} (1+z)} = \frac{\phi_{Et} \gamma_E}{1+z}$$

3.3 Post-Transition Equilibrium

We need to characterize the equilibrium after the transition ends, i.e., when $n_{Et} = 1$, $k_{Ft} = 0$. Since $n_{Et} = 1$, the profit of the E-firm is

$$\pi(k_{Et}) = \alpha (1-\psi) (k_{Et})^\alpha (A_t \chi)^{1-\alpha}$$

Note that $\pi(k_{Et})$ now features decreasing returns to scale. The average rate of return for capital investment is simply

$$\frac{\pi_{t+1}}{k_{Et+1}} \equiv \rho^E(k_{Et+1}) = \alpha (1-\psi) (k_{Et+1})^{\alpha-1} (A_{t+1} \chi)^{1-\alpha}. \tag{7}$$

The law of motion for capital is

$$K_{Et+1} = \phi_{Et} \frac{1}{1+\beta^{-1}} \psi (K_{Et})^\alpha (A_t \chi N_t)^{1-\alpha} \tag{8}$$

Finally notice that

$$\begin{aligned}
P_{t-1}^H H_{Et-1} &= (1 - \phi_{Et-1}) s_{Et-1} N_{t-1} \\
&= (1 - \phi_{Et-1}) \frac{\rho_{t-1}^E \psi}{(1 - \psi) \alpha} \frac{1}{1 + \beta^{-1}} K_{Et-1}
\end{aligned} \tag{9}$$

Note that $\rho^E(\kappa_{Et})$ is a function of κ_{Et} .

3.3.1 Housing Demand and Housing Price

We now determine the housing demand by young entrepreneurs and the equilibrium housing price. Note first that the housing demand satisfies

$$P_t^H H_{Et} = P_t^H \bar{H}.$$

Consider two cases:

Case 1: $R < \frac{P_{t+1}^H}{P_t^H} = \rho^E$ To derive the key equation on ϕ_{Et} , note that

$$\begin{aligned}
P_t^H H_{Et} &= (1 - \phi_{Et}) s_{Et} N_t \\
&= (1 - \phi_{Et}) \frac{1}{1 + \beta^{-1}} \psi (K_{Et})^\alpha (A_t \chi N_{Et})^{1-\alpha}
\end{aligned} \tag{10}$$

Case 2: $\frac{P_t^H}{P_{t-1}^H} < \rho^E$. In this case, entrepreneurs will not invest in housing, i.e. $\phi_{Et} = 0$.

Then $P_t^H = 0$. In this paper, we focus on the equilibrium with housing.

3.4 Characterizing the Equilibrium

Since all per-capita variable (except for n_{Et}) grow at the rate A_t , we detrend all per capita variables as $\hat{x}_t = x_t/A_t$.

3.4.1 Steady State

At the steady state, we have

$$\rho^{E*} = \alpha (1 - \psi) \left(\hat{k}_E^* / \chi \right)^{\alpha-1} = \alpha (1 - \psi) \frac{(1 + \beta^{-1}) (1 + z) (1 + \nu)}{\psi \phi_E^*},$$

where the second equality is derived from (25). With the no-arbitrage condition between housing and physical capital, we have

$$\alpha(1-\psi) \frac{(1+\beta^{-1})(1+z)(1+\nu)}{\psi\phi_E^*} = \frac{P_{t+1}^H}{P_t^H} = (1+z)(1+\nu)$$

This gives the share of saving of E-firm in physical capital at the steady state

$$\phi_E^* = \alpha(1-\psi)(1+\beta^{-1})/\psi \quad (11)$$

Intuitively, the larger is the returns to capital for the entrepreneur, as captured by $\alpha(1-\psi)$, the larger is the share of entrepreneurial savings in physical capital. On the other hand, the larger is ψ and ψ , which reflects a higher income of young entrepreneur and their saving propensity, the lower is the returns to physical capital and thus the lower would be the share of entrepreneurial savings in physical capital.

Note that in our economy, we need $\phi_E^* < 1$ for housing to exist. This implies that without housing the private return to physical capital by entrepreneurs will be below the balanced growth rate. This implies the following parameter restriction

$$\alpha(1-\psi)(1+\beta^{-1}) < \psi \quad (12)$$

Intuitively, a larger ψ affects the rate of returns for capital for the old entrepreneur in two way: first, it directly reduces the output share accrued to the old entrepreneur; second, by increasing the output share of the young entrepreneur, it increases the capital accumulated by the young and thus pushing down the marginal product of capital. In addition, we need to assume the returns for housing is larger than the bank deposit rate

$$\frac{P_{t+1}^H}{P_t^H} = \rho^{E*} = (1+z)(1+\nu) > R.$$

To summarize, we have the following equations at steady state,

$$\widehat{k}_E^* = \left[\frac{\psi\phi_E^*\chi^{1-\alpha}}{(1+\beta^{-1})(1+z)(1+\nu)} \right]^{\frac{1}{1-\alpha}} \quad (13)$$

$$\rho^{E*} = \alpha(1-\psi) \left(\widehat{k}_E^*/\chi \right)^{\alpha-1} \quad (14)$$

$$p^{H*}\bar{h} = p^{H*}h_E^* \quad (15)$$

$$p^{H*}h_E^* = (1-\phi_E^*) \frac{1}{1+\beta^{-1}} \psi \kappa_E^{*\alpha} \chi^{1-\alpha}. \quad (16)$$

3.4.2 Existence and Normative Implications of Bubbles

Different from the neoclassical growth model, in our economy, the old entrepreneur's returns to capital, ρ^E , is only a fraction, $1 - \psi$, of the marginal product of capital for E-firms, which is the social rate of returns to capital. This implies that housing bubbles may exist even under dynamic efficiency. The condition for dynamic efficiency is that

$$\partial y_E / \partial k_E |_{\phi_E^*=1} > (1 + z)(1 + v) \quad (17)$$

At steady state, with (13) under $\phi_E^* = 1$, (17) implies

$$\psi < \alpha(1 + \beta^{-1}). \quad (18)$$

Intuitively, the smaller ψ is, the smaller is the steady-state capital and the higher is its marginal product. Also, similar to standard OLG models, a higher α or a lower β make the economy less likely to be dynamic inefficient.

An interesting issue is the normative implication of bubbles in an economy without dynamic inefficiency. This implication is interesting because bubbles can exist in our model without dynamic inefficiency, and they reduce the aggregate resource available for aggregate consumption. In Proposition 1, we show that the following condition is sufficient for bubbles to exist and to crowd out aggregate consumption of the investors:

$$\frac{\psi}{\psi + \alpha(1 - \psi)} < \alpha(1 + \beta^{-1}) \quad (19)$$

Note that $\psi < \frac{\psi}{\psi + \alpha(1 - \psi)}$. Hence, condition (19) satisfies equation (18), which implies that the economy is dynamically efficient.

A combination of (12) and (19) gives the following parameter restriction

$$\alpha(1 - \psi)(1 + \beta^{-1}) < \psi < \alpha(1 + \beta^{-1})[\psi + \alpha(1 - \psi)] \quad (20)$$

We now derive the normative implication of bubbles under condition (20). Defining the aggregate consumption at period t of agent $j \in \{w, E\}$ as $\tilde{c}_t^j = \tilde{c}_{1t}^j + \tilde{c}_{2t}^j(1 + v)^{-1}$, we have the following

Proposition 1 : *Given (20), a housing bubble reduces aggregate consumption and welfare—measured by the lifetime utility of both the workers and the entrepreneurs at the steady state.*

Proof: see Appendix.

The intuition is clear. When the economy is dynamically efficient, a housing bubble will reduce the aggregate resource available for consumption. If the marginal product of capital is sufficiently high, then choosing housing as an alternative store of value would crowd out capital and reduce consumption.

Regarding social welfare, since the workers' wage income decreases with capital stock but the rate of return to saving (the deposit rate) is fixed, their lifetime utility decreases as a result of a housing bubble. For the entrepreneurs, the utility loss due to a fall in lifetime entrepreneurial income dominates the welfare gain arising from the income effect of a higher capital return. Hence, both workers and entrepreneurs suffer welfare losses.

Proposition 2 *Given (20), a housing bubble reduces the aggregate consumption of workers (after the transition ends) and the aggregate consumption of entrepreneurs.*

Proof: see Appendix.

Since the wage rate is a constant, it is unaffected by the presence of a bubble along the transition. Hence, the welfare of workers along transition is unaffected by the bubble. However, when the transition ends, the wage rate changes with the physical capital. So a bubble reduces the welfare of all workers in the post-transition period.

For entrepreneurs along transition, the rate of return to capital is unaffected by the presence of a bubble. Hence, given the initial capital stock, the utility of the old and young entrepreneurs alive in the first period are unchanged when a housing bubble is introduced. From the second period on, however, the income of the young entrepreneur will fall due to the crowding out of capital by the bubble. Hence, all entrepreneurs along transition will have lower welfare (since the return to capital is constant). For entrepreneurs alive in the post-transition period, the rate of return to capital is not constant but higher than that at the steady state. So the loss in income due to a reduction in capital stock is still higher than the gain from the higher rate of return to capital. Hence, they also suffer welfare losses as a result of a housing bubble.

3.4.3 The Housing Price to Output Ratio

We now characterize the dynamics of the housing price-to-income ratio, one of the key focuses of the paper. We first establish a lemma about the dynamics of the share of entrepreneurial savings in housing.

Lemma 3 *Throughout the transition and post-transition period, the share of entrepreneurial savings in physical capital is constant,*

$$\phi_{Et} = \frac{\alpha (1 + \beta^{-1}) (1 - \psi)}{\psi}, \forall t. \quad (21)$$

Proof: see Appendix.

To understand the intuition behind Lemma 3, we plug the value of ϕ_{Et} into the law of motion for capital

$$\begin{aligned} K_{Et+1} &= (1 - \psi) \alpha (K_{Et})^\alpha (A_t \chi^{n_{Et}} N_t)^{1-\alpha} \\ &= \rho_t^E K_{Et}. \end{aligned} \quad (22)$$

(22) implies that not only capital accumulated by entrepreneur is linear in current output, but is a constant fraction of output over time. The higher is the fraction of output attributable to the old entrepreneurs, as captured by $(1 - \psi) \alpha$, the larger is the share of current output going to the end-of-period capital stock. And this constant fraction of output devoted to capital accumulation is achieved by the portfolio choice of entrepreneurs according to (21), which is constant under constant marginal propensity to save in our two-period model. With Lemma 3, the following proposition captures the dynamics of housing price to output ratio.

Proposition 4 *The housing price-to-output ratio and housing price-to-wage ratio in the post-transition period are constant, while they are both increasing during the transition.*

Proof: We first prove that the ratio of housing price to aggregate output is constant in post-transition period. Since the growth rate of housing price is equal to ρ_{t+1}^E , this is equivalent to prove that the growth rate of aggregate output in post-transition period is equal to ρ_{t+1}^E .

$$\frac{Y_{t+1}}{Y_t} = \frac{Y_{t+1}}{K_{t+1}} \frac{K_{t+1}}{Y_t} = \frac{\rho_{t+1}^E}{(1 - \psi) \alpha} (1 - \psi) \alpha = \rho_{t+1}^E,$$

where the second equality is obtained by the definition of ρ_{t+1}^E and (22). Therefore, the housing price to output ratio is constant in post-transition period. Since wage is a constant fraction $(1 - \alpha) (1 - \psi)$ of aggregate output, it is straight forward that the housing price-wage ratio is constant in post-transition period.

Along the transition stage, we have $\frac{Y_{t+1}}{Y_t} = \frac{Y_{Ft+1}+Y_{Et+1}}{Y_{Ft}+Y_{Et}}$. Equation (1) implies that the growth rate of output by F-firms follows

$$\frac{Y_{Ft+1}}{Y_{Ft}} = \frac{RK_{Ft+1}/\alpha}{RK_{Ft}/\alpha} = \frac{K_{Ft+1}}{K_{Ft}} < 0$$

Therefore, we have

$$\frac{Y_{t+1}}{Y_t} < \frac{Y_{Et+1}}{Y_{Et}} = \rho_{t+1}^E = \frac{P_{t+1}^H}{P_t^H},$$

where the proof of the second equality follows the same procedure as that for the post-transition period. As a result, the housing price to output ratio will increase along the transition. \square

The intuition of Proposition 4 is as follows. In the post-transition period, the economy essentially becomes a neoclassical economy. In our simple model with full capital depreciation, the end-of-period physical capital is proportional to the current aggregate output, with the share equal to the next-period share of output going to old entrepreneur. Hence, the housing price to output ratio is constant during the post-transition period. Note that this property also holds in a neoclassical framework (with complete capital depreciation).

During the transition, however, the aggregate output growth is an average of the output growth of the E-firms and F-firms. Since F-firms keep downsizing due to labor reallocation, the aggregate output growth is less than the output growth rate of the E-firms, which equals to the returns to capital for the old entrepreneur. Therefore, the housing price will grow faster than the aggregate output (and wage rate, which is constant) along transition.

3.5 Numerical Algorithm

During transition, we have the following equation to determine labor allocated to E-firms:

$$\begin{aligned} n_{Et} &= [(1-\psi)\chi]^\frac{1}{\alpha} \left(\frac{R}{\alpha}\right)^\frac{1}{1-\alpha} \frac{k_{Et}}{\chi A_t} \\ &= [(1-\psi)\chi]^\frac{1}{\alpha} \left(\frac{R}{\alpha}\right)^\frac{1}{1-\alpha} \hat{k}_{Et}/\chi \end{aligned}$$

We check if $n_{Et} > 1$. If so, we set $n_{Et} = 1$.

Also, we have the following equations for both transitional and post-transitional periods

$$\widehat{w}_t = (1 - \psi)(1 - \alpha) \left(\widehat{k}_{Et}/n_{Et} \right)^\alpha \chi^{1-\alpha} \quad (23)$$

$$\rho_t = \alpha(1 - \psi)^{\frac{1}{\alpha}} [(1 - \alpha)/\widehat{w}_t]^{\frac{1-\alpha}{\alpha}} \chi^{\frac{1-\alpha}{\alpha}} \quad (24)$$

$$\widehat{k}_{Et+1} = \phi_{Et} \psi \widehat{k}_{Et}^\alpha (n_{Et} \chi)^{1-\alpha} / [(1 + \nu)(1 + z)(1 + \beta^{-1})] \quad (25)$$

$$\widehat{p}_t^H = \begin{cases} \widehat{p}_{t+1}^H (1 + z)(1 + \nu) / \rho_{t+1} & \text{if } \phi_{Et} < 1 \\ 0 & \text{if } \phi_{Et} = 1 \end{cases} \quad (26)$$

$$\widehat{p}_t^H h_{Et} = \widehat{p}_t^H \bar{h} \quad (27)$$

$$\widehat{p}_t^H h_{Et} = (1 - \phi_{Et}) \psi \widehat{k}_{Et}^\alpha (n_{Et} \chi)^{1-\alpha} / (1 + \beta^{-1})$$

We assume the second transition period takes T periods. At period T , the economy enters the steady state. The algorithm to solve for the transition takes the following steps:

1. guess the sequence of $\{\phi_{Et}\}_{t=1}^{T-1}$.
2. given k_{Et} , compute $\{n_{Et}, \widehat{w}_t, \rho_t, \widehat{k}_{Et+1}, \widehat{p}_t^H, h_{Et}\}$ according to the above equations.
3. Check the following condition for each period $t = 1, 2, \dots, T - 1$

$$\phi_{Et} = 1 - \frac{(1 + \beta^{-1}) \widehat{p}_t^H h_{Et}}{\psi \widehat{k}_{Et}^\alpha (n_{Et} \chi)^{1-\alpha}} \quad (28)$$

and (since ρ_{T+1}^E not known)

$$\widehat{k}_{ET+1} = \widehat{k}_E^* = \left[\frac{\phi_E^* \psi \chi^{1-\alpha}}{(1 + \beta^{-1})(1 + z)(1 + \nu)} \right]^{\frac{1}{1-\alpha}}$$

4 Numerical Results

We use the following parameter values for the numerical exercise: $\alpha = 0.3$; $\beta = 0.96^{30}$; $\psi = 0.62$ (note $\psi > \underline{\psi} = .5691$). $\chi = 4.98$. $R = 1.0147$; $z = 0.0147$; $\nu = 0$. $\bar{h} = 1$, $z = 0.0147$. These parameters satisfy the condition (20) to ensure housing bubbles exist in an economy with dynamic efficiency. We also choose $k_{E1} = 0.031$, such that the economy experiences a transition stage. Note that a smaller k_{E1} tends to prolong the transition. But it also makes

wage and thus the saving rate of the young worker smaller. At the same time, a smaller k_{E1} makes $1 - n_{Et}$ and thus k_{F1} larger. This will also tend to make smaller the bank deposit net of capital demand by F-firms, as well as the housing demand by the banking sector.

4.1 Benchmark Results

Figure 3 shows the dynamics of the benchmark economy. In Panel A, capital in E-firms increases at a faster rate during the first three periods than thereafter. This is because the marginal product of capital is constant when F-firms exist, as labor is kept reallocating from F-firms to E-firms. This can be seen in Pane B. At period 4, the transition is over, as $n_{Et} = 1$. Panel C shows that aggregate output follows a similar growing pattern to that of physical capital in E-firms.

Panel D shows the consumption pattern during transition. Notice that during transition, consumption of E-firms grows fast while that of workers is essentially flat due to a constant wage profile. As a result, most of the increase in aggregate consumption is due to the increase of consumption of entrepreneurs. In Panel E, we see that during transition, the aggregate rate of returns for capital, which is weighted average rate of returns for capital for E-firms and F-firms is increasing, since the capital share of E-firms keep increasing. However, during the post-transition stage, the aggregate returns to capital, which is simply the E-firm's rate of returns to capital, starts to decline. In Panel F, total factor productivity increases along transition, since resources are reallocated to the E-firm, which is more productive. However, in the post-transition stage, the (detrended) TFP, which is the productivity of E-firms, is constant.

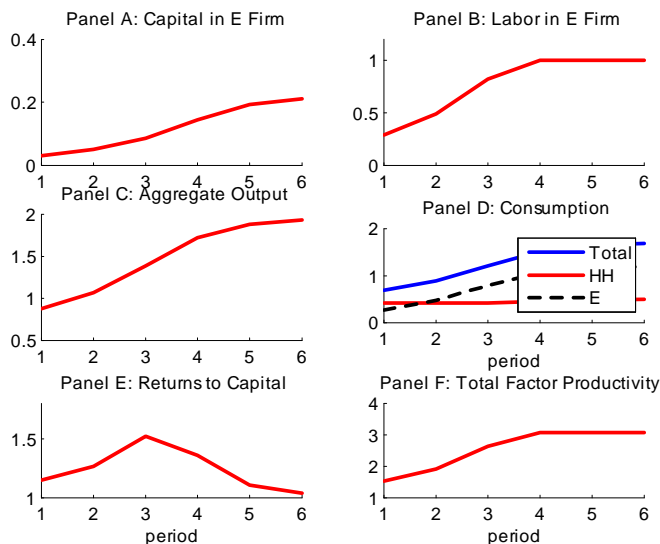


Figure 3. Macro Variables in the Benchmark Economy.

Figure 4 shows the dynamics of housing prices and share of entrepreneurial savings in housing. Panel A shows the detrended housing prices growth rate, which track the returns for capital in E-firms. Note that the rate of returns for physical capital starts to fall in period 4, when the transition is over. As a result, the growth rate of housing price is high along transition, but exhibits a declining pattern in the post-transition period. Eventually, housing prices in the long run equal the balanced growth rate of the economy. Panel B shows the ratio of housing price to income. We see that the housing price to wage ratio increases dramatically. This is because, as Panel C shows, that wage rate is constant along transition due to the presence of F-firms and labor reallocation. Similarly, the ratio of housing price to aggregate output, P_t^H/Y_t , keeps increasing during transition, but becomes a constant in the post-transition stage. Again, this is because during transition, housing prices grows at a rate faster than the growth rate of the aggregate output, as Proposition 3 argues.

Finally, in Panel D, we see that the share of E-firms' saving in housing is constant in this two-period economy. Note that even entrepreneurs along transition demand housing despite a high rate of returns for capital. This is essentially because those entrepreneurs alive during transition expect that the high capital returns driven by cheap labor and resource relocations are not sustainable in the long run, which induces future investors to seek alternative store of value for their growing wealth. This pushes up the expected rate of returns for housing even during transition. As a result, entrepreneurs born along transition in our benchmark economy have incentive to hold housing.

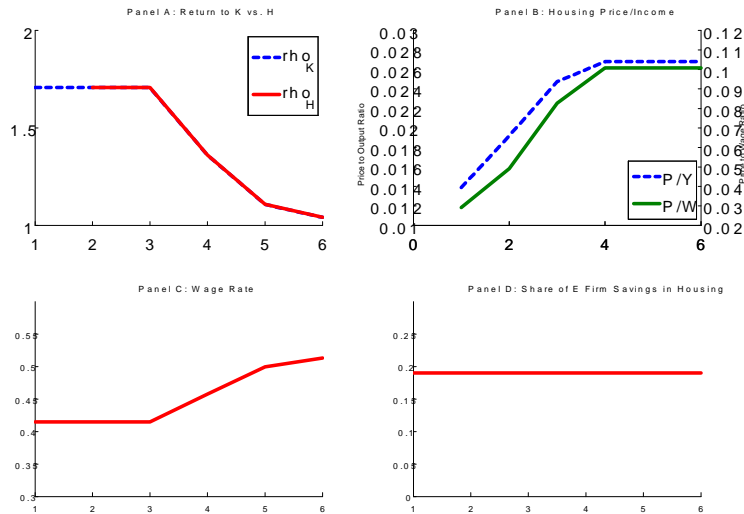


Figure 4. Housing Dynamics in the Benchmark Economy

4.2 Counterfactual Experiments

We now explore the key ingredients in our model that help to sustain a high growth rate of housing price along the transition. To this end, we conduct several counterfactual experiments, in which we shut down one of the ingredients at a time. We examine two ingredients: (i) the role of the entrepreneurial returns to capital at the steady state, (ii) the role of firm heterogeneity.

4.2.1 The Role of Bubbles for Transition and Welfare

We would like to explore the role of bubbles for the transition as welfare. Similar to all studies on bubbles, in our model, the existence of bubbles rely on the assumption that at steady state, the rate of returns for capital for entrepreneurs is lower than the balanced-growth rate (though the economy can be dynamic efficient). Therefore, to eliminate housing demand by entrepreneurs at the steady state, we impose an output subsidy to E-firms only at the steady state to equalize the rate of returns to capital for entrepreneurs at the steady state to the balanced growth rate. We keep all other parameters the same as before.¹¹ Accordingly, an E-firm's problem at steady state becomes

$$\max_{n_{Et}} (1 + \tau_{yt}) \left[(1 - \psi) (k_{Et})^\alpha (A_t \chi n_{Et})^{1-\alpha} - w_t n_{Et} \right]$$

Note that the subsidy is proportional to the net profit. Accordingly, the first-order condition for n_{Et} and the capital-labor ratio is still the same as in our benchmark economy. The profit of the entrepreneur is

$$\pi(k_{Et}) = (1 + \tau_{yt}) \rho_t^E k_{Et}$$

Figure 5 plots the transitional dynamics for both the counterfactual and the benchmark economies. Panel A shows that throughout the transition, housing price is zero. Panel B, C and D suggest an improvement of allocative efficiency in this counterfactual experiment, as both capital accumulation by E and aggregate output are higher than their counterparts in the benchmark economy. Moreover, the transition period is shorter, as labor demand by E-firms has reached 1 in period 3. Panel E and F show that the counterfactual economy generates higher aggregate consumption, and consumption for both entrepreneurs and workers. Intuitively, more entrepreneurial savings towards capital investment also increase future entrepreneurs' permanent incomes.

¹¹We find that the transitional pattern of this economy, except for the returns to capital, is equivalent to another economy in which entrepreneurs are shut down from access to housing markets.

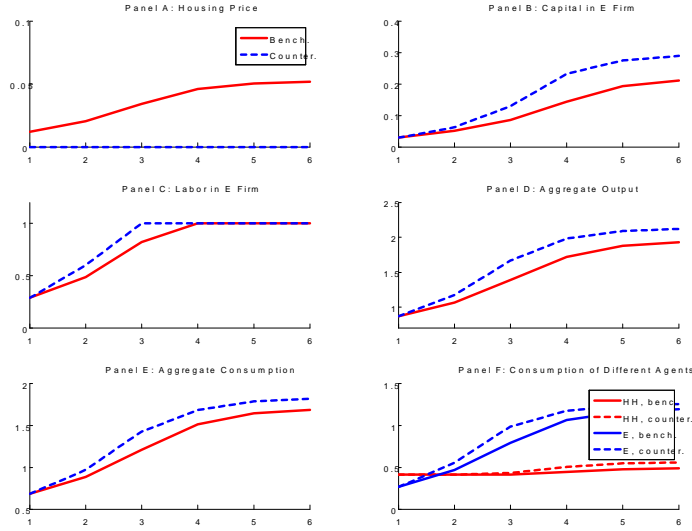


Figure 5: Transition in Economy without Dynamic Inefficiency

In summary, our experiment shows that housing bubbles crowds out physical capital, prolongs transition and reduces consumption for both entrepreneurs and workers.

4.2.2 The Role of Firm Heterogeneity

We now examine the second key ingredient of our model: heterogeneous firms in both productivity and access to financial markets. This feature allows the existence of a transition stage where labor is reallocated from F-firms to E-firms. Accordingly, the marginal product of capital for E-firms is constant along the transition. We argue that this feature is key to sustaining the persistently high growth rate of housing prices during transition.

To examine the role of heterogeneous firms, we construct an counterfactual economy where F-firms are absent. In other words, all labor is employed in E-firms at the very beginning. As a result, all E-firms are neoclassical in nature except that they are self-financed. We still keep the ingredient of dynamic inefficiency in the long run. Therefore, both the counterfactual and the benchmark economy share the same steady state.

In Panel A of Figure 6, we see that labor demand for E-firms is always 1. Accordingly, as entrepreneurs accumulate capital, the return for capital drops quickly from an initially high level to a very low level at the steady state (Panel B). In Panel C, wage rate starts to increase at the beginning of the economy. Panel D shows that housing price starts at a higher level, but overtime the growth of housing prices slows down, in contrast to a fast increase during the transition stage of the benchmark economy. Accordingly, housing price-to-aggregate output

ratio is constant (Panel E). Finally, throughout transition, aggregate consumption is now higher than the benchmark economy, though they converge to the same steady state. This implies that the negative effect of housing on aggregate consumption is particularly large in our benchmark economy. The reason is that in our benchmark economy during transition, the rate of returns to capital is very high due to labor reallocation. Therefore, the welfare loss of the crowding-out effect of housing is much larger in our benchmark economy.¹²

In summary, the presence of firm heterogeneity (in both technology and access to financial markets) helps maintain a high rate of return to capital during the transition. Accordingly, with entrepreneurial demand for housing, the equilibrium growth rate of housing prices is high along the transition. Without the presence of F-firms, the dynamics of housing prices essentially follows the growth rate of the aggregate output. As a result, the housing price-to-output ratio is constant without firm heterogeneity. Moreover the welfare loss of the economy due to housing as bubbles is much larger with firm heterogeneity.

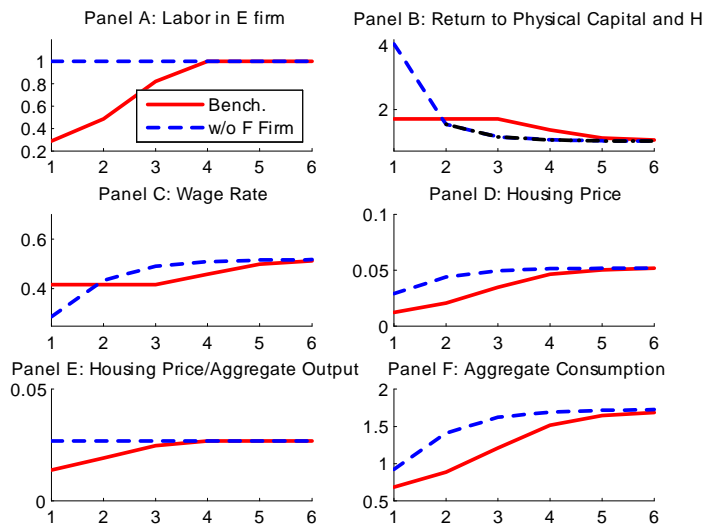


Figure 6: Transition in Economy without Firm Heterogeneity

5 Quantitative Analysis

[to be added]

¹²Note that in this counterfactual economy, housing still causes welfare loss to workers and entrepreneurs, since the economy is dynamic efficient and satisfies (19).

6 Conclusion

This paper provides an explanation to the great housing boom in China. In particular, we show in an endogenous growth model that the great housing boom can be a rational bubble arising naturally from China’s unprecedented economic transition, which features persistent and exceptional high returns to capital—driven largely by massive reallocations of cheap labor from unproductive sectors to productive sectors. Since the transition will eventually come to an end, capital returns are expected to decline sharply in the future. Based on such rational expectations, investors opt to seek alternative stores of value for their growing wealth. Given China’s underdeveloped financial market and capital controls, investors opt to speculate in the housing market in an early stage by holding the housing stock as a hedge for their wealth in addition to capital. This generates a strong speculative demand for housing investment, which rectifies the anticipated housing price boom and leads to a growing housing bubble with a rate of return equal to that of capital. Consequently, the economy exhibits an increasing housing price-to-income ratio and an increasing share of housing investment in GDP during the transition. Such a growing housing bubble crowds out capital accumulation, prolongs economic transition and reduces welfare for all agents in the economy.

There are many issues left for future research concerning the effects of housing bubble in China. For example, housing bubble reduces the private sector’s incentive to innovate. Because of the relatively low risk, low entry costs, low technology, and high profits in housing investment, the housing bubble has enticed many productive and high-tech firms in China to reallocate resources from R&D to the real estate market. In an economy transiting from labor intensive economy to capital intensive economy, such resource misallocation can be very costly: It may substantially prolong China’s economic transition and reduce China’s TFP growth, especially when its population is aging fast and labor costs rapidly rising. We plan to quantify such resource misallocation within our framework in future works.

Furthermore, the rapidly rising housing prices have caused great social concerns as more and more low-income households are excluded from the housing market—because their income growth falls behind housing price growth. The housing price growth is driven largely by upper middle income class who has enjoyed the most rapid income growth during the economic development. The inequality in wealth distribution in China has thus widened and exacerbated recently, mainly because of the rising housing prices. Again, this is an important issue for our future research.

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7 Appendix

7.1 Proof of Propositions

In this section, we prove the various propositions.

Proof of Proposition 1. To prove this proposition, consider a counterfactual economy without housing, i.e. $\phi_E = 1$. According to (13), introducing housing (i.e. $\phi_E < 1$) at the steady state would reduce physical capital. Hence, we only need to show under which condition reducing physical capital at steady state reduces aggregate consumption for both entrepreneurs and workers. Aggregating the budget constraint of the young and old entrepreneurs at period t , we get

$$\widehat{c}_t^e = \widehat{m}_t + \rho_t^E \widehat{k}_{Et} - \widehat{k}_{Et+1}(1+z)(1+v) \quad (29)$$

In our steady state analysis below, we drop the time subscript for notation concision. Taking the derivative of the right side of (29) with respect to \widehat{k}_{Et} at the steady state, we can obtain the following sufficient condition for introducing bubbles to reduce aggregate consumption for entrepreneurs.

$$\partial m / \partial k_E + \rho^E + \rho^{E'} \left(\widehat{k}_E \right) \widehat{k}_E > (1+z)(1+v) \quad (30)$$

On the left side of (30), $\partial m / \partial k_E$ is the marginal cost on the young entrepreneur's income of a reduction in physical capital. The second argument, ρ^E , refers to the marginal profit loss to the old entrepreneur of a reduction in physical capital. Finally, $\rho^{E'} \left(\widehat{k}_E \right) \widehat{k}_E$ is the marginal benefit of reducing capital in terms of an increase in marginal product. The right side of (30) is the returns for bubbles at the steady state. Using the definition of m and ρ^E , it is easy to show that (30) can be rewritten as

$$\left[\frac{\psi}{1-\psi} + \alpha \right] \rho^E > (1+z)(1+v) \quad (31)$$

At steady state, with $\phi_E = 1$, we obtain the value of ρ^E by combining (14) and (13)

$$\rho^E = \alpha (1 + \beta^{-1}) (1+z) (1+n) \frac{1-\psi}{\psi} \quad (32)$$

Plugging (32) into (31), we get

$$\alpha(1 + \beta^{-1}) > \frac{\psi}{\psi + \alpha(1 - \psi)} \quad (33)$$

Condition (33) is the same as (19).

To obtain the impact of bubbles on aggregate consumption of workers at steady state, we simply aggregate the worker's budget constraint at period t

$$\widehat{c}_t^w + \widehat{s}_t^w = \widehat{w}_t + R\widehat{s}_{t-1}^w / [(1+z)(1+n)]$$

Using $\widehat{s}^w = \widehat{w}/(1+\beta^{-1})$, at the steady state, we have

$$\widehat{c}^w = \left[1 + \left(\frac{R}{(1+z)(1+n)} - 1 \right) / (1+\beta^{-1}) \right] \widehat{w} \quad (34)$$

It is easy to show that the coefficient for \widehat{w} on the right side of (34) is positive. And since $\widehat{w} = (1-\alpha)(1-\psi) \left(\widehat{k}_E \right)^\alpha \chi^{1-\alpha}$, we have $\partial \widehat{w} / \partial \widehat{k}_E > 0$. Hence, introducing housing reduces aggregate consumption for the worker at the steady state.

Now we prove the welfare implication of housing for both the worker and entrepreneur. For the worker, simply notice that their deposit rate is fixed at R . Hence a reduction of permanent income, \widehat{w} , will reduce their lifetime utility. For the entrepreneur, the lifetime utility can be expressed as

$$\begin{aligned} & \log(m_t - s_{Et}) + \beta \log \rho_t^E s_{Et} \\ &= \log \left(\frac{\psi \rho_t^E k_{Et}}{(1+\beta)\alpha(1-\psi)} \right) + \beta \log \rho_t^E k_{Et} / \phi_{Et} \\ &= (1+\beta) \log \rho_t^E k_{Et} - \beta \log \phi_{Et} + C \end{aligned} \quad (35)$$

where C is a constant as a function of parameters. Plugging the steady-state value of ρ_t^E and k_{Et} into (35), we get

$$\begin{aligned} & (1+\beta) \log \rho^E k_E - \beta \log \phi_E + C \\ &= (1+\beta) \log \alpha \frac{1-\psi}{\psi} \frac{(1+\beta^{-1})}{\phi_E} (1+z)(1+n) \left[\frac{\psi \phi_E \chi^{1-\alpha}}{(1+\beta^{-1})(1+z)(1+\nu)} \right]^{\frac{1}{1-\alpha}} \\ & \quad - \beta \log \phi_E + C \\ &= \left[\frac{\alpha(1+\beta)}{1-\alpha} - \beta \right] \log \phi_E + \widetilde{C} \end{aligned}$$

where \widetilde{C} is another constant as a function of parameters. Hence, introducing housing, i.e. reducing ϕ_E , reduces the welfare if $\frac{\alpha(1+\beta)}{1-\alpha} > \beta$, that is, $\alpha(1+\beta^{-1}) > 1-\alpha$. Note that the

joint of participation and incentive constraint of the young entrepreneur implies $m = \psi y_E > w = (1 - \alpha)(1 - \psi)y_E$, which gives the following parameter restriction $\psi > (1 - \alpha)(1 - \psi)$, or equivalently $\frac{\psi}{\psi + \alpha(1 - \psi)} > 1 - \alpha$. Therefore, with the assumption (19), introducing housing will reduce the lifetime utility of the entrepreneur at the steady state.

Proof of Proposition 2: The proof of welfare implication for the workers and entrepreneurs along transition is straightforward. For the entrepreneur in the post-transition period, we need to prove

$$\frac{\partial \left[\partial m_t / \partial k_{Et} + \rho_t^E + \rho_t^{E'} \left(\widehat{k}_{Et} \right) \widehat{k}_{Et} \right]}{\partial \widehat{k}_{Et}} < 0. \quad (36)$$

In the post-transition period, since \widehat{k}_{Et} increases monotonically. (36) simply says that the net marginal benefit (loss) of an increase (decrease) in capital is higher when \widehat{k}_{Et} is smaller. Using (31) and the definition of ρ_t^E , the left side of (36) is

$$\begin{aligned} & \frac{\partial \left[\partial m_t / \partial k_{Et} + \rho_t^E + \rho_t^{E'} \left(\widehat{k}_{Et} \right) \widehat{k}_{Et} \right]}{\partial \widehat{k}_{Et}} \\ &= -(1 - \alpha)^2 (1 - \psi) \left(\widehat{k}_{Et} \right)^{\alpha - 2} \chi^{1 - \alpha} < 0. \end{aligned}$$

Proof of Lemma 1. We now prove that the share of entrepreneurial savings in housing is constant over time. Using the housing market clearing condition, we have

$$(1 - \phi_{Et}) \frac{1}{1 + \beta^{-1}} \psi (K_{Et})^\alpha (A_t \chi n_{Et} N_t)^{1 - \alpha} = P_t^H \bar{H} \quad (37)$$

Forwarding (37) by one period, and with $(K_{Et+1})^\alpha (A_{t+1} \chi n_{Et+1} N_{t+1})^{1 - \alpha} = K_{Et+1} \rho_{t+1}^E / [\alpha (1 - \psi)]$, equation (37) can be rewritten as

$$(1 - \phi_{Et+1}) \frac{1}{1 + \beta^{-1}} \frac{\psi \rho_{t+1}^E K_{Et+1}}{\alpha (1 - \psi)} = P_{t+1}^H \bar{H} \quad (38)$$

With the law of motion for capital (8), (38) can be rewritten as

$$(1 - \phi_{Et+1}) \frac{1}{1 + \beta^{-1}} \frac{\psi \rho_{t+1}^E}{\alpha (1 - \psi)} \frac{\phi_{Et}}{1 + \beta^{-1}} \psi (K_{Et})^\alpha (A_t \chi n_{Et} N_t)^{1 - \alpha} = P_{t+1}^H \bar{H} \quad (39)$$

Dividing (39) by (37) for all t , we have

$$\frac{1 - \phi_{Et+1}}{1 - \phi_{Et}} \frac{\phi_{Et}}{1 + \beta^{-1}} \frac{\psi \rho_{t+1}^E}{\alpha(1 - \psi)} = \frac{P_{t+1}^H}{P_t^H} = \rho_{t+1}^E, \quad (40)$$

or simply

$$\frac{1 - \phi_{Et+1}}{1 - \phi_{Et}} \frac{\phi_{Et}}{1 + \beta^{-1}} \frac{\psi}{\alpha(1 - \psi)} = 1 \quad (41)$$

Equation (41) is a first-difference equation capturing the dynamics of ϕ_{Et} . One solution to (40) is that

$$\phi_{Et} = \frac{\alpha(1 - \psi)(1 + \beta^{-1})}{\psi}, \forall.$$

7.2 Allowing Entrepreneurs to Borrow against Housing

A potential remedy for credit misallocation is to allow access of E-firms to bank credit. Note that since the lending rate is fixed and below the rate of return to housing, banks would not be willing to lend to E-firms at the low interest rate. Moreover, in China, entrepreneurs face limited enforcement in debt repayment. Hence, banks can lend to E-firms with housing (or land) as collateral.

We assume that each young entrepreneur can borrow against his housing purchase at the end of each period

$$R^E l_{Et} \leq \vartheta P_t^H h_{Et}, \quad 0 \leq \vartheta \leq 1 \quad (42)$$

where the left-hand side of (42) is the bank loan (plus interest payment) to young entrepreneurs, which has to be less than or equal to a fraction $\vartheta < 1$ of the value of housing purchase. In our benchmark economy, $\vartheta = 0$. We now consider a permanent increase in ϑ . It is easy to show that the law of motion for capital and housing now follow

$$K_{Et+1} = \frac{R^E \phi_{Et}}{R^E - \vartheta(1 - \phi_{Et})} \gamma_{Et} K_{Et} \quad (43)$$

$$P_t^H H_{Et} = \frac{(1 - \phi_{Et}) R^E}{R^E - \vartheta(1 - \phi_{Et})} \gamma_{Et} K_{Et} \quad (44)$$

where $\gamma_{Et} \equiv \frac{\rho^E \psi}{(1 - \psi)\alpha} \frac{1}{1 + \beta^{-1}}$. Clearly, a change in ϑ (expected or unexpected) affects the entrepreneurial capital and housing demand via two channels: (i) a direct effect: an increase

in ϑ increases the amount of borrowing by entrepreneurs and thus increases demand for both capital and housing; (ii) an indirect effect via ϕ_{Et} : since both the returns for physical capital and housing is larger than the borrowing cost, R^E , entrepreneurs will borrow to the credit limit, which implies an increase in the share of housing in the asset portfolio held by entrepreneurs, i.e. ϕ_{Et} fall in responses to anticipated or unanticipated increases in ϑ . Accordingly to (44), this would reinforce the positive impact on housing demand.