The Financial Accelerator and the Optimal Lending Contract*

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In the financial accelerator literature pioneered by Bernanke, Gertler and Gilchrist (1999) entrepreneurs are myopic and lenders suboptimally choose a safe rate of return on their loans. We derive the optimal lending contract for forward looking entrepreneurs and provide three main results. First, under the optimal contract we find that financial frictions do not amplify business cycle fluctuations. Second, we show that shocks to the variance of unobserved idiosyncratic productivity — so-called "risk shocks" — have little effect on the real economy under the optimal contract. Third, we find that amplification under the suboptimal contract depends on loose monetary policy: when interest rate setting follows a standard Taylor rule, the financial accelerator is significantly dampened or even reversed.

Keywords: Financial accelerator; financial frictions; risk; optimal contract; agency costs.

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1 Introduction

In one of the foundational papers in the literature on financial frictions in macroeconomic models, Bernanke, Gertler and Gilchrist (1999) derive a contract between risk averse lenders and risk neutral borrowers in the costly state verification (CSV) framework of Townsend (1979). Although this loan contract has become the standard contract for CSV models of financial frictions, it is not optimal because it assumes returns for lenders are predetermined and borrowers are myopic.

In this paper we relax these two assumptions and derive the optimal history-independent loan contract in the CSV model.[1] We allow returns to the lender to be contingent on the aggregate state of the economy following early criticism of Bernanke, Gertler and Gilchrist (1999) by Chari (2003). Chari's concerns were later formalized by Carlstrom, Fuerst and Paustian (2012). We also introduce forward looking entrepreneurs who maximize the present discounted value of all future consumption instead of next period expected consumption.[2]

Our analysis provides three main results. First, under the optimal contract we find that financial frictions do not amplify business cycles. Relative to a model with financial frictions, monetary and technology shocks generate much larger output responses when frictions are absent. Second, shocks to the cross-sectional variance of entrepreneurs' idiosyncratic productivity — what we call "risk" shocks — have little to no impact on the real economy, in contrast with the standard Bernanke, Gertler and Gilchrist contract (hereafter BGG). This is particularly important as recent work by Christiano, Motto and Rostagno (2013) emphasizes the importance of risk shocks in driving business cycles. We show here that risk shocks provide amplification only when the lending contract is suboptimal. Third, we show that the financial accelerator in the original BGG framework is dependent on three key characteristics: a suboptimal contract, loose monetary policy and extremely persistent technology shocks. We conduct a number of robustness checks in Section 5 and find that the removal of any one of these characteristics significantly weakens or eliminates the financial accelerator. Overall, our results cast doubt on the qualitative and quantitative importance of the financial accelerator in the costly state verification framework.

Our model is standard and consists of a risk averse representative household and risk neutral entrepreneurs. Entrepreneurs borrow money from the representative household and purchase capital to use in production. Entrepreneurs are identical ex ante but differ depending on the ex post realization of an idiosyncratic productivity shock. Both agents have full information

[1] To be precise, we derive the optimal one-period contract with deterministic monitoring. An excellent list of references for partial equilibrium multi-period contracts includes Monnett and Quintin (2005) for stochastic monitoring, Wang (2005) for deterministic monitoring, and Cole (2013) for self-enforcing stochastic monitoring.

[2] In an October 2013 version of their paper, Carlstrom, Fuerst and Paustian independently derive the dynamically optimal contract for forward looking households without risk shocks. Different from their paper, we introduce a frictionless benchmark to evaluate amplification and also consider the effect of risk shocks.

about the distribution of idiosyncratic shocks ex ante, so there is no adverse selection problem. Borrowers observe the realization of their idiosyncratic shock, but lenders do not: they need to pay monitoring costs to observe it.

In the BGG contract borrowers guarantee a constant safe rate of return to lenders in order to maximize returns on their equity. As a result, borrowers absorb all risk in the economy. It should be noted that this is an assumption and not an equilibrium condition. Because of this assumption, negative shocks cause a decline in entrepreneurs' net worth which leads to a tightening of financial constraints. The subsequent fall in investment and output is stronger than the effect from the initial shock. This results in the financial accelerator: the BGG contract amplifies macroeconomic fluctuations in a dynamic stochastic general equilibrium (DSGE) model.

Recent work by Carlstrom, Fuerst and Paustian (2012), hereafter CFP, shows that the BGG contract is suboptimal because the predetermined deposit rate does not provide appropriate consumption insurance for risk averse households. In CFP, risk neutral entrepreneurs find it optimal to offer lenders a contract with a negative covariance between the rate of return and the lenders' consumption. During a recession when consumption is low, entrepreneurs pay a higher borrowing rate in order to provide household consumption insurance. Entrepreneurs thus have to pay a higher interest rate on their loans exactly when their net worth is already low. Because the quantity of capital in the economy is a function of net worth, the dynamics of net worth directly affect investment, leading to a sharp decline in investment during recessions. The stabilizing effect of the insurance channel on household consumption is outweighed by a much stronger decline in investment, leading to higher volatility in the economy at large.

In BGG, CFP and the entire CSV literature entrepreneurs are myopic: they maximize their expected *next period* consumption, but expected utility depends on the expected discounted stream of *all future* consumption. We depart from the literature and embed forward looking entrepreneurs into an otherwise standard CSV framework. Our analysis provides a number of results that call the robustness of the financial accelerator into question under optimal and suboptimal contracts, for myopic and non-myopic entrepreneurs.

The intuition is as follows. When lenders' returns are predetermined, we find that to a first order approximation the lending contract is identical regardless of whether entrepreneurs are forward looking or myopic. In period t, the predetermined lending rate is chosen to satisfy the lender's Euler equation in that specific period without the possibility of revisions in period $t+1$. As a result, it does not matter whether entrepreneurs are forward looking or not, as the lender's stochastic discount factor determines the rate of return. In order to generate amplification however, this suboptimal contract must be combined with other ingredients. In our robustness exercise in Section 5, we show that contracts with a predetermined deposit rate only generate a financial accelerator when monetary policy deviates from price stability and when technology shocks are stationary.

On the other hand, when lender's returns are chosen optimally and vary with the aggregate state of the economy, the presence of forward looking entrepreneurs or myopic entrepreneurs matters greatly. In CFP, entrepreneurs sell as much insurance to the household as they can because insurance does not effect their next period expected consumption. During a recession, the provision of insurance leads to very tight financial constraints for entrepreneurs, as they face a higher lending rate due to the fall in household consumption. During a boom the opposite occurs: myopic entrepreneurs have too much capital and earn small returns on their capital. In other words myopic entrepreneurs miss good investment opportunities on a consistent basis because they do not take the future flow of capital returns into account when making investment decisions. Under the optimal contract however, forward looking entrepreneurs sell less insurance because they are concerned not only about next period expected consumption but also expected consumption in all future periods, which is impacted by insurance claims. In particular, forward looking entrepreneurs desire high net worth in states of the world where the financial premium is also high.

To provide more intuition on the role of forward looking entrepreneurs, consider the following example. Assume that ex-post there is a shock which suddenly decreases the entrepreneur's net worth. Lower net worth today means that the financial premium today and in the future will be higher. The entrepreneur desires more net worth in states with a higher financial premium because capital returns are higher and borrowing is more costly. Forward looking entrepreneurs thus find it profitable to enter into an ex-ante agreement that stipulates a lower lending rate in these states. Correspondingly, entrepreneurs prefer to pay a higher lending rate when a shock increases net worth, because the financial premium will be lower in these states. This interplay between movements in net worth and the financial premium leads risk-neutral entrepreneurs to behave in a "risk averse" manner because they want to avoid borrowing in states with a high financial premium. In contrast, if there is no costly state verification so that financial frictions are absent, non-myopic entrepreneurs will ignore concerns about the financial premium and provide as much insurance as possible, generating large amplification.

We also find that risk shocks have little effect on the real economy and give the wrong comovement between macroeconomic aggregates when contracts are optimal. This contrasts with Christiano, Motto and Rostagno (2013), who employ the BGG contract and emphasize the importance of risk shocks in generating business cycle fluctuations. Under the BGG contract, increased idiosyncratic variance causes an increase in defaults leading to a decline in the price of capital and consequently net worth. However, if returns to lenders are not predetermined and entrepreneurs are forward looking, they realize that lower net worth implies higher financial premiums and more costly borrowing in the future. Therefore, forward looking entrepreneurs desire more net worth in these states and thus negotiate lower returns to lenders, which stabilizes the response of net worth to the shock. As a result, under the optimal contract the financial accelerator is severely dampened for risk shocks.

2 The Optimal Lending Contract in Partial Equilibrium

Our main theoretical contribution in this paper is to introduce forward looking entrepreneurs into an otherwise standard CSV model of financial frictions. In this section we outline the key differences between the dynamically optimal loan contract chosen by utility maximizing entrepreneurs and the alternative loan contracts in BGG and CFP in a partial equilibrium setting. Here we assume that entrepreneurs take the price of capital and the expected return to capital as given. In Section 3 we endogenize these variables in general equilibrium.

At time t, entrepreneur j purchases capital $K_t(j)$ at a unit price of Q_t. At time $t+1$, the entrepreneur rents this capital to perfectly competitive wholesale goods producers. The entrepreneur uses his net worth $N_t(j)$ and a loan $B_t(j)$ from the representative lender to purchase capital:

$$Q_t K_t(j) = N_t(j) + B_t(j). \tag{1}$$

After buying capital, the entrepreneur is hit with an idiosyncratic shock $\omega_{t+1}(j)$ and an aggregate shock R^k_{t+1}, so that entrepreneur j is able to deliver $Q_t K_t(j) R^k_{t+1} \omega_{t+1}(j)$ units of assets. The idiosyncratic shock $\omega(j)$ is a log-normal random variable with distribution $\log(\omega(j)) \sim \mathcal{N}(-\frac{1}{2}\sigma^2_\omega, \sigma^2_\omega)$ and mean of one.

Following BGG, we assume entrepreneurs are risk neutral and die with constant probability $1-\gamma$. Upon dying, entrepreneurs consume all operational equities, which are equal to net worth minus wages. If entrepreneurs survive they do not consume anything, and they supply labor and earn wages which they later reinvest. Entrepreneur j's value function is

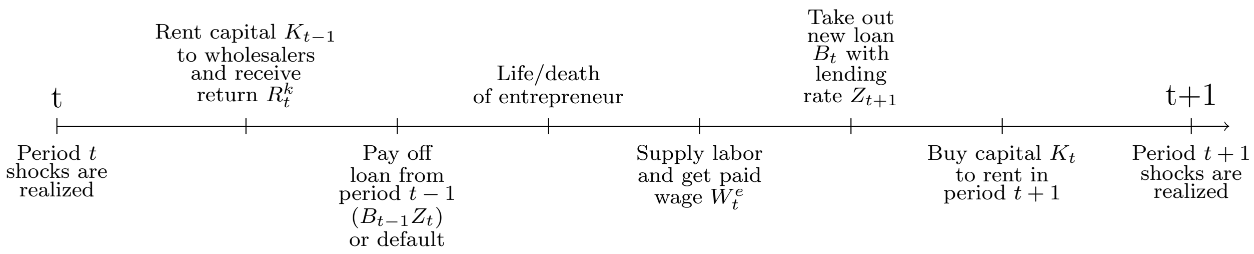
$$V^e_t(j) = (1-\gamma)\sum_{s=1}^{\infty} \gamma^s C^e_{t+s} \tag{2}$$

where C^e_{t+s} is the entrepreneur's consumption,

$$C^e_t(j) = N_t(j) - W^e_t \tag{3}$$

defined as wealth accumulated from operating firms, equal to net worth without entrepreneurial real wages W^e_t. The timeline for entrepreneurs is plotted in Figure 1.

Figure 1: Timeline for Entrepreneurs



2.1 Borrower and Lender Payoffs

The contract between the lender and borrower follows the familiar CSV framework. We assume that the lender cannot observe the realization of idiosyncratic shocks to entrepreneurs unless he pays monitoring costs μ which are a fixed percentage of total assets. Given this friction, the risk neutral borrower offers the risk averse lender a contract with an state-contingent interest rate Z_{t+1} subject to macroeconomic conditions.

The entrepreneur repays the loan only when it is profitable to do so. In particular, the entrepreneur will repay the loan only if, after repayment, he has more assets than liabilities. We define the cutoff productivity level $\bar{\omega}_{t+1}$, also known as the bankruptcy threshold, as the minimum level of productivity necessary for an entrepreneur to repay the loan:

$$\underbrace{B_t(j)Z_{t+1}(j)}_{\text{Cost of loan repayment}} = \underbrace{\bar{\omega}_{t+1}R^k_{t+1}Q_tK_t(j).}_{\text{Minimum revenue for loan repayment}} \tag{4}$$

If $\omega_{t+1}(j) < \bar{\omega}_{t+1}$ the entrepreneur defaults and enters bankruptcy; if $\omega_{t+1}(j) \geq \bar{\omega}_{t+1}$ he repays the loan. The cutoff productivity level allows us to express the dynamics of net worth for a particular entrepreneur j:

$$N_{t+1}(j) = Q_tK_t(j)R^k_{t+1}\max\left\{\omega_{t+1}(j) - \bar{\omega}_{t+1}, 0\right\} + W^e_{t+1}. \tag{5}$$

The gross rate of return for the lender, R_{t+1}, also depends on the productivity cutoff. For idiosyncratic realizations above the cutoff, the lender will be repaid the full amount of the loan $B_t(j)Z_{t+1}(j)$. For idiosyncratic realizations below the cutoff, the entrepreneur will enter bankruptcy and the lender will pay monitoring costs μ and take over the entrepreneur's assets, ending up with $(1-\mu)K_t(j)R^k_{t+1}h(\omega_{t+1}(j))$. More formally, the lender's ex post return is

$$B_t(j)R_{t+1}(j) = \begin{cases} B_t(j)Z_{t+1} & \text{if } \omega_{t+1}(j) \geq \bar{\omega}_{t+1} \\ (1-\mu)K_t(j)R^k_{t+1}\omega_{t+1}(j) & \text{if } \omega_{t+1}(j) < \bar{\omega}_{t+1} \end{cases} \tag{6}$$

Taking into account that loans to entrepreneurs are perfectly diversifiable, the lenders return on a loan R_{t+1} to entrepreneur j is defined as

$$B_t(j)R_{t+1} \equiv Q_t K_t(j) R^k_{t+1} h(\bar{\omega}_{t+1}, \sigma_{\omega,t}), \tag{7}$$

where $h(\bar{\omega}_{t+1}, \sigma_{\omega,t})$ is the share of total returns to capital that go to the lender. We define this share as

$$h(\bar{\omega}_{t+1}, \sigma_{\omega,t}) = \left\{ \underbrace{\bar{\omega}_{t+1}\Big[1 - F(\bar{\omega}_{t+1}, \sigma_{\omega,t})\Big]}_{\text{Share to lender if loan pays}} + \underbrace{(1-\mu)\int_0^{\bar{\omega}_{t+1}} \omega f(\omega, \sigma_{\omega,t}) d\omega}_{\text{Share to lender if loan defaults}} \right\} \tag{8}$$

where f is the probability density function and F is the cumulative distribution function of the log-normal distribution of idiosyncratic productivity.

In order to simplify the entrepreneur's optimization problem, we introduce the concept of leverage, κ_t, defined as the value of the entrepreneur's capital divided by net worth:

$$\kappa_t(j) \equiv Q_t K_t(j)/N_t(j). \tag{9}$$

2.2 Loan Contracts: BGG, CFP and the Optimal Contract

The differences between the BGG contract, the CFP contract and the optimal contract arise from two sources: the lender's participation constraint and the borrower's objective function.

First, the lender's participation constraint in BGG differs from CFP and the optimal contract. The participation constraint arises from the household Euler equation and stipulates the minimum rate of return that entrepreneurs must offer to lenders to receive a loan. In BGG, the participation constraint has the following form:

$$\mathbb{E}_t\Big\{\Lambda_{t,t+1}\Big\}R_{t+1} = 1, \tag{10}$$

where

$$\Lambda_{t,s} \equiv \beta^s \frac{U_{C,t+s}}{U_{C,t}} \tag{11}$$

is the household (*i.e.* shareholder) intertemporal marginal rate of substitution, also known as the household stochastic discount factor. Under this participation constraint, entrepreneurs pay a constant safe rate of return to the lenders, R_{t+1}, which ignores the risk averse representative household's desire for consumption insurance. In contrast, the participation constraint in CFP and the optimal contract is:

$$\mathbb{E}_t\Big\{\Lambda_{t,t+1}R_{t+1}\Big\} = 1. \tag{12}$$

As CFP show, the above expression implies that households prefer a state contingent rate of return that is negatively correlated with household consumption. Quite simply, households like consumption insurance. In recessions, households desire a higher rate of return because their marginal utility of consumption is high, and vice versa in booms.

Second, the borrower's objective function in BGG and CFP differs from the optimal contract. Entrepreneurs in BGG and CFP maximize next period net worth, defined in equation (5). If we substitute the expression for leverage from (9) into (5), we have the entrepreneur's objective function in BGG and CFP:

$$\kappa_t(j)N_t(j)\mathbb{E}_t\left\{R^k_{t+1}\max\left[\omega_{t+1}(j)-\bar{\omega}_{t+1},0\right]\right\}. \tag{13}$$

In contrast, under the dynamically optimal contract entrepreneurs maximize utility, given by (2). As we have mentioned before, utility maximizing entrepreneurs are concerned not only about current capital returns but also future capital returns and future financial premiums.

We now have all of the ingredients necessary to set up the entrepreneur's optimization problem and solve for the three different loan contracts: (1) the BGG contract; (2) the CFP contract; and, (3) the optimal contract.

Proposition 1 *To solve for the BGG contract, entrepreneurs choose their state contingent cutoff $\bar{\omega}_{t+1}$ and leverage $\kappa_t(j)$ to maximize next period net worth (13) subject to (5), (7) and (10). The solution to this problem is given by*

$$\kappa_t\mathbb{E}_t\left\{R^k_{t+1}g(\bar{\omega}_{t+1},\sigma_{\omega,t})\right\} = -\mathbb{E}_t\left\{\frac{g_\omega(\bar{\omega}_{t+1},\sigma_{\omega,t})}{h_\omega(\bar{\omega}_{t+1},\sigma_{\omega,t})}\right\}\frac{1}{\mathbb{E}_t\Lambda_{t,t+1}}. \tag{14}$$

where $g(\bar{\omega}_{t+1},\sigma_{\omega,t}) = \int_{\bar{\omega}_{t+1}}^{\infty}\omega f(\omega,\sigma_{\omega,t})d\omega - \bar{\omega}_{t+1}\left[1-F(\bar{\omega}_{t+1},\sigma_{\omega,t})\right]$.

Proof See Appendix B. ■

Corollary 1 *Log-linearization of the BGG optimality condition (14) and the BGG participation constraint (10) gives*

$$\mathbb{E}_t\hat{R}^k_{t+1} - \mathbb{E}_t\hat{R}_{t+1} = \nu_\kappa\hat{\kappa}_t + \nu_\sigma\hat{\sigma}_{\omega,t} \tag{15}$$

$$\hat{R}_{t+1} - \mathbb{E}_t\hat{R}_{t+1} = 0 \tag{16}$$

where the constant $\nu_\kappa = \frac{\frac{h_{\omega\omega}}{h_\omega}-\frac{g_{\omega\omega}}{g_\omega}}{-\frac{g_\omega}{g}+\frac{h_\omega}{h}+\frac{g_{\omega\omega}}{g_\omega}-\frac{h_{\omega\omega}}{h_\omega}}\frac{1}{\kappa-1}$ *and* $\nu_\sigma = \frac{-\frac{h_{\sigma\omega}}{h}\left(\frac{g_{\omega\omega}}{g_\omega}-\frac{h_{\omega\omega}}{h_\omega}-\frac{g_\omega}{g}\right)+\frac{h_\omega}{h}\left(\frac{g_{\omega\sigma}}{g_\omega}-\frac{h_{\omega\sigma}}{h_\omega}-\frac{g_{\sigma\omega}}{g}\right)}{\frac{g_{\omega\omega}}{g_\omega}-\frac{h_{\omega\omega}}{h_\omega}-\frac{g_\omega}{g}+\frac{h_\omega}{h}}\sigma_\omega$.

Proof See Appendix G. ■

Equation (15) shows that in the BGG contract the entrepreneur's leverage depends on next period's expected financial premium while (16) shows that lenders returns (deposit rate) are predetermined. We prove in Appendix E that when lenders returns are predetermined, to a first order approximation the lending contract is identical regardless of whether entrepreneurs are forward looking or myopic.

Proposition 2 *To solve for the CFP contract, entrepreneurs choose their state contingent cutoff* $\bar{\omega}_{t+1}$ *and leverage* $\kappa_t(j)$ *to maximize (13) subject to (5), (7) and (12). The solution to this problem is given by*

$$\kappa_t \mathbb{E}_t \left\{ R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right\} = -\frac{g_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{h_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})} \frac{1}{\Lambda_{t,t+1}}. \tag{17}$$

Proof See Appendix C. ■

Corollary 2 *Log-linearization of the CFP optimality condition (17) and the CFP participation constraint (12) gives*

$$\mathbb{E}_t \hat{R}^k_{t+1} - \mathbb{E}_t \hat{R}_{t+1} = \nu_\kappa \hat{\kappa}_t + \nu_\sigma \hat{\sigma}_{\omega,t} \tag{18}$$

$$\hat{R}_{t+1} - \mathbb{E}_t \hat{R}_{t+1} = \hat{R}^k_{t+1} - \mathbb{E}_t R^k_{t+1} - \tilde{\alpha}\sigma(\hat{C}_{t+1} - \mathbb{E}_t \hat{C}_{t+1}) \tag{19}$$

where $\tilde{\alpha} = -\frac{\frac{h_\omega}{h}}{\frac{h_{\omega\omega}}{h_\omega} - \frac{g_{\omega\omega}}{g_\omega}}$.

Proof See Appendix G. ■

Corollary 2 clearly illustrates the differences between the BGG contract and the CFP contract. In equation (19), lender's returns depend on capital returns and household consumption, both elements which are missing in the BGG contract. For standard calibrations, $\tilde{\alpha}$ takes a value between five and six and the risk aversion parameter σ is equal to one, so that lender's returns are very sensitive to the consumption level and the consumption insurance channel dominates the response to capital returns. When consumption is high, the lending rate declines; when consumption is low the lending rate increases. The negative covariance between the lender's consumption and returns reflects the nature of insurance, which amplifies the impact of shocks to the economy. Note that as entrepreneurs become more risk averse (as σ decreases), the impact of the consumption insurance channel declines.

Now that we have described the BGG and CFP contracts in detail, we turn our attention to the optimal contract. As we discussed above, the optimal contract takes the consumption insurance channel from CFP and adds forward looking entrepreneurs.

Proposition 3 *To solve for the optimal contract, entrepreneurs choose their state contingent cutoff* $\bar{\omega}_{t+1}$ *and leverage* $\kappa_t(j)$ *to maximize (2) subject to (3), (5), (7) and (12). The solution to this problem is given by*

$$\kappa_t \mathbb{E}_t \left\{ \Psi_{t+1} R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right\} = -\frac{g_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{h_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})} \frac{\Psi_{t+1}}{\Lambda_{t,t+1}} \tag{20}$$

where

$$\Psi_t = 1 + \gamma \kappa_t \mathbb{E}_t \Big\{ g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) R^k_{t+1} \Psi_{t+1} \Big\} \tag{21}$$

Proof See Appendix D. ■

Corollary 3 *Log-linearization of the optimal contract, (20) and (21), and the participation constraint (12) gives*

$$\mathbb{E}_t \hat{R}^k_{t+1} - \mathbb{E}_t \hat{R}_{t+1} = \nu_\kappa \hat{\kappa}_t + \nu_\sigma \hat{\sigma}_{\omega,t} \tag{22}$$

$$\hat{R}_{t+1} - \mathbb{E}_t \hat{R}_{t+1} = \hat{R}^k_{t+1} - \mathbb{E}_t R^k_{t+1} - \tilde{\alpha} \Big[\sigma(\hat{C}_{t+1} - \mathbb{E}_t \hat{C}_{t+1}) + \hat{\Psi}_{t+1} - \mathbb{E}_t \hat{\Psi}_{t+1} \Big] \tag{23}$$

$$\hat{\Psi}_{t+1} = \epsilon_N \mathbb{E}_{t+1} \Big\{ (\kappa - 1)(\hat{R}^k_{t+2} - \hat{R}_{t+2}) + \hat{R}^k_{t+2} + \nu_\Psi \hat{\sigma}_{\omega,t+1} + \hat{\Psi}_{t+2} \Big\} \tag{24}$$

where $\nu_\Psi = \frac{g_\sigma - h_\sigma \frac{g_\omega}{h_\omega}}{g} \sigma_\omega$.

Proof See Appendix G. ■

We see from (23) that under the optimal contract, the surprise to lender's returns depends not only on surprises to capital returns and consumption, as in the CFP contract, but *future* capital returns and *future* financial premiums as well. If entrepreneurs are more optimistic about expected future financial premiums or future returns to capital, they prefer to pay the lender a lower interest rate because one unit of net worth becomes more valuable. Corollary 3 thus illustrates the strong stabilizing mechanism of the optimal contract. When a crisis hits and decreases entrepreneur's net worth, expected future financial premiums will rise. But entrepreneurs will also pay lenders a smaller deposit rate, which stabilizes their net worth. As a result, the main channel for the financial accelerator, the volatility in net worth, is diminished when entrepreneurs are forward looking.

Although we have taken a partial equilibrium view here, Corollaries 1-3 are identical in the general equilibrium setting. In both partial and general equilibrium, leverage and the deposit rate are determined by the paths of capital returns and consumption. Therefore, the intuition provided by Corollaries 1-3 holds in general equilibrium.

3 The Model in General Equilibrium

We now embed the three loan contracts in a standard dynamic New Keynesian model. There are six agents in our model: households, entrepreneurs, financial intermediaries, capital producers, wholesalers and retailers. Entrepreneurs buy capital from capital producers and then rent it out to perfectly competitive wholesalers, who sell their goods to monopolistically competitive retailers. Retailers costlessly differentiate the wholesale goods and sell them to households at a markup over marginal cost. Retailers have price-setting power and are subject to Calvo price rigidities. Households bundle the retail goods in CES fashion into a final consumption good. A graphical overview of the model is provided in Figure 2 below. The dotted lines denote financial flows, while the solid lines denote real flows (goods, labor, and capital).

3.1 Households

The representative household maximizes its utility by choosing the optimal path of consumption, labor and money

$$\max \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[\frac{C_{t+s}^{1-\sigma}}{1-\sigma} + \zeta \log \left(\frac{M_{t+s}}{P_{t+s}} \right) - \chi \frac{H_{t+s}^{1+\eta}}{1+\eta} \right] \right\}, \tag{25}$$

where C_t is household consumption, M_t/P_t denotes real money balances, and H_t is household labor effort. The budget constraint of the representative household is

$$C_t = W_t H_t - T_t + \Pi_t + R_t \frac{D_t}{P_t} - \frac{D_{t+1}}{P_t} + \frac{M_{t-1} - M_t}{P_t} + \frac{B_{t-1} R_t^n - B_t}{P_t} \tag{26}$$

where W_t is the real wage, T_t is lump-sum taxes, Π_t is profit received from household ownership of final goods firms distributed in lump-sum fashion, and D_t are deposits in financial intermediaries (banks) that pay a contingent nominal gross interest rate R_t, and B_t are nominal bonds that pay a gross nominal non-contingent interest rate R_t^n.

Households maximize their utility (25) subject to the budget constraint (26) with respect to deposits, labor, nominal bonds and money, yielding four first order conditions:

$$U_{C,t} = \beta \mathbb{E}_t \left\{ R_{t+1} U_{C,t+1} \right\}, \tag{27}$$

$$U_{C,t} = \beta R_t^n \mathbb{E}_t \left\{ \frac{U_{C,t+1}}{\pi_{t+1}} \right\} \tag{28}$$

$$W_t U_{C,t} = \chi H_t^{\eta}, \tag{29}$$

$$U_{C,t} = \zeta \frac{1}{m_t} + \beta \mathbb{E}_t \left\{ \frac{U_{C,t+1}}{\pi_{t+1}} \right\}. \tag{30}$$

We define the gross rate of inflation as $\pi_{t+1} = P_{t+1}/P_t$, and real money balances as $m_t = M_t/P_t$.

3.2 Retailers

The final consumption good is made up of a basket of intermediate retail goods which are aggregated together in CES fashion by the representative household:

$$C_t = \left(\int_0^1 c_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{31}$$

Demand for retailer i's unique variety is

$$c_{it} = \left(\frac{p_{it}}{P_t}\right)^{-\varepsilon} C_t, \tag{32}$$

where p_{it} is the price charged by retail firm i. The aggregate price index is defined as

$$P_t = \left(\int_0^1 p_{it}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}. \tag{33}$$

Each retail firm chooses its price according to Calvo (1979) in order to maximize net discounted profit. With probability $1-\theta$ each retailer is able to change its price in a particular period t. Retailer i's objective function is

$$\max_{p_{it}^*} \sum_{s=0}^{\infty} \theta^s \mathbb{E}_t \left\{ \Lambda_{t+s} \frac{p_{it}^* - P_{t+s}^w}{P_{t+s}} \left(\frac{p_{it}^*}{P_{t+s}}\right)^{-\varepsilon} Y_{t+s} \right\}, \tag{34}$$

where P_t^w is the wholesale goods price. The first order condition with respect to the retailer's price p_{it}^* is

$$\sum_{s=0}^{\infty} \theta^s \mathbb{E}_t \left\{ \Lambda_{t,s} (p_{it}^*/P_{t+s})^{-\varepsilon} Y_{t+s} \left[p_{it}^* - \frac{\varepsilon}{\varepsilon-1} P_{t+s}^w \right] \right\} = 0. \tag{35}$$

From this condition it is clear that all retailers which are able to reset their prices in period t will choose the same price $p_{it}^* = P_t^* \ \ \forall i$. The price level will evolve according to

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{36}$$

Dividing the left and right hand side of (36) by the price level gives

$$1 = \left[\theta \pi_{t-1}^{\varepsilon-1} + (1-\theta)(p_t^*)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{37}$$

where $p_t^* = P_t^*/P_t$. Using the same logic, we can normalize (35) and obtain:

$$p_t^* = \frac{\varepsilon}{\varepsilon-1} \frac{\sum_{s=0}^{\infty} \theta^s \mathbb{E}_{t-1} \left\{ \Lambda_{t,s} (1/p_{t+s})^{-\varepsilon} Y_{t+s} p_{t+s}^w \right\}}{\sum_{s=0}^{\infty} \theta^s \mathbb{E}_{t-1} \left\{ \Lambda_{t,s} (1/p_{t+s})^{-\varepsilon} Y_{t+s} \right\}}, \tag{38}$$

where $p_{t+s}^w = \frac{P_{t+s}^w}{P_t}$ and $p_{t+s} = P_{t+s}/P_t$.

3.3 Wholesalers

Wholesale goods are produced by perfectly competitive firms and then sold to monopolistically competitive retailers who costlessly differentiate them. Wholesalers hire labor from households and entrepreneurs in a competitive labor market at real wage W_t and W_t^e and rent capital from entrepreneurs at rental rate R_t^r. Note that capital purchased in period t is used in period $t+1$.

Following BGG, the production function of the representative wholesaler is given by

$$Y_t = A_t K_{t-1}^{\alpha} (H_t)^{(1-\alpha)\Omega} (H_t^e)^{(1-\alpha)(1-\Omega)}, \tag{39}$$

where A_t denotes aggregate technology, K_t is capital, H_t is household labor, H_t^e is entrepreneurial labor, and Ω defines the relative importance of household labor and entrepreneurial labor in the production process. Entrepreneurs inelastically supply one unit of labor, so that the production function simplifies to

$$Y_t = A_t K_{t-1}^{\alpha} H_t^{(1-\alpha)\Omega}. \tag{40}$$

One can express the price of the wholesale good in terms of the price of the final good. In this case, the price of the wholesale good will be

$$\frac{P_t^w}{P_t} = \frac{1}{\mathcal{X}_t}, \tag{41}$$

where $\mathcal{X}_t$ is the variable markup charged by final goods producers. The objective function for wholesalers is then given by

$$\max_{H_t, H_t^e, K_{t-1}} \frac{1}{\mathcal{X}_t} A_t K_{t-1}^{\alpha} (H_t)^{(1-\alpha)\Omega} (H_t^e)^{(1-\alpha)(1-\Omega)} - W_t H_t - W_t^e H_t^e - R_t^r K_{t-1}. \tag{42}$$

Here wages and the rental price of capital are in real terms. The first order conditions with respect to capital, household labor and entrepreneurial labor are

$$\frac{1}{\mathcal{X}_t} \alpha \frac{Y_t}{K_{t-1}} = R_t^r, \tag{43}$$

$$\frac{\Omega}{\mathcal{X}_t} (1-\alpha) \frac{Y_t}{H_t} = W_t, \tag{44}$$

$$\frac{\Omega}{\mathcal{X}_t} (1-\alpha) \frac{Y_t}{H_t^e} = W_t^e. \tag{45}$$

3.4 Capital Producers

The perfectly competitive capital producer transforms final consumption goods into capital. Capital production is subject to adjustment costs, according to

$$K_t = I_t + (1-\delta) K_{t-1} - \frac{\phi_K}{2} \left(\frac{I_t}{K_{t-1}} - \delta \right)^2 K_{t-1}, \tag{46}$$

where I_t is investment in period t, δ is the rate of depreciation and ϕ_K is a parameter that governs the magnitude of the adjustment cost. The capital producer's objective function is

$$\max_{I_t} K_t Q_t - I_t, \tag{47}$$

where Q_t denotes the price of capital. The first order condition of the capital producer's optimization problem is

$$\frac{1}{Q_t} = 1 - \phi_K \left(\frac{I_t}{K_{t-1}} - \delta \right). \tag{48}$$

3.5 Lenders

One can think of the representative lender in the model as a perfectly competitive bank which costlessly intermediates between households and borrowers. The role of the lender is to diversify the household's funds among various entrepreneurs. The bank takes nominal household deposits D_t and loans out nominal amount B_t to entrepreneurs. In equilibrium, deposits will equal loanable funds ($D_t = B_t$). Households, as owners of the bank, receive a state contingent real rate of return R_{t+1} on their "deposits" — which equals the rate of return on loans to entrepreneurs.[3] Households choose the optimal lending rate according to their first order condition with respect to deposits:

$$\beta \mathbb{E}_t \left\{ \frac{U_{C,t+1}}{U_{C,t}} R_{t+1} \right\} = \mathbb{E}_t \left\{ \Lambda_{t,t+1} R_{t+1} \right\} = 1.$$

As we discussed above, the lender prefers a return that co-varies negatively with household consumption, which amplifies the financial accelerator.

3.6 Entrepreneurs

We have already described the entrepeneur's problem in detail in Section 2. Entrepreneurs choose their cutoff productivity level and leverage according to: (14) in BGG; (17) in CFP; and (20) and (21) in the dynamically optimal contract.

Wholesale firms rent capital at rate $R^r_{t+1} = \frac{\alpha Y_t}{\mathcal{X}_t K_{t-1}}$ from entrepreneurs. After production takes place entrepreneurs sell undepreciated capital back to capital goods producers for the unit price Q_{t+1}. Aggregate returns to capital are then given by

$$R^k_{t+1} = \frac{\frac{1}{\mathcal{X}_t} \frac{\alpha Y_{t+1}}{K_t} + Q_{t+1}(1-\delta)}{Q_t}. \tag{49}$$

Consistent with the partial equilibrium specification, entrepreneurs die with probability $1-\gamma$, which implies the following dynamics for aggregate net worth:

$$N_{t+1} = \gamma N_t \kappa_t R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) + W^e_{t+1}. \tag{50}$$

3.7 Goods market clearing

We have goods market clearing

$$Y_t = C_t + I_t + G_t + C^e_t + \mu G(\bar{\omega}_t, \sigma_{\omega,t-1}) R^k_t Q_{t-1} K_{t-1}, \tag{51}$$

[3] Note that lenders are not necessary in the model, but we follow BGG and CFP in positing a perfectly competitive financial intermediary between households and borrowers.

where $\mu G(\bar{\omega}) = \int_0^{\bar{\omega}} \mu f(\omega)\omega d\omega$ is the fraction of capital returns that go to monitoring costs, paid by lenders.

3.8 Monetary Policy

We assume that there is a central bank which conducts monetary policy by choosing the nominal interest rate R_t^n. In Section 4 we employ the nominal interest rate rule in BGG:

$$\log(R_t^n) - \log(R^n) = \rho^{R^n}\Big(\log(R_{t-1}^n) - \log(R)\Big) + \xi\pi_{t-1} + \epsilon_t^{R^n} \tag{52}$$

where ρ^{R^n} and ξ determine the relative importance of the past interest rate and past inflation in the central bank's interest rate rule. Shocks to the nominal interest rate are given by ϵ^{R^n}. It should be noted that the interest rule in BGG differs from the conventional Taylor rule, which targets current inflation rather than past inflation.

In Section 5, we consider the conventional Taylor rule with interest rate smoothing

$$\log(R_t^n) - log(R^n) = \rho^{R^n}\Big(\log(R_{t-1}^n) - \log(R)\Big) + \xi\pi_t + \rho^Y\Big(\log(Y_t) - \log(Y_{t-1})\Big) + \epsilon_t^{R^n}. \tag{53}$$

3.9 Shocks

The shocks in the model follow a standard AR(1) process. The AR(1) processes for technology, government spending and idiosyncratic volatility are given by

$$\log(A_t) = \rho^A \log(A_{t-1}) + \epsilon_t^A, \tag{54}$$

$$\log(G_t/Y_t) = (1-\rho^G)\log(G_{ss}/Y_{ss}) + \rho^G \log(G_{t-1}/Y_{t-1}) + \epsilon_t^G, \tag{55}$$

$$\log(\sigma_{\omega,t}) = (1-\rho^{\sigma_\omega})\log(\sigma_{\omega,ss}) + \rho^{\sigma_\omega}\log(\sigma_{\omega,t-1}) + \epsilon_t^{\sigma_\omega} \tag{56}$$

where ϵ^A, ϵ^G and ϵ^{σ_ω} denote exogenous shocks to technology, government spending and idiosyncratic volatility, and G_{ss} and $\sigma_{\omega,ss}$ denote the steady state values for government spending and idiosyncratic volatility respectively. Recall that σ_ω^2 is the variance of idiosyncratic productivity, so that σ_ω is the standard deviation of idiosyncratic productivity. Nominal interest rate shocks are defined by the BGG Rule in (52) or the Taylor rule in (53).

3.10 Equilibrium

The model has 20 endogenous variables and 20 equations. The endogenous variables are: Y, H, C, Λ, C^e, W, W^e, I, Q, K, R^n, R^k, R, p^*, $\mathcal{X}$, π, N, $\bar{\omega}$, k and Z. The equations defining these endogenous variables are: (9), (27), (29), (30), (11), (37), (38), (41), (40), (44), (45), (46), (48), (49), (50), (51), (52), (28), (A.8) and (E.3). The exogenous processes for technology, government spending and idiosyncratic volatility follow (54), (55) and (56) respectively. Nominal interest rate shocks are defined by the Taylor rule in (52).

4 Quantitative Analysis

4.1 Calibration

Our baseline calibration largely follows BGG. We set the discount factor $\beta = 0.99$, the risk aversion parameter $\sigma = 1$ so that utility is logarithmic in consumption, and the elasticity of labor is 3 ($\eta = 1/3$). The share of capital in the Cobb-Douglas production function is $\alpha = 0.35$. Investment adjustment costs are $\phi_k = 10$ to generate an elasticity of the price of capital with respect to the investment capital ratio of 0.25. Quarterly depreciation is $\delta = .025$. Monitoring costs are $\mu = 0.12$. The death rate of entrepreneurs is $1 - \gamma = .0272$, yielding an annualized business failure rate of three percent. The idiosyncratic productivity term, $\log(\omega(j))$, is assumed to be log-normally distributed with variance of 0.28. The weight of household labor relative to entrepreneurial labor in the production function is $\Omega = 0.99$.

For price-setting, we assume the Calvo parameter $\theta = 0.75$, so that only 25% of firms can reset their prices in each period, meaning the average length of time between price adjustments is four quarters. As our baseline, we follow the BGG monetary policy rule and set the autoregressive parameter on the nominal interest rate to $\rho^{R^n} = 0.9$ and the parameter on past inflation to $\xi = 0.11$. Note that in Section 5 we also consider a conventional Taylor rule where the central bank targets current inflation rather than past inflation. For the conventional Taylor rule, we set $\rho^{R^n} = 0$, $\xi = 1.5$ and $\rho^Y = 0.5$ as a benchmark, and consider an inertial interest rate rule with smoothing parameter $\rho^{R^n} = 0.5$, $\xi = 0.75$ and $\rho^Y = 0.25$. We follow BGG and set the persistence of the shocks to technology and government spending at $\rho^A = 0.999$ and $\rho^G = 0.95$. We follow Christiano, Motto and Rostagno (2013) and set the persistence of idiosyncratic volatility at $\rho^{\sigma_\omega} = 0.9706$ and the distribution of the shocks equal to $\epsilon_t^{\sigma_\omega} \sim N(0, 0.0283)$.

Following BGG, we consider a one percent technology shock and a 25 basis point shock (in annualized terms) to the nominal interest rate. For the risk shock, we allow the standard deviation of idiosyncratic productivity to increase by one percentage point, from 0.28 to 0.29.

4.2 Quantitative Comparison: BGG, CFP and the Optimal Contract

In our quantitative analysis we compare three allocations: the competitive equilibrium under the BGG contract; the competitive equilibrium under the CFP contract; and the competitive equilibrium under the optimal contract. Impulse responses for shocks to technology, the nominal interest rate and idiosyncratic volatility are found in this section.

Figure 3 shows impulse responses for a extremely persistent one percent technology shock when prices are sticky. Notice the impact of consumption insurance. Lenders in the CFP contract allocations will settle for a lower rate of return in a boom in order to ensure a higher rate of return in a recession, which amplifies the response of the economy. However, this does not occur under the optimal contract because entrepreneurs are forward looking: they act as a stabilizing influence on the economy. Forward looking entrepreneurs are reluctant to invest in new capital when a positive technology shock hits because financial premiums will be low. Asset prices will decline back to their steady state value, so entrepreneurs offer higher deposit rates to lender's in order utilize financial resources in states that promise higher capital returns. The stabilizing influence of forward looking entrepreneurs cancels out the consumption insurance channel under this calibration, such that the optimal contract and BGG output responses coincide almost exactly. In general, this coincidence does not hold outside of the particular calibration employed here.

The difference between the three allocations is very noticeable in Figure 4, which plots impulse responses for a one percent shock to the nominal interest rate when prices are sticky. Because the monetary shock is less persistent than the technology shock, the price of capital depreciates back to its steady state value very quickly after an initial rise. As a result, capital returns are positive in the first period, but negative thereafter. This leads to an even sharper difference between the response of entrepreneurs in the three models. Under the BGG contract the deposit rate does not respond to the shock at all because it is predetermined; under the CFP contract the deposit rate falls because household consumption increases in response to the shock; and under the optimal contract the deposit rate increases, because the financial premium goes down after the shock. Forward looking entrepreneurs thus stabilize consumption and output, leading to small amplification. In contrast, the CFP contract with consumption insurance leads to a decline in the lending rate following the rise in consumption, which amplifies the response of output, consumption and other macroeconomic aggregates to the interest rate shock.

In Figure 5 we plot impulse responses for a one standard deviation increase in unobserved idiosyncratic volatility σ_ω. This is what we defined earlier as a risk shock. In all three models, the household consumption response on impact is close to zero, but slightly positive for CFP and BGG and slightly negative for the optimal contract. The consumption insurance channel in CFP and the optimal contract leads to a decline in the lending rate following a risk shock. An additional factor is at work under the optimal contract: the financial premium rises because, other things equal, higher idiosyncratic variance makes default more likely. Therefore, borrowing is more expensive and returns to capital are higher. Net worth thus increases on impact under the optimal contract. Overall, risk shocks have a very small impact on the real economy in the optimal contract equilibrium, and may even boost output over a longer time horizon. Also note the negative correlation between output and consumption under the optimal contract, unlike the BGG contract where output and consumption both fall.

5 How Robust is the Financial Accelerator? Comparison with the Frictionless Model

To truly measure the strength of the financial accelerator, we need to compare the CSV model with financial frictions against a frictionless benchmark. As our frictionless benchmark, we take the model described in Section 3 and set monitoring costs and idiosyncratic productivity to zero.[4]

Our frictionless model is similar to Carlstrom and Fuerst's (1997), which also sets monitoring costs equal to zero, but different from the BGG frictionless model. BGG's frictionless model assumes a constant positive financial premium. We choose our definition because a constant positive financial premium in different aggregate states implies different profits for entrepreneurs, which distorts their decisions. We focus here only on the frictionless model with zero monitoring costs, but all of our main results hold relative to both frictionless cases.

One might ask at this point, why not use the basic New Keynesian model as a frictionless benchmark? Our reasoning is as follows. The basic New Keynesian sticky price model deviates from the CSV framework in *two* dimensions: (1) it abstracts from heterogeneity between lenders and borrowers because there are no entrepreneurs, *and* (2) it has no CSV frictions. As a result, if we use the basic New Keynesian model as a frictionless benchmark, it is impossible to isolate the impact of the CSV friction on volatility from the impact of heterogeneity. In order to isolate these two effects, we need a model that incorporates heterogeneity between lenders and borrowers but which eliminates the CSV friction. Our frictionless benchmark does exactly that, providing an exact characterization of the role of the CSV friction in generating volatility.

Before we proceed to the frictionless benchmark, let us compare the amplification response of the model with frictions to the basic New Keynesian model. Figure 6 shows that all three models with frictions generate more amplification than the basic New Keynesian model for very persistent technology shocks. In this case, forward looking entrepreneurs forecast higher capital returns in the future, which makes one unit of net worth more valuable today, leading to a large increase in net worth following the shock. For less persistent technology shocks ($\rho^A = 0.99$ for example) amplification under the optimal contract is actually lower than in the basic New Keynesian model.

Figure 6 also shows that the optimal contract delivers slightly smaller volatility than the New Keynesian model for monetary shocks. The intuition is very simple. In the wake of a positive monetary shock net worth increases and cash is abundant, so one additional unit of net worth generates a smaller consumption flow. Therefore, entrepreneurs want to increase their payments to lenders and pay out back their increase in net worth. As a result net worth does not react to the shock, which stabilizes expenditures relative to the basic New Keynesian model. Overall then, we see that amplification under the optimal contract is slightly smaller for technology shocks with persistence equal to or lower than $\rho^A = 0.99$ as well as for monetary shocks.

Now, let us consider the amplification response of the model with frictions relative to the

[4]The model without monitoring costs generates a different steady state relative to the model with monitoring costs, but the difference between the steady states is small for all variables except leverage. We correct steady state leverage in the frictionless model by increasing the share of entrepreneurs in the production function from 0.01 to 0.1. These modifications have a very small effect on equilibrium dynamics and do not alter our conclusions in any way.

frictionless benchmark with no monitoring costs. As we discussed earlier, the amplifying effect of the CSV framework depends on three characteristics: a suboptimal lending contract, extremely persistent technology shocks and loose monetary policy. The removal of any one of these characteristics eliminates the financial accelerator or even reverses the accelerator, such that financial frictions stabilize the economy in the presence of shocks.

Figure 7 plots the output response to a variety of technology shocks in a CSV model with and without monitoring costs and demonstrates the fragility of the financial accelerator to these three characteristics. The first row of plots in Figure 7 shows the response of output to an extremely persistent technology shock when prices are sticky. The model with frictions provides slightly more amplification in the suboptimal BGG lending contract, while the frictionless model generates more amplification for the CFP contract and the optimal contract.

Why do financial frictions stabilize business cycles in the latter two cases? First, entrepreneurs sell insurance to the household in order to smooth household consumption. The resulting decline in household consumption volatility leads to a rise in the volatility of entrepreneurial consumption and net worth, and entrepreneurs become a driver of the business cycle. Second, when entrepreneurs are forward looking they behave in a risk averse manner by trying to tighten the financial constraint during booms, when the financial premium is low, in order to relax it during recessions, when the financial premium is high. States with positive technology shocks promise falling asset prices in the short run after the initial reaction of asset prices on impact, and higher dividends in the long run from non-stationarity. This leads entrepreneurs to lever up and increase their net worth by a large amount, generating massive amplification in the frictionless model under the optimal contract. We also find that under the CFP contract, financial frictions stabilize business cycle, although the stabilizing effect is much smaller than under the optimal contract.

The sensitivity of the financial accelerator to stationary technology shocks is illustrated clearly in the second row of Figure 7, where we consider technology shocks with lower persistence ($\rho^A = 0.95$). In this calibration, financial frictions stabilize business cycles not only for the optimal contract and CFP but also for BGG. Why is it the case? We know, that for flexible prices model with and without financial friction deliver very similar results under BGG. Therefore, amplification should exacerbate fluctuation of markups. However, we know that for stationary technology shocks even in standard New Keynesian models, markups move procyclically and stabilize business cycles. If financial frictions exacerbate fluctuation for markups, they stabilize the model response to stationary technology shocks, since markups become even more procyclical.

Rows three and four of Figure 7 demonstrate the output response for extremely persistent technology shocks with a conventional Taylor rule and under flexible prices, respectively. The impulse responses show that the financial accelerator is not robust to more conservative monetary policy or flexible prices. As in the previous cases, the frictionless model for the CFP and optimal contracts generates higher amplification than the model with CSV frictions. Under the BGG contract, the accelerator disappears when the central bank follows a conventional Taylor rule. It is still present under flexible prices, but is extremely small quantitatively. In other words, the magnitude of the financial accelerator in the BGG case is negligible when monetary policy is more aggressive or when prices are flexible.

5.1 Sensitivity of the Financial Accelerator to Different Monetary Policy Rules

How sensitive is the financial accelerator to different monetary policy rules? Figure 8 plots output responses to a 25 basis point shock to the nominal interest rate for the BGG monetary policy rule (Row 1), the inertial Taylor rule (Row 2) and the conventional Taylor rule (Row 3). Here we see the sensitivity of the financial accelerator to different monetary policy specifications. Under the BGG policy rule, the coefficient on *past* inflation is $\xi = 0.11$, while the interest rate smoothing parameter is $\rho^{R^n} = 0.9$. Following the initial 25 basis point decrease in the nominal interest rate, there is little subsequent change in the interest rate under the BGG policy rule because the central bank is targeting *past* inflation, and also smoothing the interest rate. Any increase in inflation on impact is not taken into account until the next quarter. Under the BGG policy rule, monetary shocks are thus quite persistent, and entrepreneur's increase their net worth in the first period, which amplifies the shock. On the other hand, under conservative monetary policy asset prices and net worth are more stable, and there is no amplification.

We also calculate the quarterly inflation response to a monetary shock for the BGG policy rule and the conventional Taylor rule. For the conventional Taylor rule with a weight $\rho^{R^n} = 0.5$ on the previous interest rate, a two percent surprise to the Fed funds rate in annual terms leads to a one percent inflation response, while for the BGG monetary policy rule a one percent surprise to the Fed funds rate will lead to four percent inflation response, which significantly deviates from the flexible price equilibrium.

Overall, our simulations show that under the CFP contract and the optimal contract, financial frictions do not amplify business cycles for any calibration, while under the BGG contract they amplify business cycles only when technology shocks are extremely persistent and monetary policy is loose.

6 Related Literature

There is a large literature on the role of financial frictions in macroeconomics, particularly on how such frictions amplify and propagate shocks, which is the idea of the financial accelerator. We focus here on the CSV framework, but there are other ways of modeling financial frictions in general equilibrium models. In much of the literature, returns to lenders are predetermined by assumption and lenders are unable to use state-contingent contracts, despite the fact that they desire to do so.

One early example is Kiyotaki and Moore (1997), who show that feedback between collateral prices and loans leads to amplification. However, Krishnamurthy (2003) later proved that the amplification in Kiyotaki and Moore (1997) disappears when agents are able to use collaterilized state-contingent contracts.[5]

There is also a rich literature on pecuniary externalities which applies Kiyotaki and Moore (1997) type constraints in different environments. Topics in this vein include sudden stops for emerging economies, such as Caballero and Krishnamurthy (2001, 2003, 2004), Jeanne and Korinek (2013) and Bianchi (2011), as well as research on macroprudential policies, including Stein (2012), Jeanne and Korinek (2010) and Bianchi and Mendoza (2011). Krishnamurthy's (2003) critique of Kiyotaki and Moore (1997) also applies to this literature.

Similar concerns about the financial accelerator arise in the costly state enforcement literature. In Kiyotaki and Gertler (2010) and all other examples we are aware of, lender's returns are predetermined. Jermann and Quadrini (2012) allow both debt and equity, however they introduce adjustment costs between these instruments and rule out other instruments.

Adverse selection is another way to model financial frictions. House (2006) extends the Stiglitz and Weiss (1981) framework and shows that financial frictions amplify business cycles only when returns for lenders are predetermined. When contracts allow both debt and equity, financial frictions actually stabilize business cycles. Our results in the CSV framework are consistent with House (2006).

Our paper is also related to the growing body of medium sized DSGE models with financial frictions. To the best of our knowledge, the literature follows the BGG framework and employs myopic entrepreneurs with suboptimal contracts. Examples include Villaverde (2009, 2010) and Christiano, Motto and Rostagno (2013). Again, our results differ from the conclusions of this literature, as risk shocks under the optimal contract with non-myopic entrepreneurs have very little impact on the real economy.

7 Conclusion

This paper contributes to the literature on financial frictions in macroeconomics by introducing forward looking entrepreneurs into the costly state verification framework. In the literature, lending contracts are suboptimal and entrepreneurs are myopic. We solve for the optimal contract with forward looking entrepreneurs and show that financial frictions neither amplify nor propagate business cycles when lending contracts are optimal. In addition, we show that shocks to the variance of the unobserved productivity of entrepreneurs — so-called "risk shocks" — have little effect on the economy and generate the wrong comovement between macroeconomic aggregates when contracts are optimal.

We also investigate the robustness of the financial accelerator under the standard BGG

[5]Krishnamurthy (2003) restores financial amplification in a three period version of the model by introducing large aggregate shocks that reverses the role of lenders and borrowers, so that lenders have to post collateral. He also introduces constraints on aggregate collateral in the spirit of Holmstrom and Tirole (1998). This is a very different channel however, which has not been investigated in quantitative general equilibrium.

contract, which assumes that lenders receive a constant safe rate of return and borrowers are myopic. In this setup, we find that the accelerator depends on a combination of three things: a suboptimal lending contract, extremely persistent technology shocks, and loose monetary policy. Stationary technology shocks or a standard Taylor rule eliminate the financial accelerator or even reverse the accelerator such that financial frictions stabilize macroeconomic fluctuations. We thus conclude that the amplifying effect of financial frictions is present only under very restrictive conditions in costly state verification models.

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Figure 2: Overview of the Model

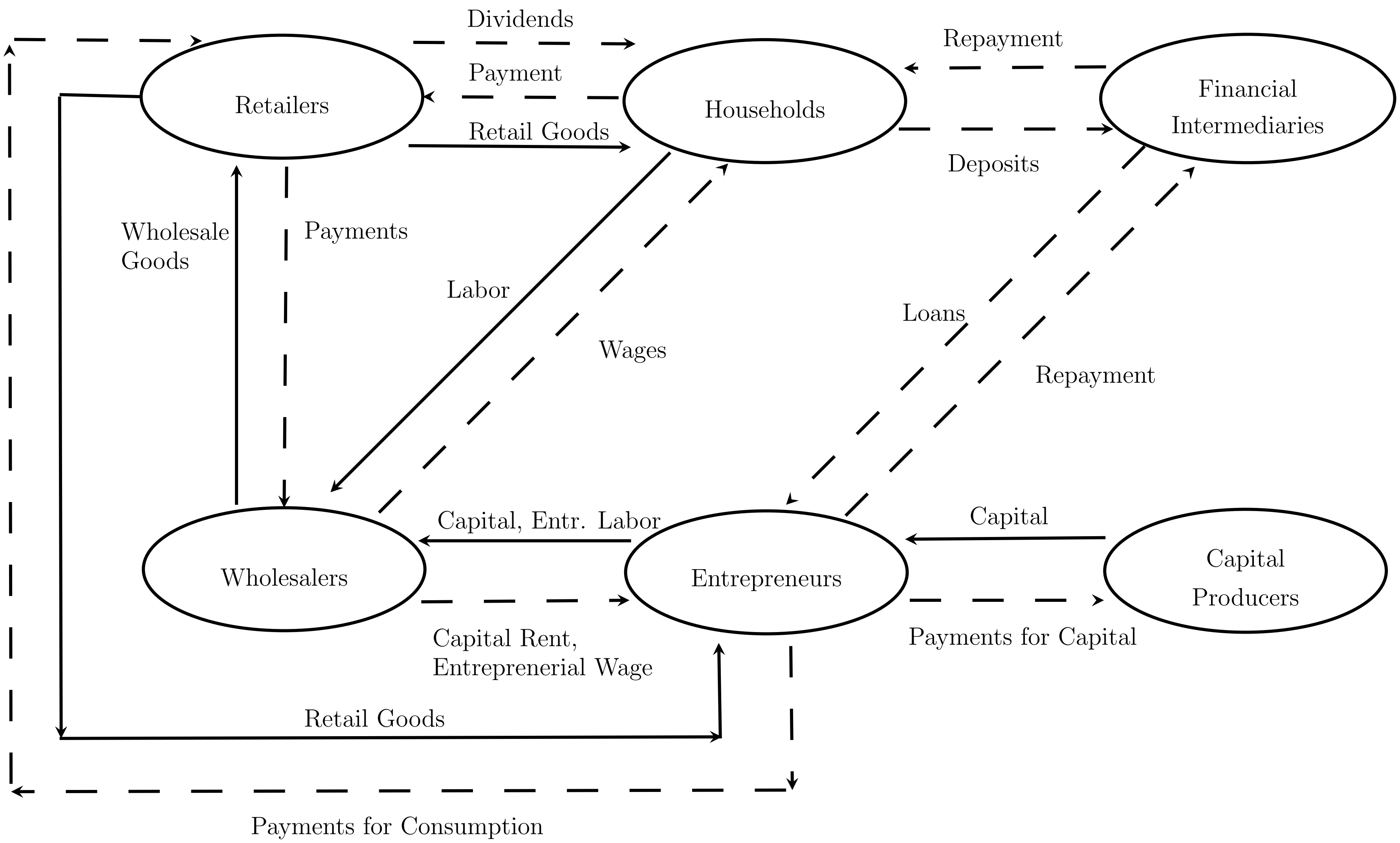
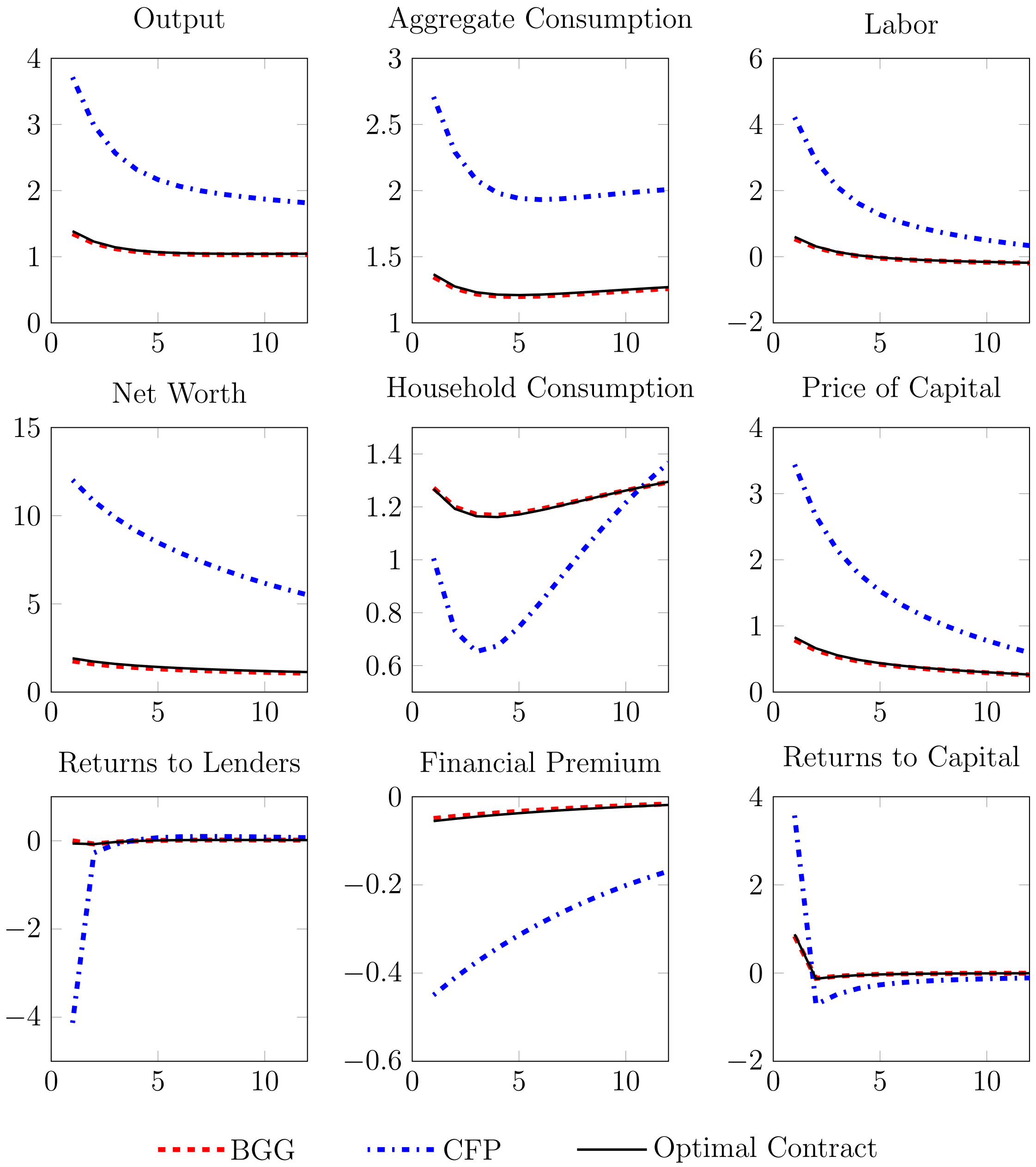
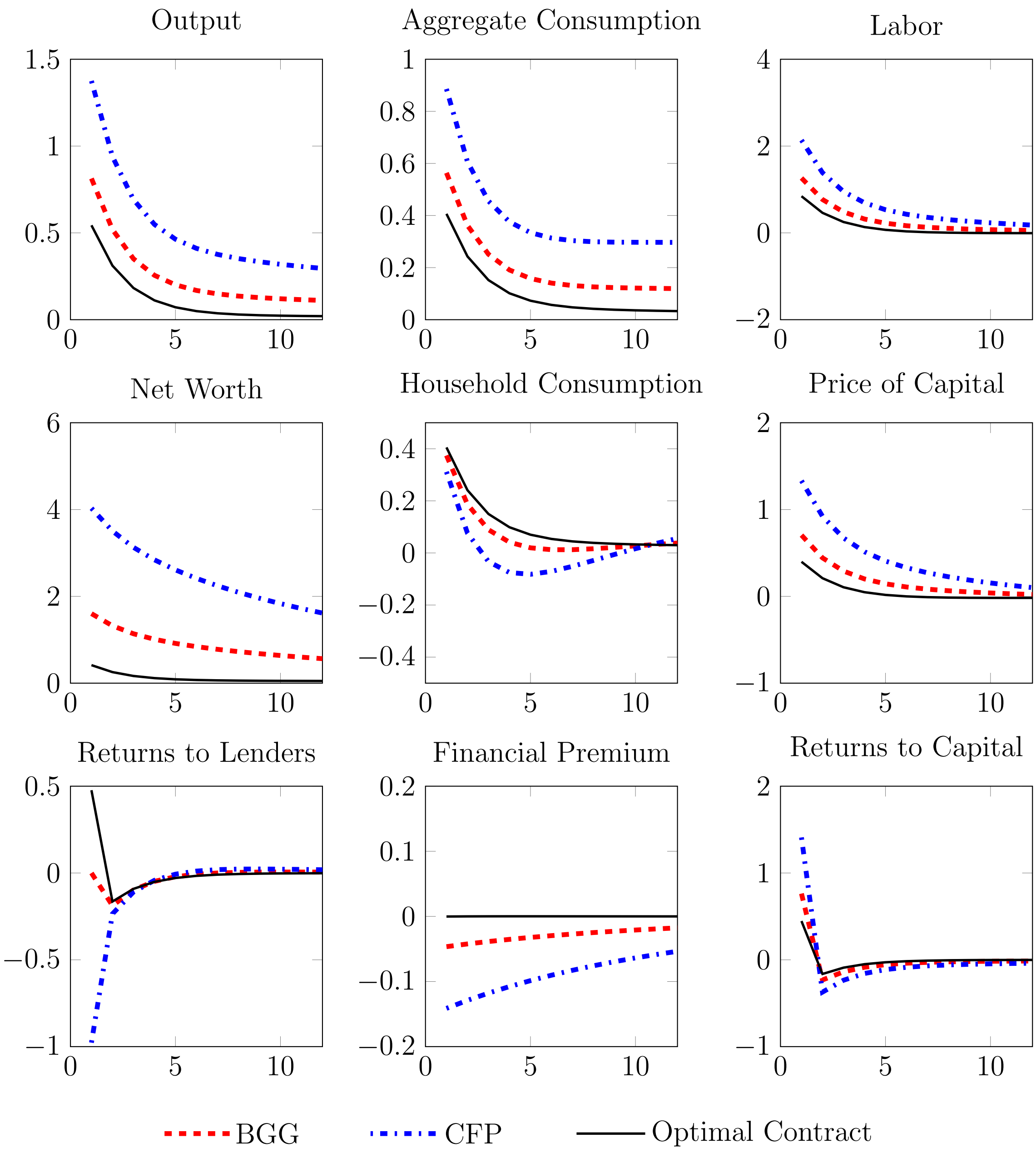


Figure 3: Technology Shock



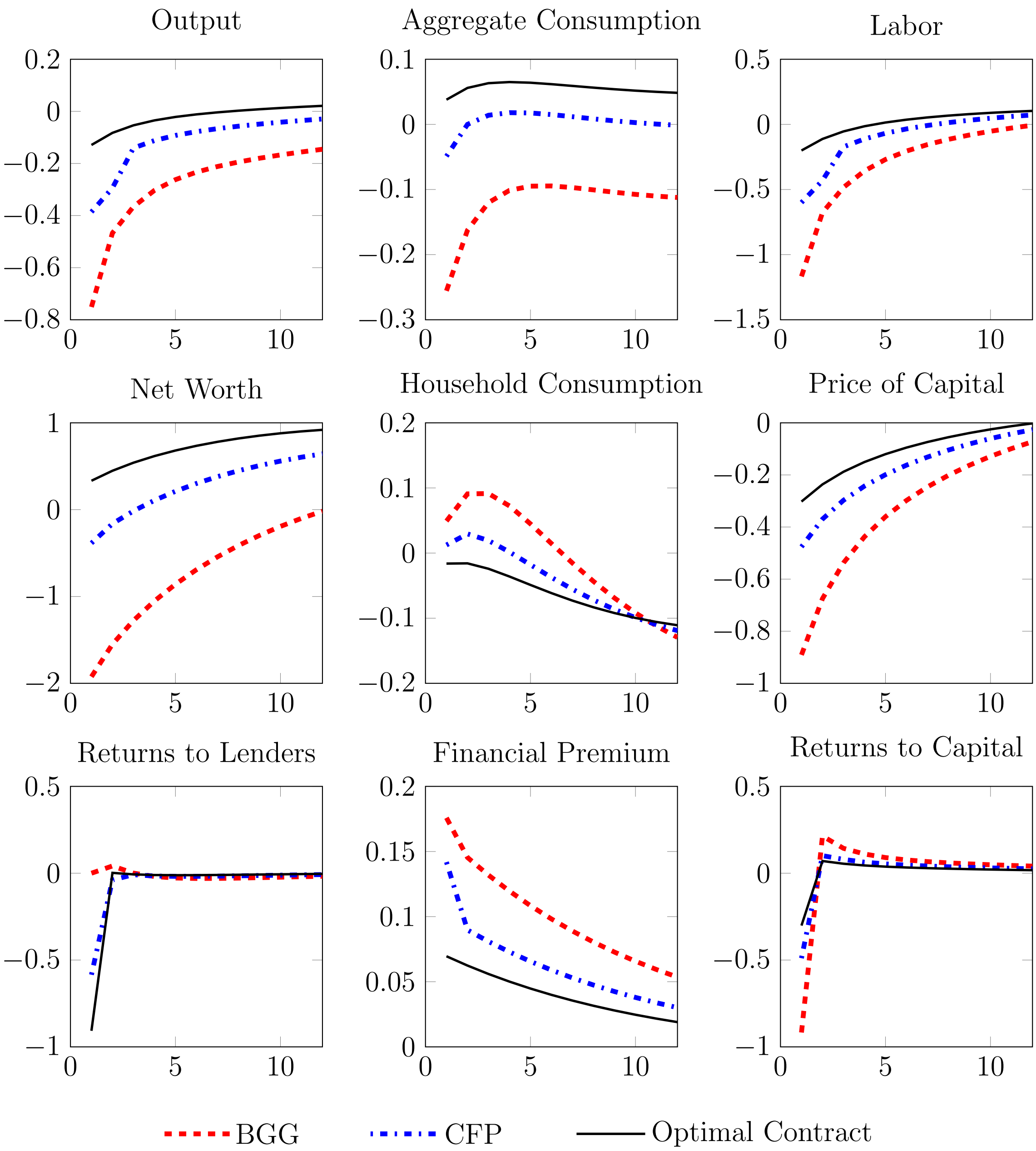
Note: All impulse responses are plotted as percent deviations from steady state.

Figure 4: Monetary Policy Shock



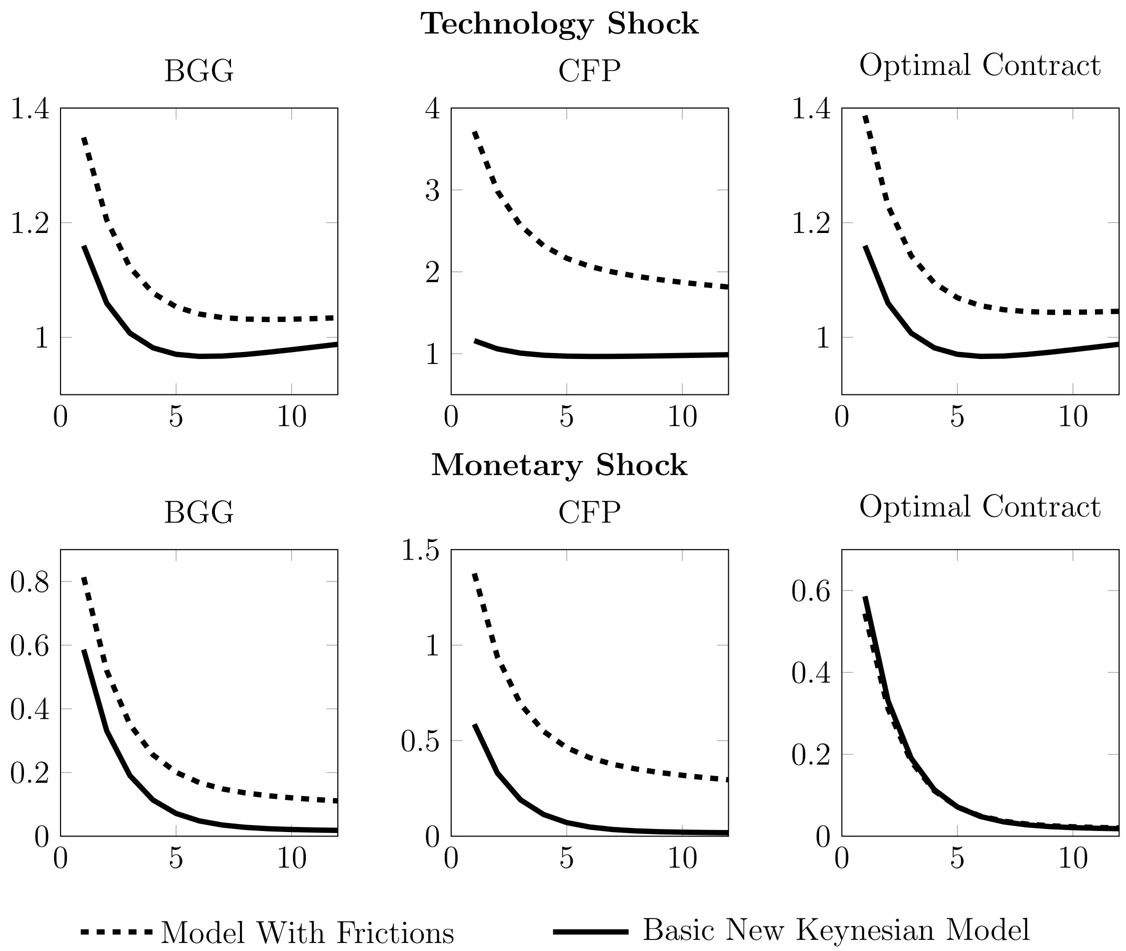
Note: All impulse responses are plotted as percent deviations from steady state.

Figure 5: Idiosyncratic Volatility Shock



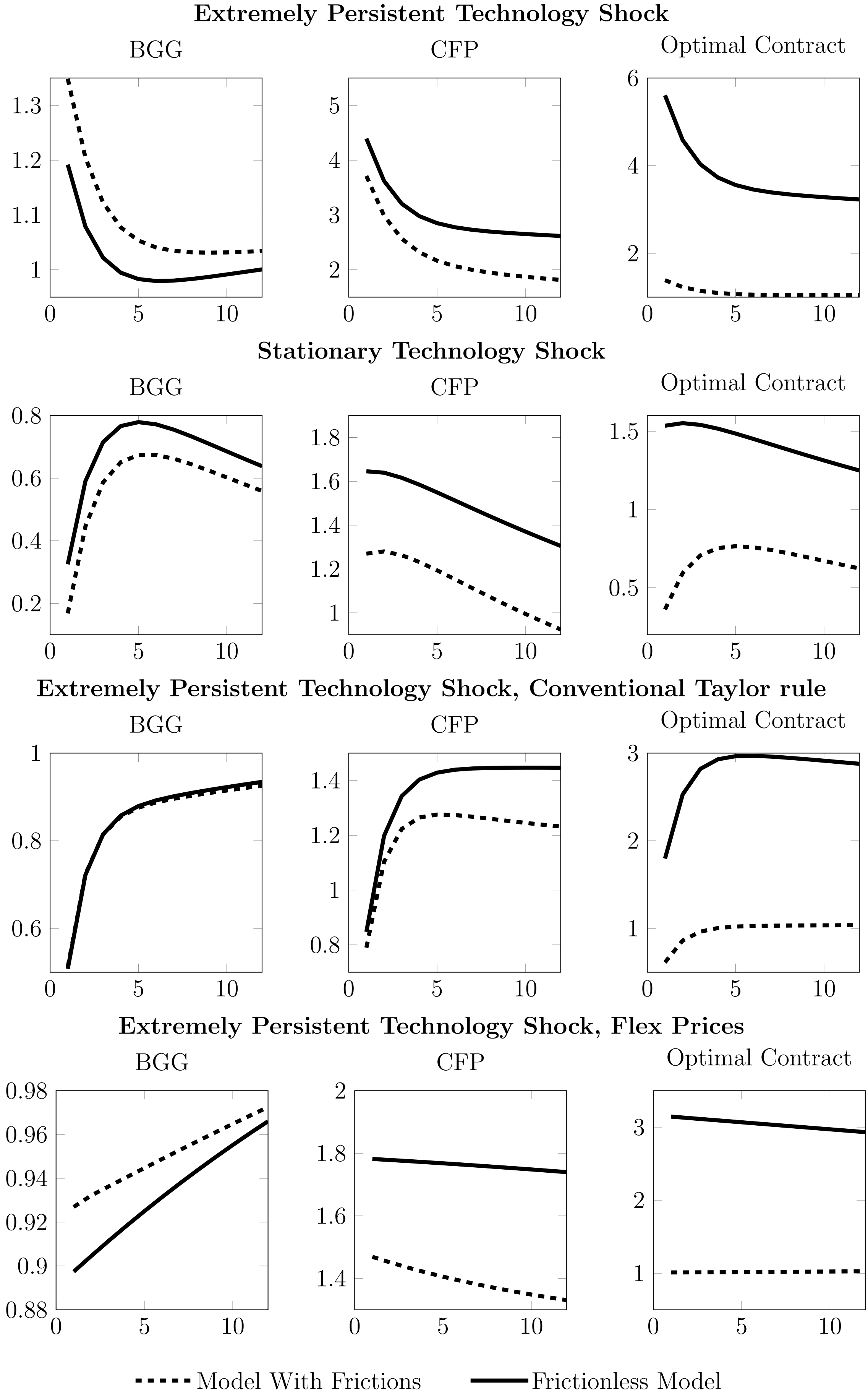
Note: All impulse responses are plotted as percent deviations from steady state.

Figure 6: Output Response Relative to the Basic New Keynesian Model



Note: All impulse responses are plotted as percent deviations from steady state.

Figure 7: Output Response Relative to the Frictionless Model, Technology Shocks

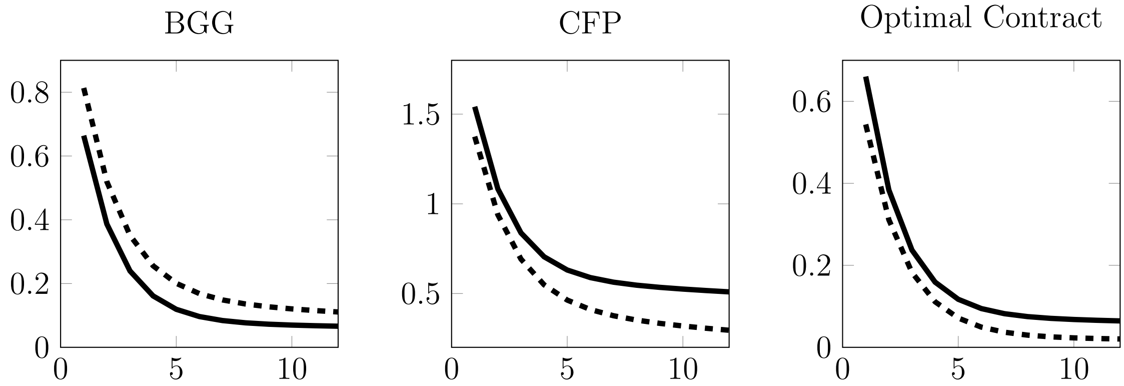


Note: All impulse responses are plotted as percent deviations from steady state.

Figure 8: Output Response Relative to the Frictionless Model, Monetary Shocks

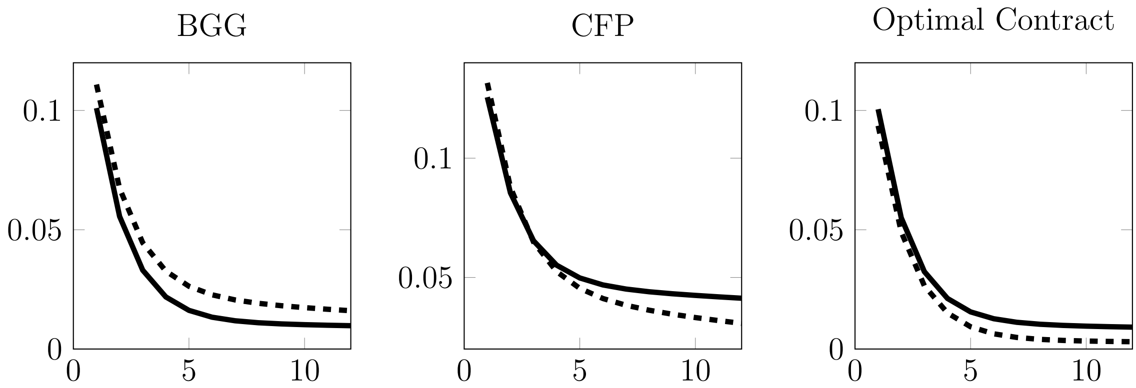
BGG Monetary Policy Rule

$$\log(R_t^n) - \log(R^n) = 0.9\Big(\log(R_{t-1}^n) - \log(R)\Big) + 0.11\pi_{t-1} + \epsilon_t^{R^n}$$



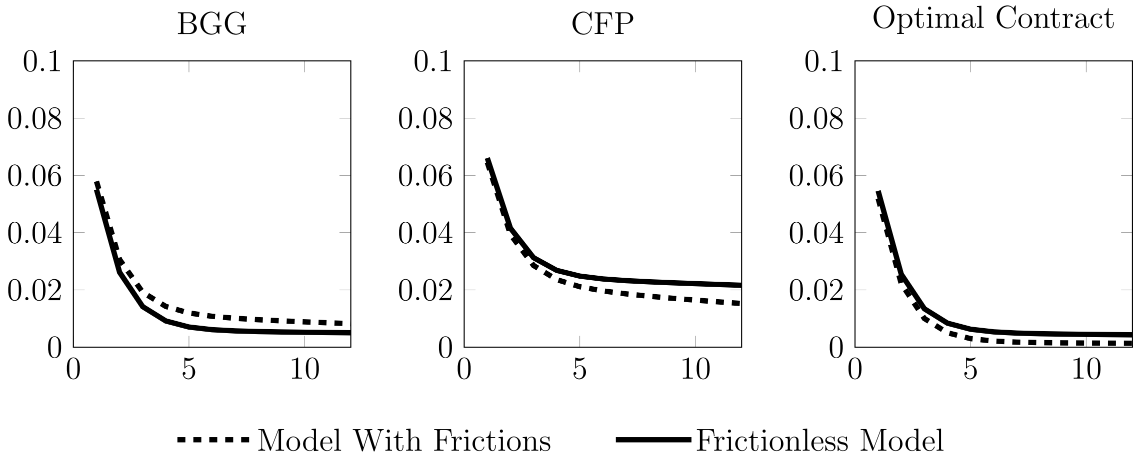
Inertial Taylor rule

$$\log(R_t^n) - log(R^n) = 0.5\Big(\log(R_{t-1}^n) - \log(R)\Big) + 0.5\Big[1.5\pi_t + 0.5\Big(\log(Y_t) - \log(Y_{t-1})\Big)\Big] + \epsilon_t^{R^n}$$



Conventional Taylor rule

$$\log(R_t^n) - log(R^n) = 1.5\pi_t + 0.5\Big(\log(Y_t) - \log(Y_{t-1})\Big) + \epsilon_t^{R^n}$$



Note: All impulse responses are plotted as percent deviations from steady state.

Technical Appendix

A Value Function Transformation

We can also formulate the optimal contract using normalized variables. For example, one can substitute leverage into the right hand side of equation (5) to obtain

$$N_{t+s}(j) = N_{t+s-1}(j)\kappa_{t+s-1}(j)R^k_{t+s}\max\{\omega_{t+s}(j) - \bar{\omega}_{t+s}, 0\} + W^e_{t+s} = N_{t+s-1}(j)R^e_{t+s}(j) + W^e_{t+s}, \tag{A.1}$$

where $R^e_t(j) = \kappa_{t-1}(j)R^k_t\max\{\omega_t(j) - \bar{\omega}_t, 0\}$ is the entrepreneur's ex post realized return. Iterating this equation backward generates

$$\begin{aligned} N_{t+s}(j) &= N_{t+s-1}(j)R^e_{t+s}(j) + W^e_{t+s} \\ &= N_t(j)\tilde{R}^e_{t,t+s} + W^e_{t+1}\tilde{R}^e_{t+1,t+s}\ldots + W^e_{t+s} \\ &= N_t(j)\tilde{R}^e_{t,t+s} + \sum_{i=1}^{s} W^e_{t+i}\tilde{R}^e_{t+i,t+s}, \end{aligned} \tag{A.2}$$

where $\tilde{R}^e_{t,t+s} = R^e_{t+1}R^e_{t+2}\ldots R^e_{t+s}$ and $\tilde{R}_{t+s,t+s} = 1$. Intuitively, $\tilde{R}^e_{t,t+s}$ is the entrepreneur's ex post accumulated rate of return on projects from period t through period $t+s$. For example, suppose the entrepreneur invests one dollar in period t and continues to reinvest his profits in new projects in each subsequent period. In period $t+s$, the entrepreneur will have accumulated $\tilde{R}^e_{t,t+s}$ from his initial one dollar investment. We can substitute (A.2) into the value function (2) and obtain

$$V^e_t(j) = (1-\gamma)\mathbb{E}_t\left\{N_t(j)R^e_{t+1} + \sum_{s=2}^{\infty}\gamma^{s-1}\left(N_t(j)\tilde{R}^e_{t,t+s} + \sum_{i=1}^{s-1} W^e_{t+i}\tilde{R}^e_{t+i,t+s}\right)\right\}. \tag{A.3}$$

Because the entrepreneur will optimize with respect to leverage and the productivity cutoff we want to express (A.3) as a function of the leverage and the productivity cutoff. In the first step, we separate terms to get

$$V^e_t(j) = (1-\gamma)N_t(j)\mathbb{E}_t\left\{\tilde{R}^e_{t,t+1}\sum_{s=1}^{\infty}\gamma^{s-1}\tilde{R}^e_{t+1,t+s}\right\} + (1-\gamma)\mathbb{E}_t\left\{\sum_{s=1}^{\infty}\gamma^{s-1}\left(\sum_{i=1}^{s-1} W^e_{t+i}\tilde{R}^e_{t+i,t+s}\right)\right\}, \tag{A.4}$$

where we used $\tilde{R}^e_{t,t+s} = \tilde{R}^e_{t,t+1}\tilde{R}^e_{t+1,t+s}$. Net worth enters the value function as a constant multiplicative term and has no effect on the entrepreneur's choice of leverage $\kappa_t(j)$ or cutoff $\bar{\omega}_{t+1}$; both enter only through $\tilde{R}^e_{t,t+1}$. Using the law of iterated expectations $\mathbb{E}_t(x_{t+1}) = \mathbb{E}_t[\mathbb{E}(x_{t+1}|\Omega_{agg,t+1})]$ and the independence of idiosyncratic productivity from aggregate productivity, we can replace the realizations of idiosyncratic productivity with their expectation and get

$$V^e_t(j) = (1-\gamma)N_t(j)\mathbb{E}_t\left[\tilde{R}^{e,agg}_{t,t+1}\sum_{s=1}^{\infty}\gamma^{s-1}\tilde{R}^{e,agg}_{t+1,t+s}\right] + (1-\gamma)\mathbb{E}_t\left[\sum_{s=2}^{\infty}\gamma^{s-1}\left(\sum_{i=1}^{s-1}W^e_{t+i}\tilde{R}^{e,agg}_{t+i,t+s}\right)\right], \tag{A.5}$$

where $\tilde{R}^{e,agg}_{t,t+s} = R^{e,agg}_{t+1}R^{e,agg}_{t+2}...R^{e,agg}_{t+s}$ with $\tilde{R}^{e,agg}_{t+s,t+s} = 1$, and $R^{e,agg}_{t+1} = \kappa_t R^k_{t+1} g(\bar{\omega}_{t+1})$ is the entrepreneur's ex post realized rate of return expressed as a function of aggregate productivity and leverage, with $g(\bar{\omega}_{t+1}) = \int_{\bar{\omega}_{t+1}}^{\infty}[\omega - \bar{\omega}_{t+1}]f(\omega)d\omega$.

Now we can reexpress value function as

$$V^e_t(j) = (1-\gamma)N_t(j)(\Psi_t - 1) + (1-\gamma)\mathbb{E}_t\left[\sum_{s=1}^{\infty}\gamma^s W^e_{t+s}(\Psi_{t+s}-1)\right], \tag{A.6}$$

where $\Psi_t = 1 + \gamma\mathbb{E}_t\left[\kappa_t R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega,t})\Psi_{t+1}\right]$. If we divide the lender's ex post returns in equation (7) by $N_t(j)$ we get

$$\Big[\kappa_t(j) - 1\Big]R_{t+1}(j) = \kappa_t(j)R^k_{t+1}h(\bar{\omega}_{t+1}, \sigma_{\omega,t}). \tag{A.7}$$

Now, if we substitute (A.7) into the Euler equation for the representative household (12), we have

$$\beta\mathbb{E}_t\Big\{U_{C,t+1}\kappa_t(j)R^k_{t+1}h(\bar{\omega}_{t+1}, \sigma_{\omega,t})\Big\} = \Big[\kappa_t(j) - 1\Big]U_{C,t}. \tag{A.8}$$

Before looking at the first order conditions to the optimization problem, it is important to notice that all entrepreneurs will choose the same leverage and state-contingent interest rate regardless of their net worth, due to the homotheticity of the problem. Thus, the entrepreneur index (j) is omitted below. We use the following notation: *BGG* refers to the contract of Bernanke, Gertler and Gilchrist (1999), *CFP* refers to the contract of Carlstrom, Fuerst and Paustian (2012), and *Optimal* refers to the optimal contract with non-myopic entrepreneurs

B BGG Contract

In the BGG contract, the lender is guaranteed a fixed rate of return. In this case, the entrepreneur's Lagrangian will be:

$$\mathcal{L}^{BGG} = (1-\gamma)\mathbb{E}_t\Bigg\{N_t\kappa_t R^k_{t+1}g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) + \lambda_{t+1}\Bigg[\beta\mathbb{E}_t\Big\{U_{C,t+1}\Big\}k_t R^k_{t+1}h(\bar{\omega}_{t+1}, \sigma_{\omega,t}) - (k_t - 1)U_{C,t}\Bigg]\Bigg\}.$$

The entrepreneur's first order conditions with respect to κ_t and $\bar{\omega}_{t+1}$ are:

$$\frac{\partial \mathcal{L}^{BGG}}{\partial \kappa_t} = N_t \mathbb{E}_t \Big\{ R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \Big\} - \mathbb{E}_t \Big\{ \lambda_{t+1} \Big\} \frac{U_{C,t}}{\kappa_t} = 0 \tag{B.1}$$

$$\begin{aligned} \frac{\partial \mathcal{L}^{BGG}}{\partial \omega_{t+1}} &= N_t \kappa_t R^k_{t+1} g_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t}) + \lambda_{t+1} \beta \mathbb{E}_t \Big\{ U_{C,t+1} \Big\} \kappa_t R^k_{t+1} h_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t}) = 0 \\ &= N_t g_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t}) + \lambda_{t+1} \beta \mathbb{E}_t \Big\{ U_{C,t+1} \Big\} h_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t}) = 0 \end{aligned} \tag{B.2}$$

If we substitute $\frac{\partial \mathcal{L}^{BGG}}{\partial \omega_{t+1}}$ into $\frac{\partial \mathcal{L}^{BGG}}{\partial \kappa_t}$, we find

$$N_t \mathbb{E}_t \Big\{ R^k_{t+1} g(\bar{\omega}_{t+1}) \Big\} = \mathbb{E}_t - \left\{ \frac{N_t g_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{h_\omega(\bar{\omega}_{t+1}) \beta \mathbb{E}_t U_{c,t+1}} \right\} \frac{U_{C,t}}{\kappa_t}. \tag{B.3}$$

Rearranging, simplifying and substituting in the stochastic discount factor yields:

$$\kappa_t \mathbb{E}_t \Big\{ R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \Big\} = -\mathbb{E}_t \left\{ \frac{g_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{h_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})} \right\} \frac{1}{\mathbb{E}_t \Lambda_{t,t+1}}. \tag{B.4}$$

C CFP Contract

In the CFP contract, the entrepreneur's Lagrangian is:

$$\mathcal{L}^{CFP} = (1-\gamma) \Bigg\{ N_t \kappa_t \mathbb{E}_t \Big[R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \Big] + \lambda_t \Big[\mathbb{E}_t \Big(\beta U_{C,t+1} k_t R^k_{t+1} h(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \Big) - (k_t - 1) U_{C,t} \Big] \Bigg\}.$$

The entrepreneur's first order conditions with respect to κ_t and $\bar{\omega}_{t+1}$ are:

$$\begin{aligned} \frac{\partial \mathcal{L}^{CFP}}{\partial \kappa_t} &= (1-\gamma) \left[N_t \mathbb{E}_t \left\{ R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right\} + \lambda_t \left(\mathbb{E}_t \left\{ \beta U_{C,t+1} R^k_{t+1} h(\bar{\omega}_{t+1}, \sigma_{\omega,t}) - U_{C,t} \right\} \right) \right] = 0, \\ \frac{\partial \mathcal{L}^{CFP}}{\partial \bar{\omega}_{t+1}} &= (1-\gamma) \left[N_t \kappa_t R^k_{t+1} g_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t}) + \lambda_t \beta U_{C,t+1} \kappa_t R^k_{t+1} h_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right] = 0. \end{aligned}$$

Rearranging these first order conditions, solving in terms of λ_t and setting them equal to each other yields:

$$\kappa_t \mathbb{E}_t \Big\{ R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \Big\} = -\frac{g_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{h_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})} \frac{1}{\Lambda_{t,t+1}}. \tag{C.1}$$

D Optimal Contract With Forward Looking Entrepreneurs

Under the optimal contract, the forward looking entrepreneur's Lagrangian has the following form (if we divide the value function by $(1-\gamma)N_t(j)$ as a scaling factor):

$$\mathcal{L}^{Optimal} = (1-\gamma)\mathbb{E}_t\Bigg\{N_t(j)(\Psi_t - 1) + \sum_{s=1}^{\infty}\gamma^s W^e_{t+s}(\Psi_{t+s}-1)$$
$$+\sum_{i=0}^{\infty}\lambda_{t+i}\Big[\beta U_{c,t+i+1}\kappa_{t+i}R_{k,t+i+1}h(\bar{\omega}_{t+i+1},\sigma_{\omega,t+i}) - (\kappa_{t+i}-1)U_{c,t+i}\Big]\Bigg\},$$

where $\Psi_t = 1 + \kappa_t\mathbb{E}_t\Big\{g(\bar{\omega}_{t+1},\sigma_{\omega,t})R_{k,t+1}\Psi_{t+1}\Big\}$. The entrepreneur's first order condition with respect to leverage κ_t is:

$$\frac{\partial\mathcal{L}^{Optimal}_t}{\partial\kappa_t} = (1-\gamma)\mathbb{E}_t\Bigg\{N_t(j)g(\bar{\omega}_{t+1},\sigma_{\omega,t})R_{k,t+1}\Psi_{t+1} + \lambda_t\Big(\beta U_{C,t+1}R^k_{t+1}h(\bar{\omega}_{t+1},\sigma_{\omega,t}) - U_{C,t}\Big)\Bigg\} = 0,$$

where we have used the fact that $\frac{\partial\Psi_t}{\partial\kappa_t} = \mathbb{E}_t\Big\{g(\bar{\omega}_{t+1},\sigma_{\omega,t})R_{k,t+1}\Psi_{t+1}\Big\}$ and $\frac{\partial\Psi_{t+i}}{\partial\kappa_t} = 0$ for $i = 1, 2, ...$ The entrepreneur's first order condition with respect to the productivity cutoff $\bar{\omega}_{t+1}$ is:

$$\frac{\partial\mathcal{L}^{Optimal}_t}{\partial\bar{\omega}_{t+1}} = (1-\gamma)\Bigg\{N_t(j)\kappa_t g_{\omega}(\bar{\omega}_{t+1},\sigma_{\omega,t})R_{k,t+1}\Psi_{t+1} + \lambda_t\Big[\beta U_{C,t+1}\kappa_t R_{k,t+1}h_{\omega}(\bar{\omega}_{t+1},\sigma_{\omega,t})\Big]\Bigg\} = 0. \tag{D.1}$$

where we have used the fact that $\frac{\partial\Psi_t}{\partial\bar{\omega}_{t+1}} = \kappa_t g'(\bar{\omega}_{t+1},\sigma_{\omega,t})R_{k,t+1}\Psi_{t+1}$ and $\frac{\partial\Psi_{t+i}}{\partial\omega_{t+1}} = 0$ for $i = 1, 2...$ We then move λ_t to the right hand side of both first order conditions and divide the equations by each other to obtain:

$$\frac{g_{\omega}(\bar{\omega}_{t+1},\sigma_{\omega,t})\mathbb{E}_{t+1}\Psi_{t+1}}{\mathbb{E}_t\left\{R^k_{t+1}g(\bar{\omega}_{t+1},\sigma_{\omega,t})\Psi_{t+1}\right\}} = \frac{\beta\kappa_t U_{C,t+1}h_{\omega}(\bar{\omega}_{t+1},\sigma_{\omega,t})}{\beta\mathbb{E}_t\Big\{U_{C,t+1}\kappa_t R^k_{t+1}h(\bar{\omega}_{t+1})\Big\} - \kappa_t U_{C,t}} = \frac{\beta\kappa_t U_{C,t+1}h_{\omega}(\bar{\omega}_{t+1},\sigma_{\omega,t})}{(\kappa_t - 1)U_{C,t} - \kappa_t U_{C,t}}$$

where we utilized the participation constraint for lenders in the final step. After rearranging and simplifying, we get

$$\kappa_t\mathbb{E}_t\Big\{\Psi_{t+1}R^k_{t+1}g(\bar{\omega}_{t+1},\sigma_{\omega,t})\Big\} = -\frac{g_{\omega}(\bar{\omega}_{t+1},\sigma_{\omega,t})}{h_{\omega}(\bar{\omega}_{t+1},\sigma_{\omega,t})}\frac{\Psi_{t+1}}{\Lambda_{t,t+1}}. \tag{D.2}$$

E BGG Contract With Forward Looking Entrepreneurs

For a predetermined lending rate, the non-myopic entrepreneur's Lagrangian has the following form if we divide the value function by $(1-\gamma)N_t(j)$ as a scaling factor:

$$\begin{aligned}\mathcal{L} =& (1-\gamma)\mathbb{E}_t\Bigg\{N_t(j)(\Psi_t - 1) + \sum_{s=1}^{\infty}\gamma^s W^e_{t+s}(\Psi_{t+s}-1)\\ &+ \sum_{i=0}^{\infty}\lambda_{t+i+1}\Big[\beta\mathbb{E}_t\Big\{U_{c,t+i+1}\Big\}\kappa_{t+i}R_{k,t+i+1}h(\bar{\omega}_{t+i+1},\sigma_{\omega,t+i}) - (\kappa_{t+i}-1)U_{c,t+i}\Big]\Bigg\},\end{aligned}$$

where $\Psi_t = 1 + \kappa_t\mathbb{E}_t\Big\{g(\bar{\omega}_{t+1},\sigma_{\omega,t})R_{k,t+1}\Psi_{t+1}\Big\}$. The entrepreneur's first order condition with respect to leverage κ_t is:

$$\frac{\partial\mathcal{L}_t}{\partial\kappa_t} = (1-\gamma)\mathbb{E}_t\Bigg\{N_t(j)g(\bar{\omega}_{t+1},\sigma_{\omega,t})R_{k,t+1}\Psi_{t+1} + \lambda_{t+1}\Bigg(\beta\mathbb{E}_t\Big\{U_{C,t+1}\Big\}R^k_{t+1}h(\bar{\omega}_{t+1},\sigma_{\omega,t}) - U_{C,t}\Bigg)\Bigg\} = 0, \tag{E.1}$$

where we have used the fact that $\frac{\partial\Psi_t}{\partial\kappa_t} = \mathbb{E}_t\Big\{g(\bar{\omega}_{t+1},\sigma_{\omega,t})R_{k,t+1}\Psi_{t+1}\Big\}$ and $\frac{\partial\Psi_{t+i}}{\partial\kappa_t} = 0$ for $i = 1,2,...$ The entrepreneur's first order condition with respect to the productivity cutoff $\bar{\omega}_{t+1}$ is:

$$\frac{\partial\mathcal{L}_t}{\partial\bar{\omega}_{t+1}} = (1-\gamma)\Bigg\{N_t(j)\kappa_t g_\omega(\bar{\omega}_{t+1},\sigma_{\omega,t})R_{k,t+1}\Psi_{t+1} + \lambda_{t+1}\Big[\beta\mathbb{E}_t\Big\{U_{C,t+1}\Big\}\kappa_t R_{k,t+1}h_\omega(\bar{\omega}_{t+1},\sigma_{\omega,t})\Big]\Bigg\} = 0. \tag{E.2}$$

where we have used the fact that $\frac{\partial\Psi_t}{\partial\bar{\omega}_{t+1}} = \kappa_t g_\omega(\bar{\omega}_{t+1},\sigma_{\omega,t})R_{k,t+1}\Psi_{t+1}$ and $\frac{\partial\Psi_{t+i}}{\partial\omega_{t+1}} = 0$ for $i = 1,2...$ One can express λ_{t+1} in the equation $\frac{\partial\mathcal{L}}{\partial\omega_{t+1}} = 0$ as a function of other variables, and substitute the result into $\frac{\partial\mathcal{L}}{\partial\kappa_t} = 0$. Then, using the participation constraint to simplify and after some rearranging, we obtain:

$$\kappa_t\mathbb{E}_t\Bigg\{\Psi_{t+1}R^k_{t+1}g(\bar{\omega}_{t+1},\sigma_{\omega,t})\Bigg\} = -\mathbb{E}_t\Bigg\{\frac{g_\omega(\bar{\omega}_{t+1},\sigma_{\omega,t})}{h_\omega(\bar{\omega}_{t+1},\sigma_{\omega,t})}\frac{\Psi_{t+1}}{\mathbb{E}_t\Lambda_{t,t+1}}\Bigg\}. \tag{E.3}$$

It is trivial to show that log-linearization of the BGG contract with myopic or non-myopic agents gives an identical optimality condition. However, this identity does not hold for higher order approximations.

F Complete Log-Linearized Model

In this section we review the whole model in its log-linearized form.

New Keynesian Components

We begin with the set of equations characterizing the standard New Keynesian components of the model. Equation (F.1) gives the Euler equation for state-contingent assets from the FOC for deposits, while (F.2) is the Euler equation for nominal bonds. The labor market clearing condition is given by (F.3), (F.4) is the New Keynesian Phillips curve, (F.5) is the production function, (F.6) gives the evolution of capital, (F.7) refers to the dynamics of the price of capital, (F.8) gives returns to capital, and (F.9) refers to goods market clearing. Shocks to technology, monetary policy, government spending and idiosyncratic risk are defined by (F.10), (F.11), (F.12) and (F.13).

$$-\sigma\Big(\mathbb{E}_t\hat{C}_{t+1}-\hat{C}_t\Big)+\mathbb{E}_t\hat{R}_{t+1}=0, \tag{F.1}$$

$$\hat{R}^n_t=\mathbb{E}_t\hat{R}_{t+1}+\mathbb{E}_t\hat{\pi}_{t+1} \tag{F.2}$$

$$\hat{Y}_t-\hat{H}_t-\hat{\mathcal{X}}_t-\sigma\hat{C}_t=\eta\hat{H}_t, \tag{F.3}$$

$$\hat{\pi}_t=-\frac{(1-\theta)(1-\theta\beta)}{\theta}\hat{\mathcal{X}}_t+\beta\mathbb{E}_t\hat{\pi}_{t+1}. \tag{F.4}$$

$$\hat{Y}_t=\hat{A}_t+\alpha\hat{K}_{t-1}+(1-\alpha)(1-\Omega)\hat{H}_t. \tag{F.5}$$

$$\hat{K}_t=\delta\hat{I}_t+(1-\delta)\hat{K}_{t-1}, \tag{F.6}$$

$$\hat{Q}_t=\delta\phi_K(\hat{I}_t-\hat{K}_{t-1}), \tag{F.7}$$

$$\hat{R}^k_{t+1}=(1-\epsilon)(\hat{Y}_{t+1}-\hat{K}_t-\hat{\mathcal{X}}_{t+1})+\epsilon\hat{Q}_{t+1}-\hat{Q}_t \tag{F.8}$$

$$Y\hat{Y}_t=C\hat{C}_t+I\hat{I}_t+G\hat{G}_t+C^e\hat{C}^e_t+\phi_\mu\hat{\phi}_{\mu,t}, \tag{F.9}$$

$$\hat{A}=\rho^A\hat{A}_{t-1}+\epsilon^A_t \tag{F.10}$$

$$\hat{R}^n_t=\rho^{R^n}\hat{R}^n_{t-1}+\xi\hat{\pi}_t+\rho^Y\hat{Y}_t+\epsilon^{R^n}_t \tag{F.11}$$

$$\hat{G}_t=\rho^G\hat{G}_{t-1}+\epsilon^G_t \tag{F.12}$$

$$\hat{\sigma}_{\omega,t}=\rho^{\sigma_\omega}\hat{\sigma}_{\omega,t-1}+\epsilon^{\sigma_\omega}_t \tag{F.13}$$

Entrepreneurial Consumption and Net Worth

The evolution of entrepreneurial net worth is given by (F.14), where (F.15) defines leverage. Entrepreneurial consumption is defined by (F.16) and the financial premium is given by (F.17).

$$\hat{N}_{t+1} = \epsilon_N(\hat{N}_t + \hat{R}_{t+1} + \kappa(\hat{R}^k_{t+1} - \hat{R}_{t+1}) + \nu_\Psi \hat{\sigma}_{\omega,t}) + (1-\epsilon_N)(\hat{Y}_t - \hat{\mathcal{X}}_t), \tag{F.14}$$

$$\hat{\kappa}_t = \hat{K}_t + \hat{Q}_t - \hat{N}_t \tag{F.15}$$

$$\hat{C}^e_{t+1} = \hat{N}_t + \hat{R}_{t+1} + \kappa(\hat{R}^k_{t+1} - \hat{R}_{t+1}) + \nu_\Psi \hat{\sigma}_{\omega,t} \tag{F.16}$$

$$\mathbb{E}_t\hat{R}^k_{t+1} - \mathbb{E}_t\hat{R}_{t+1} = \nu_\kappa \hat{\kappa}_t + \nu_\sigma \hat{\sigma}_{\omega,t} \tag{F.17}$$

Dynamics of the Lending Rate

The BGG lending rate is defined as (F.18a), the CFP lending rate is defined as (F.18b) and the optimal lending rate is defined as (F.18c). These log-linear expressions are derived in Appendix G.

$$\hat{R}_{t+1} - \mathbb{E}_t\hat{R}_{t+1} = \begin{cases} 0 & \text{(F.18a)} \\ \hat{R}^k_{t+1} - \mathbb{E}_t R^k_{t+1} - \tilde{\alpha}\sigma(\hat{C}_{t+1} - \mathbb{E}_t\hat{C}_{t+1}) & \text{(F.18b)} \\ \hat{R}^k_{t+1} - \mathbb{E}_t R^k_{t+1} - \tilde{\alpha}\Big[\sigma(\hat{C}_{t+1} - \mathbb{E}_t\hat{C}_{t+1}) + \hat{\Psi}_{t+1} - \mathbb{E}_t\hat{\Psi}_{t+1}\Big] & \text{(F.18c)} \end{cases}$$

$$\hat{\Psi}_{t+1} = \epsilon_N \mathbb{E}_{t+1}\Big\{(\kappa-1)(\hat{R}^k_{t+2} - \hat{R}_{t+2}) + \hat{R}^k_{t+2} + \nu_\Psi \hat{\sigma}_{\omega,t+1} + \hat{\Psi}_{t+2}\Big\} \tag{F.19}$$

Monitoring Costs

$$\hat{\phi}_{\mu_t} = \hat{C}^e_{t+1} + \nu_\mu\left(\frac{1}{\kappa-1}\hat{\kappa}_t - (\hat{R}_{k,t+1} - \hat{R}_{t+1})\right) + \nu_{\sigma,\mu}\hat{\sigma}_{\omega,t} \tag{F.20}$$

G Log-linearization of the Lending Contracts

Log-linearization of the Common Optimality Condition

We begin by log-linearizing the common optimality condition for each contract. The non-linear participation constraint and FOC are, respectively:

$$\beta\mathbb{E}_t\Big\{U_{C,t+1}\Big\}k_t R^k_{t+1} h(\bar{\omega}_{t+1}, \sigma_{\omega,t}) - (k_t - 1)U_{C,t} = 0, \tag{G.1}$$

$$\kappa_t \mathbb{E}_t\Big\{R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega,t})\Big\} = -\mathbb{E}_t\left\{\frac{g'(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{h'(\bar{\omega}_{t+1}, \sigma_{\omega,t})}\right\}\frac{1}{\mathbb{E}_t\Lambda_{t,t+1}}. \tag{G.2}$$

In their linearized form, these become:

$$-\sigma\Big(\mathbb{E}_t\hat{C}_{t+1}-\hat{C}_t\Big)+\hat{R}^k_{t+1}+\frac{h_\omega}{h}\bar{\omega}\hat{\omega}_{t+1}+\frac{h_{\sigma_\omega}}{h}\sigma_\omega\hat{\sigma}_{\omega,t}=\frac{1}{\kappa-1}\hat{\kappa}_t, \tag{G.3}$$

$$\hat{\kappa}_t+\mathbb{E}_tR^k_{t+1}+\frac{g_\omega}{g}\bar{\omega}\mathbb{E}_t\hat{\omega}_{t+1}+\frac{g_{\sigma_\omega}}{g}\sigma_\omega\hat{\sigma}_{\omega,t}=-\mathbb{E}_t\hat{\Lambda}_{t,t+1}+\left(\frac{g_{\omega\omega}}{g_\omega}-\frac{h_{\omega\omega}}{h_\omega}\right)\bar{\omega}\mathbb{E}_t\hat{\omega}_{t+1}+\left(\frac{g_{\omega\sigma}}{g_\omega}-\frac{h_{\omega\sigma}}{h_\omega}\right)\sigma_\omega\hat{\sigma}_{\omega,t}. \tag{G.4}$$

Now we take the expected value of the participation constraint (G.3) and obtain:

$$\mathbb{E}_t\hat{\Lambda}_{t,t+1}+\mathbb{E}_t\hat{R}^k_{t+1}+\frac{h_\omega}{h}\bar{\omega}\mathbb{E}_t\hat{\omega}_{t+1}+\frac{h_{\sigma_\omega}}{h}\sigma_\omega\hat{\sigma}_{\omega,t}=\frac{1}{\kappa-1}\hat{\kappa}_t. \tag{G.5}$$

Define $\hat{\triangle}_t=\hat{R}^k_t-\hat{R}_t$, and rewrite the system as:

$$\begin{aligned}&\frac{1}{\kappa-1}\hat{\kappa}_t-\frac{h_{\sigma_\omega}}{h}\sigma_\omega\hat{\sigma}_{\omega,t}-\mathbb{E}_t\hat{\triangle}_{t+1}=\frac{h_\omega}{h}\bar{\omega}\mathbb{E}_t\hat{\omega}_{t+1},\\&\hat{\kappa}_t+\mathbb{E}_t\hat{\triangle}_{t+1}-\left(\frac{g_{\omega\sigma}}{g_\omega}-\frac{h_{\omega\sigma}}{h_\omega}-\frac{g_{\sigma_\omega}}{g}\right)\sigma_\omega\hat{\sigma}_{\omega,t}=\left(\frac{g_{\omega\omega}}{g_\omega}-\frac{h_{\omega\omega}}{h_\omega}-\frac{g_\omega}{g}\right)\bar{\omega}\mathbb{E}_t\hat{\omega}_{t+1}.\end{aligned} \tag{G.6}$$

Now we can set these two equations equal to each other and eliminate ω:

$$\hat{\kappa}_t+\mathbb{E}_t\hat{\triangle}_{t+1}-\left(\frac{g_{\omega\sigma}}{g_\omega}-\frac{h_{\omega\sigma}}{h_\omega}-\frac{g_{\sigma_\omega}}{g}\right)\sigma_\omega\hat{\sigma}_{\omega,t}=\frac{\left(\frac{g_{\omega\omega}}{g_\omega}-\frac{h_{\omega\omega}}{h_\omega}-\frac{g_\omega}{g}\right)}{\frac{h_\omega}{h}}\left(\frac{1}{\kappa-1}\hat{\kappa}_t-\frac{h_{\sigma_\omega}}{h}\sigma_\omega\hat{\sigma}_{\omega,t}-\mathbb{E}_t\hat{\triangle}_{t+1}\right). \tag{G.7}$$

We can rearrange this to obtain:

$$\mathbb{E}_t\hat{\triangle}_{t+1}=\frac{\left(\frac{g_{\omega\omega}}{g_\omega}-\frac{h_{\omega\omega}}{h_\omega}\right)}{\left(\frac{g_{\omega\omega}}{g_\omega}-\frac{h_{\omega\omega}}{h_\omega}-\frac{g_\omega}{g}+\frac{h_\omega}{h}\right)}\frac{1}{\kappa-1}\hat{\kappa}_t+\frac{\left[-\frac{h_{\sigma_\omega}}{h}\left(\frac{g_{\omega\omega}}{g_\omega}-\frac{h_{\omega\omega}}{h_\omega}-\frac{g_\omega}{g}\right)+\frac{h_\omega}{h}\left(\frac{g_{\omega\sigma}}{g_\omega}-\frac{h_{\omega\sigma}}{h_\omega}-\frac{g_{\sigma_\omega}}{g}\right)\right]}{\left(\frac{g_{\omega\omega}}{g_\omega}-\frac{h_{\omega\omega}}{h_\omega}-\frac{g_\omega}{g}+\frac{h_\omega}{h}\right)}\sigma_\omega\hat{\sigma}_{\omega,t}. \tag{G.8}$$

This can be simplified to give the log-linear optimality condition, which is identical for all three lending contracts:

$$\mathbb{E}_t\hat{\triangle}_{t+1}=\nu_\kappa\hat{\kappa}_t+\nu_\sigma\sigma_\omega\hat{\sigma}_{\omega,t} \tag{G.9}$$

where

$$\nu_\kappa = \frac{\left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}\right)}{\left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} - \frac{g_\omega}{g} + \frac{h_\omega}{h}\right)} \frac{1}{\kappa - 1}, \tag{G.10}$$

$$\nu_\sigma = \frac{\left[-\frac{h_{\sigma_\omega}}{h}\left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} - \frac{g_\omega}{g}\right) + \frac{h_\omega}{h}\left(\frac{g_{\omega\sigma}}{g_\omega} - \frac{h_{\omega\sigma}}{h_\omega} - \frac{g_{\sigma_\omega}}{g}\right)\right]}{\left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} - \frac{g_\omega}{g} + \frac{h_\omega}{h}\right)}. \tag{G.11}$$

Log-linearization of the BGG Lending Rate

The log-linear lending rate in BGG is given by:

$$\hat{R}_{t+1} - \mathbb{E}_t \hat{R}_{t+1} = 0 \tag{G.12}$$

Log-linearization of the CFP Contract

The non-linear participation constraint and FOC are, respectively:

$$\beta \mathbb{E}_t \left\{ U_{C,t+1} k_t R^k_{t+1} h(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right\} - (k_t - 1) U_{C,t} = 0, \tag{G.13}$$

$$\kappa_t \mathbb{E}_t \left\{ R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right\} = -\frac{g'(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{h'(\bar{\omega}_{t+1}, \sigma_{\omega,t})} \frac{1}{\Lambda_{t,t+1}}. \tag{G.14}$$

In their linearized form, these become:

$$\mathbb{E}_t \hat{R}^k_{t+1} - \mathbb{E}_t \hat{R}_{t+1} + \frac{h_\omega}{h} \bar{\omega} \mathbb{E}_t \hat{\omega}_{t+1} + \frac{h_{\sigma_\omega}}{h} \sigma_\omega \hat{\sigma}_{\omega,t} = \frac{1}{\kappa - 1} \hat{\kappa}_t, \tag{G.15}$$

$$\hat{\kappa}_t + \mathbb{E}_t R^k_{t+1} + \frac{g_\omega}{g} \bar{\omega} \mathbb{E}_t \bar{\omega}_{t+1} + \frac{g_{\sigma_\omega}}{g} \sigma_\omega \hat{\sigma}_{\omega,t} = -\hat{\Lambda}_{t,t+1} + \left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}\right) \bar{\omega} \hat{\omega}_{t+1} + \left(\frac{g_{\omega\sigma}}{g_\omega} - \frac{h_{\omega\sigma}}{h_\omega}\right) \sigma_\omega \hat{\sigma}_{\omega,t}. \tag{G.16}$$

Now we plug the participation constraint (G.15) into the FOC (G.16) and obtain

$$\hat{\kappa}_t + \mathbb{E}_t R^k_{t+1} + \frac{\frac{g_\omega}{g}}{\frac{h_\omega}{h}} \left(\frac{1}{\kappa - 1} \hat{\kappa}_t - \mathbb{E}_t R^k_{t+1} + \mathbb{E}_t R_{t+1} - \frac{h_{\sigma_\omega}}{h} \sigma_\omega \hat{\sigma}_{\omega,t} \right) + \frac{g_{\sigma_\omega}}{g} \sigma_\omega \hat{\sigma}_{\omega,t} = -\hat{\Lambda}_{t,t+1} + \left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}\right) \bar{\omega} \hat{\omega}_{t+1} + \left(\frac{g_{\omega\sigma}}{g_\omega} - \frac{h_{\omega\sigma}}{h_\omega}\right) \sigma_\omega \hat{\sigma}_{\omega,t}. \tag{G.17}$$

Using $-\frac{\frac{g_\omega}{g}}{\frac{h_\omega}{h}} = (k-1)$, we can simplify the last expression to:

$$k(\mathbb{E}_t\hat{R}^k_{t+1} - \mathbb{E}_t\hat{R}_{t+1}) + \hat{\Lambda}_{t,t+1} - \mathbb{E}_t\hat{\Lambda}_{t,t+1} + \left(-\frac{g_\omega h_{\sigma_\omega}}{g h_\omega} + \frac{g_{\sigma_\omega}}{g} - \frac{g_{\omega\sigma}}{g_\omega} + \frac{h_{\omega\sigma}}{h_\omega}\right)\sigma_\omega\hat{\sigma}_{\omega,t} = \left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}\right)\bar{\omega}\hat{\omega}_{t+1}. \tag{G.18}$$

Define lender's returns by log-linearizing $R_{t+1} = \frac{\kappa_t}{\kappa_{t-1}} h(\bar{\omega}_{t+1}, \sigma_{\omega,t}) R^k_{t+1}$:

$$\hat{R}_{t+1} = -\frac{1}{k-1}\hat{k}_t + \frac{h_\omega}{h}\bar{\omega}\hat{\omega}_{t+1} + \frac{h_{\sigma_\omega}}{h}\sigma_\omega\hat{\sigma}_{\omega,t} + \hat{R}^k_{t+1}. \tag{G.19}$$

Now substitute in the expression for the cutoff and obtain

$$\begin{aligned}\hat{R}_{t+1} =& \frac{\frac{h_\omega}{h}}{\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}}\left[k(\mathbb{E}_t\hat{R}^k_{t+1} - \mathbb{E}_t\hat{R}_{t+1}) + \hat{\Lambda}_{t,t+1} - \mathbb{E}_t\hat{\Lambda}_{t,t+1} + \left(-\frac{g_\omega h_{\sigma_\omega}}{g h_\omega} + \frac{g_{\sigma_\omega}}{g} - \frac{g_{\omega\sigma}}{g_\omega} + \frac{h_{\omega\sigma}}{h_\omega}\right)\sigma_\omega\hat{\sigma}_{\omega,t}\right] \\ & -\frac{1}{k-1}\hat{k}_t + \frac{h_{\sigma_\omega}}{h}\sigma_\omega\hat{\sigma}_{\omega,t} + \hat{R}^k_{t+1}. \end{aligned} \tag{G.20}$$

Finally, substitute in the expression for leverage:

$$\begin{aligned}\hat{R}_{t+1} =& \frac{\frac{h_\omega}{h}}{\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}}\left[\hat{\Lambda}_{t,t+1} - \mathbb{E}_t\hat{\Lambda}_{t,t+1} + \left(-\frac{g_\omega h_{\sigma_\omega}}{g h_\omega} + \frac{g_{\sigma_\omega}}{g} - \frac{g_{\omega\sigma}}{g_\omega} + \frac{h_{\omega\sigma}}{h_\omega}\right)\sigma_\omega\hat{\sigma}_{\omega,t}\right] \\ & + \frac{\left[-\frac{h_{\sigma_\omega}}{h}\left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} - \frac{g_\omega}{g}\right) + \frac{h_\omega}{h}\left(\frac{g_{\omega\sigma}}{g_\omega} - \frac{h_{\omega\sigma}}{h_\omega} - \frac{g_{\sigma_\omega}}{g}\right)\right]}{\left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}\right)}\sigma_\omega\hat{\sigma}_{\omega,t} \\ & + \frac{h_{\sigma_\omega}}{h}\sigma_\omega\hat{\sigma}_{\omega,t} + (\mathbb{E}_t\hat{R}^k_{t+1} - \mathbb{E}_t\hat{R}_{t+1}) + \hat{R}^k_{t+1}. \end{aligned} \tag{G.21}$$

After some rearranging and canceling out like terms, we obtain the log-linear CFP lending rate:

$$\hat{R}_{t+1} = \frac{\frac{h_\omega}{h}}{\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}}\left[\hat{\Lambda}_{t,t+1} - \mathbb{E}_t\hat{\Lambda}_{t,t+1}\right] - (\mathbb{E}_t\hat{R}^k_{t+1} - \mathbb{E}_t\hat{R}_{t+1}) + \hat{R}^k_{t+1}. \tag{G.22}$$

This can be rewritten in the same form as Corollary 2 in the text:

$$\hat{R}_{t+1} - \mathbb{E}_t\hat{R}_{t+1} = \hat{R}^k_{t+1} - \mathbb{E}_{t+1}\hat{R}^k_{t+1} - \tilde{\alpha}\sigma\left(\hat{C}_{t+1} - \mathbb{E}_t\hat{C}_{t+1}\right) \tag{G.23}$$

where $\tilde{\alpha} = \frac{\frac{h_\omega}{h}}{\frac{h_{\omega\omega}}{h_\omega} - \frac{g_{\omega\omega}}{g_\omega}}$.

Log-linearization of the Optimal Contract

The non-linear participation constraint and FOC are, respectively:

$$\beta \mathbb{E}_t \left\{ U_{C,t+1} k_t R^k_{t+1} h(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right\} - (k_t - 1) U_{C,t} = 0 \tag{G.24}$$

$$\kappa_t \mathbb{E}_t \left\{ \Psi_{t+1} R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right\} = -\frac{g'(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{h'(\bar{\omega}_{t+1}, \sigma_{\omega,t})} \frac{\Psi_{t+1}}{\Lambda_{t,t+1}}, \tag{G.25}$$

where Ψ_t is defined as:

$$\Psi_t = 1 + \kappa_t \mathbb{E}_t \{ g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) R_{k,t+1} \Psi_{t+1} \}. \tag{G.26}$$

In their linearized form, the participation constraint and FOC become:

$$-\sigma \left(\mathbb{E}_t \hat{C}_{t+1} - \hat{C}_t \right) + \mathbb{E}_t \hat{R}^k_{t+1} + \frac{h_\omega}{h} \bar{\omega} \mathbb{E}_t \hat{\omega}_{t+1} + \frac{h_\sigma}{h} \sigma_\omega \hat{\sigma}_{\omega,t} = \frac{1}{\kappa - 1} \hat{\kappa}_t, \tag{G.27}$$

$$\begin{aligned} \mathbb{E}_t \Psi_{t+1} + \hat{\kappa}_t + \mathbb{E}_t R^k_{t+1} + \frac{g_\omega}{g} \bar{\omega} \mathbb{E}_t \bar{\omega}_{t+1} + \frac{g_{\sigma_\omega}}{g} \sigma_\omega \hat{\sigma}_{\omega,t} =& \hat{\Psi}_{t+1} - \hat{\Lambda}_{t,t+1} + \left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} \right) \bar{\omega} \hat{\omega}_{t+1} \\ &+ \left(\frac{g_{\omega\sigma}}{g_\omega} - \frac{h_{\omega\sigma}}{h_\omega} \right) \sigma_\omega \hat{\sigma}_{\omega,t}. \end{aligned} \tag{G.28}$$

where

$$\hat{\Psi}_t = \epsilon_N \left(\hat{\kappa}_t + \frac{g_\omega}{g} \bar{\omega} \mathbb{E}_t \hat{\omega}_{t+1} + \frac{g_\sigma}{g} \sigma_\omega \hat{\sigma}_{\omega,t} + \mathbb{E}_t \hat{R}^k_{t+1} + \mathbb{E}_t \hat{\Psi}_{t+1} \right). \tag{G.29}$$

Now we substitute the participation constraint into the optimality condition and obtain:

$$\begin{aligned} &\hat{\kappa}_t + \mathbb{E}_t R^k_{t+1} + \frac{\frac{g_\omega}{g}}{\frac{h_\omega}{h}} \left(\frac{1}{\kappa - 1} \hat{\kappa}_t - \mathbb{E}_t R^k_{t+1} + \mathbb{E}_t R_{t+1} - \frac{h_{\sigma_\omega}}{h} \sigma_\omega \hat{\sigma}_{\omega,t} \right) + \frac{g_{\sigma_\omega}}{g} \sigma_\omega \hat{\sigma}_{\omega,t} \\ &= -\hat{\Lambda}_{t,t+1} + \left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} \right) \bar{\omega} \hat{\omega}_{t+1} + \left(\frac{g_{\omega\sigma}}{g_\omega} - \frac{h_{\omega\sigma}}{h_\omega} \right) \sigma_\omega \hat{\sigma}_{\omega,t} + \hat{\Psi}_{t+1} - \mathbb{E}_t \hat{\Psi}_{t+1}. \end{aligned} \tag{G.30}$$

Using $-\frac{\frac{g_\omega}{g}}{\frac{h_\omega}{h}} = (k-1)$, we can simplify the previous expression to

$$\begin{aligned} \left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} \right) \bar{\omega} \hat{\omega}_{t+1} =& k (\mathbb{E}_t \hat{R}^k_{t+1} - \mathbb{E}_t \hat{R}_{t+1}) + \hat{\Lambda}_{t,t+1} - \mathbb{E}_t \hat{\Lambda}_{t,t+1} \\ &+ \left(-\frac{g_\omega h_{\sigma_\omega}}{g h_\omega} + \frac{g_{\sigma_\omega}}{g} - \frac{g_{\omega\sigma}}{g_\omega} + \frac{h_{\omega\sigma}}{h_\omega} \right) \sigma_\omega \hat{\sigma}_{\omega,t} - (\hat{\Psi}_{t+1} - \mathbb{E}_t \hat{\Psi}_{t+1}). \end{aligned} \tag{G.31}$$

Define the lender's returns by log-linearizing $R_{t+1} = \frac{\kappa_t}{\kappa_t - 1} h(\bar{\omega}_{t+1}, \sigma_{\omega,t}) R^k_{t+1}$:

$$\hat{R}_{t+1} = -\frac{1}{k-1} \hat{k}_t + \frac{h_\omega}{h} \bar{\omega} \hat{\omega}_{t+1} + \frac{h_{\sigma_\omega}}{h} \sigma_\omega \hat{\sigma}_{\omega,t} + \hat{R}^k_{t+1}. \tag{G.32}$$

Now substitute in the expression for the cutoff and obtain:

$$\hat{R}_{t+1} = \frac{\frac{h_\omega}{h}}{\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}} \Bigg[k(\mathbb{E}_t \hat{R}^k_{t+1} - \mathbb{E}_t \hat{R}_{t+1}) + \hat{\Lambda}_{t,t+1} - \mathbb{E}_t \hat{\Lambda}_{t,t+1} + \Bigg(- \frac{g_\omega h_{\sigma_\omega}}{g h_\omega} + \frac{g_{\sigma_\omega}}{g} - \frac{g_{\omega\sigma}}{g_\omega} + \frac{h_{\omega\sigma}}{h_\omega} \Bigg) \sigma_\omega \hat{\sigma}_{\omega,t}$$
$$- (\hat{\Psi}_{t+1} - \mathbb{E}_t \hat{\Psi}_{t+1}) \Bigg] - \frac{1}{k-1} \hat{k}_t + \frac{h_{\sigma_\omega}}{h} \sigma_\omega \hat{\sigma}_{\omega,t} + \hat{R}^k_{t+1}. \qquad \text{(G.33)}$$

Rearranging and canceling out like terms yields:

$$\hat{R}_{t+1} = \frac{\frac{h_\omega}{h}}{\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}} \Big[\hat{\Lambda}_{t,t+1} - \mathbb{E}_t \hat{\Lambda}_{t,t+1} - (\hat{\Psi}_{t+1} - \mathbb{E}_t \ \hat{\Psi}_{t+1}) \Big] + \hat{R}^k_{t+1} - (\mathbb{E}_t \hat{R}^k_{t+1} - \mathbb{E}_t \hat{R}_{t+1}). \quad \text{(G.34)}$$

This can be rewritten in the same form as Corollary 3 in the text:

$$\hat{R}_{t+1} - \mathbb{E}_t \hat{R}_{t+1} = \hat{R}^k_{t+1} - \mathbb{E}_{t+1} \hat{R}^k_{t+1} - \tilde{\alpha} \Big[\sigma \big(\hat{C}_{t+1} - \mathbb{E}_t \hat{C}_{t+1} \big) + \hat{\Psi}_{t+1} - \mathbb{E}_t \hat{\Psi}_{t+1} \Big] \qquad \text{(G.35)}$$

where $\tilde{\alpha} = \frac{\frac{h_\omega}{h}}{\frac{h_{\omega\omega}}{h_\omega} - \frac{g_{\omega\omega}}{g_\omega}}$. Finally, we eliminate ω in the expression for $\hat{\Psi}_t$:

$$\hat{\Psi}_t = \epsilon_N \Bigg[\hat{\kappa}_t + \frac{\frac{g_\omega}{g}}{\frac{h_\omega}{h}} \Bigg(\frac{1}{\kappa - 1} \hat{\kappa}_t - \mathbb{E}_t \hat{R}^k_{t+1} + \mathbb{E}_t \hat{R}_{t+1} - \frac{h_{\sigma_\omega}}{h} \sigma_\omega \hat{\sigma}_{\omega,t} \Bigg) + \frac{g_\sigma}{g} \sigma_\omega \hat{\sigma}_{\omega,t} + \mathbb{E}_t \hat{R}^k_{t+1} + \mathbb{E}_t \hat{\Psi}_{t+1} \Bigg] \quad \text{(G.36)}$$

and we can rearrange this expression to match Corollary 3 in the text:

$$\hat{\Psi}_{t+1} = \epsilon_N \mathbb{E}_{t+1} \Big\{ (\kappa - 1)(\hat{R}^k_{t+2} - \hat{R}_{t+2}) + \hat{R}^k_{t+2} + \nu_\Psi \hat{\sigma}_{\omega,t+1} + \hat{\Psi}_{t+2} \Big\} \qquad \text{(G.37)}$$

where $\nu_\Psi = \frac{g_\sigma - h_\sigma \frac{g_\omega}{h_\omega}}{g} \sigma_\omega$.