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BUYING FIRST OR SELLING FIRST IN HOUSING MARKETS

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JEL Classification: R21 and R31
Keywords: housing market, order of transactions, search frictions and strategic complementarities

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December 30, 2014

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1 Introduction

A large fraction of households move within the same local housing market every year. Many of these moves are by owner-occupiers who buy a new property and sell their old housing unit. However, it takes time to transact in the housing market, so a home-owner that moves may end up owning either two units or be forced to rent for some period, depending on the sequence of transactions. Either of these two alternatives may be costly.[1] There is anecdotal evidence that the incentives to "buy first" (buy the new property before selling the old property) or "sell first" (sell the old property before buying the new one) may depend on the state of the housing market.[2] If the transaction sequence decisions of moving owner-occupiers in turn affect the housing market, there are equilibrium feedbacks that could be important for housing market dynamics.

In this paper we study a tractable equilibrium model of the housing market, which explicitly features trading delays and a transaction sequence decision for moving owner-occupiers. We show that changes in the transaction sequence choice of moving home-owners can have powerful effects on the housing market and can lead to large fluctuations in the stock of houses for sale, time-on-market, trading volume, and prices.

In the model, agents continuously enter and exit a local housing market. They have a preference for owning housing over renting and consequently search for a housing unit to buy. The market is characterized by a frictional trading process in the form of search-and-matching frictions. This leads to a positive expected time-on-market for buyers and sellers, which is affected by the tightness in the market, the ratio of buyers to sellers. Once an agent becomes an owner-occupier, he may be hit by an idiosyncratic preference shock over his life cycle. In that case he becomes "mismatched" with his current house and wants to move internally in the same housing market. To do that the mismatched owner-occupier has to choose whether to buy the new housing unit first and then sell his old unit (buy first), or sell his old unit first and then buy (sell first). Given trading delays, this may lead to the agent becoming a double owner (owning two housing units) or a forced renter (owning no housing) for some time, which is costly. The expected time of remaining in such a state depends on the time-on-market for sellers and buyers, respectively.

Whenever the costs of a double owner or a forced renter are high relative to the costs of mismatch, the mismatched owner-occupier prefers buying first over selling first whenever there are *more* buyers than sellers in the market, i.e. when the market tightness is high. This behavior is intuitive once one considers how the expected time-on-market for a buyer and a seller move with the buyer-seller ratio. Whenever there are more buyers than sellers, the expected time-on-market is low for a seller and

[1] The following quote from Realtor.com, an online real estate broker, highlights this issue: "If you sell first, you may find yourself under a tight deadline to find another house, or be forced in temporary quarters. If you buy first, you may be saddled with two mortgage payments for at least a couple months." (Dawson (2013))

[2] A common realtor advice to home-owners that have to move is to "buy first" in a "hot" market, when there are more buyers than sellers and prices are high or expected to increase, and "sell first" in a "cold" market, when there are more sellers than buyers and house prices are depressed or expected to fall. Anundsen and Røed Larsen (forthcoming) provides evidence on the response of the intentions of owner-occupiers to buy first or sell first to the state of the housing market using survey data for Norway. Additional anecdotal evidence from realtors in the U.S. points to a similar dependence. In Section 2 we provide direct evidence for this link using data for the Copenhagen housing market.

high for a buyer. Consequently, if a mismatched owner buys first he expects to spend a longer time as a buyer, and hence to remain mismatched longer. However, once he buys, he expects to stay with two houses for a short time while searching for a buyer for his old property. Conversely, choosing to sell first in that case implies a short time-to-sell and a short time of remaining mismatched but a longer time of searching to buy a new unit afterwards. Because the flow costs in between the two transactions are higher than during mismatch, buying first clearly dominates selling first in that case.

Given equality between the stock of non-owner buyers and of vacant houses, whenever all mismatched owners buy first, the only sellers are the holders of vacant houses, and so the market ends with more buyers than sellers in steady state. Nevertheless, this high buyer-seller ratio is consistent with the incentives of mismatched owners to buy first. Conversely, when all mistmatched owners sell first, the only buyers are non-owners, so there are more sellers than buyers in steady state. However, a low buyer-seller ratio is consistent with the incentives of mismatched owners to sell first.

Therefore, the equilibrium effect on the buyer-seller ratio creates a strategic complementarity in the decisions of mismatched owners to buy first or sell first and as a result there can exist multiple steady state equilibria. In one steady state equilibrium (a "Buyers' market" equilibrium), mismatched owners prefer to sell first, the market tightness is low and the expected time-on-market for sellers is high. In the other steady state equilibrium (a "Sellers' market" equilibrium), mismatched owners prefer to buy first, the market tightness is high and the expected time-on-market for sellers is low.[3]

There can also exist equilibria with self-fulfilling fluctuations in prices and market tightness. Since mismatched owners are more likely to buy first (sell first) when they expect price appreciation (depreciation), they end up exerting a destabilizing force on the housing market when prices respond to changes in market tightness. For example, if agents expect prices to depreciate, they are more likely to sell first. However, this decreases the buyer-seller ratio, which in turn drags down house prices and thus confirms the agents' expectations.

Switches between the "Sellers' market" and the "Buyers' market" equilibria leads to fluctuations in the housing market. Specifically, moving from the "Sellers' market" to the "Buyers' market" equilibrium is associated with an increase in the stock of houses for sale, an increase in time-on-market for sellers, and a drop in transactions and prices. This behavior is broadly consistent with evidence on the housing cycle. Also, as we show in a simple numerical example, the fluctuations generated by the model can be substantial.

Related Literature. The paper is related to the growing literature on search-and-matching models of the housing market and fluctuations in housing market liquidity, initiated by the seminal work of Wheaton (1990).This foundational paper is the first to consider a frictional model of the housing market to explain the existence of a "natural" vacancy rate in housing markets and the negative comovement between deviations from this natural rate and house prices. In that model, mismatched

[3]Note that we derive this multiplicity under the assumption of a constant returns to scale matching function. Therefore, the strategic complementarity does not arise from increasing returns to scale as in Diamond (1982).

homeowners must also both buy and sell a housing unit. However, the model implicitly assumes that the cost of becoming a forced renter with no housing is prohibitively large, so that mismatched owners always buy first. As we show in our paper, allowing mismatched owners to endogenously choose whether to buy first or sell first has important consequences for the housing market.

The paper is particularly related to the literature on search frictions and propagation and amplification of shocks in the housing market (Diaz and Jerez (2013), Head, Lloyd-Ellis, and Sun (2014), Ngai and Tenreyro (2014), Guren and McQuade (2013), Anenberg and Bayer (2013), and Ngai and Sheedy (2014)). This literature shows how search frictions naturally propagate aggregate shocks due to the slow adjustment in stock of buyers and sellers. Additionally, they can amplify price responses to aggregate shocks, which in Walrasian models would be fully absorbed by quantity responses.[4]

Diaz and Jerez (2013) calibrate a model of the housing market in the spirit of Wheaton (1990) where mismatched owners must buy first as well as a model where they must sell first. They show that each model explains some aspects of the data on housing market dynamics pointing to the importance of a model that contains both choices. Other models of the housing market assume that the sequence of transactions is irrelevant, which implicitly assumes that the intermediate step of a transaction for an existing owner is costless (Ngai and Tenreyro (2014), Head, Lloyd-Ellis, and Sun (2014), Guren and McQuade (2013), Ngai and Sheedy (2014)).

Ngai and Sheedy (2014) model an endogenous moving decision based on idiosyncratic match quality as an amplification mechanism of sales volume. The paper shows how the endogenous participation decisions of mismatched owners in the housing market can explain why time-on-market for sellers can decrease while the stock of houses for sale increases at the same time, as was the case during the housing boom of the late 90s and early 2000s. In our model we assume that mismatched owners always participate and instead focus on their transaction sequence decisions. The implications we draw from our analysis are therefore complementary to the insights in their paper.

Anenberg and Bayer (2013) is a recent contribution that is closest to our paper, particularly in terms of motivation. The paper studies a rich quantitative model of the housing market with two segments, in which some agents are sellers in the first segment, and simultaneously choose whether to also be buyers in the second segment. Shocks to the flow of new buyers in the first segment are transmitted and amplified onto the second segment through the decisions of these agents to participate as buyers in that second segment. Therefore, unlike our paper, there is no strategic complementarity in the decisions of mismatched owners. As discussed above, the feedback between market tightness and the decisions of mismatched owners that creates this strategic complementarity is the key driver of multiplicity, self-fulfilling fluctuations, and volatility in our model. Also, in contrast to our model, buying-first in that paper is a stochastic outcome rather than an endogenous

[4] The paper is also broadly related to the Walrasian literature on house price dynamics and volatility (Stein (1995), Ortalo-Magne and Rady (2006), Gleaser, Gyourko, Morales, and Nathanson (2012)).

choice.[5,6]

The rest of the paper is organized as follows. In the next Section we present some motivating facts using individual level data from Denmark. Section 3 sets up the basic model of the housing market. Section 4 characterizes the decisions of mismatched owners and discusses the equilibrium multiplicity and the implications from equilibrium switches. Section 5 shows how the incentives of mismatched owners to buy first or sell first depend on price expectations and shows that there can exist equilibria with self-fulfilling fluctuations in house prices and tightness. Section 6 extends the equilibrium multiplicity to an environment where prices are determined by Nash bargaining. Section 7 includes additional extensions, including allowing mismatched owners to simultaneously participate as buyers and sellers. Section 8 provides a discussion on the institutional details of transacting for several countries and concludes.

2 Motivating Facts

We start by providing some motivating facts about the transaction sequence decisions of owner-occupiers for Copenhagen, Denmark. We focus on the Copenhagen urban area for the period 1992-2010. We use the Danish ownership register, which records the property ownership of individuals and legal entities as of January 1st of a given year. We combine that with a record of property sales for each year. The unique owner and property identifiers give us a matched property-owner data set, which we use to keep track of the transactions of individuals over time. We focus on individual owners who are recorded as the primary owner of a property.

We use the ownership records of individual owners over time to identify owner-occupiers who buy and sell in the Copenhagen housing market.[7] We then use the property sales record to determine the sale dates and takeover dates for the two transactions. Based on those we construct a variable that measures the time difference between the sale of the old property and the purchase of the new property. Owner-occupiers, for which this difference is negative are classified as "selling first", while those with a positive difference are classified as "buying first".

Figure 1 shows the distribution of the time difference between transaction dates (Panel 1a) and takeover dates (Panel 1b) for owner-occupiers who both buy and sell in Copenhagen during our sample period. There is substantial dispersion in the time difference between transaction dates, which suggests that a large fraction of these home-owners cannot synchronize the two transactions on the same date. Specifically, there is substantial mass even in the tails of the distribution.

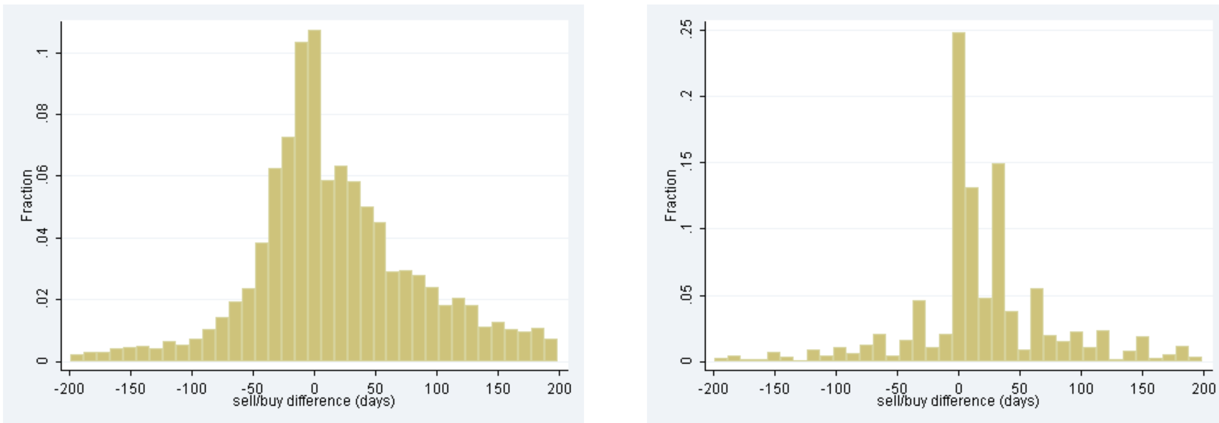
[5] Maury and Tripier (2011) study a modification of the Wheaton (1990) model, in which mismatched owners can buy and sell simultaneously, which they use to study price dispersion in the housing market. However, they do not consider the feedback from buying and selling decisions on the stock-flow process and on market tightness. This feedback is key for the mechanisms we explore in our paper.

[6] Albrecht, Anderson, Smith, and Vroman (2007) study a search model where buyers may be "desperate" if they search is unsuccessful sufficiently long, while Albrecht et al. (2012, 2014) study optimal bidding behavior in housing markets with search frictions.

[7] The Appendix contains detailed information on the data used and on the procedure for identifying owner-occupiers that buy and sell. Given the way we identify these owner-occupiers, we have a consistent count for the number of owners who buy first or sell first in a given year for the years 1993 to 2008.

Figure 1: Distribution of the time difference between "sell" and "buy" transaction dates (a) and takeover dates (b) for homeowners who both buy and sell in Copenhagen (1993-2008).

(a) (b)



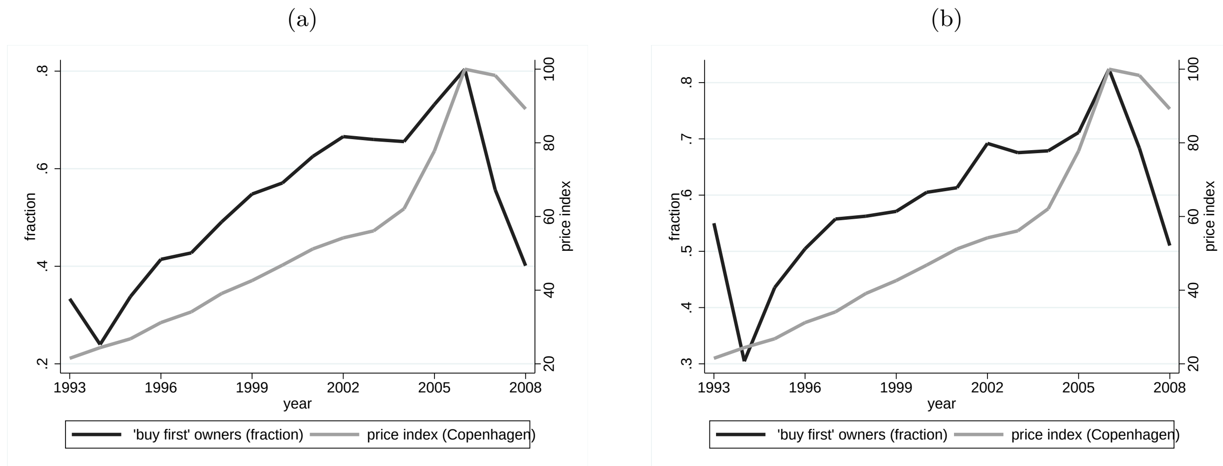
Examining the difference in takeover dates shows a similar picture. Even though the distribution is more compressed in that case since home-owners try to a greater extent to synchronize the takeover dates so that they occur on the same day or in a close interval, a large fraction of home-owners face a time difference of a month or more between the two transactions.[8] Overall, these distributions suggest that for home-owners that buy and sell in the same housing market the time difference between transactions can be substantial, confirming the anecdotal evidence cited in the introduction.

Another important observation is that the two distributions are right skewed, so home-owners tend to buy first during our sample period. This is confirmed when we examine the time series behavior of the fraction of home-owners that are identified as buying first in a given year from 1993-2008, as Figure 2 shows. Similarly to Figure 1, the left-hand panel (Panel 2a) is based on transaction dates, while the right-hand panel (Panel 2b) is based on takeover dates. Both panels also contain a price index for single family homes for the Copenhagen housing market. As the figure shows, the fraction of owners that buy first is not constant over time but exhibits wide variations going from a low of around 0.3 in 1994 to a high of 0.8 in 2006 and then back to a low of around 0.4 in 2008. This fraction tracks closely the house price index increasing over most of the sample period and peaking in the same year. It is then followed by a substantial drop as house prices start to decline after 2006. Therefore, Figure 2 suggests that the decisions to buy first may be related to the state of the housing market.

A closer examination of the period 2004-2008 strengthens this conjecture. Speficically, Figure 3 illustrates the fluctuations in key housing market variables like the for-sale stock, seller time-on-market, transaction volume and prices for Copenhagen in the period 2004-2008. It also includes our constructed fraction of buy first owners for Copenhagen in the period 2004-2008. During the first

[8] It is interesting to note that for the difference between takeover dates there are mass points around 30 day multiples. The reason for this is that in Denmark takeover dates tend to fall on the first day of a given month.

Figure 2: Fraction of owners who "buy first" and housing market conditions in Copenhagen (1993-2008). Panel (a) is based on transaction dates, and panel (b) is based on takeover dates.



Notes: The series on the fraction of "buy first" owners is from own calculations based on registry data from Statistics Denmark. See the Appendix for a description on how we identify an owner as a buyer-and-seller or as a "buy first" ("sell first") owner. We compute annual counts of the number of "buy first" and "sell first" owners by looking at the year of the first transaction for each of these owners. The fraction of "buy first" owners is then the proportion of buyer-and-seller owners that buy first. For panel (a) the identification of an owner as buy first/sell first is based on the difference in the two transaction dates. For the second, it is based on the difference in the two takeover dates. The price index is a repeat sales price index for single family houses for Copenhagen (Region Hovedstaden) constructed by Statistics Denmark.

half of this period seller time-on-market (TOM), and the for-sale stock are low while the transaction volume and the fraction of buy first owners are high. There is a switch in all of these series around the 3rd quarter of 2006 and a quick reversal during which seller time-on-market and the for-sale stock increasing rapidly, while the fraction of buy first owners drops. Transaction volume is also lower during the second half of this period. Prices increase steadily during the first half of the period and then decline.

We take these three exhibits as indication that there is a non-trivial transaction sequence choice for owner-occupiers that move in the same housing market, that the time difference between the two transactions can be substantial, and that the decision to buy first or sell first is related to the state of housing markets. These facts motivate the theoretical model we study in this paper.

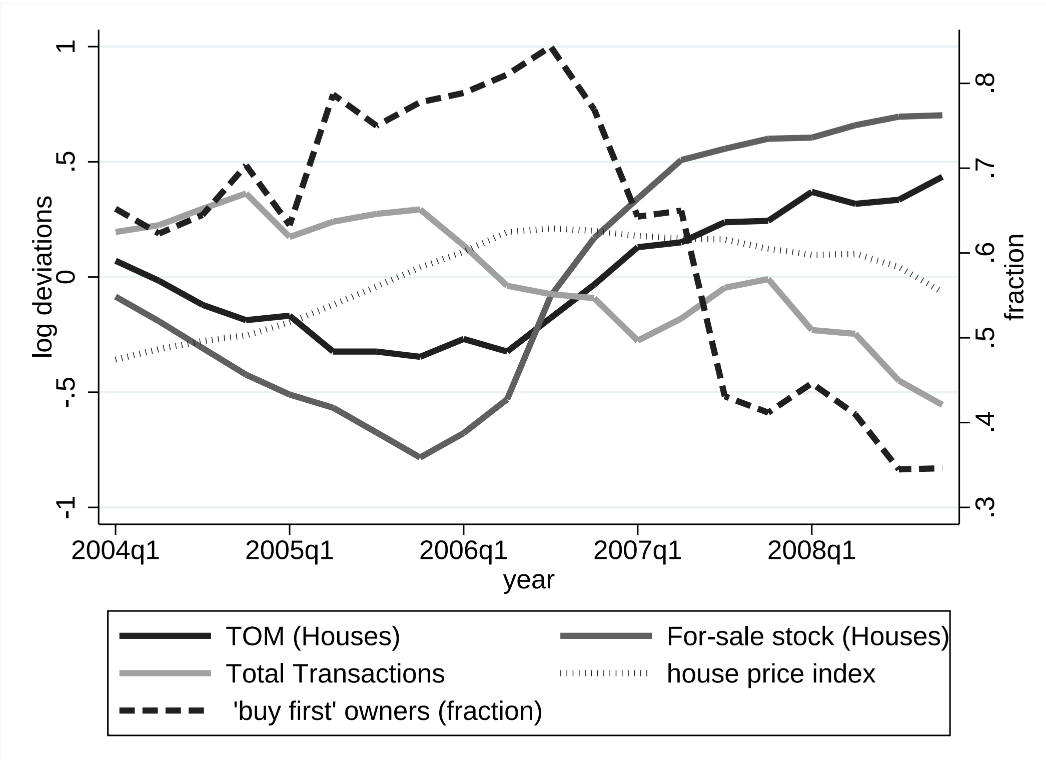
3 A Model of the Housing Market

3.1 Preferences

In this section we set up the basic model of a housing market characterized by trading frictions and re-trading shocks that will provide the main insights of our analysis. Time is continuous and runs forever, with $t \in [0, \infty)$. The housing market contains a unit measure of durable housing units that do not depreciate. In every instant there is a unit measure of agents in the economy.[9] Agents are

[9]One can think of this population size as arising from a combination of labor market conditions and limited availability of housing, which we abstract from in the model. There are alternative set-ups of the model that will

Figure 3: Housing market dynamics, Copenhagen 2004-2008



Notes: Data on seller time-on-market (TOM) and the for-sale stock is from the Danish Mortgage Banks' Federation (available at http://statistik.realkreditforeningen.dk/BMSDefault.aspx). These series are shown in log deviations from their sample mean. The total transaction volume is from Statistics Denmark. It is in log deviations from the sample mean after controlling for seasonal effects by quarter-in-year dummies. The fraction of buy first owners is from own calculations based on registry data from Statistics Denmark. See the Appendix for a description on how we identify an owner as a buyer-and-seller or as a "buy first" ("sell first") owner. We compute quarterly counts of the number of "buy first" and "sell first" owners by looking at the quarter of the first transaction for each of these owners. The fraction of "buy first" owners is then the proportion of buyer-and-seller owners that buy first.

risk neutral and discount the future at rate $r > 0$. They can borrow and lend without frictions at an interest rate of r.

Home-owners in the model receive a flow utility of $u > 0$ in every instant that they are "matched" with the housing unit they reside in. However, a matched homeowner may become dissatisfied with the housing unit he owns, in which case he becomes "mismatched" with his current housing unit. This event occurs according to a Poisson process with rate γ. In that case the homeowner obtains a flow utility of $u - \chi$, for $0 < \chi < u$.

Taste shocks of this form are standard in search theoretic models of the housing market (Wheaton (1990)). They reflect a number of realistic events that take place over the life-cycle of a household, such as marriage, divorce or retirement, changes in household size that require moving to a housing unit of a different size, or job changes that require a move to reduce commuting distances. Such shocks create potential gains from trading.[10]

Upon becoming mismatched, the agent faces a set of choices, which we denote by $X = \{b, s, sb\}$, with the corresponding action denoted by $x \in X$. First of all, he can choose to "sell first" ($x = s$), selling the unit he owns first and then buying a new one. Alternatively, he can choose to "buy first" ($x = b$), buying a new housing unit first and then selling his old one. Finally, a mismatched owner can choose to enter the housing market as both a buyer and seller ($x = sb$). In that case, the agent is on both sides of the market simultaneously and either buys first or sells first depending on whether he meets a seller or a buyer first. The agent cannot simultaneously sell and buy a unit with a single counterparty, (for example, when he meets another mismatched owner). Therefore, there is no double coincidence of housing wants among owners that want to switch houses. This is similar to the lack of double coincidence of wants used in money-search models (Kiyotaki and Wright (1993)).[11]

For much of the paper we will focus on the case where mismatched owners' choices are restricted to the first two options $x \in \{b, s\}$, so choosing $x = sb$ is prohibitively costly. The reason for this restriction is to convey the main mechanisms in the model more clearly. We extend the analysis to the full choice set in Section 7.

A mismatched owner who chooses to "buy first" may end up holding two housing units simultaneously for some period. In this case we say that he becomes a "double owner". Similarly, choosing to "sell first" may result in owning no housing. In that case the agent becomes a "forced renter". We assume that a double owner receives a flow utility of $0 \leq u_2 < u$, while a forced renter receives a flow utility of $0 \leq u_0 < u$. Both of these include some implicit costs, such as maintenance costs

lead to the same results as the ones we present here. For example, one can consider a model that features constant population growth and exogenous housing construction, so that the economy is on a balanced growth path.

[10] Rather than introducing segmentation in the housing stock, we treat all housing units as homogenous, so that a mismatched owners participate in one integrated market with other agents. Although in reality agents move across housing market segments (whether spatial or size-based) in response to a taste shock of the type we have in mind, modeling explicitly several types of housing would substantially reduce the tractability of the model. Furthermore, defining empirically distinct market segments is not straightforward as in reality households often search in several segments simultaneously (Piazzesi, Schneider, and Stroebel (2013)).

[11] Note that entering as both a buyer and seller does not mean that the agent can simultaneously sell and buy a house in the same instant, only that he chooses to receive offers both from potential buyers and sellers.

in the former case, or restrictions on the use of the rental property imposed by a landlord in the latter case.[12]

We assume that in each instant a measure g of new agents are born and enter the housing market. These new entrants start out their life without owning housing. New entrants receive a flow utility $u_n < u$, so for a sufficiently low price of housing they would prefer to become owners. Also, we assume that $u_n \geq u_0$, so it is at least as desirable for forced renters to own a house as it is for new entrants.

All agents in the economy suffer a death/exit shock with Poisson rate g. Upon such a shock, an agent exits the economy immediately and obtains a reservation utility normalized to 0. If he owns housing, his housing units are taken over by a real estate firm, which immediately places them for sale on the market.[13] Real estate firms are owned by all the agents in the economy with new entrants receiving the ownership shares of exiting agents.

Given the exit shock, agents effectively discount future flow payoffs at a rate $\rho \equiv r + g$. For notational convenience, we will directly use ρ later on. Also, we assume that agents are free to exit the economy in every instant and obtain their reservation utility of 0.

Finally, we assume that there exists a frictionless rental market with a rental price of R. Non-owners and new entrants rent a housing unit in the rental market in any given instant they do not own housing. Conversely, double owners rent out one of their units, as do real estate firms. A landlord can simultaneously rent out a unit and have it up for sale. This together with free exit from the economy imply that the equilibrium rental price can take multiple values.[14] Specifically, we will consider equilibrium rental prices in the set $[0, u_n]$.

3.2 Trading Frictions and Aggregate Variables

Heterogeneity in the housing stock naturally implies that the housing market is subject to trading frictions, and that there is no immediacy in housing transactions. To capture these trading frictions, we follow the large literature on search-and-matching models. In particular, the frictional process of matching buyers and sellers of housing units in the housing market is summarized by a standard constant returns to scale matching function $m(B(t), S(t))$, where $B(t)$ and $S(t)$ are the measure of buyers and sellers in a given instant t, respectively. The matching function gives the number of successful transactions in the housing market in a small time interval $\triangle t$. We assume that there is no directed search (Moen (1997)), and meetings are random, so different types of agents meet with probabilities that are proportional to their mass in the population of sellers or buyers. We define the market tightness in the housing market as the buyer-seller ratio, $\theta(t) \equiv \frac{B(t)}{S(t)}$. Additionally, $\mu(\theta(t)) \equiv m\left(\frac{B(t)}{S(t)}, 1\right) = \frac{m(B(t),S(t))}{S(t)}$ is defined as the Poisson rate with which a seller successfully

[12] For tractability we also assume that a double owner does not experience mismatching shocks. This ensures that the maximum holdings of housing by an agent will not exceed two units in equilibrium.

[13] For simplicity, we assume that agents are not compensated for their housing upon exiting the economy. We extend our results in Section 7.2 to a case where exiting agents are compensated for their housing by the real estate firms.

[14] In particular, if $\mathcal{R}$ is the set of possible equilibrium rental prices, we have that $[0, u_n] \subset \mathcal{R}$. However, the equilibrium rental price R may be higher than u_n because of the additional value from homeownership that a new entrant anticipates.

transacts with a buyer. Similarly, $q\left(\theta\left(t\right)\right) \equiv \frac{m(B(t),S(t))}{B(t)} = \frac{\mu(\theta(t))}{\theta(t)}$ is the rate with which a buyer meets a seller and transacts.

Beside the market tightness $\theta\left(t\right)$, which will be relevant for agents' equilibrium payoffs, we keep track of the following aggregate stock variables:

- $B_n\left(t\right)$ - new entrants;
- $O\left(t\right)$ - matched owners;
- $B_1\left(t\right)$ - mismatched owners who buy first;
- $S_1\left(t\right)$ - mismatched owners who sell first;
- $S_2\left(t\right)$ - double owners;
- $B_0\left(t\right)$ - forced renters;
- $A\left(t\right)$ - housing units that are sold by real-estate firms.

Therefore, the total measure of buyers is $B\left(t\right) = B_n\left(t\right) + B_0\left(t\right) + B_1\left(t\right)$ and the total measure of sellers is $S\left(t\right) = S_1\left(t\right) + S_2\left(t\right) + A\left(t\right)$.

Since the total population is constant and equal to 1 in every instant, it follows that

$$B_n + B_0 + B_1 + S_1 + S_2 + O = 1. \tag{1}$$

Also, since the housing stock does not shrink or expand over time, the following housing ownership condition holds in every instant,

$$O + B_1 + S_1 + A + 2S_2 = 1. \tag{2}$$

Summing up, the life-cycle of an agent in the model proceeds as follows. An agent begins his life as a new entrant. With rate $q\left(\theta\right)$, he becomes a matched owner. Once matched he becomes mismatched with rate γ. A mismatched owner chooses to either buy first or sell first. An owner who buys first becomes a double owner with rate $q\left(\theta\right)$, who in turn sells and reverts to being a matched owner with rate $\mu\left(\theta\right)$. An owner who sells first becomes a forced renter with rate $\mu\left(\theta\right)$ and after that moves to being a matched owner with rate $q\left(\theta\right)$. In every stage of life an agent exits the economy with rate g.

3.3 House price determination

We begin our analysis by assuming that the house price p is fixed and does not vary with the market tightness θ. However, similarly to the literature on rigid wages in search-and-matching models (Hall (2005), Gertler and Trigari (2009)), in the equilibria we consider, the price p lies in the bargaining set of all actively trading counterparty pairs. We progressively relax this assumption by assuming that p varies with θ in a reduced form way in Section 5.2 and by assuming that prices are determined by symmetric Nash bargaining in Section 6. The main insights of our analysis hold through in those cases as well.

Given that the price is assumed to lie in the bargaining sets of counterparties, it can also be considered as the market clearing price in a competitive market with frictional entry of counterparties. In particular, similarly to Duffie, Garleanu, and Pedersen (2005) or Rocheteau and Wright (2005), the total measure of participants in that competitive market is determined by the matching function $M(B, S)$ with buyers and sellers of different types entering according to their fractions in the population of buyers and sellers, respectively. Once in the market, there is anonymity, and all agents are price takers. The transaction price leaves trading counterparties (weakly) better off from transacting at that price. However, given the limited heterogeneity of agents, there are generically many market clearing prices that leave agents (weakly) better off from transacting, which generates some indeterminacy in the price level. We resolve this indeterminacy by selecting some price p from the set of market clearing prices and examining equilibria in that case.[15]

4 Steady State Equilibria

We start by characterizing steady state equilibria of this economy.[16] We first discuss the value functions of different types of agents in a candidate steady state equilibrium.

4.1 Value functions

We have the following set of value functions for different agents in this economy:

- V^{B1} - value function of a mismatched owner who buys first;
- V^{S1} - value function of an owner who sells first;
- V^{B0}- value function of a forced renter;
- V^{S2} - value function of a double owner;
- V^{Bn} - value function of a new entrant;
- V - value function of matched owner;
- V^{A} - value function of a real-estate firm that holds one housing unit.

[15]Moreover, one particular choice for this fixed price can be the outcome of bargaining between buyer and seller, in which the buyer has full bargaining power but does not know the type of the seller. As shown in the Appendix, take-it-or-leave-it offers from buyers under private information about the seller's type can generate a fixed price that is equal to the present discounted value of rental income. Under certain conditions, a unique fixed price can therefore be microfounded as resulting from bargaining between heterogenous buyers and sellers.

[16]Informally, in a steady state equilibrium, agents (most importantly mismatched owners) make choices that maximize their discounted payoffs given the market tightness θ, and aggregate variables and agent values are constant over time. Finally, the house price, p, is such that it is privately optimal for agents to transact. A formal definition of a steady state equilibrium of this economy and some parametric restrictions can be found in the Appendix.

Given these notations, we have a standard set of Bellman equations for the agents' value functions in a steady state equilibrium.[17]

First of all, for a mismatched owner who chooses to buy first (a "buyer first" for short) we have

$$\rho V^{B1} = u - \chi + q(\theta) \max\left\{-p + V^{S2} - V^{B1}, 0\right\}, \tag{3}$$

where $u - \chi$ is the flow utility from being mismatched. Upon matching with a seller, a buyer first purchases a housing unit at price p, in which case he becomes a double owner, incurring a utility change of $V^{S2} - V^{B1}$.

A double owner has a flow utility of $u_2 + R$ while searching for a counterparty. Upon finding a buyer, he sells his second unit and becomes a matched owner. Therefore, his value function satisfies the equation[18]

$$\rho V^{S2} = u_2 + R + \mu(\theta)\left(p + V - V^{S2}\right). \tag{4}$$

The value function of a mismatched owner who chooses to sell first (a "seller first" for short) is analogous to that of a "buyer first" apart from the fact that a "seller first" enters on the seller side of the market first and upon transacting becomes a forced renter. Therefore,

$$\rho V^{S1} = u - \chi + \mu(\theta) \max\left\{p + V^{B0} - V^{S1}, 0\right\}. \tag{5}$$

Finally, for a forced renter we have

$$\rho V^{B0} = u_0 - R + q(\theta)\left(-p + V - V^{B0}\right). \tag{6}$$

The remaining value functions are straightforward and are given in the Appendix. Importantly, given the assumption that a real-estate firm can rent out a housing unit without costs, in any steady state equilibrium

$$\rho p \geq R, \tag{7}$$

as otherwise real estate agents do not find it optimal to sell housing. However, the condition can hold with a strict inequality, since agents are assumed to derive some value from owning housing rather than renting.

4.2 Optimal choice of mismatched owners

In a steady state equilibrium, the optimal decision of mismatched owners depends on the simple comparison

$$V^{B1} \gtreqless V^{S1}. \tag{8}$$

[17]Note that we will abstract from steady state equilibria, in which a mismatched owners that is indifferent between some action mixes over these actions over time. This restriction is without loss of generality.

[18]We present these value functions assuming that they always trade at the price p, since that will always be the case in the steady state equilibria we consider. For example, for the case of an owner with two units we have $V + p \geq \frac{u_2+R}{\rho}$. The Appendix provides a set of sufficient conditions for this to hold.

We can substitute for V^{B0} and V^{S2} from equations (6) and (4) into the value functions for a "buyer first" and "seller first", V^{B1} and V^{S1} to obtain

$$V^{B1} = \max\left\{\frac{u-\chi}{\rho}, \frac{u-\chi}{\rho+q(\theta)} + \frac{q(\theta)(u_2-(\rho p-R))}{(\rho+\mu(\theta))(\rho+q(\theta))} + \frac{q(\theta)\mu(\theta)}{(\rho+\mu(\theta))(\rho+q(\theta))}V\right\}, \tag{9}$$

and

$$V^{S1} = \max\left\{\frac{u-\chi}{\rho}, \frac{u-\chi}{\rho+\mu(\theta)} + \frac{\mu(\theta)(u_0+(\rho p-R))}{(\rho+\mu(\theta))(\rho+q(\theta))} + \frac{q(\theta)\mu(\theta)}{(\rho+\mu(\theta))(\rho+q(\theta))}V\right\}. \tag{10}$$

There are several important observations to make. First, even though the flow utility from having two housing units is u_2, the effective utility flow is $u_2-(\rho p-R)$, and similarly the effective utility flow from being a forced renter is $u_0+(\rho p-R)$. Therefore, even if the utility flows, u_0 and u_2, are equal it is still at least as costly to become a double owner compared to a forced renter. The reason is that a double owner faces a potentially lower rental income than the user cost of owning a housing unit, while a forced renter benefits from this possibility. Therefore, even with frictionless financing and a frictionless rental market, in an environment with search-and-matching frictions owning two units may be more costly than going through a phase of forced renting.

Therefore, we define the effective utility flow for a forced renter as $\tilde{u}_0 \equiv u_0+\triangle$, and for a double owner as $\tilde{u}_2 \equiv u_2-\triangle$, where $\triangle \equiv \rho p-R$ is the "ownership premium" that an agent who owns a housing unit must pay relative to renting. Also, if $u_0 = u_2-2\triangle$, then $\tilde{u}_0=\tilde{u}_2$, so the effective utility flows of a double owner and a forced renter are the same. This particular case will serve as an important benchmark.

Secondly, and more importantly, search-and-matching frictions may also affect the value of "selling first" versus "buying first" through the expected times on the market for a buyer ($\frac{1}{q(\theta)}$) and a seller ($\frac{1}{\mu(\theta)}$). To show this, suppose that it is optimal for both a "buyer first" and a "seller first" to transact and consider the difference $D(\theta) \equiv V^{B1}-V^{S1}$. We have that

$$D(\theta) = \frac{\mu(\theta)}{(\rho+q(\theta))(\rho+\mu(\theta))}\left[\left(1-\frac{1}{\theta}\right)(u-\chi-\tilde{u}_2)-\tilde{u}_0+\tilde{u}_2\right]. \tag{11}$$

In the benchmark case, where $\tilde{u}_0=\tilde{u}_2=c$ equation (11) simplifies to

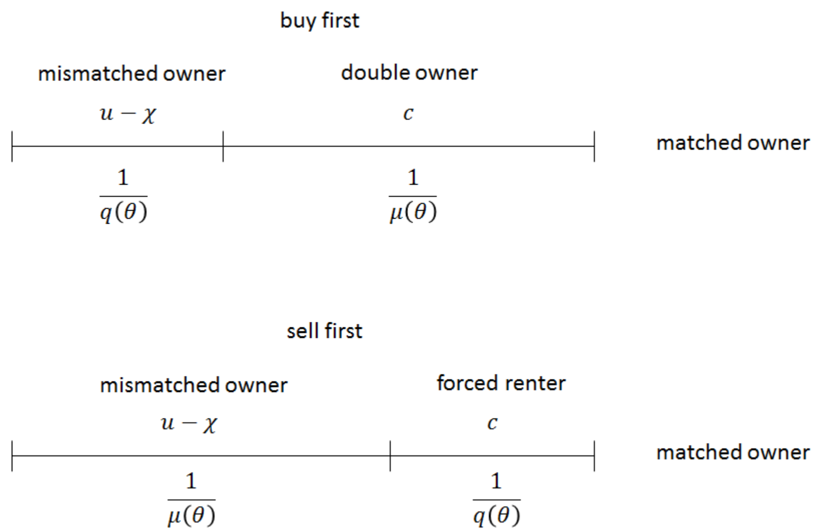
$$D(\theta) = \frac{(\mu(\theta)-q(\theta))(u-\chi-c)}{(\rho+q(\theta))(\rho+\mu(\theta))}. \tag{12}$$

Furthermore, in the limit where the effective discount rate is arbitrarily small ($\rho\to 0$) it becomes

$$D(\theta) = \left(\frac{1}{q(\theta)}-\frac{1}{\mu(\theta)}\right)(u-\chi-c). \tag{13}$$

Therefore, buying first versus selling first depends on the difference in the expected time on the market for a buyer versus a seller, $\frac{1}{q(\theta)}-\frac{1}{\mu(\theta)}$.

Figure 4: Buying first versus selling first when $\theta < 1$.



If the expected time on the market for buyers is *higher* than the expected time on the market for sellers, then the value of a "buyer first" is higher than the value of a "seller first", provided that the utility flow from being mismatched is higher than the utility flow of a double owner or a non owner (so $u - \chi > c$).

The behavior of mismatched owners seems counter-intuitive at first. After all, if the expected time on the market for a buyer is higher than that for a seller, why would entering as a "buyer first" be preferred to entering as a "seller first". The reason for the counter-intuitive behavior is that a mismatched owner has to undergo two transactions on both sides of the market before he becomes a matched owner. Given this, a mismatched owner wants to minimize the expected time in the situation that is relatively more costly. If it is more costly to be a double owner or a forced renter than to be mismatched, then a mismatched owner would care more about the expected time on the market for the second transaction.

For example, consider the schematic representation of a mismatched owner's expected payoffs in Figure 4 in the case when $\theta < 1$. If the agent enters as a "buyer first" (top part of Figure 4), he has a short expected time on the market as a buyer. However, he anticipates a long expected time on the market in the next stage when he is a double owner and has to dispose of his old housing unit. In contrast, entering as a "seller first" (bottom part of Figure 4) implies a long expected time on the market until the agent sells his property but a short time on the market when the agent is a forced renter and has to buy a new property. If $u - \chi > c$, then it is more costly to be stuck in the second stage for a long time (as a double owner or forced renter) rather than to remain mismatched and searching.

Therefore, entering as a "seller first" is strictly preferred to entering as a "buyer first" whenever $\theta < 1$. Market tightness is increasing in the number of buyers that enter the market and decreasing in the number of sellers that enter the market. As we discuss in the next section, entering as a "seller first" decreases market tightness. Therefore, in equilibrium there will be a form of strategic complementarity in mismatched owners' actions.

Is it reasonable to assume that the costs of being a mismatched owner are lower than the costs of a double owner or forced renter? Anecdotal evidence points to the mismatch state as not particularly costly for the majority of homeowners. In rare instances is the alternative of a household having to permanently reside in a unit which they are not perfectly satisfied with, worse than a situation, in which households are forced to permanently rent (despite preferring to own) or to permanently own two housing units. Therefore, our analysis focuses on this more empirically relevant and realistic case, as we summarize in the following parametric restriction:

Assumption A1: $u - \chi \geq \max\{\tilde{u}_0, \tilde{u}_2\}$**.**

We now formally characterize the optimal action of a mismatched owner given a steady state market tightness θ. We adopt the notation $\theta = \infty$ for the case where the buyer-seller ratio is unbounded.

We define

$$\tilde{\theta} \equiv \frac{u - \chi - \tilde{u}_2}{u - \chi - \tilde{u}_0}. \tag{14}$$

Note that if $\tilde{u}_2 = \tilde{u}_0$, then $\tilde{\theta} = 1$, while if $\tilde{u}_2 > \tilde{u}_0$, then $\tilde{\theta} < 1$, and vice versa if $\tilde{u}_2 < \tilde{u}_0$.[19]

The following lemma fully characterizes the incentives of mismatched owners to buy first or sell first given steady state market tightness θ:

Lemma 1. *Let $\tilde{\theta}$ be as defined in (14). Then for $\theta \in (0, \infty)$, $V^{B1} > V^{S1} \iff \theta > \tilde{\theta}$ and $V^{B1} = V^{S1} \iff \theta = \tilde{\theta}$. For $\theta = 0$ and $\theta = \infty$, $V^{B1} = V^{S1} = \frac{u-\chi}{\rho}$.*

Proof. See Appendix. □

Lemma 1 shows that, in general, as θ increases, the incentives to buy first are strengthened. For high values of θ buying first dominates selling first. For low values of θ, selling first dominates buying first.

4.3 Steady state flows and stocks

We turn next to a description of the steady state equilibrium stocks and flows in this model. The full set of equations for the stocks are included in the Appendix. Here we just make some important observations on the stock-flow process in the model. First, combining the population and housing ownership conditions (1) and (2) we get that

$$B_n(t) + B_0(t) = A(t) + S_2(t). \tag{15}$$

[19] In what follows we will additionally assume that at $\theta = \tilde{\theta}$, both $V^{S1} > \frac{u-\chi}{\rho}$ and $V^{B1} > \frac{u-\chi}{\rho}$, so that a mismatched owner is strictly better off from transacting at $\theta = \tilde{\theta}$. This removes uninteresting steady state equilibria in which mismatched owners never transact. Assumption A2 in the Appendix gives a sufficient condition for this.

Therefore, in every instant in this economy the stock of new entrants and forced renters must equal the stock of double owners and real estate firms. In other words, the stock of agents that do not own housing but demand housing must equal the stock of vacant houses.

This identity implies that in a candidate steady state equilibrium where all mismatched owners buy first (so that there are no forced renters), the market tightness, denoted by $\overline{\theta}$ satisfies

$$\overline{\theta} = \frac{B_n + B_1}{A + S_2} = \frac{B_n + B_1}{B_n} > 1.$$

Similarly, if $\underline{\theta}$ denotes the market tightness in a candidate steady state where all mismatched owners sell first (so there are no double owners), we have that

$$\underline{\theta} = \frac{B_n + B_0}{A + S_1} = \frac{A}{A + S_1} < 1.$$

Therefore, $\underline{\theta} < 1 < \overline{\theta}$. This points to the possible wide variations in market tightness from changes in the behavior of mismatched owners. In Lemma 2 we show that $\overline{\theta}$ solves

$$\left(\frac{1}{q(\theta) + g} + \frac{1}{\gamma}\right)\theta + \left(\frac{1}{q(\theta) + g} - \frac{1}{\mu(\theta) + g}\right) = \frac{1}{g} + \frac{1}{\gamma}, \tag{16}$$

and $\underline{\theta}$ solves

$$\left(\frac{1}{\mu(\theta) + g} + \frac{1}{\gamma}\right)\frac{1}{\theta} = \frac{1}{g} + \frac{1}{\gamma}. \tag{17}$$

These two equations arise from the flow conditions and population and housing conditions if all mismatched agents enter as "buyers first" and "sellers first", respectively.

Lemma 2. *Consider equations (16) and (17). Each has a unique solution, denoted by $\overline{\theta}$ and $\underline{\theta}$, respectively. $\overline{\theta}$ satisfies the stock-flow conditions when all mismatched owners buy first, and $\underline{\theta}$ satisfies the stock-flow conditions if they all sell first. Furthermore, $\overline{\theta} > 1$, $\underline{\theta} < 1$, and $\overline{\theta}$ is increasing in γ and $\underline{\theta}$ is decreasing in γ.*

Proof. See Appendix. □

To see what equations (16) and (17) imply for the market tightnesses, it is illustrative to consider a limit economy with small flows, where $g \to 0$ and $\gamma \to 0$ but the ratio $\frac{\gamma}{g} = \kappa$ is kept constant in the limit. In particular, in the limit as $\gamma \to 0$, $g \to 0$, and $\frac{\gamma}{g} = \kappa$, given equations (16) and (17), we have that

$$\lim_{\gamma \to 0, g \to 0, \frac{\gamma}{g} = \kappa} \overline{\theta} = 1 + \kappa, \tag{18}$$

and

$$\lim_{\gamma \to 0, g \to 0, \frac{\gamma}{g} = \kappa} \underline{\theta} = \frac{1}{1 + \kappa}. \tag{19}$$

Therefore, in this limit economy $\overline{\theta} = \frac{1}{\underline{\theta}}$, and so the larger is κ, the wider is the variation in market tightness if mismatched owners switch from buying first to selling first and vice versa. This particular feature of the stock-flow process for our model will imply that the behavior of mismatched owners can have important effects on transaction volume, time-on-market for sellers, and ultimately, on prices.

4.4 Equilibrium characterization

We now combine the observations on the optimal choice of mismatched owners and the steady state stocks from the previous two sections to characterize equilibria.

We first characterize the steady state equilibria in the benchmark case where there is symmetry in the flow payoffs of a double owner and a forced renter, so $\tilde{u}_0 = \tilde{u}_2 = c$. There always exists a steady state equilibrium with $\theta = 1$. In that case from Lemma 1, mismatched owners are indifferent between buying first and selling first. Also, if one half of them buy first and the other half sell first, the stock-flow conditions of the model are satisfied given $\theta = 1$. We summarize this observation in the following

Proposition 3. *Consider the above economy and suppose that $\tilde{u}_0 = \tilde{u}_2 = c$. Then there exists a steady state equilibrium with $\theta = 1$. In that equilibrium mismatched owners are indifferent between entering as a "buyer first" and a "seller first".*

Proof. See Appendix. □

Nevertheless, this is not the only equilibrium in this case. In particular, there is a steady state equilibrium with $\theta = \overline{\theta}$, in which all mismatched owners buy first. There is also an equilibrium with $\theta = \underline{\theta}$, in which all mismatched owners sell first.

By definition, $\theta = \overline{\theta}$ is the steady state market tightness level that satisfies the stock-flow conditions of this economy if all mismatched owners buy first. However, $\overline{\theta} > 1$, so by Lemma 1 mismatched owners choose to buy first. Similarly, $\underline{\theta} < 1$ satisfies the stock-flow conditions and by Lemma 1 mismatched owners sell first. We summarize this observation in the following

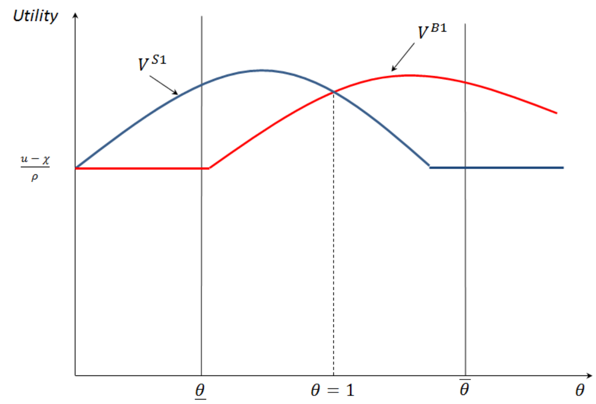
Proposition 4. *Consider the above economy and suppose that $\tilde{u}_0 = \tilde{u}_2 = c$. Then there exists a steady state equilibrium with $\theta = \underline{\theta}$, in which mismatched owners sell first. There also exists a steady state equilibrium with $\theta = \overline{\theta}$, in which mismatched owners buy first.*

Proof. See Appendix. □

Therefore, multiple steady state equilibria are possible. In one equilibrium mismatched owners are strictly better off selling first, even though the equilibrium market tightness $\theta < 1$, so that there are more sellers than buyers in the market. Conversely, in the other equilibrium mismatched owners are better off buying first, even though the equilibrium market tightness $\theta > 1$, so that there are more buyers than sellers in the market.

Given the steady state value of θ in the two steady state equilibria, we call the equilibrium with $\theta < 1$ a "Buyers' market" equilibrium, and the one with $\theta > 1$ a "Sellers' market" equilibrium. In

Figure 5: Equilibrium multiplicity with $\tilde{u}_0 = \tilde{u}_2 = c$.



the former, the expected time on the market is lower for buyers than for sellers and vice versa for the latter.

Figure 5 illustrates this equilibrium multiplicity and the equilibrium value functions of mismatched owners.[20] It is easy to see from the figure that the steady state equilibrium with $\theta = 1$ is unstable in the following sense: a small perturbation in θ around the equilibrium value of $\theta = 1$ will make mismatched agents either strictly better off from entering as "buyers first" or "sellers first", driving the value of θ away from $\theta = 1$ and towards $\underline{\theta}$ or $\overline{\theta}$, respectively.

Apart from this benchmark case there can be multiple equilibria more generally when $\tilde{u}_0 \neq \tilde{u}_2$. However, if the payoff asymmetry is sufficiently strong, there will be a unique equilibrium. In particular, if $\tilde{u}_0$ is sufficiently low compared to $\tilde{u}_2$, there is a unique equilibrium in which mismatched owners buy first and vice versa when $\tilde{u}_2$ is sufficiently low compared to $\tilde{u}_0$. Whether there is equilibrium uniqueness or multiplicity depends on a comparison of $\underline{\theta}$ and $\overline{\theta}$ on the one hand, against the value of $\tilde{\theta}$ at which a mismatched owner is indifferent between buying first and selling first as defined in condition (14), on the other hand. Therefore, we have the following

Proposition 5. *Consider the above economy. Let $\tilde{\theta}$ be defined by condition (14), and $\overline{\theta}$ and $\underline{\theta}$ be defined by (16) and (17) with $\overline{\theta}, \underline{\theta} \in (0, \infty)$.*

1. *If $\tilde{\theta} \in [\underline{\theta}, \overline{\theta}]$, then there exists a steady state equilibrium with $\theta = \overline{\theta}$, in which mismatched owners buy first. There also exists a steady state equilibrium with $\theta = \underline{\theta}$ in which mismatched owners sell first.*

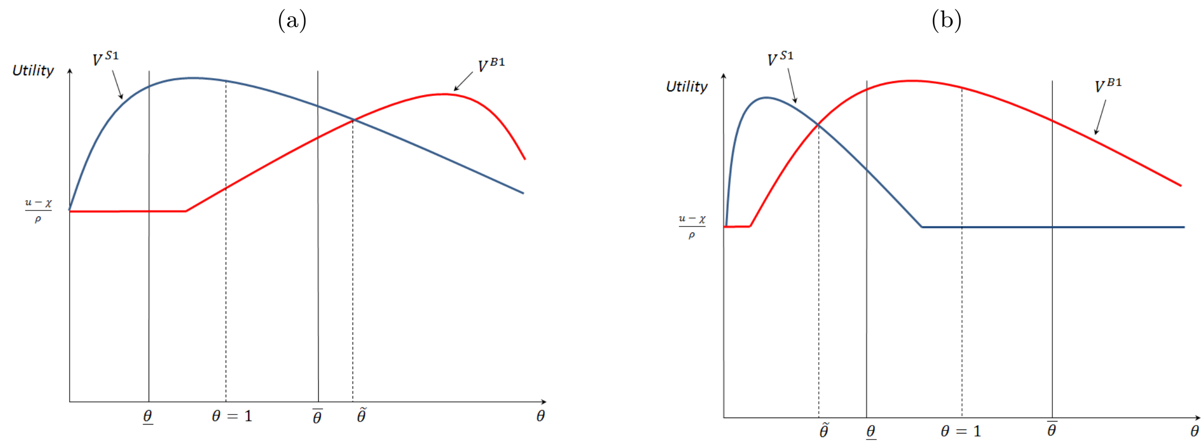
2. *If $\tilde{\theta} < \underline{\theta}$, no steady state equilibrium exists in which all mismatched owners sell first.*

3. *If $\tilde{\theta} > \overline{\theta}$, no steady state equilibrium exists in which all mismatched owners buy first.*

Proof. See Appendix. $\square$

[20]For illustrative purposes, in the figures below we assume that V^{B1} and V^{S1} as defined in (9) and (10) are single peaked, i.e. there is a $\hat{\theta}^{B1}$, s.t. V^{B1} is increasing for $\theta < \hat{\theta}^{B1}$ and decreasing for $\theta > \hat{\theta}^{B1}$ and similarly for V^{S1}.

Figure 6: Examples with unique equilibria in the case of $\tilde{\theta} > \overline{\theta}$ (a) and $\tilde{\theta} < \underline{\theta}$ (b).



Therefore, in general, either both symmetric equilibria exist, or only one of them, depending on the flow payoffs $\tilde{u}_0$ and $\tilde{u}_2$. Specifically, Figure 6 shows examples in which only one symmetric equilibrium exists. In Figure 6a $\tilde{\theta} > \overline{\theta}$, so that a "Sellers' market" equilibrium with $\theta = \overline{\theta}$ and in which all mismatched owners buy first, does not exist. Figure 6b shows the opposite case when $\tilde{\theta} < \underline{\theta}$, so that a "Buyers' market" equilibrium with $\theta = \underline{\theta}$ does not exist.

Therefore, shocks to the payoffs of mismatched owners can lead to equilibrium switches.[21] Apart from payoff shocks, equilibrium switches may occur because of changes in agents' believes. We show this possibility in Section 5.2 where we allow the house price to respond to a change in tightness. Before that we discuss the implications of equilibrium switches for transaction volume, time-on-market, and the stock of houses for sale.

4.5 Equilibrium switches

The previous section showed that there can be multiple steady state equilibria in an environment where mismatched owners face costs associated with double ownership or forced renting. Given this multiplicity, there could be equilibrium switches due to payoff shocks or shocks to agents' expectations.

In this section we briefly discuss some implications of such equilibrium switches for the housing market. We will discuss the implications of our model for the limit economy with small flows introduced in Section 4.3. Specifically, in the limit where flows where $g \to 0$ and $\gamma \to 0$ and $\frac{\gamma}{g} = \kappa$, $\overline{\theta} = 1 + \kappa$ and $\underline{\theta} = \frac{1}{1+\kappa} = \frac{1}{\overline{\theta}}$. Suppose that the economy starts in a "Sellers' market" equilibrium

[21] As an example of such a payoff shock, suppose that the payoff of a double owner, u_2, includes costs associated with obtaining a mortgage that allows him to finance the downpayment on his new property prior to the sale of his old property. When financial markets function normally, these costs are relatively low. Suppose that in that case $\tilde{u}_2 > \tilde{u}_0$ and $\tilde{\theta} < \underline{\theta}$. Therefore, in that case the "Buyers' market" equilibrium in which mismatched owners sell first, does not exist. Now suppose that there is a shock to financial markets so that obtaining a bridging mortgage becomes very costly, and thus $\tilde{u}_2 < \tilde{u}_0$ and $\tilde{\theta} > \overline{\theta}$. As a result, after the shock buying first is no longer optimal and the "Sellers' market" equilibrium in which mismatched owners buy first no longer exists.

with market tightness $\theta = \overline{\theta}$. In that case

$$\overline{\theta} = \frac{\overline{B}}{\overline{S}} = \frac{B_n + B_1}{A + S_2} = \frac{B_n + B_1}{B_n}, \tag{20}$$

where $\overline{B}$ and $\overline{S}$ denote the stocks of buyers and sellers in the "Sellers' market" equilibrium. Suppose that the whole stock of mismatched owners, B_1, decide to sell first rather than buy first. In that case, the new market tightness becomes

$$\theta^{'} = \frac{B'}{S'} = \frac{B_n}{B_n + B_1} = \frac{1}{\overline{\theta}},$$

where B' and S' denote the stocks of buyers and sellers immediately after the switch. Therefore, θ' is the reciprocal of the pre-switch tightness. In the limit economy, that reciprocal value is exactly $\underline{\theta}$. Therefore, there is an immediate switch from the "Sellers' market" to the "Buyers' market" buyer-seller ratio and there is no dynamic adjustment in θ. After the switch there is adjustment only in the non-payoff relevant aggregate stock variables.

What are the implications of this switch? First of all, clearly average time-on-market for sellers, $\frac{1}{\mu(\theta)}$ increases. Secondly, consider the ratio of the stock of sellers before and after the switch. That ratio is exactly

$$\frac{\overline{S}}{S'} = \frac{B_n}{B_n + B_1} = \frac{1}{\overline{\theta}}. \tag{21}$$

Therefore, there is an increase in the stock of houses for sale since previous buyers are now sellers. Finally, transaction volume may also fall depending on the shape of the matching function. Specifically, suppose that we have a Cobb-Douglas matching function, so $m(B,S) = \mu_0 B^{\alpha} S^{1-\alpha}$, for $0 < \alpha < 1$, and consider the ratio of transaction volumes before and after the switch. This ratio is given by

$$\frac{m\left(\overline{B}, \overline{S}\right)}{m\left(B', S'\right)} = \frac{\mu\left(\overline{\theta}\right)\overline{S}}{q\left(\underline{\theta}\right)B'} = \frac{\mu\left(\overline{\theta}\right)B_n}{q\left(\underline{\theta}\right)B_n} = \frac{\mu\left(\overline{\theta}\right)}{q\left(\underline{\theta}\right)}. \tag{22}$$

The latter ratio is

$$\frac{\mu\left(\overline{\theta}\right)}{q\left(\underline{\theta}\right)} = \frac{\mu_0 \overline{\theta}^{\alpha}}{\mu_0 \underline{\theta}^{\alpha-1}} = (1+\kappa)^{2\alpha-1}.$$

Hence, transaction volume falls after the equilibrium switch if $\alpha > \frac{1}{2}$ and increases if $\alpha < \frac{1}{2}$. Genesove and Han (2012) estimate a value of $\alpha = 0.84$. At that value, transaction volume would drop after the switch from a "Sellers' market" equilibrium to a "Buyers' market" equilibrium.

Consequently, a switch from a "Sellers' market" equilibrium to a "Buyers' market" equilibrium implies a behavior for key housing market variables like the for-sale stock, average time-to-sell, and transaction volume that is broadly consistent with evidence on housing cycles (Diaz and Jerez (2013), Guren (2013)). This behavior is also consistent with the evidence on the housing cycle for Copenhagen as shown in Figure 3.

4.6 Quantitative relevance

In this section we provide a numerical example to assess the quantitative relevance of our mechanism. We use data from the Copenhagen housing market to determine plausible values for the rate of mismatch, γ, and the exit rate, g.

We use two quantities to determine these parameters. The first is the fraction of owners that we identify as both buying and selling, and who are recorded as owning two properties. Those correspond to the stock of double owners in our model. The fraction of such owners is fairly low at around 0.04% during the 90s but quickly increases and reaches a high of 0.3% in 2006. During the period 2005-2006, which we take as the period where the market is in the "Sellers' market" steady state with mismatched owners buying first, the average fraction of such owners is 0.22%. The second quantity is the fraction of owners that exit the market within a year. During the period 2005-2006 the average fraction of owners that exit within a year is 4.7%.

We assume that the matching function is Cobb-Douglas $\mu(\theta) = \mu_0 \theta^{\alpha}$ with $\alpha = 0.84$, following Genesove and Han (2012). We also calibrate the matching efficiency parameter μ_0 to match an average time-on-market for a seller of around 3 month, which corresponds to the time-on-market for single-family homes in Copenhagen during 2005-2006. As already mentioned, we assume that the housing market is in a "Sellers' market" equilibrium during 2005-2006.

Matching the fractions of double owners and owners that exit the market, we find a value of $\gamma = 0.01$ and a value of $g = 0.05$. This implies an average duration for homeowners of around 17 years and an annual turnover rate of 6%. Both of these numbers are reasonable given estimates for UK and US housing markets.

Given these calibrated parameters, the implied market tightness in a "Sellers' market" equilibrium is $\overline{\theta} = 1.196$. The market tightness in a "Buyers' market" equilibrium is $\underline{\theta} = 0.836$. Note that these values are almost reciprocal since the estimated values of γ and g are small. This suggests that the focus on the limit economy in Section 4.5 is reasonable.

Therefore, going back to our equilibrium switching example in the previous section, switching from a "Sellers' market" equilibrium to the "Buyers' market" equilibrium decreases the market tightness by around 30%. This is associated with a 20% increase in the stock of houses for sale, and a 36% increase in the average time-to-sell. Looking at the ratio of transactions before and after the switch, we find that transactions fall by around 14%.[22]

5 House Price Fluctuations

Up to now we considered a constant house price p that agents are willing to transact at. In this section, we first examine the implications of expected changes in the house price for the behavior of mismatched owners. We then construct dynamic equilibria with self-confirming fluctuations in prices and tightness.

[22]In Section 6 we also discuss the quantitative implications for house prices in the model with Nash bargaining.

5.1 Exogenous house price movements

We first show that expected future changes in the house price affect the incentives of mismatched owners to buy first or sell first. Consider the following partial equilibrium example. Suppose that $u_0 = u_2$ and the house price $p = \frac{R}{\rho}$, so the effective flow payoffs of a forced renter and double owner are equal, $\tilde{u}_0 = \tilde{u}_2 = c$. Consider a simple exogenous process for the house price p. With rate λ the house price p changes to a permanent new level p_N.[23]

We compare the utility from buying first relative to selling first for a mismatched owner before the price change. For the value functions prior to the price change we have expressions similar to those in Section 4.1 but with an additional term reflecting the price uncertainty.[24] For example, the value function of a mismatched owner who buys first and transacts satisfies

$$\begin{aligned} V^{B1} = \frac{u-\chi}{\rho+q(\theta)+\lambda} + q(\theta)\frac{c+\lambda(p_N-p)+\mu(\theta)V}{(\rho+q(\theta)+\lambda)(\rho+\mu(\theta)+\lambda)} + \\ + \frac{\lambda}{\rho+q(\theta)+\lambda}\left(\frac{q(\theta)v^{S2}}{\rho+\mu(\theta)+\lambda}+\overline{V}_N\right), \end{aligned} \tag{23}$$

where $v^{S2} = \frac{c}{\rho+\mu(\theta)} + \frac{\mu(\theta)}{\rho+\mu(\theta)}V$, and $\bar{V}_N = \max\left\{V_N^{B1}, V_N^{S1}\right\}$, with V_N^{B1} and V_N^{S1} denoting the value of buying first and selling first after the price change.

Importantly, the value of buying first depends on the expected price change $\lambda(p_N - p)$. Specifically, expected price appreciations increase the value from buying first. The intuition for this dependence is that in the presence of search frictions and trading delays, the choice of buying first or selling first exposes a mismatched owner to price risk. For example, if he buys first, once he buys a new housing unit at the current price p, he must sell later. However, he may end up selling his old housing unit at the new price of p_N later on. If he expects house prices to appreciate, so $p_N > p$, this expected capital gain increases the value of buying first for any value of the buyer-seller ratio θ.

Similarly, the value from selling first is decreasing in the expected price change.[25] Selling first also exposes a mismatched owner to price risk in the presence of trading delays. If he sells his housing unit at the current price p, the agent must buy a housing unit but may end up buying at a price of p_N. Therefore, a seller first is effectively short in the housing market and must later cover his short position. Naturally, a higher future price $p_N > p$ leads to a lower value for the agent.

[23] Since we assume that $p = \frac{R}{\rho}$, one can think of a permanent change in the equilibrium rental rate to R_N, which leads to a house price change to $p_N = \frac{R_N}{\rho}$.

[24] We assume that θ remains constant over time, so the only change occurs in the house price p. Also, for this exercise, we implicitly assume that $\gamma \to 0$, so that V is independent of the house price p.

[25] This value function is given by

$$\begin{aligned} V^{S1} = \frac{u-\chi}{\rho+\mu(\theta)+\lambda} + \mu(\theta)\frac{c-\lambda(p_N-p)+q(\theta)V}{(\rho+\mu(\theta)+\lambda)(\rho+q(\theta)+\lambda)} + \\ + \frac{\lambda}{\rho+\mu(\theta)+\lambda}\left(\frac{\mu(\theta)v^{B0}}{\rho+q(\theta)+\lambda}+\overline{V}_N\right), \end{aligned} \tag{24}$$

where $v^{B0} = \frac{c}{\rho+q(\theta)} + \frac{q(\theta)}{\rho+q(\theta)}V$ and $\bar{V}_N = \max\left\{V_N^{B1}, V_N^{S1}\right\}$.

The price risk associated with the transaction sequence decision creates asymmetry in the payoff from buying first or selling first. Specifically, at $\theta = 1$, the difference between the two value functions $D(\theta) = V^{B1} - V^{S1}$ takes the form

$$D(1) = \frac{\mu(1)}{(\rho + q(1) + \lambda)(\rho + \mu(1) + \lambda)} 2\lambda(p_N - p). \tag{25}$$

An expected price decrease, leads to a higher value of V^{S1} relative to V^{B1}, even if matching rates for a buyer and a seller are the same. Consequently, $V^{S1} > V^{B1}$ even for some values of $\theta > 1$. If the expected price decrease is sufficiently large, so that even at $\theta = \overline{\theta}$, $D(\overline{\theta}) < 0$, then selling first will dominate buying first for any value of θ that is consistent with equilibrium. Similarly, a sufficiently large expected price increase, will imply that $D(\underline{\theta}) > 0$, so buying first will dominate selling first for any value of θ that is consistent with equilibrium. We summarize these observations in the following

Proposition 6. *Consider the modified economy with an exogenous house price change. Then for every $\lambda > 0$, there exists a $\underline{p} < p$, such that for $p_N < \underline{p}$, a mismatched owner strictly prefers selling first to buying first for $1 < \theta \leq \overline{\theta}$. Similarly, there exists a $\overline{p} > p$, such that for $p_N > \overline{p}$, a mismatched owner strictly prefers buying first to selling first for $\underline{\theta} \leq \theta < 1$.*

Proof. See Appendix. □

Proposition 6 has two implications. First, expectations about price movements influence mismatched owners' incentives to buy first or sell first. Sufficiently large expected price increases induce mismatched owners to buy first even if the market tightness θ is low and vice versa for price decreases.

Secondly, the actions of mismatched owners are destabilizing for the housing market in the following sense. Suppose that the house price is an increasing function of market tightness θ. Then, if mismatched owners anticipate that the price will decrease for some exogenous reason, they will prefer to sell first rather than buy first. However, that behavior will tend to decrease the market tightness, which in turn would lower the house price even further.

In the next section we show that this particular behavior of mismatched owners can lead to price fluctuations due to self-fulfilling expectations about housing market conditions.

5.2 Self-fulfilling house price fluctuations

We use the insight from the previous section to construct dynamic equilibria with self-fulfilling fluctuations in house prices and tightness. Similarly to Section 4.5 we look at a limit economy with small flows where the market tightness when mismatched owners buy first and the tightness when they sell first are reciprocal to one another, so $\overline{\theta} = \frac{1}{\underline{\theta}}$.

We assume that the house price p is increasing in the market tightness θ, that is $p = f(\theta)$, with $f(\theta)$ a strictly increasing function of θ. We take this relationship as exogenous and reduced-form to illustrate the equilibrium consequences of the interaction of this feedback between housing prices and market liquidity conditions with the transaction decisions of mismatched owners.

We consider equilibria, in which a mismatched owner chooses to buy first or sell first depending on the realization of a two-state Markov chain $X(t) \in \{0,1\}$. $X(t)$ starts in $X(t) = 0$ and with Poisson rate λ transitions permanently to $X(t) = 1$. The realization of $X(t)$ plays the role of a sunspot variable that helps coordinate mismatched owners' actions.

We assume that if $X(t) = 0$, mismatched owners anticipate that other mismatched owners will buy first, and if $X(t) = 1$, they anticipate that other mismatched owners will sell first. Therefore, we will index equilibrium variables in both of these cases by the realization of the state $X(t)$, for example, the market tightness if $X(t) = 0$ is $\theta(t) = \theta_0$ and the price is $p(t) = p_0$.

For the "Sellers' market" regime ($X(t) = 0$) in which the economy starts 1) mismatched owners prefer to buy first and the market tightness is $\theta_0 = \overline{\theta}$, and 2) agents expect that with rate λ, the economy permanently switches to a "Buyers' market" regime with market tightness $\theta_1 = \underline{\theta}$. In that second regime, 1) mismatched owners strictly prefer to sell first, and 2) agents expect that the economy will remain in the "Seller's market" regime forever.

We describe these equilibria in the following

Proposition 7. *Consider the limit economy with $g \to 0$, $\gamma \to 0$ and $\frac{\gamma}{g} = \kappa$, and with the sunspot process described above. Suppose p_0, p_1, u_0, u_2 and R are such that $p_0 > p_1$, $\tilde{\theta}_0 = \frac{u-\chi-(u_2+R-\rho p_0)}{u-\chi-(u_0-R+\rho p_0)} < \overline{\theta}$ and $\tilde{\theta}_1 = \frac{u-\chi-(u_2+R-\rho p_1)}{u-\chi-(u_0-R+\rho p_1)} > \underline{\theta}$. Then there is a $\overline{\lambda}$, such that for $\lambda < \overline{\lambda}$, there exists a dynamic equilibrium characterized by two regimes $x \in \{0,1\}$. In the first regime, $\theta_0 = \overline{\theta}$, p_0, and mismatched owners buy first. In the second regime, $\theta_1 = \underline{\theta}$, $p_1 < p_0$, and mismatched owners sell first. The economy starts in regime 0 and transitions to regime one with rate λ.*

Proof. See Appendix. □

The transition between the two regimes is broadly consistent with our motivating Figure 2. When the house price is high, owners prefer to buy first. A decline in the house price is associated with a reversal of the incentives of owners and they prefer to sell first. Additionally, as discussed in Section 4.5, the stock of houses for sale and average time-to-sell increases, while transaction volume drops.

6 A Model with Prices Determined by Nash Bargaining

A key assumption for our analysis so far has been the fixed price of housing units, which lies below the reservation price of buyers and above the reservation price of sellers. In Section 5.2 we allowed for a simple reduced-form relation between this price and the market tightness θ. Apart from showing the existence of equilibria with fluctuations in θ, Proposition 7 indicates that the results on steady state equillibria from Section 4 would also hold in the case of a dependence between p and θ (i.e. when $\lambda = 0$). Nevertheless, this relation is still exogenous.

To endogenize this relation and to show that our main results are robust to alternative assumptions on house price determination, in the Appendix we describe a modification of our housing model with random matching between buyers and sellers of different types, in which counterparties split the surplus from trading according to symmetric Nash Bargaining. Therefore, there is no longer a single transaction price p but prices depend on the types of the trading counterparties.[26] Here, we provide a short treatment for the results we obtain in that case.

Again, we consider a limit economy with small flows where $g \to 0$ and $\gamma \to 0$ but the ratio $\frac{\gamma}{g} = \kappa$ is kept constant in the limit. Though the results of equilibrium multiplicity under Nash bargaining hold more generally, the limit economy is characterized by very tractable expressions for the fractions of buyers and sellers of different types, and for the equilibrium market tightness in a "Sellers' market" and "Buyers' market" equilibria. As shown in Section 4.3, in that limit economy, $\overline{\theta} = 1 + \kappa$ and $\underline{\theta} = \frac{1}{1+\kappa}$. Therefore, depending on the value of κ, in the limit economy the two steady state market tightnesses can be arbitrarily close to 1. Furthermore, in the limit economy the fractions of buyers and sellers of each type are simple functions of the market tightness θ. In particular, in a "Buyers' market" equilibrium the fraction of sellers that are real estate firms is given by

$$\lim_{\gamma \to 0, g \to 0, \frac{\gamma}{g} = \kappa} \frac{A}{S} = \underline{\theta} = \frac{1}{1+\kappa}, \tag{26}$$

and so is the fraction of buyers that are new entrants, $\frac{B_n}{B}$. Conversely, in a "Sellers' market" equilibrium the fraction of sellers that are real estate firms is $\frac{A}{S} = \frac{1}{\overline{\theta}}$ and so is the fraction of new entrants, $\frac{B_n}{B}$. Since agents match randomly with counterparties of different types, these fractions determine the probabilities of meeting a particular trading partner in that limit economy.

In the Appendix we describe the steady state value functions in a "Sellers' market" and "Buyers' market" equilibrium under a set of parametric assumptions.[27] Here we just show the difference in the value functions in a "Sellers' market" and "Buyers' market" equilibrium when the value of κ is close to 0, so $\overline{\theta}$ and $\underline{\theta}$ are close to 1. In particular, for values of κ sufficiently close to 0, in a "Buyers' market" equilibrium with random matching and symmetric Nash bargaining, the difference between the value from buying first and selling first, $D(\underline{\theta}) \equiv V^{B1} - V^{S1}$, is given by

$$\lim_{g \to 0, \lambda \to 0, \frac{\lambda}{g} = \kappa} D(\underline{\theta}) = \frac{\frac{1}{2}\mu(\underline{\theta})}{r + \frac{1}{2}\mu(\underline{\theta})} \left(\frac{u_2 - u}{r + \frac{1}{2}\mu(\underline{\theta})} + \frac{B_n}{B} \frac{u_n - u_0}{r + \frac{1}{2}q(\underline{\theta})} \right). \tag{27}$$

Therefore, a decrease in $\underline{\theta}$ (equivalently, an increase in κ) leads to a decrease in $D(\underline{\theta})$ since the expression in parenthesis, $\frac{u_2 - u}{r + \frac{1}{2}\mu(\underline{\theta})} + \frac{B_n}{B} \frac{u_n - u_0}{r + \frac{1}{2}q(\underline{\theta})}$ decreases. This decrease comes from two effects.

[26] Additionally, we assume that the flow utility of a new entrant u_n is strictly higher than the flow utility of a forced renter u_0.

[27] These assumptions ensure that the surplus from trading between a mismatched owner who buys first and a real estate firm is always positive when θ is close to 1, regardless of the equilibrium, and similarly that the surplus from trading between a mismatched owner who sells first and a new entrant is also positive. Additionally, the surplus between a mismatched owner that buys first and one that sells first is assumed to be negative around $\theta = 1$. Finally, there is an assumption that ensures that as $\theta \to 1$ the value of buying first equals the value from selling first in both equilibria.

First, $\mu\left(\underline{\theta}\right)$ decreases and $q\left(\underline{\theta}\right)$ increases, so the first term in the parenthesis becomes more negative (given that $u_2 < u - \chi < u$) and the second term decreases (since $u_n > u_0$). This effect is tightly linked to the main mechanism in our basic set-up discussed in Section 4.2, relating mismatched owners' incentives to buy first or sell first to a change in the market tightness θ. Specifically, as before, a decrease in $\underline{\theta}$ decreases the value from buying first given a higher expected time-on-market for double owners, while it increases the value from choosing to selling first, given a shorter expected time-on-market for forced renters.

Additionally, the fraction of new entrants ($\frac{B_n}{B} = \underline{\theta}$) decreases as $\underline{\theta}$ falls, which decreases the second term further. This additionally strengthens the incentives to sell first. The reason for this additional effect, which is not present in our basic set-up with a single transaction price, is a compositional effect on the buyer side of the market. In particular, as $\underline{\theta}$ falls there are relatively fewer new entrants and relatively more forced renters. Since forced renters have a lower outside option compared to new entrants (given the lower flow utility), there is a higher surplus from transacting with a forced renter, which provides an additional incentive for mismatched owners to sell first.

The same forces operate in a "Sellers' market" equilibrium where mismatched owners buy first. Therefore, we can show the following result on the existence of multiple steady state equilibria in the model with Nash bargaining:

Proposition 8. *Consider the economy with prices determined by symmetric Nash bargaining. Suppose that conditions B1, B2, and B3 given in the Appendix hold and consider the limit economy, in which $g \to 0$, $\lambda \to 0$, and $\frac{\lambda}{g} = \kappa$. There is a $\kappa^* > 0$, such that, for $\kappa < \kappa^*$, there exists a steady state equilibrium, in which all mismatched owners buy first and the equilibrium market tightness converges to $\overline{\theta} = 1 + \kappa$. Also, there exists a steady state equilibrium, in which all mismatched owners sell first and the equilibrium market tightness converges to $\underline{\theta} = \frac{1}{1+\kappa}$.*

Proof. See Appendix. □

How much can prices fluctuate across the two equilibria in the model with Nash bargaining? To assess this, we use the calibrated values from our simple calibration in Section 4.6 and conduct numerical experiments. These reveal that average transaction prices can decrease by up to 10% across the two equilibria given the calibrated values for γ, g, and the matching function and the implied market tightnesses. This constitutes around one half of the observed decline in house prices in Copenhagen in the period 2007-2012.

7 Additional Extensions

7.1 Allowing for Entry as both Buyer and Seller

Up to now, we assumed that there is a trade-off in the decision of a mismatched owner to enter the housing market as a buyer or as a seller. In this section, we allow for the possibility that mismatched owner can choose to be both a buyer and a seller at the same time, and extend our main result about equilibrium multiplicity. Importantly, the main mechanisms studied above carry over, since the decision to enter as both a buyer and a seller depends ultimately on the value from entering as a buyer only and the value from entering as a seller only.

We denote by SB the measure of mismatched owners who enter as both a seller and a buyer in the housing market.[28] We assume that there is a small search cost $\epsilon > 0$ from participating on each side of the market, so that entering as both a buyer and a seller implies that the mismatched owner must incur 2ϵ, while entering as a buyer or as a seller only implies a search cost of ϵ. Therefore, if an agent is considering entering as both a buyer and a seller but expects that he would never transact (either as a buyer or as a seller), then he is better off from entering only on one side of the market, either as a buyer only or a seller only.

The value function V^{SB} satisfies the following equation in a steady state equilibrium

$$\rho V^{SB} = u - \chi - 2\epsilon + \mu(\theta) \max\left\{0, p + V^{B0} - V^{SB}\right\} + q(\theta) \max\left\{0, -p + V^{S2} - V^{SB}\right\}. \tag{28}$$

We solve for the value function to obtain

$$V^{SB} = \max\left\{\tilde{V}^{SB}, V^{B1} - \epsilon, V^{S1} - \epsilon\right\}, \tag{29}$$

where

$$\tilde{V}^{SB} = \frac{u - \chi - 2\epsilon}{\rho + \mu(\theta) + q(\theta)} + \frac{q(\theta)}{\rho + \mu(\theta) + q(\theta)} v^{S2} + \frac{\mu(\theta)}{\rho + \mu(\theta) + q(\theta)} v^{B0},$$

with

$$v^{B0} \equiv \frac{\tilde{u}_0}{\rho + q(\theta)} + \frac{q(\theta)}{\rho + q(\theta)} V, \tag{30}$$

and

$$v^{S2} \equiv \frac{\tilde{u}_2}{\rho + \mu(\theta)} + \frac{\mu(\theta)}{\rho + \mu(\theta)} V. \tag{31}$$

Condition (29) shows that if a mismatched owner anticipates not transacting as a buyer (seller), he is better off from entering only as a buyer (seller). Below we disregard the search cost ϵ, but assume that if $\tilde{V}^{SB} < \max\left\{V^{B1}, V^{S1}\right\}$ the owner either buys first or sells first.

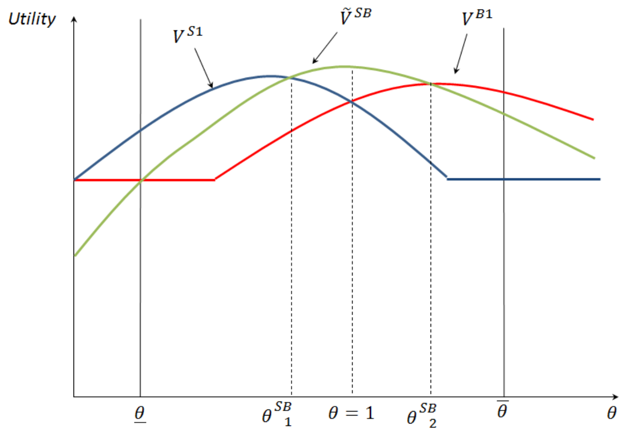
We can re-write $\tilde{V}^{SB}$ as

$$\begin{aligned} \tilde{V}^{SB} &= \frac{\rho + \mu(\theta)}{\rho + \mu(\theta) + q(\theta)} V^{S1} + \frac{q(\theta)}{\rho + \mu(\theta) + q(\theta)} v^{S2} \\ &= \frac{\rho + q(\theta)}{\rho + \mu(\theta) + q(\theta)} V^{B1} + \frac{\mu(\theta)}{\rho + \mu(\theta) + q(\theta)} v^{B0}, \end{aligned} \tag{32}$$

that is the value of simultaneous selling and buying can be written as a weighted average of the value of selling first and v^{S2} or the value of buying first and v^{B0}. Therefore, $\tilde{V}^{SB} \leq V^{S1} \iff v^{S2} \leq V^{S1}$ and $\tilde{V}^{SB} \leq V^{B1} \iff v^{B0} \leq V^{B1}$. We denote by θ_1^{SB} the value of θ for which $v^{S2} = V^{S1}$ and by θ_2^{SB} the value of θ for which $v^{B0} = V^{B1}$. Note that $\tilde{V}^{SB} < V^{S1}$ for $\theta < \theta_1^{SB}$, and $\tilde{V}^{SB} < V^{B1}$ for

[28] Note that the definition of equilibrium requires a straightforward extension to accommodate this particular type of mismatched owners in the economy.

Figure 7: Equilibrium multiplicity when entry as both a buyers and seller is allowed ($\tilde{u}_0 = \tilde{u}_2$).



$\theta > \theta_2^{SB}$. We now show the main result of this Section:

Proposition 9. *Consider the above economy. Let θ^{S2} be defined as the value of θ, at which $v^{S2} = \frac{u-\chi}{\rho} = V^{B1}$ and θ^{B0} be defined as the value of θ, at which $v^{B0} = \frac{u-\chi}{\rho} = V^{S1}$. Suppose that $\theta^{B0} < \theta^{S2}$. Then it is never optimal for a mismatched owner to enter as both a buyer and a seller. Suppose that $\theta^{B0} \geq \theta^{S2}$. If $\theta_1^{SB} \leq 1$, then there exists a steady state equilibrium with market tightness $\theta = 1$, in which mismatched owners enter as both a buyer and a seller. If $\underline{\theta} < \theta_1^{SB}$ and $\overline{\theta} > \theta_2^{SB}$, there can also exist "Buyers' market" and "Sellers' market" equilibria as described in Proposition 5.*

Proof. See Appendix. □

Figure 7 below shows the equilibrium multiplicity in this case and also denotes the values for θ_1^{SB} and θ_2^{SB}. Note that if $\underline{\theta} > \theta_1^{SB}$, then a "Buyers' market" equilibrium does not exist, since entering as a seller first only is dominated by entering as both a buyer and a seller. Similarly, if $\overline{\theta} < \theta_2^{SB}$, then a "Sellers' market" equilibrium does not exist, since entering as a buyer first is dominated by entering as both a buyer and a seller. Therefore, there could be a unique equilibrium in which $\theta = 1$. However, if $\underline{\theta}$ and $\overline{\theta}$ are sufficiently away from $\theta = 1$ or equivalently, θ_1^{SB} and θ_2^{SB} are sufficiently close to $\theta = 1$, there will again be multiplicity.

Also, Proposition 9 shows that the equilibrium with $\theta = 1$ need not always exist but rather that it exists under some more special conditions. For example, consider a case where there is a strong asymmetry in flow payoffs, so $\tilde{u}_0 \ll \tilde{u}_2$. Clearly, in that case it is never optimal to try to sell before buying, since becoming a forced renter (even for a short time) is too costly.

7.2 Homeowners compensated for their housing unit upon exit

In this section we show that our main results continue to hold under the alternative assumption that homeowners are compensated for the value of their housing units when they exit the economy.

Suppose that upon exit homeowners receive bids for their housing unit(s) from a set of competitive real estate firms. Therefore, given that the value of a housing unit to a real estate firm is $V^A(\theta)$, homeowners receive $V^A(\theta)$ for each housing unit that they own. Again, we consider a steady state equilibrium with a fixed market tightness θ. We define $\tilde{u}_0(\theta, g) \equiv u_0 + \triangle - gV^A(\theta)$ and $\tilde{u}_2(\theta, g) = u_2 - \triangle + gV^A(\theta)$. Note that $V^A(\theta)$ is (weakly) increasing in θ, so $\tilde{u}_2$ is increasing in θ and $\tilde{u}_0$ is decreasing in θ;

Given this definition, the difference between the values from buying first and selling first (assuming a mismatched owner transacts in both cases), $D(\theta) \equiv V^{B1} - V^{S1}$, can be written as

$$D(\theta) = \frac{\mu(\theta)}{(\rho + q(\theta))(\rho + \mu(\theta))} \left[\left(1 - \frac{1}{\theta}\right)(u - \chi - \tilde{u}_2(\theta, g)) - \tilde{u}_0(\theta, g) + \tilde{u}_2(\theta, g) \right].$$

Let $\tilde{\theta}$ be defined implicitly by

$$\tilde{\theta} \equiv \frac{u - \chi - \tilde{u}_2\left(\tilde{\theta}, g\right)}{u - \chi - \tilde{u}_0\left(\tilde{\theta}, g\right)},$$

whenever that equation has a solution.[29] Note that in the limit as $g \to 0$, assumption A1 will hold. Therefore, for g sufficiently close to zero, we will have that $u - \chi > \max\{\tilde{u}_0(\theta, g), \tilde{u}_2(\theta, g)\}$, for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, and so a version of Lemma 1 will hold in this case as well. Given this result one can then easily construct multiple steady state equilibria as in Proposition 5.

8 Institutional Details and Concluding Comments

In this section we compare the process of housing sales in several countries, and then provide brief concluding comments.

8.1 Institutional Details

Actual housing markets in different countries differ in their institutional characteristics. Naturally, our model of the housing market abstracts from many of these peculiarities. As a result, the fit between the model and the way that houses are bought and sold may vary across countries. Nonetheless, we think our model captures essential elements of housing transactions for many countries, including Denmark, Norway, the Netherlands, and the United States. In these countries, the institutional set-up for the process of housing transactions is such that home-owners are concerned about the order of buying and selling, at least to some extent. In principle, the same issue occurs in the United Kingdom, but here the phenomenon of housing chains (see Rosenthal (1997)) may provide a way to accommodate the risks associated with moving in the owner-occupied housing market. Because of the widespread usage of housing chains in the UK, our model may be less suited for capturing the way houses are bought and sold in that country. Instead it describes more closely the housing markets of countries where housing chains are rare or non-existent.[30]

[29] Note that the above equation for $\tilde{\theta}$, whenever it has a solution, has a unique solution for any $g \geq 0$, since given the properties of $\tilde{u}_0$ and $\tilde{u}_2$, it follows that the right hand side of this expression is (weakly) decreasing in θ. Furthermore, the right hand side is strictly decreasing in g for any $\theta > 0$, so by the implicit function theorem, $\tilde{\theta}$ is decreasing in g.

[30] There is anecdotal evidence that innovations in mortgage financing in recent years may have decreased the importance of housing chains in the UK market as well.

Additionally, in England and Wales buyers and sellers are not legally bound to an agreed transaction until late in the process, so that both sides easily renege on offers (Rosenthal (1997)). As a result, if a household is not able to complete a second transaction as fast as desired, it may just withdraw from a first transaction in order to avoid the costly period in between. As shown in the Appendix in Table 1 for buyers and Table 2 for sellers, commitment to an agreed transaction is significantly larger outside the UK. The tables show whether the law requires a grace period, what the penalty is for reneging during and after this grace period, which conditions that allow to dissolve a contract are usually included in the contract, and for what period parties can still refer to these conditions. In Denmark (where our transactions data are from) for instance, only buyers enjoy a grace period of 6 days, in which they can cancel the transaction at a cost of one percent of the transaction price. Afterwards, buyers are liable for the full amount, while sellers can be taken to court if they do not transfer the house. Sometimes purchase offers allow for contingencies such as the ability to secure financing or the approval of one's own lawyer, but referral to these conditions requires proof and is restricted in time. The picture that emerges from the tables is that in Denmark, Norway, the Netherlands, and the United States it may be very costly to renege on a transaction once a purchase offer has been made or a conditional contract has been signed. For that reason, and because the period between two transactions can involve substantial costs, whether first to buy or first to sell should be an important decision for households in these countries.

8.2 Concluding Comments

The transaction sequence decision of owner-occupiers are affected by housing market conditions, such as the expected time-on-market for buyers and sellers and expectations about future house price appreciation. However, these choices in turn exert important feedback effects on the housing market, particularly on the buyer-seller ratio and from there on the time-to-trade for buyer and sellers. This makes the choice whether to buy-before-selling or sell-before-buying strategic complements, resulting in multiple equilibria. Switches between such equilibria can be associated with large changes in the stock of units for sale, average time-on-market, transactions, and prices. All of these are broadly consistent with key stylized facts about housing cycles.

The tractable equilibrium model that we study in this paper to show these effects is deliberately simplified and so lacks heterogeneity in many important dimensions. In particular, it lacks heterogeneity in the costs of being a double owner versus a forced renter, which are likely to vary substantially across households and also over time in response to aggregate shocks. In addition, we assumed constancy of the rate of mismatch and entrance and exit of the market, but endogenous fluctuations in γ and g are likely to increase propagation of aggregate shocks. Enriching the model along these dimensions will allow for a detailed quantitative model of the housing market, which can then be taken to the data. We view this as an important and promising step for future research.

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Appendix

A. Data Description

We use two data sets. The first (EJER) is an ownership register which contains the owners (private individuals and legal entities) of properties in Denmark as of the end of a given calendar year. The data set contains unique identifiers for owners (which, unfortunately, cannot be matched with other datasets beyond EJER for different years). It also contains unique identifiers for each individual property. The second data set (EJSA) contains a record of all property sales in a given calendar year. The majority of transactions include information on the sale price, take over and sale dates. Furthermore, they contain the property identifiers used in the EJER data-set, which allows for linking of the two datasets. The first data set is available from 1986 (recording ownership in 1985) until 2010 (recording ownership at the end of 2009), while the second is available from 1992 to 2010. Therefore, we effectively use data from 1991 (for ownership as of January 1, 1992) to 2009 (for ownership as of January 1, 2010).

We focus on the Copenhagen urban area (Hovedstadsområdet). We take the definition of the Copenhagen urban area as containing the following municipalities (by number): 101, 147, 151, 153, 157, 159, 161, 163, 165, 167, 173, 175, 183, 185, 187, 253, 269.[31]

We restrict attention to private owners and also to the primary owner of a property in a given year (whenever a property has more than one owners). Furthermore, we examine transactions where the new owner is a private individual and which have a non-missing sale date. We drop properties that are recorded to transact more than once in a given year. We also remove property-year observations for which no owner is recorded. This leaves us with a total of 3312520 property-year observations. These comprise 199812 unique properties and 345943 unique individual owners over our sample period.

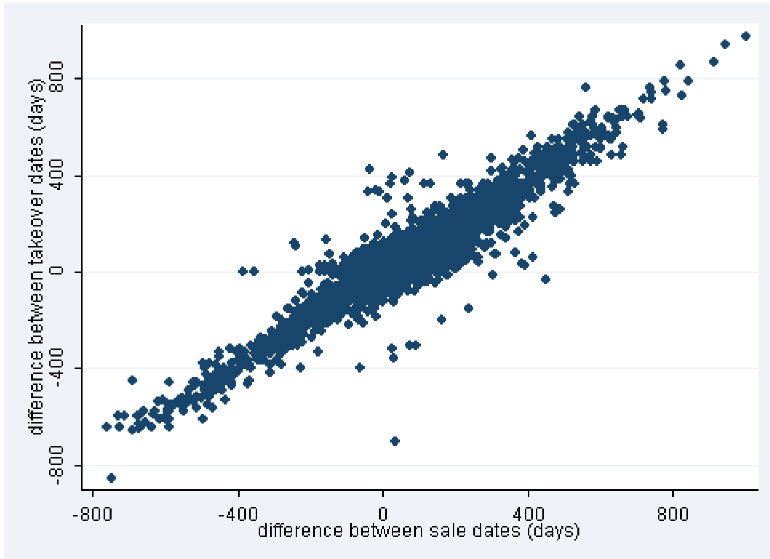
To identify an individual owner as a buyer-and-seller we rely on the information from the ownership register across consecutive years. First of all, we use the information on ownership over consecutive years to determine the counterparties for each recorded transaction in our sample. We then identify an individual owner as a buyer-and-seller if he is recorded to buy a new property and sell an old property within the same year or over two consequtive years. An old property is defined as a property which an individual is registered as owning over at least 2 consequtive years.[32] In principal, adding back those agents does not substantially change the pattern we uncover. Also, we do not count individuals that are recorded as holding two properties for two or more consequtive years, which we treat as purchases for investment purposes.

We conduct this for individuals that are recorded as owning at most 2 properties at the end of any calendar year in our sample. This comprises the large majority of individual owners in our

[31] Due to a reform in 2007, which merged some municipalities and created a new one, we omit municipality 190 for consistency.

[32] We make this restriction in order not to misclassify as a buyer-and-seller an individual who acquires a house, for example as a bequest (which is not recorded as a transaction), which he ends up selling quickly and then buys a new house with the proceeds from the sale.

Figure 8: Difference in sale dates vs. difference in takeover dates, Copenhagen (1993-2008)



sample. In particular, in a given year in our sample from 1991-2009 there are on average only around 0.4% of individual owners who own more than two properties in the Copenhagen. Therefore, the majority of individuals hold at most 1 or 2 properties over that period. In particular, on average, around 1.6% of individual owners hold two properties at the end of a calendar year in our sample. Interestingly, around 5% of the recorded owners of two properties at the end of a calendar year are also identified as a buyer-and-seller according to our identification procedure described above with that number going up to almost 14% at the peak of the housing boom in 2006.

For each individual owner that has been identified as buyer-and-seller, we compute the time period (in days) between the sale of the old property and the purchase of the new property. Similarly, we compute the time period (in days) between the takeover date that of the buyer-and-seller's old property by the new owner and the takeover date for his new property. We then denote a buyer-and-seller for which the time period is negative (sale date is before purchase date) as "selling first" and a buyer-and-seller for which the time period is positive (sale date is after purchase date) as "buying first". We also do the same classification but based on takeover dates rather than sale dates. Given the way we identify a buyer-and-seller, we have a consistent count for the number of owners who "buy first" vs. "sell first" in a given year for the years 1993 to 2008.

In principle, and as Figures 1 and 2 show, working with either of the two identifications produces similar results. This is not surprising given that the time difference between the sale dates and takeover dates are highly correlated with a correlation coefficient of 0.9313. Figure 8 visualizes this strong correlation by plotting a scatter plot of the two time differences.

B. Institutional details

Table 1: Institutional details for buyers

Country	Grace period	Penalty for reneging during grace period	Penalty for reneging after grace period	Possible conditions to dissolve contract (requires proof)	Period to refer to dissolving conditions
Denmark	6 days	1% of price	Liable for full amount	May include as conditions: • Ability to secure financing • Lawyer reservation	As specified in the purchase offer
Norway	None	N/A	Liable for full amount	May include as condition: • Ability to secure financing	As specified in the purchase offer
Netherlands	3 days	None	Standard contract: • End contract: >10% price • Demand fulfillment: >0.3% sale price per day • Court	Standard contract: • Ability to secure financing • Applicability for national mortgage insurance • Structural inspection	As specified in the contract: usually not for more than a few weeks after signing it
United States	None (3 days in NJ only)	N/A	Losing the Earnest Money Deposit, ranging from \$500 to 10% of the price	Standard purchase offer: • Ability to secure financing • Appraisal • Structural inspection	Usually not for more than a few weeks after signing the offer, e.g. 17 days in CA
England & Wales	Pull out until exchange of (unconditional) contracts	Occasionally one loses holding deposit, ranging from £500 to £1000	10% price	None	N/A

Table 2: Institutional details for sellers

Country	Grace period	Penalty for reneging during grace period	Penalty for reneging after grace period	Possible conditions to dissolve contract
Denmark	None	N/A	Court	None
Norway	None	N/A	Court	None
Netherlands	3 days	N/A	Standard contract: • End contract: >10% price • Demand fulfillment: >0.3% sale price per day • Court	None
United States	None (3 days in NJ only)	N/A	Court	None
England & Wales	Pull out until exchange of contracts	None	Court	None

The information in Tables 1 and 2 is based on:

- http://boligejer.dk/koebsaftale/ for Denmark
- http://www.eiendomsrettsadvokaten.no/advokathjelp/kjop-og-salg-av-eiendom/bolig-eiendom-kjop-salg-eierskifte-forsikring-avhending-avhendingslov-opplysning-undersokelse-tinglysning-budgivning-skjote-kontraktsinngaelse/ for Norway
- http://www.eigenhuis.nl/juridisch/ for the Netherlands
- http://www.realtor.com and https://www.doorsteps.com/ for the United States
- http://hoa.org.uk/advice/guides-for-homeowners for England and Wales

C. Equilibrium concept and parameter restrictions for the basic model

First of all, the steady state value functions for a new entrant, a matched owner, and a real estate firm satisfy the following equations:

$$\rho V^{Bn} = u_n - R + q(\theta)\left(-p + V - V^{Bn}\right), \tag{33}$$

$$\rho V = u + \gamma\left(\max\left\{V^{B1}, V^{S1}\right\} - V\right), \tag{34}$$

and

$$\rho V^{A} = R + \mu(\theta)\left(p - V^{A}\right). \tag{35}$$

Importantly, in every steady state equilibrium, V satisfies $V \geq \tilde{V}$, where $\tilde{V} = \frac{u}{\rho+\gamma} + \frac{\gamma}{\rho+\gamma}V^m$, with $V^m = \frac{u-\chi}{\rho}$. Hence, $\tilde{V}$ is the value of a matched owner who never transacts. Therefore, $V \geq \tilde{V} = \frac{u}{\rho} - \frac{\gamma}{\rho+\gamma}\frac{\chi}{\rho}$ in any steady state equilibrium.

Parameter restrictions

Sufficient conditions for new entrants, forced renters and double owners to prefer transacting and becoming matched owners are given by

$$\frac{u_n - R}{\rho} \leq \tilde{V} - p, \tag{36}$$

$$\frac{u_0 - R}{\rho} \leq \tilde{V} - p, \tag{37}$$

and

$$\frac{u_2 + R}{\rho} \leq \tilde{V} + p. \tag{38}$$

Since $u_n \geq u_0$, we can disregard (37), as it is implied by (36). Conditions (36) and (38) imply restrictions for the values of the house price, p, that are sufficient for these agents to be willing to transact at p, namely $p \in \left[\frac{u_2}{\rho} - \tilde{V} + \frac{R}{\rho}, \tilde{V} - \frac{u_n}{\rho} + \frac{R}{\rho}\right]$.

From (35) a real estate firm is willing to transact iff $p \geq \frac{R}{\rho}$. Therefore, equilibrium is defined for a house price p, that satisfies

$$p \in \left[\max\left\{\frac{u_2}{\rho} - \tilde{V}, 0\right\} + \frac{R}{\rho}, \tilde{V} - \frac{u_n}{\rho} + \frac{R}{\rho}\right]. \tag{39}$$

For $u - \chi \geq \max\{u_0, u_2\}$, which is the condition we will use to characterize equilibria, it follows that $\frac{u_2}{\rho} - \tilde{V} < 0$ and so the set for prices is given by

$$p \in \left[\frac{R}{\rho}, \tilde{V} - \frac{u_0}{\rho} + \frac{R}{\rho}\right]. \tag{40}$$

Finally, a sufficient conditon for $V^{S1} > \frac{u-\chi}{\rho}$ and $V^{B1} > \frac{u-\chi}{\rho}$ at $\theta = \tilde{\theta}$, with $\tilde{\theta}$ as defined in (14) is

Assumption A2: $\frac{u-\chi}{\rho} < \frac{u-\chi}{\rho+\mu(\tilde{\theta})} + \frac{\mu(\tilde{\theta})}{(\rho+\mu(\tilde{\theta}))(\rho+q(\tilde{\theta}))}\tilde{u}_0 + \frac{\mu(\tilde{\theta})q(\tilde{\theta})}{(\rho+\mu(\tilde{\theta}))(\rho+q(\tilde{\theta}))}\left(\frac{u}{\rho} - \frac{\gamma}{\rho+\gamma}\frac{\chi}{\rho}\right)$.
Note that $\frac{u}{\rho} - \frac{\gamma}{\rho+\gamma}\frac{\chi}{\rho} \leq V$, $\forall\theta$, so the right hand side of this expression is lower than the value of V^{S1} at $\theta = \tilde{\theta}$.

Steady state flow conditions

Before moving to our formal definition, it is necessary to describe the flow conditions that the aggregate stock variables defined in Section 3.2 must satisfy. We have that in a steady state equilibrium, given a market tightness θ, the steady state values of B_n, B_0, B_1, S_1, S_2, O, and A must satisfy the following system of flow conditions:

$$g = \left(q\left(\theta\right) + g\right) B_n, \tag{41}$$

$$\mu\left(\theta\right) S_1 = \left(q\left(\theta\right) + g\right) B_0, \tag{42}$$

$$\mu\left(\theta\right) S_2 + q\left(\theta\right)\left(B_n + B_0\right) = \left(\gamma + g\right) O, \tag{43}$$

$$\gamma x_b O = \left(q\left(\theta\right) + g\right) B_1, \tag{44}$$

$$\gamma x_s O = \left(\mu\left(\theta\right) + g\right) S_1, \tag{45}$$

$$q\left(\theta\right) B_1 = \left(\mu\left(\theta\right) + g\right) S_2, \tag{46}$$

$$g\left(O + B_1 + S_1 + 2S_2\right) = \mu\left(\theta\right) A, \tag{47}$$

$$x_b + x_s = 1, \tag{48}$$

where x_b, and x_s are the equilibrium fractions of mis-matched owners that “buy first”, and “sell first”, respectively. Apart from these conditions, the aggregate variables must satisfy the population constancy and housing ownership conditions (1) and (2). Finally, the equilibrium market tightness θ, satisfies

$$\theta = \frac{B}{S} = \frac{B_n + B_0 + B_1}{S_1 + S_2 + A}. \tag{49}$$

Equilibrium definition

We define a steady state equilibrium for this economy in the following way:

Definition 10. A steady state equilibrium consists of a house price p, equilibrium rental rate R, value functions V^{Bn}, V^{B0}, V^{B1}, V^{S2}, V^{S1}, V, V^A, market tightness θ, fractions of mismatched owners that choose to buy first and sell first, x_b, and x_s, and aggregate stock variables, B_n, B_0, B_1, S_1, S_2, O, and A such that:

1. The house price $p \in \left[\frac{R}{\rho}, \tilde{V} - \frac{u_0}{\rho} + \frac{R}{\rho}\right]$;
2. The equilibrium rental rate $R \in [0, u_0]$;
3. The value functions satisfy equations (3)-(6) and (33)-(35) given θ, p, and R;
4. Mismatched owners choose $x \in \{b, s\}$, to maximize $\overline{V} = \max\left\{V^{B1}, V^{S1}\right\}$ and the fractions x_b, and x_s reflect that choice, i.e.

$$x_b = \int_i I\{x_i = b\}\, di,$$

   where $i \in [0,1]$ indexes the i-th mismatched owner, and similarly for x_s;
5. The market tightness θ solves (49) given B_n, B_0, B_1, S_1, S_2, O, and A;
6. The aggregate stock variables B_n, B_0, B_1, S_1, S_2, O, and A, solve (41)-(47) given θ and mismatched owners' optimal decisions reflected in x_b and x_s.

D. Proofs

Proof of Lemma 1

Proof. First of all, note that the function $D(\theta)$, defined in (11) crosses zero only at $\theta = \tilde{\theta}$. To see this, notice that

$$\lim_{\theta \to 0} D(\theta) = \frac{\tilde{u}_2 - (u - \chi)}{\rho} < 0,$$

and

$$\lim_{\theta \to \infty} D(\theta) = \frac{u - \chi - \tilde{u}_0}{\rho} > 0.$$

Away from these two limiting values, $D(\theta) > 0$, whenever

$$\left(1 - \frac{1}{\theta}\right)(u - \chi - \tilde{u}_2) - \tilde{u}_0 + \tilde{u}_2 > 0,$$

which is equivalent to $\tilde{\theta} < \theta$. Therefore, $D(\theta) > 0$ iff $\theta \in \left(\tilde{\theta}, \infty\right)$ and $D(\theta) < 0$ iff $\theta \in \left(0, \tilde{\theta}\right)$. Therefore, $D(\theta) = 0$, iff

$$\left(1 - \frac{1}{\theta}\right)(u - \chi - \tilde{u}_2) - \tilde{u}_0 + \tilde{u}_2 = 0,$$

or $\theta=\tilde{\theta}$. Note that $D(\theta)$ fully summarizes the incentives of a mismatched owner to buy first/sell first apart from $\theta=0$ and $\theta\to\infty$. To see this, let

$$\begin{aligned}\tilde{V}^{B1} &= \frac{u-\chi}{\rho+q(\theta)}+\frac{q(\theta)\tilde{u}_2}{(\rho+\mu(\theta))(\rho+q(\theta))}+\frac{q(\theta)\mu(\theta)}{(\rho+\mu(\theta))(\rho+q(\theta))}V-\frac{u-\chi}{\rho}\\ &= \frac{q(\theta)}{\rho+q(\theta)}\left(\frac{\tilde{u}_2}{\rho+\mu(\theta)}+\frac{\mu(\theta)}{\rho+\mu(\theta)}V-\frac{u-\chi}{\rho}\right),\end{aligned}$$

and

$$\begin{aligned}\tilde{V}^{S1} &= \frac{u-\chi}{\rho+\mu(\theta)}+\frac{\mu(\theta)\tilde{u}_0}{(\rho+\mu(\theta))(\rho+q(\theta))}+\frac{q(\theta)\mu(\theta)}{(\rho+\mu(\theta))(\rho+q(\theta))}V-\frac{u-\chi}{\rho}\\ &= \frac{\mu(\theta)}{\rho+\mu(\theta)}\left(\frac{\tilde{u}_0}{\rho+q(\theta)}+\frac{q(\theta)}{\rho+q(\theta)}V-\frac{u-\chi}{\rho}\right).\end{aligned}$$

The functions $\tilde{V}^{B1}$ and $\tilde{V}^{S1}$ give the difference between the value of transacting and never transacting for a buyer first and seller first, respectively.

By Assumption A2, at $\tilde{\theta}$, $V^{S1}>\frac{u-\chi}{\rho}$ and $V^{B1}>\frac{u-\chi}{\rho}$, so at $\theta=\tilde{\theta}$, $\tilde{V}^{B1}>0$, and $\frac{\tilde{u}_2}{\rho+\mu(\theta)}+\frac{\mu(\theta)}{\rho+\mu(\theta)}V-\frac{u-\chi}{\rho}>0$. Furthermore, this latter inequality holds for any $\theta>\tilde{\theta}$, and so $\tilde{V}^{B1}>0$ for any $\theta>\tilde{\theta}$. Therefore, for any $\theta>\tilde{\theta}$, a mismatched owner who buys first is better off transacting than not transacting. Similarly, for $\theta<\tilde{\theta}$ the mismatched owner who sells first is better off transacting than not transacting.

Therefore, for $\theta\in(0,\theta)$, if $D(\theta)>0$, a mismatched owners is better off buying first (and transacting) compared to selling first (and transacting or not transacting) and similarly, if $D(\theta)<0$, a mismatched owner is better off selling first (and transacting) compared to buying first (and transacting or not transacting). At $D(\theta)=0$, he is indifferent between buying first (and transacting) and selling first (and transacting).

Finally, clearly if $\theta\to\infty$, $\tilde{V}^{B1}\to 0$, so $V^{B1}\to\frac{u-\chi}{\rho}=V^{S1}$. Similarly, if $\theta\to 0$, $\tilde{V}^{S1}\to 0$, and so $V^{S1}\to\frac{u-\chi}{\rho}=V^{B1}$. □

Proof of Lemma 2

Proof. For the case where mismatched owners buy first ($x_s=0$), the stock-flow conditions are

$$g=(q(\theta)+q)B_n,$$

$$\gamma O=(q(\theta)+g)B_1,$$

$$q(\theta)B_1=(\mu(\theta)+g)S_2,$$

$$g=(\mu(\theta)+g)A,$$

$$B_n+B_1+S_2+O=1,$$

and

$$B_n = A + S_2.$$

It follows that $B_n = \frac{g}{q(\theta)+g}$ and $A = \frac{g}{\mu(\theta)+g}$, or $A = \frac{q(\theta)+g}{\mu(\theta)+g} B_n$, so $S_2 = \frac{g}{q(\theta)+g} - \frac{g}{\mu(\theta)+g}$. Therefore, from the equation for θ, we have that $B_1 = (\theta - 1) B_n$ and so $O = \frac{1}{\gamma} (q(\theta) + g)(\theta - 1) B_n$. Substituting into the population constancy condition, we have that

$$\theta B_n + B_n - \frac{q(\theta) + g}{\mu(\theta) + g} B_n + \frac{1}{\gamma} (q(\theta) + g)(\theta - 1) B_n = 1,$$

which, after substituting for B_n and re-arranging we can write as

$$\left(\frac{1}{q(\theta) + g} + \frac{1}{\gamma} \right) \theta + \left(\frac{1}{q(\theta) + g} - \frac{1}{\mu(\theta) + g} \right) = \frac{1}{g} + \frac{1}{\gamma}.$$

This is exactly equation (16). At $\theta = 1$, the left-hand side equals

$$\frac{1}{q(1) + g} + \frac{1}{\gamma} < \frac{1}{g} + \frac{1}{\gamma}.$$

Furthermore, note that $\left(\frac{1}{q(\theta)+g} + \frac{1}{\gamma} \right) \theta$ is strictly increasing in θ and also unbounded. Similarly, $\left(\frac{1}{q(\theta)+g} - \frac{1}{\mu(\theta)+g} \right)$ is strictly increasing in θ as well. Therefore, the left-hand side of (16) is strictly increasing in θ, unbounded, and lower than the right-hand side for $\theta = 1$. Therefore, it has a unique solution for $\theta > 1$. We call this solution $\overline{\theta}$. Furthermore, by the Implicit Function Theorem, it immediately follows that $\overline{\theta}$ is increasing in γ.

For the case where mismatched owners sell first ($x_s = 1$) the stock-flow conditions become

$$g = (q(\theta) + q) B_n,$$

$$\mu(\theta) S_1 = (q(\theta) + g) B_0,$$

$$\gamma O = (\mu(\theta) + g) S_1,$$

$$g = (\mu(\theta) + g) A,$$

$$B_n + B_0 + S_1 + O = 1,$$

and

$$B_n + B_0 = A.$$

It follows that $A = \frac{g}{\mu(\theta)+g} = B_0 + B_n$, $S_1 = \frac{1-\theta}{\theta} A$ and $O = \frac{1}{\gamma} (\mu(\theta) + g) \frac{1-\theta}{\theta} A$. Therefore, substituting for these in the population constancy condition, we have that

$$\frac{1}{\theta} A + \frac{1}{\gamma} (\mu(\theta) + g) \frac{1 - \theta}{\theta} A = 1.$$

Substituting for A, we obtain an equation for θ of the form

$$\left(\frac{1}{\mu(\theta)+g}+\frac{1}{\gamma}\right)\frac{1}{\theta}=\frac{1}{g}+\frac{1}{\gamma},$$

which is equation (17). At $\theta=1$, the left-hand side equals

$$\frac{1}{\mu(1)+g}+\frac{1}{\gamma}<\frac{1}{g}+\frac{1}{\gamma}.$$

Note also that $\left(\frac{1}{\mu(\theta)+g}+\frac{1}{\gamma}\right)\frac{1}{\theta}$ is strictly decreasing in θ and goes to 0 as $\theta\to\infty$. Also it asymptotes to ∞ as $\theta\to 0$. Therefore, the equation has a unique solution for $\theta<1$. We call this solution $\underline{\theta}$. By the Implicit Function Theorem, it immediately follows that $\underline{\theta}$ is decreasing in γ.

□

Proof of Proposition 3

Proof. By Lemma 1, for $\tilde{u}_0=\tilde{u}_2=c$, mismatched owners are indifferent between buying first and selling first at $\theta=1$. Also, by Assumption A2, they are strictly better off from transacting than not transacting. Finally, to show that the stock-flow conditions are satisfied, suppose that $x_s=x_b=\frac{1}{2}$. We have

$$\gamma\frac{1}{2}O=(q(\theta)+g)B_1, \tag{50}$$

and

$$\gamma\frac{1}{2}O=(\mu(\theta)+g)S_1. \tag{51}$$

At $\theta=1$, $\mu(\theta)=q(\theta)=\mu(1)$, so $B_1=S_1$. Also, $B_n=A=\frac{g}{\mu(1)+g}$ and $B_0=S_2=S_1\frac{\mu(1)}{\mu(1)+g}$. Finally, population constancy implies that

$$2S_1\frac{\mu(1)}{\mu(1)+g}+2S_1+2S_1\frac{\mu(1)+g}{\gamma}=\frac{\mu(1)}{\mu(1)+g},$$

which is satisfied for some $S_1\in\left(0,\frac{1}{2}\right)$. □

Proof of Proposition 4

Proof. By Lemma 2 $\overline{\theta}$ satisfies the stock-flow conditions when all mismatched owners buy first, and similarly $\underline{\theta}$ satisfies the stock-flow conditions if they sell first. Then by Lemma 1 their actions are optimal given these market tightnesses. □

Proof of Proposition 5

Proof. Clearly, Lemma 2 that determines the values of $\overline{\theta}$ and $\underline{\theta}$ is independent of the agents' payoffs. With regard to Item 1, a direct application of Lemma 1 shows that if $\tilde{\theta}\in\left[\underline{\theta},\overline{\theta}\right]$, then at $\theta=\underline{\theta}$ a mismatched owner is (weakly) better off from selling first and at $\theta=\overline{\theta}$, he is (weakly) better off buying first. Consequently, agents' actions are optimal given θ and the steady state value of θ is consistent with agents' actions. Considering Item 2, by the same logic a steady state equilibrium in which mismatched owners buy first and $\theta = \overline{\theta}$ exists. To see that it is the only symmetric steady state equilibrium, remember from Lemma 1 that mismatched owners only sell first for $\theta < \tilde{\theta}$, which contradicts $\tilde{\theta} < \underline{\theta}$. The same logic applies to Item 3. □

Proof of Proposition 6

Proof. Consider the difference between the two value functions, $D(\theta) = V^{B1} - V^{S1}$ assuming that the mismatched owner transacts in both cases.

$$D(\theta) = \frac{\mu(\theta)\left(1-\frac{1}{\theta}\right)\left(u-\chi-c+\lambda\left(\overline{V}_N - v^{B0}\right)\right)}{(\rho+q(\theta)+\lambda)(\rho+\mu(\theta)+\lambda)} + \frac{\frac{\lambda\mu(\theta)\left(1-\frac{1}{\theta}\right)q(\theta)}{(r+\mu(\theta))(r+q(\theta))}[\rho V - c] + \mu(\theta)\left(1+\frac{1}{\theta}\right)\lambda(p_N - p)}{(\rho+q(\theta)+\lambda)(\rho+\mu(\theta)+\lambda)}. \tag{52}$$

Consider the case of $1 < \theta \leq \overline{\theta}$, so $\overline{V}_N = V_N^{B1}$. If $\overline{V}_N = V_N^{B1}$, this difference simplifies further to

$$D(\theta) = \frac{\mu(\theta)\left[\left(1-\frac{1}{\theta}\right)\left(1+\frac{\lambda}{\rho+q(\theta)}\right)(u-\chi-c) + \left(1+\frac{1}{\theta}\right)\lambda(p_N-p)\right]}{(\rho+q(\theta)+\lambda)(\rho+\mu(\theta)+\lambda)}. \tag{53}$$

Suppose that $p_N < p$ and define θ_{B1}^{PR} as the solution to

$$\frac{\theta_{B1}^{PR}-1}{\theta_{B1}^{PR}+1}\left(1+\frac{\lambda}{\rho+q\left(\theta_{B1}^{PR}\right)}\right) = \frac{\lambda(p-p_N)}{(u-\chi-c)}. \tag{54}$$

Therefore, θ_{B1}^{PR} is the value of θ that leaves a mismatched owner indifferent between buying first and selling first he anticipates a price change of $p_N - p$ and a market tightness of $\theta > 1$ after the price change. Note that θ_{B1}^{PR} is increasing in $p - p_N$ if $\theta_{B1}^{PR} \geq 1$. Therefore, a sufficient condition for mismatched owners to prefer to sell first, given $1 < \theta \leq \overline{\theta}$, is that $\theta_{B1}^{PR} > \overline{\theta}$.

Similarly, consider the case of $\underline{\theta} \leq \theta < 1$, so $\overline{V}_N = V_N^{S1}$. In that case the difference in value functions can be written as

$$D(\theta) = \frac{\mu(\theta)\left[\left(1-\frac{1}{\theta}\right)\left(1+\frac{\lambda}{\rho+\mu(\theta)}\right)(u-\chi-c) + \left(1+\frac{1}{\theta}\right)\lambda(p_N-p)\right]}{(\rho+q(\theta)+\lambda)(\rho+\mu(\theta)+\lambda)}. \tag{55}$$

Suppose that $p_N > p$ and define θ_{S1}^{PR} as the solution to

$$\frac{\theta_{S1}^{PR}-1}{\theta_{S1}^{PR}+1}\left(1+\frac{\lambda}{\rho+\mu\left(\theta_{S1}^{PR}\right)}\right) = \frac{\lambda(p-p_N)}{(u-\chi-c)}. \tag{56}$$

Similarly, to the case of θ_{B1}^{PR}, θ_{S1}^{PR} is increasing in $p - p_N$ if $\theta_{S1}^{PR} \leq 1$. Then, a sufficient condition for mismatched owner to prefer to buy first, given $\underline{\theta} \leq \theta < 1$ is that $\theta_{S1}^{PR} < \underline{\theta}$. □

Proof of Proposition 7

Proof. First, we consider the second regime $X(t) = 1$. In that regime the equilibrium market tightness, $\theta_1 = \underline{\theta}$ and agents' payoffs are as in Section 4.1. Therefore, by Lemma 1 and given the assumption for $\tilde{\theta}_1$, mismatched owners prefer to sell first at $\underline{\theta}$, and so $\underline{\theta}$ is consistent with the behavior of mismatched owners.

Second, consider the first regime. The value function of a mismatched owner who buys first in the first regime (and transacts) is given by

$$V_0^{B1} = \frac{u-\chi}{\rho+q\left(\overline{\theta}\right)+\lambda} + \frac{q\left(\overline{\theta}\right)}{\rho+q\left(\overline{\theta}\right)+\lambda}\left(V_0^{S2}-p_0\right) + \frac{\lambda}{\rho+q\left(\overline{\theta}\right)+\lambda}V^{S1},$$

where

$$V_0^{S2} \quad = \quad v^{S2}\left(\overline{\theta},p_0\right) + \frac{\lambda}{\rho+\mu\left(\overline{\theta}\right)+\lambda}\left(v^{S2}\left(\underline{\theta},p_1\right) - v^{S2}\left(\overline{\theta},p_0\right) + p_1 - p_0\right) + p_0,$$

with

$$v^{S2}\left(\theta,p\right) = \frac{u_2+R-\rho p}{\rho+\mu\left(\theta\right)} + \frac{\mu\left(\theta\right)}{\rho+\mu\left(\theta\right)}V,$$

and V^{S1} is given in (10), since the third term arises since in the second regime a mismatched owner sells first. For the value of selling first we have

$$V_0^{S1}\left(\overline{\theta}\right) = \frac{u-\chi}{\rho+\mu\left(\overline{\theta}\right)+\lambda} + \frac{\mu\left(\overline{\theta}\right)}{\rho+\mu\left(\overline{\theta}\right)+\lambda}\left(V_0^{B0}+p_0\right) + \frac{\lambda}{\rho+\mu\left(\overline{\theta}\right)+\lambda}V^{S1},$$

where

$$V_0^{B0} = v^{B0}\left(\overline{\theta},p_0\right) + \frac{\lambda}{\rho+q\left(\overline{\theta}\right)+\lambda}\left(v^{B0}\left(\underline{\theta},p_1\right) - v^{B0}\left(\overline{\theta},p_0\right) + p_0 - p_1\right) - p_0,$$

with

$$v^{B0}\left(\theta,p\right) = \frac{u_0-R+\rho p}{\rho+q\left(\theta\right)} + \frac{q\left(\theta\right)}{\rho+q\left(\theta\right)}V.$$

Consider the difference $D_0\left(\overline{\theta}\right) = V_0^{B1}\left(\overline{\theta}\right) - V_0^{S1}\left(\overline{\theta}\right)$. And note that

$$\lim_{\lambda\to 0} D_0\left(\overline{\theta}\right) = \frac{\mu\left(\theta\right)}{\left(\rho+q\left(\theta\right)\right)\left(\rho+\mu\left(\theta\right)\right)}\left[\left(1-\frac{1}{\theta}\right)\left(u-\chi-\tilde{u}_2\right) - \tilde{u}_0 + \tilde{u}_2\right] > 0,$$

by the assumption on $\tilde{\theta}_0$. Since $V_0^{B1}\left(\overline{\theta}\right)$ and $V_0^{S1}\left(\overline{\theta}\right)$ are continuous in λ, it follows that $D_0\left(\overline{\theta}\right)$ is continuous in λ as well, so that $D_0\left(\overline{\theta}\right) > 0$ will also be the case for λ sufficiently close to 0. Therefore, there exists a $\overline{\lambda}$ such that for $\lambda < \overline{\lambda}$, $V_0^{B1}\left(\overline{\theta}\right) > V_0^{S1}\left(\overline{\theta}\right)$ and mismatched owners prefer to buy first. Finally, $\overline{\theta}$ is consistent with the behavior of mismatched owners. $\square$

Proof of Proposition 9

Proof. To show the first part, suppose that $\theta^{B0} < \theta^{S2}$. It follows that $v^{S2} < \frac{u-\chi}{\rho} < V^{S1}$ for $\theta < \theta^{B0}$, and so $\theta_1^{SB} > \theta^{B0}$. Also, since $V^{S1} > v^{B0}$ for $\theta > \theta^{B0}$, it follows that θ_1^{SB} lies to the right of the value of θ, at which v^{S2} and v^{B0} cross. Similarly, $\theta_2^{SB} < \theta^{S2}$ and θ_2^{SB} lies to the left of the point where v^{S2} and v^{B0} cross. Therefore, $\theta_1^{SB} > \theta_2^{SB}$ and so $\tilde{V}^{SB} < \max\left\{V^{B1}, V^{S1}\right\}$ for any θ and it is never optimal for a mismatched owner to enter as both a buyer and a seller.

To show the second part, suppose that $\theta^{S2} < \theta^{B0}$. It follows that $v^{S2} < \frac{u-\chi}{\rho} < V^{S1}$ for $\theta > \theta^{S2}$, and so $\theta_1^{SB} > \theta^{S2}$. Also, since $V^{S1} < v^{B0}$ for $\theta < \theta^{B0}$, it follows that θ_1^{SB} lies to the left of the value of θ, at which v^{S2} and v^{B0} cross. Similarly, $\theta_2^{SB} < \theta^{B0}$ and θ_2^{SB} lies to the right of the point where v^{S2} and v^{B0} cross. Therefore, $\theta_1^{SB} < \theta_2^{BS}$ and $\tilde{V}^{SB} \geq \max\left\{V^{B1}, V^{S1}\right\}$ for $\theta \in \left[\theta_1^{BS}, \theta_2^{BS}\right]$. In that case, depending on the value of θ_1^{SB}, it is possible for a steady state equilibrium to exist, in which agents enter as both buyers and sellers.

In an equilibrium where agents enter as both buyers and sellers, we have the following flow conditions and housing and population conditions:

$$O + SB + 2S_2 = 1 - A,$$

and

$$B_n + B_0 + O + SB + S_2 = 1.$$

From these equations, it follows that $B_n + B_0 = A + S_2$. Therefore, $\theta = \frac{B_n+B_0+SB}{B_n+B_0+SB} = 1$. Given $\theta = 1$, one can solve for the aggregate stock variables given the flow conditions.

Additionally, note that if $\underline{\theta} < \theta_1^{SB}$, then $\tilde{V}^{SB} < \max\left\{V^{B1}, V^{S1}\right\}$ at $\underline{\theta}$. Therefore, if $V^{S1} \geq V^{B1}$, there will exist a "Buyers' market" equilibrium. This is ensured if $\tilde{\theta} \geq \underline{\theta}$. Similarly, if $\overline{\theta} > \theta_1^{SB}$, then $\tilde{V}^{SB} < \max\left\{V^{B1}, V^{S1}\right\}$ at $\overline{\theta}$. Therefore, if $V^{B1} \geq V^{S1}$, there will exist a "Sellers' market" equilibrium. This is ensured if $\tilde{\theta} \leq \overline{\theta}$. □

E. A model with prices determined by Nash bargaining

In this section we describe a modified version of our benchmark model. Matching is still random, but prices are determined by symmetric Nash bargaining between trading counterparties. Given random matching with counterparties of different types, buyers meet a particular seller type with a probability equal to their proportion in the population of sellers, and similarly for sellers. As in our benchmark model, we assume that mismatched owners can buy first or sell first.

We focus on steady state equilibria with value functions, the market tightness θ, and the stocks of different agent types constant over time. We denote the transaction price from a meeting between a seller of type $i \in \{S_1,\ S_2,\ A\}$ and a buyer of type $j \in \{B_n,\ B_0,\ B_1\}$ by p_{ij}, and similarly the bilateral match surplus is denoted by Σ_{ij}. These surpluses between a mismatched owner who buys first and different seller types are given by

$$\Sigma_{S1B1} = V^{S2} - V^{B1} + V^{B0} - V^{S1},$$

$$\Sigma_{S2B1} = V^{S2} - V^{B1} + V - V^{S2} = V - V^{B1},$$

$$\Sigma_{AB1} = V^{S2} - V^{B1} - V^{A},$$

while the surpluses between a forced renter and sellers are given by

$$\Sigma_{S1B0} = V - V^{B0} + V^{B0} - V^{S1} = V - V^{S1},$$

$$\Sigma_{S2B0} = V - V^{B0} + V - V^{S2} = 2V - V^{B0} - V^{S2},$$

$$\Sigma_{AB0} = V - V^{B0} - V^{A},$$

and similarly for the surpluses between a new entrant and sellers. With symmetric Nash bargaining between agents, we have that prices satisfy conditions of the form

$$p_{S1B1} + V^{S2} - V^{B1} = \frac{1}{2}\Sigma_{S1B1},$$

with similar conditions for the other prices p_{ij}. Simplifying this condition, we obtain that

$$p_{S1B1} = \frac{1}{2}V^{S2} - \frac{1}{2}V^{B0} - \frac{1}{2}\left(V^{B1} - V^{S1}\right). \tag{57}$$

Similarly, for the other prices we have the following set of equations:

$$p_{S2B1} = V^{S2} - \frac{1}{2}V - \frac{1}{2}V^{B1}, \tag{58}$$

and

$$p_{AB1} = \frac{1}{2}V^{S2} - \frac{1}{2}V^{B1} + \frac{1}{2}V^{A}. \tag{59}$$

For new entrants and forced renters, we have that

$$p_{S1j} = \frac{1}{2}V + \frac{1}{2}V^{S1} - V^{j}, \tag{60}$$

$$p_{S2j} = \frac{1}{2}V^{S2} - \frac{1}{2}V^{j}, \tag{61}$$

$$p_{Aj} = \frac{1}{2}V - \frac{1}{2}V^{j} + \frac{1}{2}V^{A}, \tag{62}$$

where $j \in \{B_n,\ B_0\}$. Using the surplus and price conditions above, the steady state value functions of agents are

$$\rho V^{S1} = u - \chi + \frac{1}{2}\mu\left(\theta\right)\left(\frac{B_1}{B}\max\left\{\Sigma_{S1B1}, 0\right\} + \frac{B_0}{B}\max\left\{\Sigma_{S1B0}, 0\right\} + \frac{B_n}{B}\max\left\{\Sigma_{S1Bn}, 0\right\}\right), \tag{63}$$

$$\rho V^{B1} = u - \chi + \frac{1}{2} q(\theta) \left(\frac{S_1}{S} \max\{\Sigma_{S1B1}, 0\} + \frac{S_2}{S} \max\{\Sigma_{S2B1}, 0\} + \frac{A}{S} \max\{\Sigma_{AB1}, 0\} \right), \quad (64)$$

$$\rho V^{S2} = u_2 + R + \frac{1}{2} \mu(\theta) \left(\frac{B_1}{B} \max\{\Sigma_{S2B1}, 0\} + \frac{B_0}{B} \max\{\Sigma_{S2B0}, 0\} + \frac{B_n}{B} \max\{\Sigma_{S1Bn}, 0\} \right), \quad (65)$$

$$\rho V^{B0} = u_0 - R + \frac{1}{2} q(\theta) \left(\frac{S_1}{S} \max\{\Sigma_{S1B0}, 0\} + \frac{S_2}{S} \max\{\Sigma_{S2B0}, 0\} + \frac{A}{S} \max\{\Sigma_{AB0}, 0\} \right), \quad (66)$$

$$\rho V^{Bn} = u_n - R + \frac{1}{2} q(\theta) \left(\frac{S_1}{S} \max\{\Sigma_{S1Bn}, 0\} + \frac{S_2}{S} \max\{\Sigma_{S2Bn}, 0\} + \frac{A}{S} \max\{\Sigma_{ABn}, 0\} \right), \quad (67)$$

and

$$\rho V^{A} = R + \frac{1}{2} \mu(\theta) \left(\frac{B_1}{B} \max\{\Sigma_{AB1}, 0\} + \frac{B_0}{B} \max\{\Sigma_{AB0}, 0\} + \frac{B_n}{B} \max\{\Sigma_{S1Bn}, 0\} \right). \quad (68)$$

Apart from these value functions and price conditions, there is also a set of flow conditions that determines the stock of agents of different types in the population as well as the population constancy and housing conditions.

A steady state equilibrium of this economy will be defined similarly to Definition 10 but with equilibrium prices satisfying conditions (57)-(62) rather than being exogenously fixed within the bargaining set of agents.

There are several observations to make about the match surpluses. $\Sigma_{S1B0} = V - V^{S1} > 0$ and $\Sigma_{S2B1} = V - V^{B1} > 0$ given that $\chi > 0$. Also, it will be the case that $\Sigma_{S2B0} = 2V - V^{B0} - V^{S2} > 0$ and $\Sigma_{AB0} = V - V^{B0} - V^{A} > 0$, since forced renters have the lowest outside options, so the surplus from trading between any seller and a forced renter must be positive. Similarly, we assume that u_n is sufficiently small so that $\Sigma_{ABn} = V - V^{NB} - V^{A} > 0$. This would imply that $\Sigma_{S2Bn} > 0$, since matching between a double owner and a new entrant creates a higher value than matching between a real estate seller and new entrant.

We will show our main analytical result for the model with Nash bargaining under the following three parametric assumptions. Note that these assumptions are only sufficient for having equilibrium multiplicity with symmetric Nash bargaining.

Assumption B1: $u_2 - u_0 = u - u_n$.

Assumption B2: $r(u_2 - (u - \chi)) + \frac{1}{2}\mu_0 \chi > 0$.

Assumption B3: $r(u_2 - u_0) \geq 2\left[r(u_2 - (u - \chi)) + \frac{1}{2}\mu_0\chi\right]$.

The first assumption essentially implies that the flow costs of being a double owner or forced renter are the same. Specifically, assuming that $u_2 = u - \psi_2$ for some $\psi_2 \geq 0$ and $u_0 = u - \psi_0$, for some $\psi_0 \geq 0$, this assumption implies that $\psi_0 = \psi_2 = \psi$. This assumption is similar to the symmetric restriction made in Section 4.4 in the case without Nash bargaining. It ultimately implies that at $\theta = 1$, $V^{B1} = V^{S1}$.

Assumption B2 ensures that $\Sigma_{AB1} > 0$ and $\Sigma_{S1Bn} > 0$ around $\theta = 1$ in a candidate "Buyers' market" and "Sellers' market" equilibrium and is easily satisfied provided that the matching efficiency μ_0 is sufficiently high. Finally, Assumption B3 ensures that $\Sigma_{S1B1} < 0$ around $\theta = 1$ in a candidate "Buyers' market" and "Sellers' market" equilibrium. Finally, we maintain the assumption that $\psi > \chi$. This assumption is necessary for $\Sigma_{S1B1} < 0$ around $\theta = 1$ in a candidate "Buyers' market" and "Sellers' market" equilibrium.

As before, $\overline{\theta}$ and $\underline{\theta}$ are the solutions to equations (16) and (17), respectively, and denote the steady state market tightnesses The first equation arises from the steady state flow conditions whenever all mismatched owners buy first, whereas the second arises from the steady state flow conditions when all mismatched owners sell first.

We can re-write these two equations as

$$\frac{\gamma + g + q(\theta)}{q(\theta) + g}\theta + \gamma\frac{\mu(\theta) - q(\theta)}{(q(\theta) + g)(\mu(\theta) + g)} = 1 + \frac{\gamma}{g},$$

and

$$\frac{\gamma + g + \mu(\theta)}{\mu(\theta) + g}\frac{1}{\theta} = 1 + \frac{\gamma}{g},$$

respectively.

Using the above two equations, we have that in the limit as $g \to 0$, $\gamma \to 0$ and $\frac{\gamma}{g} = \kappa$,

$$\overline{\theta} \to 1 + \kappa,$$

and

$$\underline{\theta} \to \frac{1}{1 + \kappa}.$$

Therefore, depending on the value of κ, in this limit the two steady state market tightnesses $\overline{\theta}$ and $\underline{\theta}$ can be arbitrarily close to 1.

We utilize this observation to construct multiple equilibria for the case of Nash bargaining in a limit economy, in which the steady state market tightnesses in each equilibrium are close to 1. This is done in Proposition 8 in Section 6. Here we provide a proof of this result.

Proof of Proposition 8

Proof. We first show the existence of a "Buyers' market" equilibrium in which no mismatched owners buy first ($B_1 = 0$) and $\theta = \underline{\theta} < 1$. Assume that in this limit $\Sigma_{S1Bn} > 0$, for which we will provide a sufficient condition later. First, note that given the flow conditions for the stocks of agents, we have the following relations:

$$A = \frac{g}{g + \mu(\theta)} = B_0 + B_n = B,$$

and

$$B_n = \frac{g}{g + q(\theta)}.$$

Therefore,

$$\frac{B_n}{B} = \frac{g + \mu(\theta)}{g + q(\theta)},$$

and

$$\frac{B_0}{B} = \frac{q(\theta) - \mu(\theta)}{g + q(\theta)}.$$

Also,

$$S_1 = \frac{g + q(\theta)}{\mu(\theta)} B_0 = \frac{q(\theta) - \mu(\theta)}{\mu(\theta)} B,$$

so given that $A = B$, it follows that

$$\frac{S_1}{A} = \frac{1}{\theta} - 1.$$

This in turn means that

$$\frac{A}{S} = \frac{A}{S_1 + A} = \theta.$$

In the limit we consider, we have that $\frac{A}{S} = \underline{\theta} = \frac{1}{1+\kappa}$, $\frac{S_1}{S} = \frac{\kappa}{1+\kappa}$, $\frac{B_n}{B} = \underline{\theta} = \frac{1}{1+\kappa}$, and $\frac{B_0}{B} = \frac{\kappa}{1+\kappa}$.

Turning to the steady state value functions, note first that

$$\rho V^{B0} = u_0 - R + \frac{1}{2} q(\theta) \left(\theta \Sigma_{AB0} + (1 - \theta) \Sigma_{S1B0}\right),$$

and

$$\rho V^{Bn} = u_n - R + \frac{1}{2} q(\theta) \left(\theta \Sigma_{ANB} + (1 - \theta) \Sigma_{S1NB}\right),$$

so

$$V^{Bn} - V^{B0} = \frac{u_n - u_0}{\rho + \frac{1}{2} q(\theta)}.$$

Also,

$$\rho V^A = R + \frac{1}{2} \mu(\theta) \left(\frac{g + \mu(\theta)}{g + q(\theta)} \left(V - V^{NB} - V^A\right) + \frac{q(\theta) - \mu(\theta)}{g + q(\theta)} \left(V - V^{B0} - V^A\right) \right),$$

or

$$\left(\rho+\frac{1}{2}\mu\left(\theta\right)\right)V^{A}=R+\frac{1}{2}\mu\left(\theta\right)\left(V-V^{B0}-\frac{g+\mu\left(\theta\right)}{g+q\left(\theta\right)}\frac{u_n-u_0}{\rho+\frac{1}{2}q\left(\theta\right)}\right).$$

In the limit

$$V^{A}=\frac{R}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}+\frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}\left(V-V^{B0}-\underline{\theta}\frac{u_n-u_0}{r+\frac{1}{2}q\left(\underline{\theta}\right)}\right).$$

Similarly, in the limit

$$V^{S2}=\frac{u_2}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}+\frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}V+V^{A}.$$

This in turn implies that

$$V-V^{S2}=\frac{rV-u_2}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}-V^{A}=\frac{u-u_2}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}-V^{A}.$$

Turning to the value functions of mismatched owners, an owner that sells first has a value function given by

$$\rho V^{S1}=u-\chi+\frac{1}{2}\mu\left(\theta\right)\left(V-V^{S1}-\frac{g+\mu\left(\theta\right)}{g+q\left(\theta\right)}\frac{u_n-u_0}{\rho+\frac{1}{2}q\left(\theta\right)}\right),$$

which can be re-written as

$$V^{S1}=\frac{u-\chi}{\rho+\frac{1}{2}\mu\left(\theta\right)}+\frac{\frac{1}{2}\mu\left(\theta\right)}{\rho+\frac{1}{2}\mu\left(\theta\right)}V-\frac{\frac{1}{2}\mu\left(\theta\right)}{\rho+\frac{1}{2}\mu\left(\theta\right)}\frac{g+\mu\left(\theta\right)}{g+q\left(\theta\right)}\frac{u_n-u_0}{\rho+\frac{1}{2}q\left(\theta\right)}.$$

For the value function of a deviating mismatched owner who buys first, we have that

$$\rho V^{B1}=u-\chi+\frac{1}{2}q\left(\theta\right)\left(\theta\max\left\{\Sigma_{AB1},0\right\}+\left(1-\theta\right)\max\left\{\Sigma_{S1B1},0\right\}\right).$$

Supposing that in the limit we consider $\Sigma_{AB1}>0$ and $\Sigma_{S1B1}<0$, we have that

$$\left(r+\frac{1}{2}\mu\left(\underline{\theta}\right)\right)V^{B1}=u-\chi+\frac{1}{2}\mu\left(\underline{\theta}\right)\left(V^{S2}-V^{A}\right).$$

Consider the difference between the utilities from buying first compared to selling first. In the limit we consider, we have that

$$\left(r+\frac{1}{2}\mu\left(\underline{\theta}\right)\right)\left(V^{B1}-V^{S1}\right)=\frac{1}{2}\mu\left(\underline{\theta}\right)\left(V^{S2}-V^{A}-V+\underline{\theta}\frac{u_n-u_0}{\rho+\frac{1}{2}q\left(\underline{\theta}\right)}\right).$$

Substituting for $V^{S2}-V^{A}-V$, we get that

$$V^{B1}-V^{S1}=\frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}\left(\frac{u_2-u}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}+\underline{\theta}\frac{u_n-u_0}{r+\frac{1}{2}q\left(\underline{\theta}\right)}\right).$$

Note that at $\underline{\theta} = 1$ (i.e. for $\kappa = 0$),

$$\frac{u_2 - u}{r + \frac{1}{2}\mu(\underline{\theta})} + \underline{\theta}\frac{u_n - u_0}{r + \frac{1}{2}q(\underline{\theta})} = 0,$$

given Assuption B1. As $\underline{\theta}$ moves away from 1 toward 0 (κ moves towards infinity), we have that $\frac{u_2-u}{r+\frac{1}{2}\mu(\underline{\theta})} + \underline{\theta}\frac{u_n-u_0}{r+\frac{1}{2}q(\underline{\theta})}$ decreases, so $V^{B1} < V^{S1}$ for $\underline{\theta} < 1$. Therefore, it is not optimal for a mismatched owner to deviate and buy first in an equilibrium in which mismatched owners sell first and $\theta < 1$.

Finally, we verify that our conjectures for the surpluses Σ_{S1Bn}, Σ_{AB1}, and Σ_{S1B1} are correct. We have that in the limit we consider

$$\begin{aligned}
\Sigma_{S1Bn} &= V - V^{Bn} + V^{B0} - V^{S1} = V - V^{S1} - \frac{u_n - u_0}{r + \frac{1}{2}q(\underline{\theta})} \\
&= \frac{\chi}{r + \frac{1}{2}\mu(\underline{\theta})} + \frac{\frac{1}{2}\mu(\underline{\theta})(\underline{\theta} - 1) - r}{r + \frac{1}{2}\mu(\underline{\theta})}\frac{u_n - u_0}{r + \frac{1}{2}q(\underline{\theta})} \\
&= \frac{r(\chi + u_0 - u_n) + \frac{1}{2}q(\underline{\theta})\chi + \frac{1}{2}\mu(\underline{\theta})(\underline{\theta} - 1)(u_n - u_0)}{\left(r + \frac{1}{2}q(\underline{\theta})\right)\left(r + \frac{1}{2}\mu(\underline{\theta})\right)}.
\end{aligned}$$

Therefore, at $\underline{\theta} = 1$, $\Sigma_{S1Bn} > 1$ if

$$r(\chi + u_0 - u_n) + \frac{1}{2}\mu_0\chi > 0.$$

Note that given Assumption B1, this is equivalent to

$$r(u_2 - (u - \chi)) + \frac{1}{2}\mu_0\chi > 0,$$

which holds by Assumption B2. Therefore, by continuity of the value functions with respect to θ, it follows that there is a $\kappa_1 > 0$, such that for $\kappa < \kappa_1$, $\Sigma_{S1Bn} > 0$. Similarly, in the limit we consider

$$\begin{aligned}
\Sigma_{AB1} &= V^{S2} - V^{B1} - V^{A} = V^{S2} - V^{A} - \frac{u - \chi}{r + \frac{1}{2}\mu(\underline{\theta})} - \frac{\frac{1}{2}\mu(\underline{\theta})}{r + \frac{1}{2}\mu(\underline{\theta})}\left(V^{S2} - V^{A}\right) \\
&= \frac{r\left(V^{S2} - V^{A}\right) - (u - \chi)}{r + \frac{1}{2}\mu(\underline{\theta})} = \frac{\frac{r}{r+\frac{1}{2}\mu(\underline{\theta})}u_2 + \frac{\frac{1}{2}\mu(\underline{\theta})}{r+\frac{1}{2}\mu(\underline{\theta})}u - (u - \chi)}{r + \frac{1}{2}\mu(\underline{\theta})} \\
&= \frac{r(u_2 - (u - \chi)) + \frac{1}{2}\mu(\underline{\theta})\chi}{\left(r + \frac{1}{2}\mu(\underline{\theta})\right)^2}.
\end{aligned}$$

At $\underline{\theta} = 1$, $\Sigma_{AB1} > 0$ if $r(u_2 - (u - \chi)) + \frac{1}{2}\mu_0\chi > 0$, which is our parametric Assumption B2. Therefore, by continuity of the value functions with respect to $\underline{\theta}$, it follows that there is a $\kappa_2 > 0$, such that for $\kappa < \kappa_2$, $\Sigma_{AB1} > 0$. Finally, in the limit we consider

$$
\begin{aligned}
\Sigma_{S1B1} &= V^{S2} - V^{B1} + V^{B0} - V^{S1} \\
&= V^{S2} - V^{B1} + \frac{rV^{B0} - (u-\chi) + R}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} - V^{A} \\
&= \Sigma_{AB1} + \frac{rV^{B0} - (u-\chi) + R}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)}.
\end{aligned}
$$

At $\underline{\theta} = 1$,

$$
\begin{aligned}
\frac{rV^{B0} - (u-\chi) + R}{r + \frac{1}{2}\mu_0} &= \frac{\frac{r}{r+\frac{1}{2}\mu_0}\left(u_0 - R\right) + \frac{\frac{1}{2}\mu_0}{r+\frac{1}{2}\mu_0}u - \frac{\frac{1}{2}\mu_0}{r+\frac{1}{2}\mu_0}rV^{A} - (u-\chi) + R}{r + \frac{1}{2}\mu_0} \\
&= \frac{ru_0 + \frac{1}{2}\mu_0 u - \frac{1}{2}\mu_0\left(rV^{A} - R\right) - \left(r + \frac{1}{2}\mu_0\right)(u-\chi)}{\left(r + \frac{1}{2}\mu_0\right)^2}.
\end{aligned}
$$

Substituting for Σ_{AB1}, we get

$$
\Sigma_{S1B1} = \frac{r\left(u_0 + u_2 - 2\left(u-\chi\right)\right) + \mu\chi - \frac{1}{2}\mu_0\left(rV^{A} - R\right)}{\left(r + \frac{1}{2}\mu_0\right)^2}.
$$

Therefore, a sufficient condition for $\Sigma_{S1B1} < 0$ at $\underline{\theta} = 1$ is

$$
r\left(u_0 + u_2 - 2\left(u-\chi\right)\right) + \mu_0\chi \leq 0,
$$

or

$$
r\left(u_2 - u_0\right) \geq 2\left[r\left(u_2 - \left(u-\chi\right)\right) + \frac{1}{2}\mu_0\chi\right],
$$

which is our parametric assumption B3. Again by continuity of the value functions with respect to $\underline{\theta}$, we have that there is a $\kappa_3 > 0$, s.t. for $\kappa < \kappa_3$, $\Sigma_{S1B1} < 0$. Taking $\underline{\kappa} = \min\{\kappa_1, \kappa_2, \kappa_3\}$, we have that for $\kappa < \underline{\kappa}$, there is a "Buyers' market" equilibrium with a market tightness given by $\underline{\theta} = \frac{1}{1+\kappa}$.

We follow the same steps to show the existence of a "Sellers' market" equilibrium in which no mismatched owners sell first ($S_1 = 0$) and $\theta = \overline{\theta} > 1$. Again, we assume that $\Sigma_{AB1} > 0$ and show a sufficient condition for that later. First, note that given the flow conditions for the stocks of agents, we have the following relations:

$$
A = \frac{g}{g + \mu\left(\theta\right)},
$$

and

$$
B_n = \frac{g}{g + q\left(\theta\right)} = A + S_2 = S.
$$

Therefore,

$$
\frac{A}{S} = \frac{\frac{g}{g+\mu(\theta)}}{\frac{g}{g+q(\theta)}} = \frac{g + q\left(\theta\right)}{g + \mu\left(\theta\right)},
$$

and

$$\frac{S_2}{S} = \frac{\mu(\theta) - q(\theta)}{g + \mu(\theta)}.$$

Looking at the buyer side, note that

$$q(\theta) B_1 = (g + \mu(\theta)) S_2,$$

so

$$\frac{B_1}{S} = \frac{B_1}{B_n} = \theta - 1.$$

Therefore,

$$\frac{B_n}{B} = \frac{1}{\theta}.$$

In the limit we consider, we have that $\frac{A}{S} = \frac{1}{\overline{\theta}}$, $\frac{S_2}{S} = \frac{\overline{\theta}-1}{\overline{\theta}}$, $\frac{B_n}{B} = \frac{1}{\overline{\theta}}$ and $\frac{B_1}{B} = \frac{\overline{\theta}-1}{\overline{\theta}}$. Similarly to the sell first case, we have that

$$\left(\rho + \frac{1}{2}\mu(\theta)\right) V^A = R + \frac{1}{2}\mu(\theta)\left(\frac{1}{\theta}\left(V - V^{NB}\right) + \frac{\theta - 1}{\theta}\left(V^{S2} - V^{B1}\right)\right),$$

and

$$\left(\rho + \frac{1}{2}\mu(\theta)\right) V^{S2} = u_2 + \left(\rho + \frac{1}{2}\mu(\theta)\right) V^A + \frac{1}{2}\mu(\theta) V.$$

Therefore, as in the "sell first" case,

$$V - V^{S2} = \frac{\rho V - u_2}{\rho + \frac{1}{2}\mu(\theta)} - V^A.$$

Also, as in the previous case,

$$V^{Bn} - V^{B0} = \frac{u_n - u_0}{\rho + \frac{1}{2}q(\theta)}.$$

Turning to the value functions of a mismatched owner, we have that

$$\rho V^{B1} = u - \chi + \frac{1}{2}q(\theta)\left(\frac{g + q(\theta)}{g + \mu(\theta)}\left(V^{S2} - V^{B1} - V^A\right) + \frac{\mu(\theta) - q(\theta)}{g + \mu(\theta)}\left(V - V^{B1}\right)\right),$$

so in the limit we consider,

$$\left(r + \frac{1}{2}q\left(\overline{\theta}\right)\right) V^{B1} = u - \chi + \frac{1}{2}q\left(\overline{\theta}\right)\frac{1}{\overline{\theta}}\frac{u_2 - u}{r + \frac{1}{2}\mu\left(\overline{\theta}\right)} + \frac{1}{2}q\left(\overline{\theta}\right) V.$$

For the value function of a deviating agent who chooses to "sell first", we have that

$$\rho V^{S1} = u - \chi + \frac{1}{2}\mu(\theta)\left(\frac{1}{\theta}\max\{0, \Sigma_{S1Bn}\} + \frac{\theta - 1}{\theta}\max\{0, \Sigma_{S1B1}\}\right).$$

Assume that $\Sigma_{S1Bn} > 0$ and $\Sigma_{S1B1} < 0$. Then in the limit,

$$\left(r + \frac{1}{2}q\left(\overline{\theta}\right)\right) V^{S1} = u - \chi + \frac{1}{2}q\left(\overline{\theta}\right) V + \frac{1}{2}q\left(\overline{\theta}\right) \frac{u_0 - u_n}{r + \frac{1}{2}q\left(\overline{\theta}\right)}.$$

Therefore, the difference between $V^{B1} - V^{S1}$ satisfies

$$\left(r + \frac{1}{2}q\left(\overline{\theta}\right)\right)\left(V^{B1} - V^{S1}\right) = \frac{1}{2}q\left(\overline{\theta}\right)\left(\frac{1}{\overline{\theta}} \frac{u_2 - u}{r + \frac{1}{2}\mu\left(\overline{\theta}\right)} + \frac{u_n - u_0}{r + \frac{1}{2}q\left(\overline{\theta}\right)}\right).$$

At $\overline{\theta} = 1$, we have that

$$\frac{1}{\overline{\theta}} \frac{u_2 - u}{r + \frac{1}{2}\mu\left(\overline{\theta}\right)} + \frac{u_n - u_0}{r + \frac{1}{2}q\left(\overline{\theta}\right)} = 0,$$

by Assumption B1. As $\overline{\theta}$ increases, we have that $\frac{1}{\overline{\theta}} \frac{u_2 - u}{r + \frac{1}{2}\mu(\overline{\theta})} + \frac{u_n - u_0}{r + \frac{1}{2}q(\overline{\theta})}$ increases, so $V^{B1} > V^{S1}$ for $\overline{\theta} < 1$. Therefore, it is not optimal for a mismatched owner to deviate and sell first in an equilibrium in which mismatched owners buy first and $\theta > 1$.

Finally, we verify that our conjectures for the surpluses Σ_{AB1}, Σ_{S1Bn}, and Σ_{S1B1} are correct in the buy first case as well. Very similar to the sell first case, in the limit we consider

$$\begin{aligned}
\Sigma_{AB1} &= V^{S2} - V^{B1} - V^{A} = V^{S2} - V^{A} - V + V - \frac{u - \chi}{r + \frac{1}{2}q\left(\overline{\theta}\right)} - \frac{\frac{1}{2}q\left(\overline{\theta}\right)}{r + \frac{1}{2}q\left(\overline{\theta}\right)}\left[\frac{1}{\overline{\theta}}\left(V^{S2} - V^{A} - V\right) + V\right] \\
&= \frac{\left(r + \frac{1}{2}q\left(\overline{\theta}\right)\frac{\overline{\theta} - 1}{\overline{\theta}}\right)}{r + \frac{1}{2}q\left(\overline{\theta}\right)} \frac{u_2 - u}{r + \frac{1}{2}\mu\left(\overline{\theta}\right)} + \frac{\chi}{r + \frac{1}{2}q\left(\overline{\theta}\right)} \\
&= \frac{r\left(u_2 - (u - \chi)\right) + \frac{1}{2}\mu\left(\overline{\theta}\right)\chi + \frac{1}{2}q\left(\overline{\theta}\right)\frac{\overline{\theta} - 1}{\overline{\theta}}\left(u_2 - u\right)}{\left(r + \frac{1}{2}\mu\left(\overline{\theta}\right)\right)\left(r + \frac{1}{2}q\left(\overline{\theta}\right)\right)}.
\end{aligned}$$

Note that at $\overline{\theta} = 1$, Σ_{AB1} in the buy first case is the same as the sell first case. Therefore, there is a $\kappa_4 > 0$, such that for $\kappa < \kappa_4$ and $\overline{\theta} = 1 + \kappa$, $\Sigma_{AB1} > 0$. Similarly,

$$\begin{aligned}
\Sigma_{S1Bn} &= V - V^{Bn} + V^{B0} - V^{S1} = V - V^{S1} - \frac{u_n - u_0}{r + \frac{1}{2}q\left(\overline{\theta}\right)} \\
&= \frac{\chi}{r + \frac{1}{2}q\left(\overline{\theta}\right)} - \frac{r}{r + \frac{1}{2}q\left(\overline{\theta}\right)} \frac{u_n - u_0}{r + \frac{1}{2}q\left(\overline{\theta}\right)} \\
&= \frac{r\left(\chi + u_0 - u_n\right) + \frac{1}{2}q\left(\overline{\theta}\right)\chi}{\left(r + \frac{1}{2}q\left(\overline{\theta}\right)\right)^2},
\end{aligned}$$

which at $\overline{\theta} = 1$ is again the same as for the sell first case. Therefore, there is a $\kappa_5 > 0$, such that for $\kappa < \kappa_5$, $\Sigma_{S1Bn} > 0$. Finally,

$$\begin{aligned}
\Sigma_{S1B1} &= V^{S2} - V^{B1} + V^{B0} - V^{S1} \\
&= V^{S2} - V^{B1} + \frac{rV^{B0} - (u-\chi) + R + \frac{1}{2}q\left(\overline{\theta}\right)\left(\overline{\theta}-1\right)\left(V^{S2} - V^{B1} - V^{A}\right)}{r + \frac{1}{2}q\left(\overline{\theta}\right)} - V^{A} \\
&= \left(1 + \frac{\frac{1}{2}q\left(\overline{\theta}\right)\left(\overline{\theta}-1\right)}{r + \frac{1}{2}q\left(\overline{\theta}\right)}\right)\Sigma_{AB1} + \frac{rV^{B0} - (u-\chi) + R}{r + \frac{1}{2}q\left(\overline{\theta}\right)}.
\end{aligned}$$

At $\overline{\theta} = 1$, showing that $\Sigma_{S1B1} < 0$ in the buy first case therefore follows the sell first case, so that $\Sigma_{S1B1} < 0$ for $\kappa < \kappa_6$, for some $\kappa_6 > 0$. Taking $\overline{\kappa} = \min\{\kappa_4, \kappa_5, \kappa_6\}$, we have that for $\kappa < \overline{\kappa}$, there is a "Sellers' market" equilibrium with a market tightness given by $\overline{\theta} = 1 + \kappa$. Finally, taking $\kappa^* = \min\{\overline{\kappa}, \underline{\kappa}\}$, we arrive at the desired result. □

F. A fixed price as the outcome of take-it-or-leave-it offers under private information

In this section we show that a fixed price equal to the present discounted value of rental income can be microfounded as the outcome of bargaining under private information about types, with full bargaining power for buyers. Suppose therefore in this section that buyers make take-it-or-leave-it offers, but do not know the type of the seller. However, buyers do know the fractions of the types in the economy. Because of heterogeneity among sellers, their reservation prices vary. Matching is still random, so that buyers cannot direct their search to the seller type with the lowest reservation price but meet a particular seller type with a probability equal to their proportion in the population of sellers. The question is then whether buyers, upon meeting a seller, make an offer that only sellers with a low reservation price would accept (and thus trade only if they have met a seller of this type), or make an offer that all sellers would accept (and therefore trade for sure).

We consider the symmetric case with $\tilde{u}_0 = \tilde{u}_2 = c$ (which for $p = \frac{R}{\rho}$ amounts to $u_0 = u_2 = c$), so that $\tilde{\theta} = 1$. In addition, we maintain Assumptions A1 and A2 and assume that $u_n < u - \chi$, so that both mismatched owners and new entrants are strictly better off to enter the market. As in the model with symmetric Nash bargaining, we focus on steady state equilibria with value functions, market tightness θ, and the stocks of different agent types constant over time. Moreover, although results hold more generally, we again consider a limit economy with small flows where $g \to 0$ and $\gamma \to 0$ but the ratio $\frac{\gamma}{g} = \kappa$ is kept constant in the limit. Remember that in this case $\overline{\theta} \to 1 + \kappa$ and $\underline{\theta} \to \frac{1}{1+\kappa}$. We will show that under these conditions both in a "buy first" and in a "sell first" equilibrium no buyer has an incentive to deviate from targeting both types of sellers by demanding a lower price than the unique prevailing price $p = \frac{R}{\rho}$.

Still denoting the value of a matched owner that remains passive upon mismatch by $\tilde{V}$, note first that at $\tilde{\theta} = 1$ Assumption A2 can be simplified to

$$\frac{u-\chi}{\rho} < \frac{u-\chi}{\rho+\mu_0} + \frac{\mu_0}{\left(\rho+\mu_0\right)^2}c + \frac{{\mu_0}^2}{\left(\rho+\mu_0\right)^2}\tilde{V},$$

$$\Leftrightarrow \frac{u-\chi}{\rho} < \frac{c}{\rho+\mu_0} + \frac{\mu_0}{\rho+\mu_0}\tilde{V},$$

$$\Leftrightarrow 0 < \rho\left(c-(u-\chi)\right) + \mu_0\left(\rho\tilde{V} - (u-\chi)\right),$$

which, for future reference, is not greater than $\rho\left(c-(u-\chi)\right) + \mu_0\left(\rho V - (u-\chi)\right)$.

Under the unique price to be proven, the value functions are given by equations (3)-(6) and (33)-(35), given θ and R. We first show that in an equilibrium in which mismatched owners "buy first", buyers have no incentive to demand a lower price than $p = \frac{R}{\rho}$. In such an equilibrium there are two types of sellers: double owners and real estate agents. As before, the lowest price that a real estate agent would be willing to accept is $p^A = V^A = \frac{R}{\rho}$. The lowest price that a double owner would be willing to accept is $p^{S2} = V^{S2} - V$. Substituting these prices in the value functions, in an equilibrium with price dispersion $p^{S2} < p^A$, since

$$\rho\left(V^{S2} - V - V^A\right) = u_2 + R + \mu(\theta)\left(p^{S2} + V - V^{S2}\right) - \rho V - R - \mu(\theta)\left(p^A - V^A\right),$$

$$\Leftrightarrow \rho\left(V^{S2} - V - V^A\right) = u_2 - \rho V < 0,$$

$$\Leftrightarrow V^{S2} - V < V^A.$$

For that reason, under full information buyers would like to buy from a double owner, but the question is whether under private information they will make an offer that only double owners would accept. Note that for any $p \geq p^{S2}$ double owners are willing to sell, while for $p < p^{S2}$ they are not. As a result, since buying a house is preferred to being passive, among all possible deviations no offer is more profitable than demanding $V^{S2} - V$. The proof can therefore be restricted to this deviating offer. Note also that a deviating mismatched owner that sells first has zero mass, so that its presence doesn't affect the take-it-or-leave-it offers that buyers make.

First considering new entrants, for them to demand p^A it must be the case that

$$V - V^{Bn} - p^A \geq \frac{S_2}{S}\left(V - V^{Bn} - p^{S2}\right).$$

Substituting prices and using that $S = S_2 + A$ yields

$$\frac{A}{S}\left(V - V^{Bn} - \frac{R}{\rho}\right) \geq \frac{S_2}{S}\left(V - V^{S2} + \frac{R}{\rho}\right). \tag{69}$$

From the value functions we have that

$$\rho\left(V - V^{Bn} - \frac{R}{\rho}\right) = \rho V - u_n + R - q(\theta)\left(V - V^{Bn} - \frac{R}{\rho}\right) - R,$$

$$\Leftrightarrow (\rho + q(\theta)) \left(V - V^{Bn} - \frac{R}{\rho} \right) = \rho V - u_n,$$

and

$$\rho \left(V - V^{S2} + \frac{R}{\rho} \right) = \rho V - u_2 - R - \mu(\theta) \left(V - V^{S2} + \frac{R}{\rho} \right) + R,$$

$$\Leftrightarrow (\rho + \mu(\theta)) \left(V - V^{S2} + \frac{R}{\rho} \right) = \rho V - u_2. \tag{70}$$

Moreover, in the limit we consider, we know from the section on Nash bargaining that $\frac{A}{S} = \frac{1}{\overline{\theta}}$ and $\frac{S_2}{S} = \frac{\overline{\theta}-1}{\overline{\theta}}$, so that (69) amounts to

$$\frac{1}{\overline{\theta}} \left(\rho + \mu \left(\overline{\theta} \right) \right) \left(\rho V - u_n \right) \geq \frac{\overline{\theta} - 1}{\overline{\theta}} \left(\rho + q \left(\overline{\theta} \right) \right) \left(\rho V - u_2 \right).$$

where both sides are positive, but where the right-hand side can be made arbitrarily close to zero by moving closer to $\overline{\theta} = 1$. Therefore, it follows that there is a $\kappa_7 > 0$, such that for $\kappa < \kappa_7$, new entrants in a "buy first" equilibrium demand $p = \frac{R}{\rho}$ upon meeting a selller. Substituting u_0 for u_n, the same condition holds for a deviating mismatched owner that sells first, and then becomes a forced renter. Therefore, there is a $\kappa_8 > 0$, such that for $\kappa < \kappa_8$, forced renters in a "buy first" equilibrium make the same offer.

For mismatched owners that buy first to demand p^A it must be the case that

$$V^{S2} - V^{B1} - p^A \geq \frac{S_2}{S} \left(V^{S2} - V^{B1} - p^{S2} \right),$$

$$\Leftrightarrow \frac{A}{S} \left(V^{S2} - V^{B1} - \frac{R}{\rho} \right) \geq \frac{S_2}{S} \left(V - V^{S2} + \frac{R}{\rho} \right). \tag{71}$$

Rearranging the value functions yields

$$\rho \left(V^{S2} - V^{B1} - \frac{R}{\rho} \right) = u_2 + R + \mu(\theta) \left(V - V^{S2} + \frac{R}{\rho} \right) - R - (u - \chi) - q(\theta) \left(V^{S2} - V^{B1} - \frac{R}{\rho} \right),$$

$$\Leftrightarrow (\rho + q(\theta)) \left(V^{S2} - V^{B1} - \frac{R}{\rho} \right) = u_2 - (u - \chi) + \mu(\theta) \left(V - V^{S2} + \frac{R}{\rho} \right). \tag{72}$$

Substituting the steady state fractions and (72) into (71), in the limit we consider we have that

$$\frac{1}{\overline{\theta}} \left(u_2 - (u - \chi) + \mu \left(\overline{\theta} \right) \left(\frac{R}{\rho} + V - V^{S2} \right) \right) \geq \frac{\overline{\theta} - 1}{\overline{\theta}} \left(\rho + q \left(\overline{\theta} \right) \right) \left(V - V^{S2} + \frac{R}{\rho} \right),$$

$$u_2 - (u - \chi) \geq \left[\left(\overline{\theta} - 1 \right) \left(\rho + q \left(\overline{\theta} \right) \right) - \mu \left(\overline{\theta} \right) \right] \left(V - V^{S2} + \frac{R}{\rho} \right).$$

Substituting (70) yields

$$\left(\rho + \mu \left(\overline{\theta} \right) \right) \left(u_2 - (u - \chi) \right) \geq \left[\left(\overline{\theta} - 1 \right) \left(\rho + q \left(\overline{\theta} \right) \right) - \mu \left(\overline{\theta} \right) \right] \left(\rho V - u_2 \right),$$

$$\Leftrightarrow \rho\left(u_2-(u-\chi)\right)+\mu\left(\overline{\theta}\right)\left(\rho V-(u-\chi)\right)\geq\left(\overline{\theta}-1\right)\left(\rho+q\left(\overline{\theta}\right)\right)\left(\rho V-u_2\right).$$

The left-hand side is positive for any $\overline{\theta}\geq 1$ by Assumption A2. Moving $\overline{\theta}$ towards 1 can make the right-hand side arbitrarily close to zero, so that there exists a $\kappa_9>0$, such that for $\kappa<\kappa_9$, mismatched owners that buy first in a "buy first" equilibrium demand $p=\frac{R}{\rho}$ upon meeting a selller. Taking $\overline{\kappa}'=\min\{\kappa_7,\kappa_8,\kappa_9\}$, we have that for $\kappa<\overline{\kappa}'$, all buyers demand $p=\frac{R}{\rho}$ upon meeting a selller in a "buy first" equilibrium with a market tightness given by $\overline{\theta}=1+\kappa$.

Secondly, we show that in an equilibrium in which mismatched owners sell first, buyers have no incentive to demand a lower price than $p=\frac{R}{\rho}$. In such an equilibrium there are two types of sellers: mismatched owners that sell first, and real estate agents. The lowest price that a real estate agent would be willing to accept is still $p^A=V^A=\frac{R}{\rho}$. The lowest price that a mismatched owner would be willing to accept is $p^{S1}=V^{S1}-V^{B0}$. It must be the case that $V^{B0}-V^{S1}+p^A\geq 0$, because mismatched owners don't remain passive by Assumption A2. It follows that $p^{S1}\leq p^A$, so that with full information buyers would like to buy from a mismatched owner. Again the question is whether under private information buyers will make an offer that only mismatched owners would accept. Similar to the "buy first" equilibrium, the proof can be restricted to the deviation of demanding $V^{S1}-V^{B0}$.

First considering forced renters, for them to demand p^A it must be the case that

$$V-V^{B0}-p^A\geq\frac{S_1}{S}\left(V-V^{B0}-p^{S1}\right),$$

$$\Leftrightarrow\frac{A}{S}\left(V-V^{B0}-\frac{R}{\rho}\right)\geq\frac{S_1}{S}\left(V^{B0}-V^{S1}+\frac{R}{\rho}\right). \tag{73}$$

Rearranging the value functions yields

$$\rho\left(V-V^{B0}-\frac{R}{\rho}\right)=\rho V-u_0+R-q(\theta)\left(V-V^{B0}-\frac{R}{\rho}\right)-R,$$

$$\Leftrightarrow(\rho+q(\theta))\left(V-V^{B0}-\frac{R}{\rho}\right)=\rho V-u_0, \tag{74}$$

and

$$\rho\left(V^{B0}-V^{S1}+\frac{R}{\rho}\right)=u_0-R+q\left(\theta\right)\left(-\frac{R}{\rho}+V-V^{B0}\right)-(u-\chi)-\mu\left(\theta\right)\left(\frac{R}{\rho}+V^{B0}-V^{S1}\right)+R,$$

$$\Leftrightarrow(\rho+\mu(\theta))\left(V^{B0}-V^{S1}+\frac{R}{\rho}\right)=u_0-(u-\chi)+q\left(\theta\right)\left(V-V^{B0}-\frac{R}{\rho}\right). \tag{75}$$

Substituting (75), (73) therefore amounts to

$$\frac{A}{S}\left(\rho+\mu(\theta)\right)\left(V-V^{B0}-\frac{R}{\rho}\right)\geq\frac{S_1}{S}\left(u_0-(u-\chi)+q\left(\theta\right)\left(V-V^{B0}-\frac{R}{\rho}\right)\right),$$

$$\Leftrightarrow \left[\frac{A}{S}\left(\rho+\mu(\theta)\right)-\frac{S_1}{S}q(\theta)\right]\left(V-V^{B0}-\frac{R}{\rho}\right)\geq\frac{S_1}{S}\left(u_0-(u-\chi)\right).$$

Substituting (74) yields

$$\left[\frac{A}{S}\left(\rho+\mu(\theta)\right)-\frac{S_1}{S}q(\theta)\right]\left(\rho V-u_0\right)\geq\frac{S_1}{S}\left(\rho+q(\theta)\right)\left(u_0-(u-\chi)\right),$$

$$\Leftrightarrow \frac{A}{S}\left(\rho+\mu(\theta)\right)\left(\rho V-u_0\right)\geq\frac{S_1}{S}\rho\left(u_0-(u-\chi)\right)+\frac{S_1}{S}q(\theta)\left(\rho V-(u-\chi)\right).$$

From the section on Nash bargaining we know that $\frac{A}{S}=\underline{\theta}$ and $\frac{S_1}{S}=1-\underline{\theta}$. Substituting these steady state fractions, we have that

$$\underline{\theta}\left(\rho+\mu\left(\underline{\theta}\right)\right)\left(\rho V-u_0\right)\geq\left(1-\underline{\theta}\right)\left[\rho\left(u_0-(u-\chi)\right)+q\left(\underline{\theta}\right)\left(\rho V-(u-\chi)\right).\right]$$

Again, by moving towards $\underline{\theta}=1$ the right-hand side can be made arbitrarily close to zero while the left-hand side remains positive. Therefore, there exists a $\kappa_{10}>0$, such that for $\kappa<\kappa_{10}$, forced renters in a "sell first" equilibrium demand $p=\frac{R}{\rho}$ upon meeting a selller. Substituting u_n for u_0 the same condition holds for a new entrant, so that there is a $\kappa_{11}>0$, such that for $\kappa<\kappa_{11}$, new entrants make the same offer.

Finally, for a deviating mismatched owner that buys first to demand p^A it must be the case that

$$V^{S2}-V^{B1}-p^A\geq\frac{S_1}{S}\left(V^{S2}-V^{B1}-p^{S1}\right),$$

$$\Leftrightarrow \frac{A}{S}\left(V^{S2}-V^{B1}-\frac{R}{\rho}\right)\geq\frac{S_1}{S}\left(V^{B0}-V^{S1}+\frac{R}{\rho}\right). \tag{76}$$

Rearranging the value functions yields

$$\rho\left(V^{S2}-V^{B1}-\frac{R}{\rho}\right)=u_2+R+\mu\left(\theta\right)\left(\frac{R}{\rho}+V-V^{S2}\right)-u-\chi+q\left(\theta\right)\left(-\frac{R}{\rho}+V^{S2}-V^{B1}\right)-R,$$

$$\Leftrightarrow \left(\rho+q(\theta)+\mu\left(\theta\right)\right)\left(V^{S2}-V^{B1}-\frac{R}{\rho}\right)=u_2-(u-\chi)+\mu\left(\theta\right)\left(V-V^{B1}\right),$$

with

$$\mu\left(\theta\right)\left(V-V^{B1}\right)=\mu\left(\theta\right)\left(V-\frac{u-\chi}{\rho}\right)-\mu\left(\theta\right)q\left(\theta\right)\left(V^{S2}-V^{B1}-\frac{R}{\rho}\right).$$

From (75) we know that

$$\left(\rho+\mu(\theta)\right)\left(V^{B0}-V^{S1}+\frac{R}{\rho}\right)=u_0-(u-\chi)+q\left(\theta\right)\left(V-V^{B0}-\frac{R}{\rho}+V^{S1}-V^{S1}\right),$$

$$\Leftrightarrow \left(\rho+q(\theta)+\mu(\theta)\right)\left(V^{B0}-V^{S1}+\frac{R}{\rho}\right)=u_0-(u-\chi)+q\left(\theta\right)\left(V-V^{S1}\right),$$

with

$$q(\theta)\left(V - V^{S1}\right) = q(\theta)\left(V - \frac{u-\chi}{\rho}\right) - \mu(\theta)\, q(\theta)\left(V^{B0} - V^{S1} + \frac{R}{\rho}\right).$$

Therefore, (76) simply amounts to

$$\frac{A}{S}\left(u_2 - (u-\chi) + \mu(\theta)\left(V - \frac{u-\chi}{\rho}\right)\right) \geq \frac{S_1}{S}\left(u_0 - (u-\chi) + q(\theta)\left(V - \frac{u-\chi}{\rho}\right)\right).$$

Substituting the steady state fractions, we have that

$$\underline{\theta}\left(u_2 - (u-\chi) + \mu(\underline{\theta})\left(V - \frac{u-\chi}{\rho}\right)\right) \geq (1-\underline{\theta})\left(u_0 - (u-\chi) + q(\underline{\theta})\left(V - \frac{u-\chi}{\rho}\right)\right).$$

The left-hand side is positive for any $0 < \underline{\theta} \leq 1$ by Assumption A2. Moving $\overline{\theta}$ towards 1 can make the right-hand side arbitrarily close to zero, so that there exists a $\kappa_{12} > 0$, such that for $\kappa < \kappa_{12}$, deviating mismatched owners that buy first in a "sell first" equilibrium demand $p = \frac{R}{\rho}$ upon meeting a selller. Taking $\underline{\kappa}' = \min\{\kappa_{10}, \kappa_{11}, \kappa_{12}\}$, we have that for $\kappa < \underline{\kappa}'$, all buyers demand $p = \frac{R}{\rho}$ upon meeting a selller in a "sell first" equilibrium with a market tightness given by $\underline{\theta} = \frac{1}{1+\kappa}$. Finally, taking $\kappa' = \min\{\overline{\kappa}', \underline{\kappa}'\}$, we have that both in a "buy first" and in a "sell first" equilibrium, the take-it-or-leave-it offer that buyers make is equal to $p = \frac{R}{\rho}$.