# Vacancies in Housing and Labor Markets[1]

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**Abstract**

The housing and business cycles are clearly tied together and this has been addressed by the recent literature. Our analysis of the housing and labor markets is based on a DMP model for interdependent housing and labor markets that gives rise naturally to vacancy rates in housing and labor markets. We estimate the model using data at the MSA level on housing vacancies from the US Census Bureau's Housing Vacancy Survey (HVS), starting in 1986, data on job vacancies from the Help-Wanted Index starting in 1951, and from the Job Opening and Labor Turnover Survey (JOLTS) since December 2000. We estimate a Beveridge curve for labor markets, while allowing for spillovers from the housing market, as well as a novel Beveridge curve for housing markets. We then specify VAR models for housing and job vacancies and estimate, based on the VAR results, impulse response functions to study how shocks to either the housing or labor markets will propagate themselves in the other market. The results show that shocks to the owner and rental vacancies have negative and significant impacts on job vacancies.

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# Contents

# 1 Introduction

The housing and business cycles are clearly tied together and this has become even more apparent given the recent Great Depression. Figure 1 shows the national growth rate for real house prices and GDP and the unemployment rate. The house price index tracks quite well with both the real GDP growth rate and the unemployment rate; the correlation is 0.5 and -0.5, respectively. This is an indication that the national business and housing cycles are closely synchronized. In fact, Leamer (2007) has made the claim that housing *IS* the business cycle. He shows that, at the national level, residential investment is a much better predictor of recessions than aggregate business activity (though this same relationship is not as strong at the MSA level [see Ghent and Owyang 2010)].

Recent work has looked at homeownership and how this impacts the labor market. One notable example is the so called "Oswald hypothesis" (Oswald 1977a, b; Coulson and Fisher 2009) that asserts that homeownership and unemployment rates are positively related given the relatively higher moving costs for homeowners. However, as Beugnot *et al.* (2014) show theoretically, even if the unemployment and homeownerships rates are positively correlated, individuals would be better off in economies where homeownership is promoted, and the costs from higher homeownership rates, if any, are principally associated with mobility costs, which are higher for homeowners. A second way that this link has been established is through the literature that estimates quality of life based on hedonic models of wages and house prices. We propose a new approach based on the link between housing and labor market vacancies.

Underlying vacancy rates in both the housing and labor markets are search models that give rise to natural vacancy rates. The vacancy rate is a long-established concept in labor markets and is central to the study of frictions in those markets as established under the DMP framework. Vacancies of dwelling units in housing markets are also rigorously grounded in search models (Wheaton 1990; Ngai and Sheedy 2013) and studies of housing market adjustment through search have been long-standing (Ioannides 1975; Genovese and Han 2012). We develop here a joint search model of the labor and housing markets.

One additional feature in the housing market is that there are both rental and homeownership vacancy rates. These are linked via the housing tenure choice, that is via households' choice of renting versus owner-occupancy. We investigate both vacancy rates in our analysis of labor and housing market vacancies.

Vacancies in housing markets are not as well understood from an empirical standpoint as those in labor markets. In part, this has to do with the fact that there is no housing market counterpart to the Beveridge curve, a firmly established tool in the study of labor markets that tracks the joint movement of unemployment and job vacancy rates. Whereas the Beveridge curve is simply an "accounting relationship" that links vacancy and unemployment rates in steady state situations, shifts in the job creation condition allows us to track the movement of the economy during the business cycle as the points where the Beveridge curve and the job creation condition intersect; see Figures 2 and 3 in Pissarides (2011). Furthermore, when the economy undergoes restructuring, the Beveridge curve itself may shift, which is in fact evident from Figure 4 below.

Our theoretical model gives rise to an empirical model that includes the Beveridge Curve and its counterpart in the housing market. To do so, we develop a concept in the housing market that corresponds to unemployment in the labor market, that of the unfulfilled demand on the part of renters for owner occupied housing. We find that the cyclical movement along this curve is in the opposite direction as that on the labor market Beveridge curve. We believe that we are the first to develop the housing market version of the Beveridge curve. Our theoretical model of housing and labor market vacancies explicitly captures the interdependence of the two markets through these two curves. That is, vacancies in the housing market can shift the labor market Beveridge curve and vice versa.

We take our theoretical model to the data that are obtained from the US Census Bureau's Housing Vacancy Survey (HVS), the national version of the American Housing Survey (NAHS), the Help-Wanted Index from the Conference Board, and BLS's Opening and Labor Turnover Survey (JOLTS). We estimate models at the national level as well as for 37 CBSAs. Annual national-level data on housing vacancies are available starting in 1950 and MSA-level data are measured back as far as 1986. Monthly data on job vacancies begin in 1951. Following Barnichon (2010), we combine the early print index with the recent online index to construct a consistent index of job openings for 1951-2014 at both the national and CBSA level.

Our estimates at the national level show that housing market vacancies do shift the labor market Beveridge Curve. Furthermore, we estimate the "unfulfilled homeownership rate" from a housing tenure choice equation for multiple waves of the NAHS. We define a renter household to be an "unfulfilled owner" if its predicted probability of owning is greater than or equal to 0.5. We then use this prediction to estimate the housing market version of Beveridge Curve. Our results are consistent with the theoretical prediction that the cyclical movement along this curve is opposite to that of the labor market Beveridge curve.

Finally, using the data at the CBSA level, we develop a VAR model of housing and labor market vacancies and estimate impulse response functions to study how shocks to either the housing or labor markets will propagate themselves in the other market (impulse response functions). The results show that shocks to the owner and rental vacancies have negative and significant impacts on job vacancies. This is consistent with the thinking that during the Great Recession of 2007-2009 the downturn in the housing market led to the subsequent decline in the labor market and the real economy.

Pissarides (2013) wonders whether the housing market crash might be an appealing explanation for the current recession. Since homeowners are known to be less mobile than renters, the extraordinary expansion of home ownership in recent years might have contributed to the decline in residential mobility. He argues, however, that there is little evidence of "house-lock effect", namely that falling house prices and the negative equity in many houses are factors behind the fall in mobility. Pissarides argues nonetheless that composition effects due to the shift to more home ownership could still be significant. He speculates that if a secular decline in mobility, whatever its origin might be, is to persist, we should expect future recessions in the US to definitely impact the labor Beveridge curve.

Section 2 provides a review of the recent literature that employs search models in the empirical study of housing markets. Sections 3 and 4 develop the theoretical model that captures the interdependence of the housing and labor markets with frictions. Section 5 describes the data and Section 6 presents the results.

## 2 Literature Review

There have been a few papers in the literature that employ search models in the empirical study of housing markets, though very few of them examine both the housing and labor market by means of the full complement of ideas proposed here. Both Head and Lloyd-Ellis (2012; 2014) and Rupert and Wasmer (2012) develop models of joint housing-labor search, which are complementary to one another. Rupert and Wasmer (2012) develop a theory of the relationship between unemployment and housing market frictions that focuses on the trade-off between commuting time and location decisions within a single labor market. Jobs break up with an exogenous probability, and new job offers arrive at a rate that is specified in the usual way for labor markets with frictions as a function of labor market tightness, the ratio of vacancies to unemployment. Since the only housing amenity that the model considers is commuting distance, which also indexes wage offers, the job acceptance decision is solely in terms of the maximum acceptable distance [*ibid.*, Eq. (6)]. Workers reject job offers that are too far from their location. Since labor income net of commuting depends on commuting distance, the value of employment is conditional on commuting distance.

Rupert and Wasmer show that higher arrival rate of housing opportunities makes workers less choosy about jobs. That is, with more opportunities, workers are prepared to live further away, because they know that opportunities to move closer are more frequent. Also, a higher arrival rate of housing opportunities increases the vacancy rate and thus the rate of job creation, because of the higher rate of job acceptance by workers. As a result, improved opportunities to move decrease unemployment because of workers' being more likely to accept offers and firms' increasing vacancies and thus labor market tightness. In a variation of the model, workers receive demographic ("family") shocks, which change the valuation of the current dwelling and allows workers to sample from the existing stock of vacancies, as opposed to just new vacancies when their jobs break up. Job separations now reflect the possibility that workers may not find an acceptable offer and thus vacancy, and the distribution of commuting distances occupied by workers is suitably adjusted [ *ibid.,* Eq. (20) ]. In equilibrium, the distribution of commuting distances occupied by workers is a linear combination of the distribution function of new vacancies, weighted by the rate at which new job opportunities arrive, of the distribution function of all vacancies, weighted by the rate at which demographic shocks arrive, and of the distribution of job offers over commuting distances, conditional on their coming from acceptable commuting distances, weighted by the rate of total separations. It is thus clear that labor turnover and frictions, including demographic shocks, have profound effects on individuals' locations. With job and housing vacancy searches being jointly indexed by commuting distance, the housing search process is subsumed into the job search. The housing market is not modelled, however, and therefore the impact of location decisions on housing prices is not reflected in the model. In effect, the spatial distribution of new and existing vacancies play the role of housing supply, but demand is not rationed by housing price.

In contrast, Head and Lloyd-Ellis (2012) focus on frictions in the housing market and the role of housing markets in generating frictions between labor markets. They do not allow for frictions in the labor market, which is assumed to be Walrasian. Unlike Rupert and Wasmer, Head and Lloyd-Ellis do distinguish between ownership and renting, a trade-off that plays a significant role in their paper. They abstract from the local urban geography and treat labor and housing markets separately. Bellman equations are defined separately for employed and unemployed renters and owners and are conditional on city type. All those four types possibly coexist in each city. The economy consists of many identical cities of two types, with the types differing in terms of the average productivity of labor. The housing market is intermediated by real estate managers. The value of vacant homes differs across the two city types. Their Bellman equations for dwelling units reflect the capital gains associated with offers by either employed or unemployed workers, adjusted by the rate at which offers are made. New housing units may be constructed, with unit costs that increase with city population to reflect congestion.

Head and Lloyd-Ellis solve for the steady-state equilibrium values of the Bellman equations. Among notable results are the following: the rent differential across the two city types is determined by unemployed renters who are assumed to move costlessly between cities, even if they do not receive an offer. In contrast, the differential in the value of houses is determined by the marginal (unemployed) home-owners who must first receive an outside offer and then incur the (endogenous) liquidity cost of selling their house. This result suggests that anchoring the opportunity cost of owning housing calculations on rent differentials must account for basic characteristics of labor turnover across different city types.

The key frictions modelled by Head and Lloyd-Ellis pertain to the illiquidity of housing. Because home-owners accept job offers from other cities at a lower rate than do renters, a link is generated between home-ownership and unemployment both at the city level and in the aggregate. When they calibrate the model to match aggregate U.S. statistics on mobility, housing, and labor flows, the model predicts that the effect of home-ownership on aggregate unemployment is small. When unemployment is high, however, changes in the rate of home-ownership can have economically significant effects.

In a sequence of papers, Ngai and Sheedy emphasize the frictions associated with buying and selling homes. Ngai and Sheedy (2015) emphasize the dynamic impact of the fact that the majority of housing transactions involve households moving from one house to another, whereby they put their existing homes on the market and plan to buy new homes. This is motivated by households' desire to improve match quality, and consequently they cause a cleansing effect on the quality distribution. Moving may be triggered by an event, like a demographic shock to a household that causes a reassessment of its housing demand. Ngai and Sheedy (2013) emphasizes sellers' decisions, namely when to put a house up for sale and when to agree to a sale.

Ngai and Sheedy do not take a position on the correlation between residential moves and job changes. Using data from the PSID for 1991-1993, Ioannides and Kan (1996) report that for 1974-1983, the proportion of job changes for household heads was 15% per year, that for residential moves was 15.6%, and that for moves and job changes was 6%, which amounts to more than 40% of the movers. Thus, the correlation between moving and a job change is substantial. Furthermore, nearly two-thirds of movers did so to rent, and one-third to own. Using data from the CPS for 2004, Ioannides and Zanella (2008), Table 1, report that 17% of moves occur for work-related reasons and 52.7% for housing- and neighborhood- related reasons.

To the best of our knowledge, ours is the first paper to introduce a Beveridge curve for housing markets in a manner that is consistent with the original definition for the labor market. Peterson (2009) introduces a Beveridge curve for housing markets based on a relationship between the vacancy rate for housing and the rate of household formation, which he intends as a "long-run supply" relationship. Peterson argues that the rate of capital formation being a decreasing function of the housing vacancy rates can be explained by the following: one, the marginal cost of a new house is decreasing in the growth rate of the housing stock; and two, the probability of selling a new house is decreasing in the vacancy rate. Whereas the former assumption is counterintuitive, in view of urban congestion, the latter does agree with intuition. Also, we recently became aware of Limnios (2014), who explores whether frictions in the housing market can help explain frictions in the labor market, but he works with the rental housing market only.

# 3 Theoretical Model

We adopt a dynamic model of the housing market with frictions, due to Head and Lloyd-Ellis (2012). The model allows us to examine housing vacancies while also accounting for owning and renting. We extend their model to study frictions in the labor market by using the solutions of the Bellman equations to structure labor market bargaining. This leads naturally to interactions between frictional housing and labor markets. The remainder of the presentation of the theoretical model is structured as follows. First we introduce the preference structure, which we employ in studying both the housing and labor markets with frictions. This is followed by the development of the Bellman equations for owners and renters, the specification of housing supply, separately for the owner-occupied and rental stock, and the determination of the value of vacant housing, which reflects critically the illiquidity of housing. The Bellman equations are solved after we have determined the relative numbers of agents in different states, employed and unemployed owners and employed and unemployed renters. Finally, we extend the model to allow for turnover in the ownership market and for frictions in the rental market (which is not allowed by Head and Lloyd-

Ellis 2012). The latter development allows us to introduce a Beveridge curve for the housing market. Next we turn to the labor market with frictions, where we start with the presentation of model from Pissarides (1985), as presented in Pissarides (2000), and then use the solutions for the Bellman equations obtained earlier to characterize the determination of wages and job vacancy rates. It follows naturally that there are spillovers from the housing market to the labor market through the wage curve. For most of the analysis, we examine the model at a steady state.

## 3.1 Preferences

Let $W^j, U^j, j = R, H$ denote expected lifetime utility, conditional on being employed, unemployed, for a renter, homeowner, respectively, which is expressed in real terms, and under the assumption of unrestricted borrowing or lending at a fixed rate of interest, $\rho$. These are generated by utility flow, denoted by $\pi^j$, and defined in terms of preference over consumption $c$ , labor supply, $l$, and housing consumption, $z$, in a typical period:

$$\pi^j(c, l, z) = c^j - l + z^j, j = R, H. \tag{1}$$

We assume that labor is supplied at the unit level, that is a person is either employed in area $i$ earning $w_i$, or unemployed, receiving $b_i < w_i$. We suppress the location-specific subscript $i$, unless it is strictly necessary for clarity.

We adapt the definition of utility flow (1) above by accounting for the fact that housing unit costs are different for renters and owners and depend on local conditions. Let: consumption be the numeraire, its price is set equal to 1, and let $\kappa$ be rent per unit of rental housing. Ignoring commuting costs, the quantity of housing consumed by renters in a particular area is given by rent expenditure/$\kappa$. Let $p_h$ be the annual user cost of owner-occupied housing. The latter is defined in standard fashion [Poterba (1986); Henderson and Ioannides (1987)], given the housing price index, $p$, as the annualized user cost of housing per unit of housing value. It consists of: mortgage payments at a rate of interest $\iota$, times the portion of the value of a dwelling unit that is financed, $1 - \text{equ}$, adjusted for the tax deductability of mortgage interest associated with the portion of the value of owner-occupied housing that is leveraged, by multiplying by 1 minus the marginal US income tax rate, $1-\tau$. Property taxes, denoted by rate $\tau_p$ here, are also deductible for US income tax purposes. In addition, allowing for maintenance and depreciation, at rates maint and depr, respectively, and deducting the rate of expected housing price appreciation, $\text{appr}^e$, yield the annualized user cost of housing as:

$$p_h = \left[(1-\tau)[\iota(1-\text{equ})+\tau_p] + \text{depr} + \text{maint} - \text{appr}^e\right]. \tag{2}$$

A dwelling unit of value $V$ generates an annualized user cost of $p_h V$.[2] The respective quantity of housing consumed, that is, housing services, is given by $\frac{p_h V}{p}$.

Suppose that there are no taxes, nor maintenance and depreciation, nor appreciation, and an individual borrows at the real rate of interest $\rho$ to finance staying in a house of value $V_i$, and thus incurring annual housing costs equal to the opportunity cost of housing of value $V$, $\rho V$. Equivalently, since housing is durable, services from it emanate at the real rate of interest, $\rho$, from the actual stock, and are given by $\rho\frac{V}{p}$, which reflects the user cost concept introduced above.

Under the assumption of perfect capital markets, with individuals' being able to borrow against their expected future income at rate $\rho$, the Bellman equations for $W^j, U^j, j =$ H(omeowner), R(enter), may be written as follows. Let $w$ denote the wage rate. The spending in a particular period is equal to the wage rate plus dissaving. This plus spending on non-housing consumption equals the wage rate (or unemployment compensation, as appropriate) plus dissaving. Therefore, the flow of utility for a homeowner, ignoring the disutility of work and allowing for institutional considerations entering through the definition of $p_h$, is equal to:

$$\pi^H(w) = w - \rho V + \text{DISSAVING} + \frac{p_h V}{p}, \tag{3}$$

where $-\rho V$ denotes the dissaving associated with holding housing of value $V$. For a renter:

$$\pi^R(w) = w - \text{rent exp} + \text{DISSAVING} + \frac{\text{rent exp}}{\kappa}. \tag{4}$$

For an unemployed individual, $b$ takes the place of $w$ in the above expressions.

[2] This definition maintains consistency between the rental and the ownership sectors. However, this could be modified so as to be based on transactions values instead of the vacant unit value. Also property taxes rates, maintenance and depreciation rates as well as housing price appreciation rates may be area-specific.

In the simplest possible case at the steady state, where housing circumstances do not change with renters and owners maintaining their housing tenure status for ever, we have the Bellman equations for the conditional value functions, first for owners:

$$\rho W^H = \pi^H(w) + \delta[U^H - W^H]; \tag{5}$$

$$\rho U^H = \pi^H(b) + \mu[W^H - U^H]; \tag{6}$$

and correspondingly for renters:

$$\rho W^R = \pi^R(w) + \delta[U^R - W^R]; \tag{7}$$

$$\rho U^R = \pi^R(b) + \mu[W^R - U^R]; \tag{8}$$

where the flow utilities are specified in (3)- (4) above, except that the term DISSAVING is of course dropped when we integrate from the flow to the stock (the value functions), under the assumption of unrestricted borrowing. This is implicit in expressing the conditions for the value functions in (5–8) above. From now on, we will use $\pi^j, j = H, R$ without the term DISSAVING. Also, in contrast to Head and Lloyd-Ellis, our definition of $\pi^H(w)$ in (3) above makes it dependent on $V$, which is endogenous. We will ignore this endogeneity from now on, when we derive the equilibrium value of $V$ below.

The associated steady-state unemployment rate is given by: $\frac{\delta}{\delta+\mu}$. The job destruction rate is typically assumed to be exogenous. It may vary across MSAs because of differences in their industrial compositions. The job finding rate is typically specified in terms of the the job matching process and labor market tightness, to which we come further below. It can reflect individual characteristics, which is relevant at the empirical stage. Housing spells of homeowners are assumed to last for ever, if job market events and housing tenure events are independent. At such a steady state, we could assume that housing units for renters and owners are perfect substitutes.

## 3.2 Frictions

Both housing and labor markets are subject to frictions. The individual (or household, the two terms will be used interchangeably) is subject to the risk of job loss: jobs break up at a Poisson rate $\delta$, and the unemployed individual finds a job at a rate $\mu$, per unit of time. Regarding the housing market with frictions, we account for matching of dwelling units and individuals via search, which leads to the determination of vacancy rates for houses. In the simplest possible case where rental units may be found instantaneously and thus costlessly, while units for owner-occupancy involve a matching process, frictions correspond naturally to the ownership market. Consequently, the values of vacant units as assets may differ from the transaction prices at which they change owners.

Specifically, let $\gamma$ denote the rate at which new dwelling units sold by construction firms match with potential buyers, which Head and Lloyd-Ellis specify as the product of the rate at which buyers find dwelling units, $\lambda$, and the ratio of buyers to vacant units on the market, $\phi$:

$$\gamma = \lambda\phi. \tag{9}$$

Clearly, $\phi$ and thus $\gamma$ may vary across areas, and we will introduce a subscript $i$, when it is necessary for clarity. This definition may be generalized by specifying a matching function between individuals and dwelling units.[3] It may also be generalized to account for the time it takes owner-occupied houses to be transferred from one household to another, when turnover in owner-occupied units is allowed. See section 3.6 below. Matching in the housing markets involves frictions and makes houses to some extent illiquid; their value when vacant depends

[3] Let $I_{b,t}, I_{h,t}$, denote the stock of buyers searching for houses, the stock of sellers searching for buyers, respectively. Let the matching process be specified in the standard fashion in terms of the Poisson rate of contacts generated, denoted by

$$\Gamma_t = \Gamma\left(I_{b,t}, I_{h,t}\right).$$

So, in general, the rate of arrivals of contacts to the typical dwelling unit in MSA $i$ is:

$$\gamma = \frac{1}{I_{h,t}}\Gamma\left(I_{b,t}, I_{h,t}\right),$$

which under the assumption of constant returns to scale, this may be written as:

$$\Gamma\left(\frac{I_{b,t}}{I_{h,t}}, 1\right) = \Gamma(\phi, 1).$$

This differs from the Head and Lloyd-Ellis assumption, (9) above, only because of the nonlinearity, but is consistent with the assumptions typically made about matching models. Parameter $\lambda$ is subsumed in this formulation.

on the speed with which a buyer can be found for a vacant house. The model highlights this effect. We specify further below $\phi$, the ratio of buyers to vacant units in the market. In section 3.5.1 below, we allow for frictions in the rental housing market as well.

## 3.3 Housing Supply

The total number of individuals $N$, which below we will allow to grow at rate $\nu$, is distributed over the four different states, employed and unemployed homeowners and employed and unemployed renters, $N^{WH}, N^{UH}, N^{WR}, N^{UR}$. Their numbers sum up to total population:

$$N^{WR} + N^{UR} + N^{WH} + N^{UH} = N. \tag{10}$$

It is more convenient to work with the relative numbers of agents, $n^{WH} = \frac{N^{WH}}{N}$. So, we have:

$$n^{WH} + n^{UH} + n^{WR} + n^{UR} = 1. \tag{11}$$

With new individuals entering the market at a rate $\nu$, we need to allow for the construction of new dwelling units. Following Glaeser *et al.* (2014), and Head and Lloyd-Ellis, *op. cit.*, assuming free entry in the housing construction-real estate business implies that for rental housing units the present value of rents equals the asset value of their construction costs:

$$\frac{\kappa}{\rho} = c_0 + c^R \frac{R}{N},$$

where $c_0$ denotes fixed construction cost, and $c^R \frac{R}{N}$ variable costs that depend linearly on the rental housing stock, $R$, relative to population, $N$. The entire stock of rental units are occupied as soon as they are produced, that is, the rental housing market is not subject to frictions. Since all rental units are occupied, $R = N^{WR} + N^{UR}$, the above may be rewritten instead in terms of $n^{WR}, n^{UR}$ :

$$\frac{\kappa}{\rho} = c_0 + c^R \left( n^{WR} + n^{UR} \right). \tag{12}$$

For owner-occupied units, their supply costs when vacant must compensate their producers:

$$V = c_0 + c^H \frac{H}{N}, \tag{13}$$

where $c^H \frac{H}{N}$, variable costs, that depend linearly on the housing stock for ownership, $H$, relative to the population, in order to express the cost of land due to congestion. We could also allow for costly conversion of dwelling units from one mode of tenure to another. Note that these supply equations link "prices," that is rents and values of vacant units, to their respective stocks relative to the numbers of individuals. Next, we relate these supply equations to demand conditions.

### 3.4 The Value of Vacant Housing

For asset equilibrium, the value of a vacant dwelling in particular city reflects the fact that it may be purchased by either employed renters or unemployed renters. The return per unit of time to holding an asset of value $V$ is equal to the probability per unit of time that it may be sold either to an employed renter, at price $P^W$, or an unemployed renter, at price $P^U$, whichever of the two bids is higher:

$$\rho V = \gamma \mathcal{E} \max_j \left\{ P^j - V \right\}, j = W, U. \tag{14}$$

This arbitrage equation for $V$ is written out by recognizing that the event $P^W > P^U$ occurs with probability equal to the proportion of employed among all renters, $\alpha = \frac{N^{WR}}{R}$, and correspondingly, the event $P^W < P^U$ occurs with probability $1 - \alpha = \frac{N^{UR}}{R}$.

Consistent with the literature of markets with frictions, a seller and a buyer of a dwelling unit engage in Nash bargaining and split the surplus from the transaction, with $\sigma$ of $V$ going to the seller and $1 - \sigma$ of $W^H - W^R$ to the buyer. So, the prices paid by employed and unemployed households satisfy:

$$P^W = \sigma V + (1 - \sigma) \left[ W^H - W^R \right]; P^U = \sigma V + (1 - \sigma) \left[ U^H - U^R \right]. \tag{15}$$

Thus, we may solve for $V$ from (14) and (15):

$$V = \frac{(1 - \sigma)\gamma}{\rho + (1 - \sigma)\gamma} \left[ \alpha [W^H - W^R] + (1 - \alpha)[U^H - U^R] \right]. \tag{16}$$

Recall that we assume here that once individuals purchase their dwelling units and become homeowners they remain so for ever. Their conditional value functions are given by

(5–6) above. Renters, on the other hand, are faced with opportunities, at a rate $\gamma$, to purchase dwelling units and become homeowners. Thus, the respective Bellman equations, the counterparts of (7-8), become:

$$\rho W^R = \pi^R(w) + \delta[U^R - W^R] + \gamma \left[W^H - P^W - W^R\right]; \tag{17}$$

$$\rho U^R = \pi^R(b) + \mu[W^R - U^R] + \gamma \left[U^H - P^U - U^R\right]. \tag{18}$$

We may modify the model to allow for interdependence between employment and housing tenure mode transitions, but for the moment, such transitions are assumed to be independent. However, conditions in the housing market have a profound effect on the conditional value functions.

Next, we use (16) in order to express the transaction prices, $P^W, P^R$, in terms of the conditional value functions, $W^H, W^R$, and $U^H, U^R$. We substitute back into the Bellman equations, (5-6) for owners, and (17-18) for renters, solve for those quantities, namely for $W^H, W^R, U^H, U^R$, as functions of the real wage rate and unemployment compensation, on one hand, and of labor market and housing market tightness, on the other. Labor market tightness enters the employment rate for owners and renters, as we discuss in more detail below. Housing market tightness enters via the rate at which prospective home-owners make contacts with housing construction firms, defined in (9) above, $\gamma = \lambda\phi$. Here, $\lambda$ is a matching parameter and $\phi$ is the ratio of numbers of individuals who are potential buyers (all renters) to vacant units, employed and unemployed ones, over the number of all sellers. The number of sellers is equal to the stock of dwelling units for owner-occupancy that is not owned and is thus vacant. If $H$ denotes the total housing stock, then we have:

$$\gamma = \lambda\phi = \lambda \frac{N^{WR} + N^{UR}}{H - N^{WH} - N^{UH}}. \tag{19}$$

## 3.5 Solution for Housing Market Flows

Given wages and other parameters, which are determined from the labor markets with frictions model, we may solve the model as follows. Given (16), the transaction prices for owner-occupied units, $P^W, P^R$, may be expressed in terms of the four conditional value functions, $W^H, W^R, U^H, U^R$. There are seven unknowns, namely:

- the total stock for owner-occupancy, $H$;
- the rental stock, $R$;
- the rent, $\kappa$;
- and the four numbers of agents in different states, $N^{WR}, N^{UR}, N^{WH}, N^{UH}$.

By solving equ. (5), (6), (17) and (18) for the conditional value functions, we may express the value of a vacant home, $V$, in terms of the four unknown numbers of agents, $N^{WR}, N^{UR}, N^{WH}, N^{UH}$, in different states, the unknown housing rent, $\kappa$, and the wage rate.

It is more convenient to think of the model as set in a steady state, with the number of individuals growing at an exogenous rate $\nu$. Along the steady state, all absolute quantities grow at the same rate. This leads to four flow relationships in terms of relative numbers of agents: First, all $N$ individuals distribute themselves over the four states, so that their respective relative numbers sum up to 1: (11) holds.

Second, the change in the number of employed renters in a given city $i$, $\frac{dN^{WR}}{dt}$ equals the number of unemployed renters that become employed, $\mu N^{UR}$, minus the measure of employed renters whose jobs are destroyed, $\delta N^{WR}$, and minus those who cease to be renters because they become owners, $\lambda N^{WR}$. That is:

$$\frac{dN^{WR}}{dt} = \mu N^{UR} - (\delta + \lambda) N^{WR}.$$

By Dividing by $N$, imposing the condition that for a steady state, $\frac{dN^{WR}}{dt} = \nu N^{WR}$, and rewriting, we have:

$$(\nu + \delta + \lambda) n^{WR} - \mu n^{UR} = 0. \tag{20}$$

Third, the change in the relative number of unemployed homeowners, $\nu n^{UH}$, is equal to minus those unemployed homeowners who find jobs, $\mu n^{UH}$, plus the number of those employed homeowners who lose their jobs, plus the number of unemployed renters who become homeowners, $\lambda n^{UR}$. Rewriting, we have:

$$(\mu + \nu) n^{UH} - \delta n^{WH} - \lambda n^{UR} = 0. \tag{21}$$

Fourth, the increase in the number of employed homeowners, $\nu n^{WH}$, is equal to the number of unemployed homeowners getting jobs, $\mu n^{UH}$, plus the number of employed renters who become homeowners, $\lambda n^{WR}$, minus those employed homeowners who become unemployed. Rewriting, we have:

$$\nu n^{WH} + \delta n^{WH} - \lambda n^{WR} - \mu n^{UH} = 0. \tag{22}$$

By rewriting the above equations in matrix form, we have:

$$\begin{bmatrix} \delta+\lambda+\nu & -\mu & 0 & 0 \\ 0 & -\lambda & -\delta & \mu+\nu \\ -\lambda & 0 & \delta+\nu & -\mu \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} n^{WR} \\ n^{UR} \\ n^{WH} \\ n^{UH} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}. \tag{23}$$

The matrix on the l.h.s. of (23) depends on the parameters $(\delta, \lambda, \mu, \nu)$ only. This yields the solution:

$$n^{WR} = \frac{\mu\nu}{(\lambda+\nu)(\lambda+\nu+\delta+\mu)}, n^{UR} = \frac{\nu(\lambda+\nu+\delta)}{(\lambda+\nu)(\lambda+\nu+\delta+\mu)}; \tag{24}$$

$$n^{WH} = \frac{\lambda\mu(\lambda+\nu+\delta)+\lambda\mu(\mu+\nu))}{(\lambda+\nu)(\delta+\mu+\nu)(\lambda+\nu+\delta+\mu)}, n^{UH} = \frac{\lambda\delta(\lambda+\nu+\delta+\mu)+\lambda\nu(\delta+\lambda+\nu))}{(\lambda+\nu)(\delta+\mu+\nu)(\lambda+\nu+\delta+\mu)}. \tag{25}$$

Thus, the share of employed renters is given by $\alpha = \frac{n^{WR}}{n^{WR}+n^{UR}}$, or

$$\alpha = \frac{\mu}{\lambda+\nu+\delta+\mu}; \quad \gamma = \lambda\frac{\frac{\nu}{\lambda+\nu}}{h - \frac{\lambda}{\lambda+\nu}}, \tag{26}$$

where $h = \frac{H}{N}$, is the ratio of the stock for owner-occupancy to the population. In equilibrium, the denominator above must be positive. The model implies an equilibrium homeownership rate,

$$\mathrm{hr} = n^{WH} + n^{UH} = \frac{\lambda}{\lambda+\nu}. \tag{27}$$

The equilibrium rental rate is thus equal to $\frac{\nu}{\lambda+\nu}$. We note that these rates depend critically on the rate of growth of the population. In contrast, the equilibrium unemployment rate does not depend on population growth. Clearly, in order to arrive at a more general model, we need to specify churning within the housing market. We go some way further in this direction below in sections 3.5.1 and 3.6. In section 3.5.1, we also relax the assumption that all renters seek to become homeowners, which boosts the equilibrium rental rate.

The conditional value functions for homeowners may be obtained by solving (5–6). Thus, we have:

$$W^H = \frac{1}{\rho(\delta+\mu+\rho)}\left[(\rho+\mu)\pi^H(w^H) + \delta\pi^H(b)\right]. \tag{28}$$

$$U^H = \frac{1}{\rho(\delta+\mu+\rho)}\left[\mu\pi^H(w^H) + (\delta+\rho)\pi^H(b)\right]. \tag{29}$$

These expressions allow for the possibility that bargaining between firms and workers may lead to wage rates that are different between renters and owners, $w^H, w^R$, respectively, from now on.

We can solve for the conditional value functions for renters, (17-18), after we have expressed the transaction prices for vacant units in terms of the conditional value functions. Recall (15), which gives give transaction prices via Nash bargaining. From (15) and (16), we have instead of (17), (18), respectively:

$$\rho W^R = \pi^R(w) + \delta[U^R - W^R]$$

$$+\gamma_i\frac{\sigma\rho+\sigma(1-\sigma)\gamma_i(1-\alpha_1)}{\rho+(1-\sigma)\gamma_i}\left[W^H - W^R\right] - \gamma_i\frac{\sigma(1-\sigma)\gamma_i(1-\alpha_1)}{\rho+(1-\sigma)\gamma_i}\left[U^H - U^R\right]; \tag{30}$$

$$\rho U^R = \pi^R(b) + \mu[W^R - U^R]$$

$$+\gamma_i\frac{\sigma\rho+\sigma(1-\sigma)\gamma_i\alpha_1}{\rho+(1-\sigma)\gamma_i}\left[U^H - U^R\right] - \gamma_i\frac{\sigma(1-\sigma)\gamma_i\alpha_1}{\rho+(1-\sigma)\gamma_i}\left[W^H - W^R\right]. \tag{31}$$

These equations in concise matrix notation become:

$$\begin{bmatrix} \rho+\delta+\sigma\gamma_i\frac{\rho+(1-\sigma)\gamma_i(1-\alpha_i)}{\rho+(1-\sigma)\gamma_i} & -\left(\delta+\frac{\sigma(1-\sigma)\gamma_i^2(1-\alpha_i)}{\rho+(1-\sigma)\gamma_i}\right) \\ -\left(\mu+\frac{\sigma(1-\sigma)\gamma_i^2\alpha_i}{\rho+(1-\sigma)\gamma_i}\right) & \rho+\mu+\sigma\gamma_i\frac{\rho+(1-\sigma)\gamma_i\alpha_i}{\rho+(1-\sigma)\gamma_i} \end{bmatrix}\begin{bmatrix} W^R \\ U^R \end{bmatrix}$$

$$= \begin{bmatrix} \pi^R(w^R) + \frac{\gamma_i\sigma\rho W^H}{\rho+(1-\sigma)\gamma_i} + \frac{\gamma_i^2\sigma(1-\sigma)(1-\alpha_i)}{\rho+(1-\sigma)\gamma_i}\left[W^H - U^H\right] \\ \pi^R(b) + \frac{\gamma_i\sigma\rho U^H}{\rho+(1-\sigma)\gamma_i} - \frac{\gamma_i^2\sigma(1-\sigma)\alpha_i}{\rho+(1-\sigma)\gamma_i}\left[W^H - U^H\right] \end{bmatrix} \tag{32}$$

While a full solution of (32) is straightforward and would allow us to compute agents' expected lifetime utility for the purpose of welfare analysis, it is helpful to solve for $W^R - U^R$, which is a crucial input to the bargaining problem between firms and unemployed renters.

$$W^R(w^R;b) - U^R(w^R;b) = \frac{1}{D}\frac{\rho(\rho+\gamma)}{\rho+(1-\sigma)\gamma}\left(w^R - b + \gamma\sigma\rho\left[W^H(w^H;b) - U^H(w^H;b)\right]\right), \tag{33}$$

where $D$ denotes the determinant of the matrix on the LHS of (32). Naturally, parameters and all variables reflecting housing market conditions through $\alpha$ and $\gamma$ that enter the matrix in (32).

To recapitulate, the conditional value functions $(W^H, U^H, W^R, U^R)$ have been solved in terms of the wage rates, $w^H, w^R$, the housing market tightness, $\gamma$, which in view of (26) depends on $h$, and the labor market tightness that enters via the employment rate, $\mu$. From (12) and in view of (24), the rent $\kappa$ is determined as a function of the share of renters $n^{WR} + n^{UR} = \frac{\nu}{\lambda+\nu}$, and thus is exogenous. Finally, from (16), $V$ may be expressed, via the conditional value functions, in terms of labor market tightness and the wage rates, $w^H, w^R$. These derivations when used in (13) yield an equation for the relative stock of owner-occupied units, $h$. Finally, the equilibrium is fully determined once the wage rates are set. We turn to this next, which requires looking at the labor market with frictions. In contrast, Head and Lloyd-Ellis assume frictionless labor markets.

### 3.5.1 Allowing for Frictions in Rental Markets

Underlying the model so far is an assumption that the rental housing market is frictionless. However, the concept of vacancies does apply to rental units, and in fact data on rental market vacancies are also available. We use such data below. The purpose of this extension is rationalize rental market vacancies. To the variables denoting the relative numbers of individuals in different labor market states, $(n^{WR}, n^{UR}, n^{WH}, n^{UH})$, and the housing stock for owner-occupancy $H$, we need to add the stock of vacant rental units $v^R$. We complete the model of frictions by also allowing for mismatching of renters, that is, given their circumstances some renters would rather be owning. Let the probability of mismatch be denoted by $\text{msm}^R$.

The first three equations in (23) hold with the modification that instead of $\lambda$, the rate at which prospective buyers find dwelling units, we now have $\lambda(1-\text{msm}^R)$. In addition, the rental housing stock satisfies

$$\frac{R}{N} = n^{WR} + n^{UR} + \frac{v^R}{N}. \tag{34}$$

At the steady state, the increase in the rental housing stock must be equal to the decrease in rental vacancies, due to vacancies that are matched, $\gamma \upsilon^R$ plus the number of renters who are not mismatched. That is:

$$\nu \frac{R}{N} = \gamma \frac{\upsilon^R}{R} \frac{R}{N} + (1 - \text{msm}^R) \left( n^{WR} + n^{UR} \right). \tag{35}$$

### 3.5.2 A Housing Beveridge Curve

The Beveridge Curve for labor markets is a well-established and a widely researched concept. The intuitive similarities between housing and labor markets allows us to exploit analogies in order to obtain a Beveridge Curve for housing. Analogous to vacancies in labor markets, which is unsatisfied demand by firms for workers, there correspond prospective buyers and prospective renters in housing markets, which is unsatisfied demand by individuals for housing. Analogous to unemployed individuals, which is unsatisfied demand for employment by individuals, there correspond vacant dwelling units for sale and for renting.

We work first with the rental market. By substituting for $\frac{R}{N}$ from (34) in (35), we have for the rental vacancy rate $\frac{\upsilon^R}{R}$:

$$\frac{\frac{\upsilon^R}{R}}{1 - \frac{\upsilon^R}{R}} = \frac{1 - \text{msm}^R - \nu}{\nu - \gamma}. \tag{36}$$

Therefore, the rental vacancy rate $\frac{\upsilon^R}{R}$ is decreasing in the rate of mismatch. Note that the relationship is in fact logistic, a prediction which is confirmed by the data, as we see further below.

The counterpart of this relationship for the ownership market may be obtained by solving for $\frac{H - N^{WH} - N^{UH}}{H} = \frac{h - n^{WH} - n^{UH}}{h}$. Note that once we allow for mismatch among renters giving rise to unsatisfied ownership demand, the solutions for $n^{WH}$ and $n^{UH}$ depend on $\lambda(1 - \text{msm}^R)$ and thus on the incidence of mismatch. Working with the previous solution, and recalling (27), we have that the equilibrium homeownership rate is:

$$n^{WH} + n^{UH} = \frac{\lambda(1 - \text{msm}^R)}{\lambda(1 - \text{msm}^R) + \nu}.$$

The equilibrium homeownership rate decreases with the probability of mismatch. That is, an increase, due to mismatch of renters, in the number of individuals searching to buy homes reduces the homeownership rate.[4] Therefore, the owner vacancy rate, $\frac{\upsilon^H}{H}$, can be expressed symmetrically with the owner vacancy rate above, $\frac{\frac{\upsilon^H}{H}}{1-\frac{\upsilon^H}{H}}$, as:

$$\frac{\frac{\upsilon^H}{H}}{1-\frac{\upsilon^H}{H}} = \left[\frac{n^{WH}+n^{UH}}{h}\right]^{-1} - 1 = h^{-1}\left[1+\frac{\nu}{\lambda(1-\mathrm{msm}^R)}\right]. \tag{37}$$

It is thus increasing in the rate of mismatch, a prediction which is confirmed by the data, as we see further below, for our proposed measure of mismatch.

We see that our proposed Beveridge curve for housing depends critically on the rate of mismatch for renters, which is at the heart of the interaction between the rental and ownership markets. It may also express cyclical effects through its dependence on credit conditions.

## 3.6 Residential Moving and Intra-city Turnover

In the model so far, owners stay for ever in the houses they buy. Yet, it is common experience that owners move, as their circumstances change, by selling their homes to buy other homes or to rent. We generate turnover by homeowners by adopting a modeling trick due to Wheaton (1990), also used by Head and Lloyd-Ellis and Ngai and Sheedy, namely that homeowners suffer a taste, or demographic shock. That is, we assume with probability equal to $Prob_m$ home-owners experience housing taste shocks. Upon experiencing a shock, the service flow a home-owner receives from their current home falls permanently to $\pi^H - \epsilon$, cet. par., while the service flow potentially available to them from other houses remains equal to $\pi^H$. Analytically, this is analogous to the exogenous job destruction rate that so simplifies the DMP model. This creates a mismatch, and all such *mismatched* owners immediately become potential buyers, search for a new house and match with vacant houses via the same technology as renters. Once they find a new house, they immediately sell their old house to

[4]In view of the generalization of the matching model in footnote 3 above, the rate at which buyers find dwelling units, $\lambda$, may be written in terms of the matching function $\Gamma(.,.)$, and the ratio of potential buyers to vacant units, $\phi$. That is:

$$\lambda = \Gamma(1, \phi^{-1}).$$

a real estate firm at the market price for vacant units. The real estate firm sells the new house at a price that reflects the usual bargaining outcome:

$$P_i^{WH} = (1-\sigma)\left[W^H - \tilde{W}^H\right] + \sigma V_i, P_i^{UH} = (1-\sigma)\left[U^H - \tilde{U}^H\right] + \sigma V_i,$$

where $\tilde{W}^H, \tilde{U}^H$ denote the conditional value functions for mismatched owners. We need to express this new set of possibilities by rewriting the Bellman equations for home-owners as follows:

$$\rho W^H = \pi^H(w^H) + \delta(1 - Prob_m)[U^H - W^H] + Prob_m \delta_m \left[W^H - \tilde{W}^H\right];$$

$$\rho U^H = \pi^H(b) + \mu(1 - Prob_m)[W^H - U^H] + Prob_m \delta_m \left[U^H - \tilde{U}^H\right],$$

and for the mismatched owners, the conditional value functions satisfy:

$$\rho \tilde{W}^H = \pi^H(w^H) - \epsilon + \delta[\tilde{U}^H - \tilde{W}^H] + \gamma_i \left[W^H - P_i^{WH} + V_i - \tilde{W}^H\right];$$

$$\rho \tilde{U}^H = \pi^R(b) - \epsilon + \mu[\tilde{W}^H - \tilde{U}^H] + \gamma_i \left[W^H - P_i^{UH} + V_i - \tilde{U}^H\right].$$

To these new set of possibilities, there correspond steady state unemployment and ownership rates.

# 4 The Labor Market with Frictions

The model so far has taken as given the wage rate and the employment rate. The treatment that follows completes the analysis by employing the same model to examine symmetrically the labor market under frictions. Since housing market conditions enter the analysis, it follows that housing market outcomes show up as determinants of wages and the unemployment rate. We embed the above model of individuals into the Diamond, Mortensen and Pissarides (DMP) model, by following the compact approach of Pissarides (1985), as presented in Pissarides (2000). This setup, which does not address wage dispersion, is viewed as the canonical equilibrium model of search unemployment. Albeit simple, the model is flexible enough to be useful for both confronting data and analyzing policy issues.

### 4.1 Labor market flows

Consider a labor market in a steady state with a fixed number of labor force participants, $L$ who are either employed or unemployed. Time is continuous and agents have infinite time horizons. Jobs are destroyed at the exogenous rate $\delta$; all employed workers thus lose their jobs and enter unemployment at the same rate. Unemployed workers enter employment at the rate $\mu$ which is endogenously determined, as we see shortly below. Frictions in the labor market are summarized by a matching function of the form

$$M = \mathcal{M}(uL, vL), \tag{38}$$

where $uL$ is the number of unemployed workers, a stock, and $vL$ the number of job vacancies, also a stock. The matching function is taken as increasing in both arguments, concave and exhibiting constant returns to scale.

Unemployed workers find jobs at the rate

$$\mu = \frac{\mathcal{M}(uL, vL)}{uL} = \mathcal{M}\left(1, \frac{v}{u}\right) = \mu(\theta),$$

where $\theta \equiv \frac{v}{u}$ is a measure of labor market tightness. Firms fill vacancies at the rate

$$q = \frac{\mathcal{M}(uL, vL)}{vL} = \mathcal{M}\left(\left(\frac{v}{u}\right)^{-1}, 1\right) = q\left(\frac{v}{u}\right) = q(\theta). \tag{39}$$

It follows that:

$$\mu'(\theta) > 0,\ q'(\theta) < 0.$$

Why? By definition:

$$\mu(\theta) = \theta q(\theta).$$

Differentiating with respect to $\theta$ we have:

$$\mu'(\theta) = q(\theta) + \theta q'(\theta).$$

From the definition of $q$ the second term above becomes:

$$\theta q'(\theta) = -\theta \frac{\partial \mathcal{M}}{\partial (uL)} \left(\frac{v}{u}\right)^{-2} = -\frac{\partial \mathcal{M}}{\partial (uL)} (\frac{v}{u})^{-1} < 0.$$

Greater labor market tightness reduces the rate at which firms fill their vacancies. Therefore,

$$\mu'(\theta) = \mathcal{M}\left(\theta^{-1}, 1\right) - \theta^{-1}\frac{\partial \mathcal{M}}{\partial (uL)} > 0.$$

The inequality follows from the concavity of $\mathcal{M}(\cdot, \cdot)$ by a simple geometric argument, provided that $\lim_{ul \to 0} \mathcal{M}(uL, \upsilon L) \to \infty$. See Figure 2. The intuition is straightforward: the tighter the labor market, the easier it is for workers to find a job, and the more difficult for firms to fill a vacancy. A steady state in the labor market requires that the unemployment rate is unchanging over time. This occurs when the inflow from employment into unemployment, $\delta(1-u)L$, equals the outflow from unemployment to employment, $\mu(\theta)uL$. The steady-state unemployment rate is thus given as:

$$u = \frac{\delta}{\delta + \mu(\theta)}. \tag{40}$$

Since $\mu(\theta)$ is increasing in its argument, this equation also implies a negative relationship, at the *steady state*, between unemployment and vacancies known as the Beveridge curve, after the British economist William Beveridge (1879–1963). It is depicted on an unemployment rate – vacancy rate space, $(u, \upsilon)$ space.

The full dynamic equation for the unemployment rate readily follows: At any point in time, $(1-u)L$ people are employed. Of these, per unit of time, $(1-u)L\delta dt$ people lose their jobs and enter unemployment. Again, during time $dt$, $uL\mu(\theta)$ are finding jobs, thus reducing the ranks of the unemployed. Consequently,

$$du = (1-u)L\delta dt - uL\mu(\theta)\, dt.$$

Rewriting this as a differential equation we have:

$$\dot{u} = (1-u)L\delta - uL\mu(\theta). \tag{41}$$

The Beveridge curve follows if we impose the condition that the unemployment rate remains constant, equilibrium unemployment $\dot{u} = 0$. In an important sense, this is a mechanical accounting relationship, the consequences of flow balance.

A deterioration of matching efficiency, i.e., a decline in job finding given a certain level of tightness, involves an outward shift of the Beveridge curve in the $(u, \upsilon)$ space. An increase in the job destruction rate, possibly induced by faster sectoral reallocation of jobs, is also associated with an outward shift of the Beveridge curve. On the other hand, since other model parameters, such as the productivity of a match between worker and employer (due to technology or aggregate-demand factors), do not appear in this relation, movements in these parameters imply movements along the curve. These differences between model parameters allow us to gain insights into which fundamental factors are the likely determinants of unemployment and vacancies.[5] The Beveridge curve, computed using U.S. monthly data on unemployment and vacancies, has been reported by BLS and is based on its Job Openings and Labor Turnover Survey (JOLTS) [`www.bls.gov/ljt`]. The movements in the unemployment rate, $u$, measured here as unemployment divided by the labor force, and $\upsilon$, the job openings (vacancy) rate, measured here as openings divided by employment plus openings. Monthly observations are used to track the business cycle.

During the Great Recession, a marked outward shift in the Beveridge curve has been observed. Earlier recessions were also associated with such shifts, though not as pronounced.[6] The reasons for this shift are not yet fully understood. However, it is clear that the curve is turning around, exactly as predicted by Pissarides' theory. We come to that shortly below. This feature of the observed Beveridge curve has consequences for the housing market, and it is one of the aims of the present paper to explore it fully.

## 4.2 The Behavior of Workers at Bargaining

Our benchmark model assumes exogenous search effort. Workers' decisions can influence unemployment only through their impact on wage setting. Workers care about their expected present values of incomes and recognize that these values depend on labor market transition rates as well as wages while employed and unemployment benefits while unemployed.

We work with the value functions developed before. Recall $U^j, j = H, R$ denote the

[5] Equation (40) is a steady-state relation, and thus it is not immediate that it can be used to analyze time-series data. However, if the adjustments to steady state are rather quick, the equation is a good approximation over shorter time horizons.

[6] For recent discussions of shifts in the Beveridge curve for labor markets, see Elsby *et al.* (2014) and Diamond and Sahin (2014).

expected present value of income of an unemployed worker and $W^j, j = H, R$, the corresponding present value of an employed worker. Equ. (28-29) for owners, and Equ. (32), for renters, are solved for the quantities, namely for $W^H, W^R, U^H, U^R$, as functions of the real wage rate, unemployment compensation, and labor market and housing market tightness. The solutions for the conditional value functions enter the bargaining model. Here we are faced with a modeling choice, namely whether or not to make wage determination conditional on whether the individual is a renter or an owner. That is, in modeling bargaining between prospective workers and employers whether we assume that the benefit from employment for the worker is either $W^H - U^H$ or $W^R - U^R$. If we assume that bargaining is conditional of housing tenure status, then the resulting labor market tightness will also be similarly conditional. The spirit of bargaining theory in the context of search models suggests that the tenure status ought to affect bargaining, because the utility of the threat point does. Of course, there is no reason that otherwise equivalent labor should be discriminated in employment decisions, unless of course they have different reservation wages. If we were to posit that the benefit from employment, for the purpose of bargaining, WS, is the expected surplus, it is expressed as follows:

$$\mathrm{WS} = \frac{n^{UR}}{n^{UR} + n^{UH}}[W^R - U^R] + \frac{n^{UH}}{n^{UR} + n^{UH}}[W^H - U^H]. \tag{42}$$

Still, the logic of the bargaining model requires that the transactions price be dependent on the buyer's being employed or unemployed, as expressed by (15) above, so in wage bargaining the same logic requires that we treat firms' bargaining with workers separately for homeowners and renters. A way to get around this awkward step would be to assume competitive search, as modeled by Diaz and (2013) and Moen (1997).

## 4.3 The Behavior of Firms at Bargaining

Jobs are created by firms that decide to open new positions. Job creation involves some costs and firms care about the expected present value of profits, net of hiring costs. The unit value of a firm's output is $p_g$. Assume, as is standard in this literature, that firms are small, in the sense that each firm has only one job that is either vacant or occupied by a worker.

There is a flow cost, $p_g c$, associated with a vacancy, per unit of time. Note it is defined in terms of the value of the output. Let $V_u$ denote that expected present value of having a vacancy unfilled and $V_f$ the corresponding value of having a job occupied, a vacancy filled, by a worker. A job vacancy is an asset from which the firm expects to make money. A job vacancy is filled at the rate $q(\theta)$, whereas an occupied job is destroyed at the rate $\delta$. The value functions associated with a vacancy and a filled job satisfy, respectively, the following equations:

$$\rho V_u = -p_g c + q(\theta)(V_f - V_u). \tag{43}$$

$$\rho V_f = p_g - w + \delta(V_u - V_f), \tag{44}$$

where $p_g$ denotes the price of the good produced, which for consistency with the earlier part of the paper can be set equal to 1, as the good is the numeraire. The l.h.s. of (43) is the opportunity cost per unit of time of a vacancy. Its r.h.s. is the expected return, when costs are incurred per unit of time, $p_g c$, plus the expected capital gain from a job vacancy's getting filled, $q(\theta)(V_f - V_u)$. Similarly, the l.h.s. of (44) is the opportunity cost per unit of time of a filled vacancy. Its r.h.s. is the expected return, which consist of output minus the wage rate, profit per unit of time, $p_g - w$, plus the expected capital gain from a job's becoming vacant, $\delta(V_u - V_f)$.

Hiring by firms is done indirectly by firms' opening vacancies. Firms open vacancies as long as it is profitable to do so. As firms open up vacancies, the value of a vacancy decreases. At the free entry equilibrium, $V_u = 0$.

So, solving (43) in terms of $V_u$ and setting $V_u = 0$, yields an equation for $V_f$ :

$$V_f = \frac{p_g c}{q(\theta)}. \tag{45}$$

Solving (44) for $V_f$ and setting $V_u = 0$, yields a second equation for $V_f$, $V_f = \frac{p_g - w}{\rho + \delta}$. From these two equations for $V_f$ we have:

$$w = p_g - (\rho + \delta)\frac{p_g c}{q(\theta)}. \tag{46}$$

Here, $p_g$ is the value of the marginal product of labor. Once filled, each job produces a unit of output per unit of time. It is equal to the wage rate plus the capitalized value of the firm's hiring cost. A vacancy once created is expected to last for $q(\theta)^{-1}$ periods of time, generating costs $\frac{pc}{q(\theta)}$. Each vacancy is created with probability $\delta$ per unit of time and the hiring cost incurs an interest cost at a rate $\rho$. So, So, the capitalized value of the firm's hiring cost is given by $(\rho + \delta)\frac{p_g c}{q(\theta)}$. From this relationship, $w$ as a function of $\theta$ in $(\theta, w)$ space is downwardsloping.

Equ. (46) is referred to as the *job creation* condition. It plays the role of the demand for labor in the standard model of a labor market without frictions, where the quantity of labor is represented by labor market tightness, $\theta = \frac{v}{u}$, the ratio of the vacancy rate to the unemployment rate. Note that in equilibrium, from Equ. (46), that given $p_g, w$, the incentives to create vacancies are reduced by a higher real interest rate, a higher job destruction rate and a higher vacancy cost. Vacancy creation is encouraged by improved matching efficiency that exogenously increases the rate at which the firm meets job searchers.

## 4.4 Wage bargaining

Since the labor market is characterized by frictions and bilateral meetings, the standard wage determination mechanism is not appropriate. The main approach that has been used by the markets with frictions literature assumes that there is bargaining between the employer and the worker. So suppose that wages are set through individual worker-firm bargaining. The logic of our model requires that we distinguish between homeowners and renters in their bargaining with employers.[7]

### 4.4.1 Homeowners' bargaining and labor market equilibrium

The expected capital gain for an unemployed homeowner from becoming employed is equal to $W^H - U^H$. A firm, on the other hand, gives up $V_u = 0$, in order to gain $V_f$. Following

[7] Our approach to both housing and labor markets is based on the original formulations of the markets with frictions. But those formulations can be extended by means of competitive search models, along the lines of Diaz and Jerez (2013), which is applied to housing markets, and Moen (1997), who aims at job market applications.

generalized Nash bargaining, the wage rate is determined so as to split the total surplus,

$$\text{Total Surplus}^H = W^H - U^H + V_f - V_u, \tag{47}$$

in order to

$$\max_{w^H} : \left(W^H - U^H\right)^{1-\sigma_L} (V_f - V_u)^{\sigma_L}, \tag{48}$$

where $1 - \sigma_L$ is a measure of the worker's relative bargaining power. With free entry of vacancies, $V_u = 0$, and thus: $V_f = \frac{p-w}{\rho+\delta}$. Note that the threat points in the Nash bargaining are taken to be what the worker and the firm would receive upon separation from each other. As Hall and Milgrom (2008) note, the job-seeker then returns to the market and the employer waits for another applicant. A consequence is that the bargained wage is a weighted average of the applicant's productivity on the job and the value of unemployment. That latter value, in turn, depends in large part on the wages offered by other jobs.[8]

The first-order condition for the maximization of the total surplus is:

$$W^H - U^H = (1 - \sigma_L)\left[W^H - U^H + V_f\right],$$

which yields $\sigma_L(W^H - U^H) = (1-\sigma_L)V_f = (1-\sigma_L)\frac{p-w^H}{\rho+\delta}$. Following Pissarides (2000), we do not work with the reduced forms for $W^H$ and $U^H$, to express $W^H - U^H$ and instead follow the logic of the Nash equilibrium and solve for $W^H$ from (5), substitute into the l.h.s. of the above equation and solve for $w^H$ to get:

$$w^H = (1 - \sigma_L)p_g + \sigma_L \rho U^H.$$

Next, we solve for $U^H$ from (6) using the alternative expression for $W^H - U^H$, obtained from $\sigma_L(W^H - U^H) = (1 - \sigma_L)V_f = (1 - \sigma_L)\frac{p_g c}{q(\theta)}$. Solving for $\rho U^H$ we have: $\rho U^H = b + \theta p_g c \frac{1-\sigma_L}{\sigma_L}$.

[8]Some researchers have made alternative assumptions about the threat points. Hall and Milgrom (2008) assume that the threat point is to delay and postpone bargaining and agreement instead of threatening to walk out of the deal, as Pissarides does. "The bargainers have a joint surplus, arising from search friction, that glues them together." Hall and Milgrom (2008). They assume that the threats are to extend bargaining rather than to terminate it. The result is to loosen the tight connection between wages and outside conditions of the Mortensen–Pissarides model. When the labor market is hit with productivity shocks, the Hall–Milgrom bargaining model delivers greater variation in employer surplus, employer recruiting efforts, and employment than does the Nash bargaining model.

This yields:

$$w^H = \sigma_L b + (1 - \sigma_L) p_g (1 + c\theta^H), \tag{49}$$

the wage curve for owners. Not surprisingly, it does not depend on housing market conditions: once individuals become homeowners, they stay as homeowners. Of course, this would change if we were to modify the model and allow for turnover for homeowners as well, as in Section (3.6) above.

The two equations, the wage curve and the job creation condition, (49) and(50), the latter being the same as (46) but reproduced here for clarity,

$$w^H = p_g - (\rho + \delta) \frac{p_g c}{q(\theta^H)}, \tag{50}$$

play the role of the labor supply and demand curves, respectively. Solving them jointly determines the wage rate and labor market tightness in the labor market for homeowners, $(\theta^H, w^H)$. It might sound odd that labor market conditions depend on housing tenure status, but the logic of the model is that bargaining is adapted to all parties' specific circumstances. One way to rationalize this is to consider that owners would likely have smaller separation rate $\delta$ that renters. Thus, firms are prepared to invest more in employment relationships with owners.

### 4.4.2 Renters' bargaining and labor market equilibrium

Following the same logic, we formulate the bargaining problem for renters in order to obtain the wage curve for renters. Because in the model renting is a transitional state, as individuals look forward to becoming owners, the wage curve reflects conditions both for renters and owners. The bargaining model is defined as maximizing $\max_{w^R} : \left(W^R - U^R\right)^{1-\sigma_L} (V_f - V_u)^{\sigma_L}$, subject to a total surplus condition, like (47), yields:

$$\sigma_L (W^R - U^R) = (1 - \sigma_L) V_f = (1 - \sigma_L) \frac{p_g - w^R}{\rho + \delta}. \tag{51}$$

Following Pissarides (2000), we do not work with the reduced forms for $W^R, U^R$ to write for $W^R - U^R$ from (33). Instead we follow the logic of Nash equilibrium and solve for $W^R$ from

(30):

$$\left[\rho+\delta+\gamma_i\frac{\sigma\rho+\sigma(1-\sigma)\gamma_i(1-\alpha_1)}{\rho+(1-\sigma)\gamma_i}\right]W^R=\pi^R(w)+\left[\delta+\gamma_i\frac{\sigma(1-\sigma)\gamma_i(1-\alpha_1)}{\rho+(1-\sigma)\gamma_i}\right]U^R$$

$$+\gamma_i\frac{\sigma\rho+\sigma(1-\sigma)\gamma_i(1-\alpha_1)}{\rho+(1-\sigma)\gamma_i}W^H-\gamma_i\frac{\sigma(1-\sigma)\gamma_i(1-\alpha_1)}{\rho+(1-\sigma)\gamma_i}U^H.$$

By substituting into (51), we solve the resulting equation for $w^R$, in terms of $(W^H, U^H)$ and $U^R$. Next, we solve from (31) for $U^R$, in terms of $(W^H, U^H)$ and by using the alternative version of the maximization condition for the surplus, $\sigma_L(W^R-U^R)=(1-\sigma_L)\frac{pc}{q(\theta)}$. Finally, we use the solutions (28-29) in the resulting expression for the wage equation.

A short cut (that uses the reduced form solution and is not fully accurate) would be:

$$w^R=b+\frac{\sigma_L}{1-\sigma_L}p_g\left[(\rho+\delta+\gamma\sigma)\frac{c}{q(\theta^R)}+c\theta^R\right]-\frac{\gamma\rho}{\delta+\rho+\mu(\theta^H)}(w^H-b), \tag{52}$$

the wage curve for renters. Since $q(\theta^R)$ is decreasing in $\theta^R$, the wage curve, which plays the role of the supply curve here, is increasing in $\theta^R$. Equation (52) and the job creation condition for renters,

$$w^R=p_g-(\rho+\delta)\frac{p_g c}{q(\theta^R)}, \tag{53}$$

when solved as a simultaneous system determine $(\theta^R, w^R)$ labor market tightness and the wage rate for renters.

Not surprisingly, the wage curve for renters does depend on housing market conditions: renters become homeowners at the first opportunity, and thereafter stay as homeowners. Forward-looking agents anticipate this prospect. We note that the result implies a spill-over effect from the housing market. An increase in $\theta^H$ in labor market tightness in the labor market for homeowners increases the employment rate for homeowners and shifts upwards the wage curve for renters. A decrease in housing market tightness, $\gamma$, shifts downwards the wage curve in this labor market. This causes, *cet. par.,* a decrease in labor market tightness $\theta^R$, which increases unemployment and decreases vacancies. This, in turn, shifts the Beveridge curve upwards, exactly as it was observed during the downturn associated with the Great Recession of 2007-2009 in the US. Therein lies the power of the Beveridge curve tool in that it allows us to track structural shifts in the overall economy as well as at sectoral or regional levels.

Before we move on with the empirics, some remarks are in order. One, at first sight, it might be puzzling that wage rates for owners and renters are different, since individuals are identical with respect to their productivity. Yet differences in wages between renters and owners follows directly from the model. The economic interpretation rests on the fact that firms anticipate different turnover patterns between renters and owners, due to different mobility costs. There are, of course, numerous ways in which the model can be extended, in addition to developing fully the case of turnover by owners and its implications for wage determination, unemployment and labor and housing vacancy rates. A particularly interesting feature that is worth exploring is to allow for correlation between residential moves and job changes. As discussed in section 2 above, more than one-third of moves are also associated with job change.

# 5 Data

Annual data at the national level on homeownership and rental vacancies is available from the Census Bureau starting in 1950. Data on housing vacancies at the MSA level come from the Census Bureau's Housing Vacancy Survey (HVS). The HVS is a regular part of the Current Population Survey (CPS). Units that are found to be vacant or were otherwise not interviewed are included in the HVS.[9] These data are available from 1986-present on

[9]The definition of a vacant housing unit as given by the Census Bureau is "A housing unit is vacant if no one is living in it at the time of the interview, unless its occupants are only temporarily absent. In addition, a vacant unit may be one which is entirely occupied by persons who have a usual residence elsewhere. New units not yet occupied are classified as vacant housing units if construction has reached a point where all exterior windows and doors are installed and final usable floors are in place. Vacant units are excluded if they are exposed to the elements, that is, if the roof, walls, windows, or doors no longer protect the interior from the elements, or if there is positive evidence (such as a sign on the house or block) that the unit is to be demolished or is condemned. Also excluded are quarters being used entirely for nonresidential purposes, such as a store or an office, or quarters used for the storage of business supplies or inventory, machinery, or agricultural products. Vacant sleeping rooms in lodging houses, transient accommodations, barracks, and other quarters not defined as housing units are not included in the statistics in this report. A vacant unit for rent consists of "units offered for rent and those offered both for rent and sale."

an annual basis for the largest 75 MSAs (though there are less than 75 MSAs in the early years). These data are somewhat problematic since MSA definitions change over time.

Data on monthly job vacancies starting in 1951 come from the Help-Wanted Index for the fifty largest MSAs; these are an aggregate of ads carried by the press that is provided by the Conference Board.[10] However, it is known that since the mid- to late-1990s, this "print"-based measure of vacancy posting has become increasingly unrepresentative as advertising over the internet has become more prevalent. Figure 2 plots the National print Help-Wanted Index starting in 1977 (note that it coincides with the composite index until 1994). One can see the drop off around 2000. Barnichon (2010) builds a vacancy posting index that captures the behavior of total"print" and "online"-help-wanted advertising, by combining the print with the online Help-Wanted Index published by the Conference Board since 2005. Figure 2 includes our version of the combined index. It closely replicates Barnichon's index which goes through 2009 and extends it through June 2014.

Figure 3 plots the job vacancy data (the composite Help-Wanted Index divided by the size of the labor force) along with ownership and rental vacancy rates for 1956-2014. The correlations are given below the figure. There is a reasonably strong negative correlation between job vacancies and both rental and homeownership vacancy rates. This is explained by the observations that there tend to be more job vacancies when the labor market is hot, as workers can be more selective, and fewer rental or home ownership vacancies, when the housing market is hot as renters are motivated to enter the ownership market (though there is more churning). We believe this fact has not been noticed before. A potential causal mechanism is that as vacancies increase in the housing market, job vacancies decrease since this opens up more residential locations and allows workers to make better job matches (given the joint residential location and job matching decision process).

We now have the same job vacancy data at the MSA level for 40 MSAs, We have applied a similar procedure to create a combined Help-Wanted Index (HWI) for each of the MSAs.[11]

[10] Pissarides (1986) for Britain, and Blanchard and Diamond (1989) for the US have used the Help-Wanted Index in studying labor market adjustment.

[11] We have data for 51 MSAs for the HW online index (HWOL) and 49 MSAs for the print index. But

Details about the construction of our MSA-level combined HWI are given in the Appendix. Summary statistics for the composite HWI for 1986 - present are given in Table 1.

Additional data on monthly job vacancies starting in December 2000 are available from the Bureau of Labor Statistics in the Job Openings and Labor Turnover Survey (JOLTS). These data are only provided at the level of the four Census regions for total nonfarm employment as well as aggregated by a number of industrial categories.

We use the National version of the American Housing Survey (NAHS) to estimate renters unfulfilled desire to be home ownners. The NAHS is an unbalanced panel of more than 50,000 housing units that are interviewed every two years and contains detailed information on dwelling units and their occupants through time, including the current owner's evaluation of the unit's market value. For now, we use the NAHS for survey years 1997-2011. Summary statistics for all the variables used in this calculation are given in Table 1.

# 6 Empirics

## 6.1 Beveridge Curve: labor market

Recall that the Beveridge curve plots job vacancies versus the unemployment rate. Movements along the Beveridge curve indicate positions in the business cycle: higher unemployment and lower vacancies in periods of recession, and lower unemployment and higher vacancies in periods of expansion. Shifts in the Beveridge curve can arise from a variety of reasons: changes in efficiency of the job matching process, skill mismatch, changes in the labor force participation rates, and others, such as economic and policy uncertainty. See Pissarides (2011) for details on shifts in the US and UK Beveridge curves.

Figure 4 plots the National Beveridge Curve for 1951-2014. The job vacancy rate, vjobs,

Austin, Buffalo, Honolulu, Las Vegas, Orlando, Portland, Providence, San Jose, Tucson, and Virginia Beach are included in the online data but not in the print data whereas Albany, Allentown, Dayton, Knoxville, Omaha, Syracuse, Toledo, and Tulsa are included in the print data but not in the online data. Also, Houston is missing the print index for 1996.9 to 2003.7 so it is excluded.

is the composite help wanted index divided by the labor force. The data are split into five episodes that are determined by apparent shifts in the Beveridge curve. Figure 5 plots the Beveridge curve for the most recent episode using the monthly JOLTS data. The curve begins in the upper left corner in January 2001 in a strong period of low unemployment and a high job vacancy rate. It then moves south east and ends up in the southeast corner at the end of 2009 in a weak period of high unemployment and low job vacancies. There appears to be an outward shift in the Beveridge curve over the next six months followed by a north-west movement to a stronger economic position in 2014.

Our theory establishes spillovers between the wage curves for owners and renters, where the latter depends on conditions in the housing market. This is evident in the last term on the r.h.s. of Eq. (52), via $\gamma$, the rate at which new dwelling units sold by construction firms are matched with potential buyers, and $\mu(\theta^H)$, the employment rate in the labor market for homeowners. A larger $\gamma$ causes a downward shift of the wage curve for renters, thus increasing the respective labor market tightness and causing a downward movement along the corresponding Beveridge curve. The prediction for the full effect requires that we solve jointly the two wage-job creation curve systems for the labor market for renters and owners.

Denote the unemployment rate in MSA $i$ and time $t$ as $\text{unempl}_{i,t}$, the job vacancy rate as $\text{vjobs}_{i,t}$, the ownership vacancy rate as $\text{vown}_{i,t}$, and the rental vacancy rate as $\text{vrent}_{i,t}$. Then the augmented Beveridge curve is specified as

$$\text{vjobs}_{i,t} = \alpha_{0i} + \alpha_1 \text{unempl}_{i,t}^{-1} + \alpha_2 \text{vown}_{i,t} + \alpha_3 \text{vrent}_{i,t} + \epsilon_{i,t}. \tag{54}$$

We estimate this model using the national composite help wanted index for 1951-2014 where the jobs vacancy rate is this index divided by the size of the labor force. The results for 1951–2014 are given in Table 2. Column (1) of Table 2 is for the regression of the jobs vacancy rate on the inverse of the unemployment rate. The ownership and rental vacancy rates are then added to produce the augmented Beveridge curve. We also include a flag for the 1951-1955 period since the housing vacancy rates are only observed starting in 1956. The results are given in Column (2) of Table 2. This significantly increases the fit of the regression and increases the (magnitude) of the elasticity of the jobs vacancy rate with respect to the unemployment rate to -0.50 (see Column 2). Both housing vacancy rates are significant. Surprisingly the corresponding coefficient estimates are opposite in sign. The coefficient for the rental vacancy rate has a smaller $p-$value and an elasticity near $-1$. This is in line with the negative relationship between labor and housing market vacancy rates that we hypothesized earlier. We next add in indicators of the five episodes as given in Figure 4. These are all significant and slightly increase (the magnitude) of the unemployment rate elasticity. Now only the rental vacancy rate is significant and its elasticity has shrunk (in magnitude) to $-0.40$. See columns 3 and 4 of Table 2.

## 6.2 Beveridge Curve regressions: housing market

Our theory aims at a symmetric treatment of the labor and housing markets over and above the presence of spillovers. In particular, we would like to develop a housing market Beveridge curve. Whereas the vacancy rate concept applies equally well to the housing market, hitherto there is no obvious counterpart of unemployment in the housing market. In the labor market, the unemployment rate measures the extent of the unfulfilled desire of labor market participants to work. We posit a counterpart concept in terms of an unfulfilled desire on the part of renters to become homeowners. Individuals are prevented from owning homes due to the inability to get a mortgage because of down payment constraints, poor credit, or because of discrimination in the mortgage credit market.

We proceed by estimating a housing tenure choice equation and then use these results to predict the probability of owning. We define renters with a probability of homeownership greater than or equal to 0.5 as unfulfilled owners. The number of this group is denoted as $N_{u,rent}$ and the number of owners as $N_{own}$. The unfulfilled homeownership rate is defined as:

$$\text{uhr} = \frac{N_{u,rent}}{N_{u,rent} + N_{own}}. \tag{55}$$

We note that $N_{u,rent}$ corresponds naturally to the concept of mismatched renters, introduced in section 3.5.2 above. In terms of the notation of the earlier part of the paper, the unfulfilled homeownership rate is given by:

$$\text{uhr} = \frac{\nu \text{msm}^R}{\nu \text{msm}^R + \lambda}. \tag{56}$$

Therefore the properties of the vacancy rates for owners and renters, $\upsilon^H$ and $\upsilon^R$, defined respectively above in (37) and (36), with respect to the probability of mismatch for renters are inherited by the unfulfilled homeownership rate.

To obtain uhr, we estimate the following equation for the propensity for household head $i$ in MSA $m$, and year $t$ to be a homeowner

$$\text{own}^*_{i,m,t} = \alpha_0 + \alpha_1 \frac{\text{index}^{\text{value}}_{mt}}{\text{index}^{\text{rent}_{mt}}}$$
$$+\alpha_2 \text{income}^p_{imt} + \alpha_2 \text{income}^T_{imt} + \mathbf{X}_{imt}\alpha_4 + \epsilon_{it} \tag{57}$$

Where the discrete indicator $\text{own}_{i,m,t}$ is defined as $\text{own}_{i,m,t} = 1, \text{if}$

$$\epsilon_{it} \geq -\left(\alpha_0 + \alpha_1 \frac{\text{index}^{\text{value}}_{imt}}{\text{index}^{\text{rent}_{imt}}} + \alpha_2 \text{income}^p_{imt} + \alpha_2 \text{income}^T_{imt} + \mathbf{X}_{imt}\alpha_4\right) \tag{58}$$

and renting otherwise:

$$\text{own}_{i,m,t} = 0, \text{otherwise}$$

where $\text{index}^{\text{value}}_{imt}, \text{index}^{\text{rent}}_{imt}$ are rental and house value indices (to be explained below). The value to rent ratio is included in the housing tenure equation to capture the relative cost of owning versus renting. The variables $\text{income}^p_{imt}, \text{income}^T_{imt}$ are permanent and transitory annual household income. Due to mortgage market imperfections, they have different impacts. In the absence of suitable data, $\text{income}^p_{imt}$ proxies for wealth. The vector $\mathbf{X}_{i,m,t}$ includes socioeconomic characteristics, like individual education, gender, race, age, and household size.

We generate the auxiliary variables $\text{index}^{\text{rent}}_{mt}, \text{index}^{\text{value}}_{mt}$ from the following hedonic equations, for renters and owners, respectively:

$$\ln(\text{rent}_{imt}) = \alpha_{0,m} + \alpha_1 \mathbf{Y}_{1,i,m,t} + \epsilon_{1,i,t}, i = \text{renter}, \tag{59}$$

where $\text{rent}_{imt}$ is reported monthly rent paid, and

$$\ln(\text{price}_{imt}) = \beta_{0,m} + \beta_1 \mathbf{Y}_{1,i,m,t} + \beta_2 \mathbf{Y}_{2,i,m,t} + \epsilon_{2,i,t}, i = \text{owner} \tag{60}$$

where $\text{price}_{imt}$ is the respondent's estimate of the property's market price, $\mathbf{Y}_{1,i,m,t}$ denotes a vector of dwelling unit characteristics, and $\mathbf{Y}_{2,i,m,t}$ property tax and lot size. The intercepts of the above hedonic equations vary by MSA, $m$. Then the rent and value indices are calculated as follows:

$$\text{index}_{mt}^{\text{value}} = 100 \times \exp[\beta_{0,m}]; \tag{61}$$

$$\text{index}_{mt}^{\text{rent}} = 100 \times \exp[\alpha_{0,m}] \tag{62}$$

The predicted values of the permanent and transitory components of household incomes, $\text{income}_{imt}^{p}, \text{income}_{imt}^{T}$ are obtained as the predicted value and residual, respectively, from the following equation:

$$\ln\left(\text{income}_{imt}\right) = \gamma_{0,m} + \gamma_1 \mathbf{Z}_{i,m,t} + \epsilon_{2.m,t} \tag{63}$$

where $\text{income}_{imt}$ denotes reported household income and $\mathbf{Z}_{i,m,t}$ denotes a vector that includes functions of education, age, race, and gender.

In order to calculate our unfulfilled homeownership rate variable, $\text{uhr}_t$, defined in (55) above, we estimate the tenure choice equation by probit, where the probability of owning is given by $\Phi(\mathcal{X}_{i,m,t}\hat{\alpha})$, where $\Phi(\cdot)$ denotes the cumulative normal distribution, and the vector $\mathcal{X}_{i,m,t}$ includes all characteristics used so far. We define renter $i$, $\text{own}_{i,m,t} = 0$, as an unfulfilled renter,

$$\text{u-rent}_{i,m,t} = 1, \text{if } \Phi(\mathcal{X}_{i,m,t}\hat{\alpha}) \geq 0.5; \text{and } \ \text{own}_{i,m,t} = 0. \tag{64}$$

It follows that the unfulfilled homeownership rate is given by:

$$\text{uhr}_t = 100 \times \frac{N_{u-rent}}{N_{u-rent,t} + N_{own,t}},$$

where $N_{u-rent,t} = \sum_i \ \text{u-rent}_{i,m,t}$, and $N_{own,t}$ all self-reported owners.

We estimate the unfulfilled homeownership rate $\text{uhr}_t$ using the NAHS. The US housing Beveridge curves are plotted in Figure 6 with the rental and owner vacancy rates on the left and right axis, respectively. Note that both curves follow very similar patterns, and they are roughly logistic, exactly as predicted by our theoretical model in section 3.5.2 above. Each starts in the southeast corner. This is a sign of a hot market when vacancy rates are low and there is an increased renter willingness to get into the ownership market, increasing the unfulfilled ownership vacancy rate. There is then movement to the northwest corner when the housing market is at its low point in 2009-2011. The housing Beveridge curve then moves southeast in 2011, reflecting improvement in the market. We believe that this is the first time that Beveridge curves for the housing market have been drawn.[12]

Next, we specify and estimate the augmented housing Beveridge curves for the ownership and rental markets

$$vx_{i,t} = \beta_0 + \beta_1 \mathrm{uhr}_{i,t}^{-1} + \beta_2 \mathrm{vjobs}_{i,t} + \varepsilon_{i,t}, \tag{65}$$

where $x =$ own, rent. Results for the standard Beveridge curve are given in Columns (1) and (3) of Table 3. Of course, this is really only illustrative at this point since we only have 8 observations. The coefficient estimates for $\mathrm{uhr}_{i,t}^{-1}$ are positive and significant in both cases but the elasticities seem unusually large (approximately -6.0 and -3.0 for the ownership and rental models, respectively). The augmented housing Beveridge curve results are given in Columns (2) and (4). The coefficient estimate for $\mathrm{vjobs}_{i,t}$ is negative in both cases but not significant.

## 6.3 VAR Models and Impulse Response Functions for Owner, Rental, and Job Vacancy Rates

We next specify and estimate VAR models of job and housing ownership and rental vacancy rates using the data at the CBSA level. The purpose is to establish the interrelationship between the two markets and then to calculate how shocks in one market propagate themselves in the other market using an impulse response function. These models are extensions of the augmented labor and housing market Beveridge curves that include lags of the explanatory and dependent variables, the CBSA house price index as an additional control variable, and CBSA fixed effects. We have data for 37 CBSAs for 1991–2012.

[12] Peterson (2009) defines a long-run "Beveridge Curve" in the housing market as the rate of household formation as a decreasing function of the vacancy rate for housing, which he finds to be true for the owner-occupied market, the rental market, and the total market for housing irrespective of ownership status. He sees this as a long-run supply condition that he explains by assuming that, one, the cost to produce new housing is decreasing in the growth rate of the housing stock, and two, the likelihood of selling a new house is decreasing in the vacancy rate. The first condition clashes with a long-held stylized fact of urban congestion; the second one is, however, consistent with the search model.

First, we check for unit roots in each of the time series (using xtunitroot in Stata). All variables, including the unemployment rate (and its inverse) and the owner house price index are found to have a unit root. So we run the VAR regressions in first differences. Next, we test for Granger Causality. These three regressions for owner, rental, and job vacancy rates include two lags of these variables, fixed effects, and time dummies. Whether we run these tests in levels or in first differences, the only (Granger) causality runs from owner and renter vacancy rates to job vacancy rates.

The VAR equations are reduced form (there are no contemporaneous variables included as explanatory variables). That is:

$$\Delta \text{ vx}_{i,t} = \alpha_{0,x} + \sum_{j=1,2} \alpha_{1,j,x} \Delta \text{vown}_{i,t-j} + \sum_{j=1,2} \alpha_{2,j,x} \Delta \text{vrent}_{i,t-j}$$

$$+ \sum_{j=1,2} \alpha_{3,j,x} \Delta \text{vjobs}_{i,t-j} + \sum_{j=1,2} \alpha_{4,j,x} \Delta \mathbf{X}_{i,t-j} + u_{t,x} + v_{i,x} + \varepsilon_{it,x}, \qquad (66)$$

where vx = own, rent, job vacancy rates, that is, $o, r, j$, $\mathbf{X}_{i,t-j}$ is a two-vector containing the inverse of the unemployment rate and the house price index. First, we estimate these three equations (66) and get coefficient estimates with two lags included. The results are given in Table 4 below.

After estimating these three equations, we want to look at responses to shocks to vjobs, vown, and vrent. We do so by adding a one standard deviation increase in (values given in Table X) and following the changes in $\Delta$ vx$_{i,t}$, $x = o, r, j$ over time. This produces three sets of impulse response functions (IRFs); with shocks to the first-differences in owner, rental, and job vacancy rates. Note that this means that the ordering of the variables in not necessary. IRFs for the three cases are given below; Figure 7. These are cumulative in the levels of vown, vrent, vjobs. These results reinforce the Granger causality findings. The responses to the owner and rental vacancy rates due to a shock to job vacancies are small and not significantly different from zero. The responses to the owner vacancy rates due to a shock to rental vacancies are small and not significantly different from zero (and vice versa). But the shocks to the owner and rental vacancies do have negative and significant impacts on job vacancies. In the case of the shock to owner vacancies, there is a long-term negative and significant impact of about $-0.04$ in the job vacancies variable. In the case of the shock to rental vacancies, there is a negative and significant impact for the first few periods but the long-term impact of about $-0.15$ is not significant. The RMSE from the VAR equation for job vacancies is 0.27 so the ratio of the long-term impact from the shock to the owner vacancy rate is 0.15, a reasonably large impact.

These results reinforce the thinking on the Great Recession where it was the downturn in the housing market that resulted in the subsequent decline in the real economy.

# 7 Conclusions

This paper explores theoretically and empirically the potential interdependence between the housing and business cycles by analyzing the housing and labor markets by means of a DMP-type model. The model gives rise naturally to vacancy rates in housing and labor markets. We estimate the model using data at the MSA level on housing vacancies from the US Census Bureau's Housing Vacancy Survey (HVS) starting in 1986 and data on job vacancies from the Help-Wanted Index that starts in 1951 and from the Job Opening and Labor Turnover Survey (JOLTS) since December 2000. We also estimate a Beveridge curve for labor markets, while allowing for spillovers from the housing market, which are predicted by our model. We develop a novel Beveridge curve for the housing market, which is based on a housing market counterpart for the concept of unemployment that we propose, namely that of the unfulfilled homeownership rate. Our estimated housing Beveridge curves for the rental and ownership markets confirm our theoretical predictions. Our estimates accounts for interdependence between those markets, and so do a number of VAR-type models for housing and job vacancies. The results from the VAR models can be used to study how shocks to either the housing or labor markets will propagate themselves in the other market. This can help explain the strong relationship between the housing and labor markets during the Great Recession of 2007-2009.

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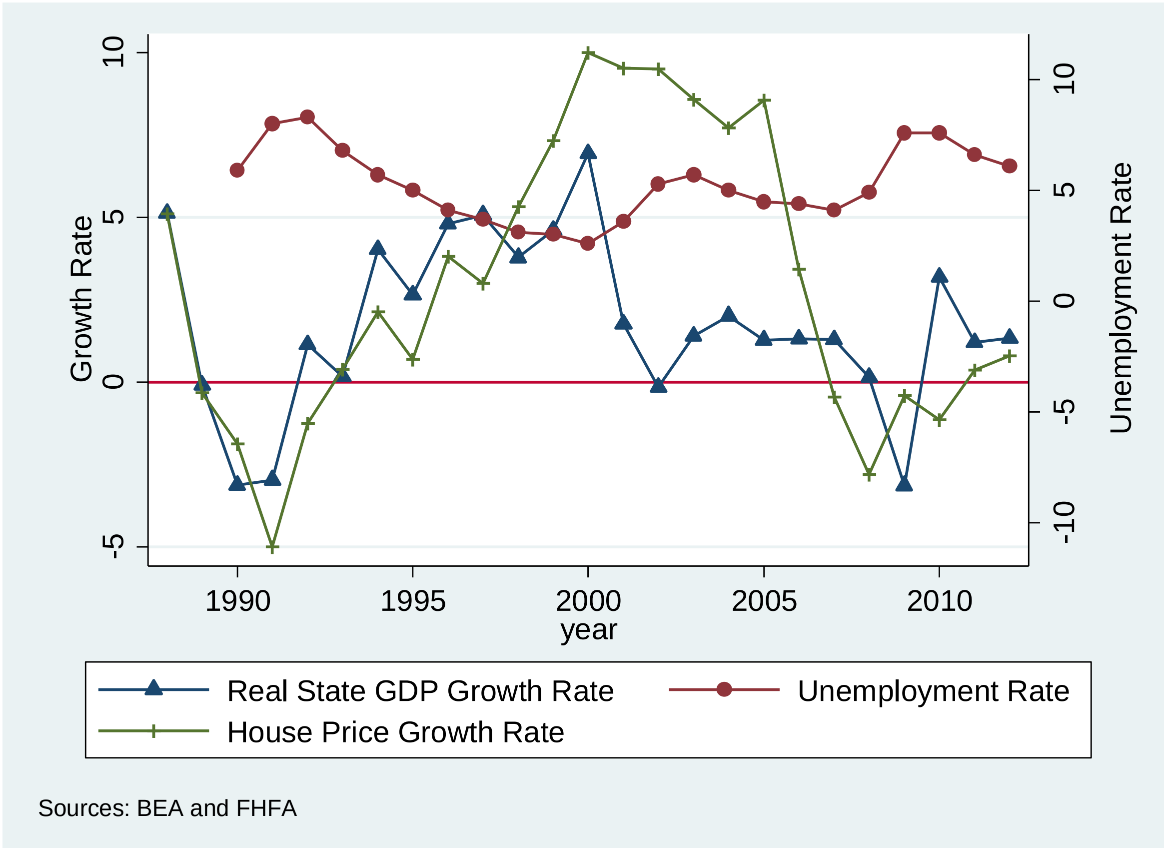
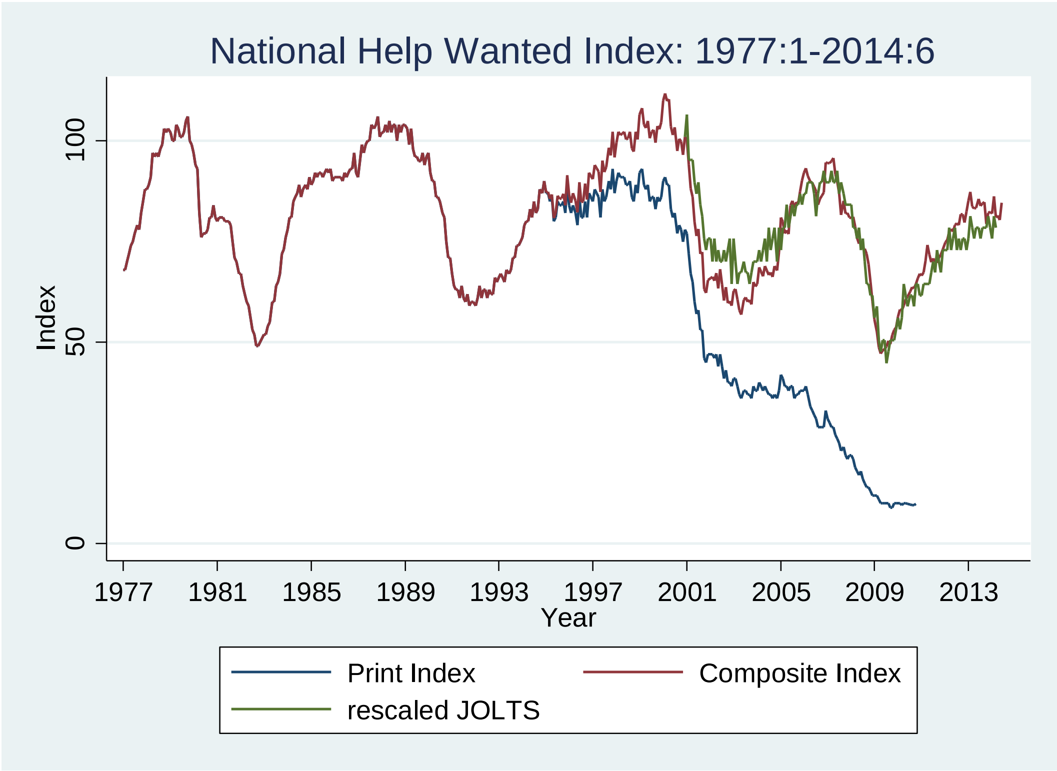
# 9 Figures

**Figure 1**



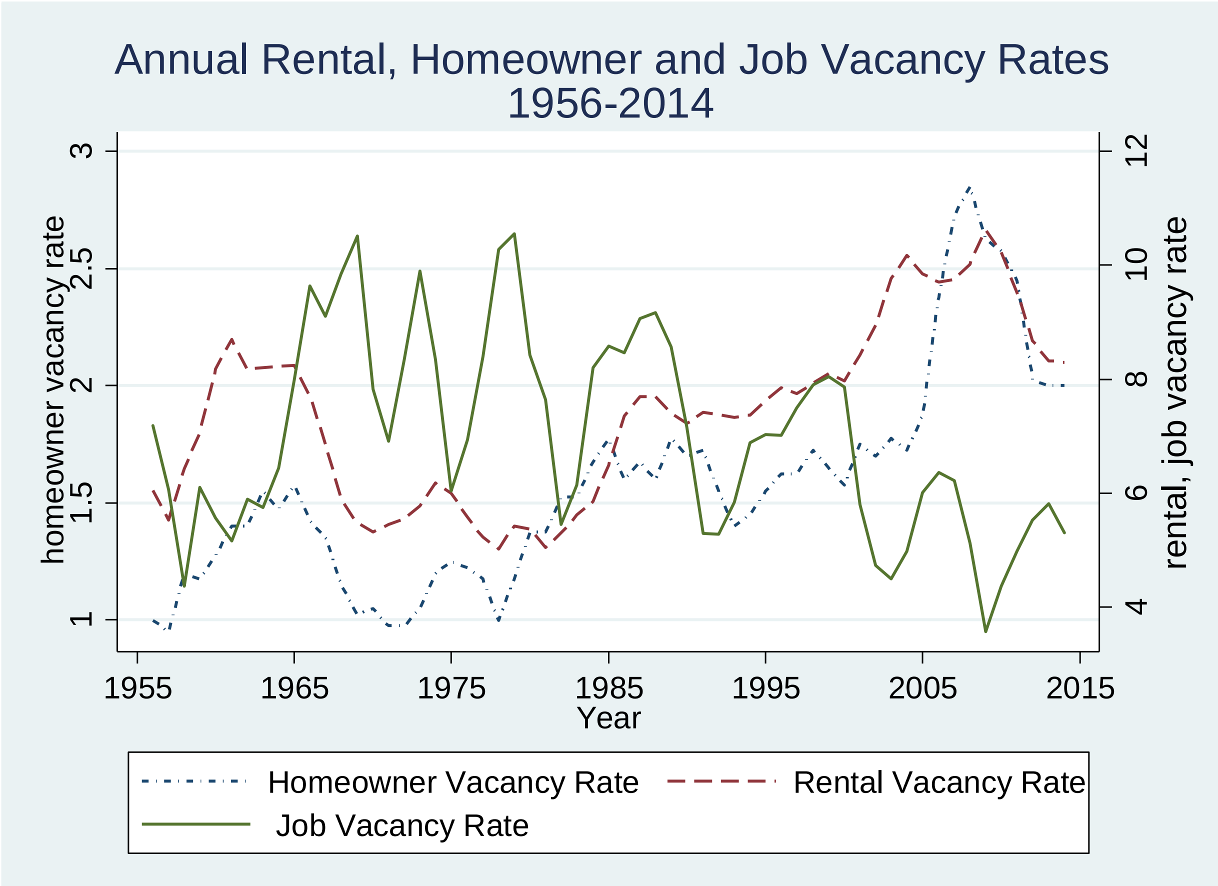
10
5
0
-5
Growth Rate
10
5
0
-5
-10
Unemployment Rate
1990
1995
2000
2005
2010
year
Real State GDP Growth Rate
Unemployment Rate
House Price Growth Rate
Sources: BEA and FHFA

**Figure 2**



National Help Wanted Index: 1977:1-2014:6
Index
0
50
100
1977
1981
1985
1989
1993
1997
2001
2005
2009
2013
Year
Print Index
Composite Index
rescaled JOLTS

**Figure 3**




**Correlations**

| | Rental Vacancy | Home Vacancy | Job Vacancy |
|---|---|---|---|
| Home vacancy | 0.805 | | |
| Job Vacancy | -0.591 | -0.495 | |
| Unemployment Rate | 0.003 | 0.294 | -0.468 |

**Figure 4**

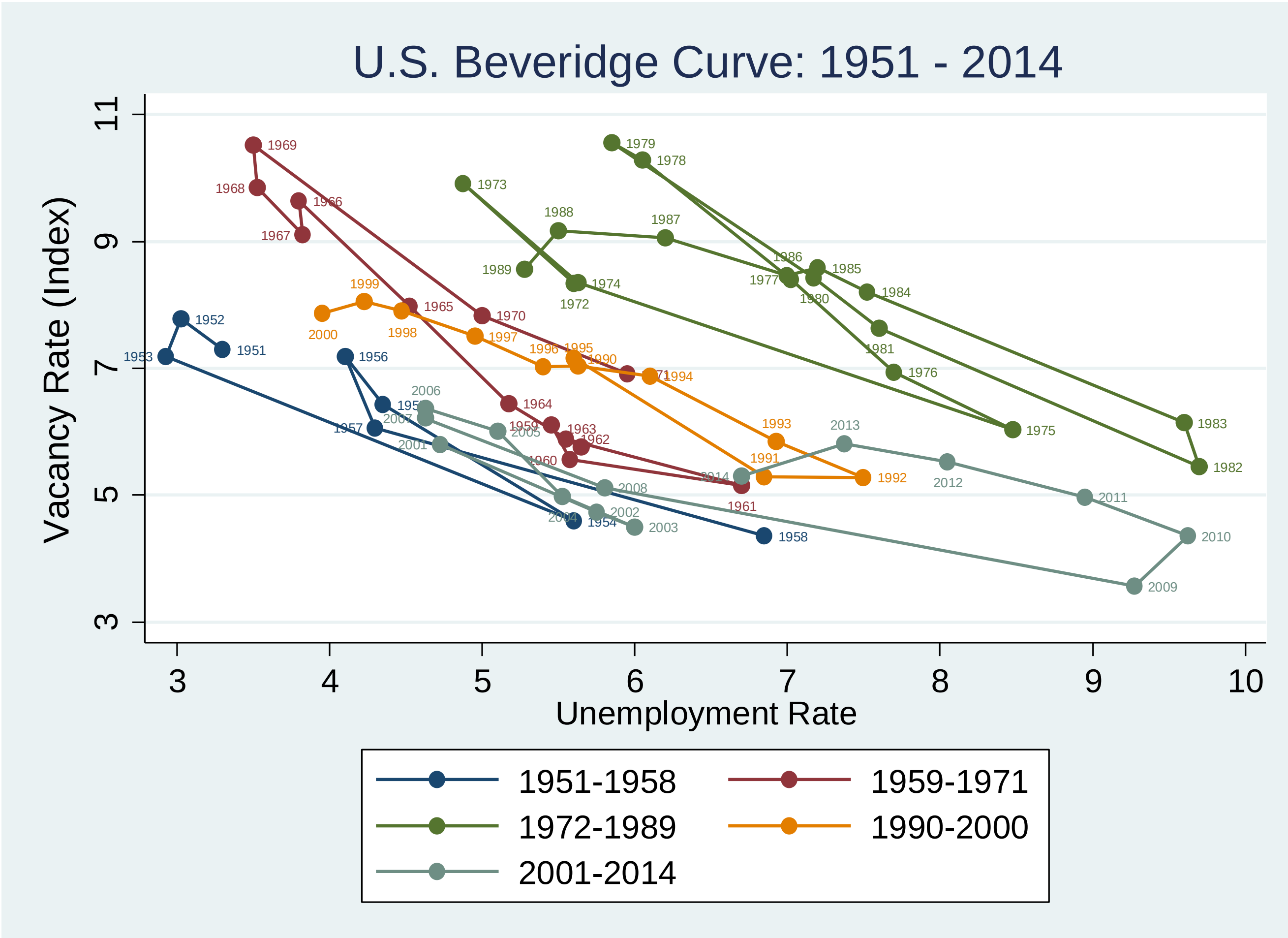

U.S. Beveridge Curve: 1951 - 2014
Vacancy Rate (Index)
Unemployment Rate
1951-1958
1959-1971
1972-1989
1990-2000
2001-2014

**Figure 5**

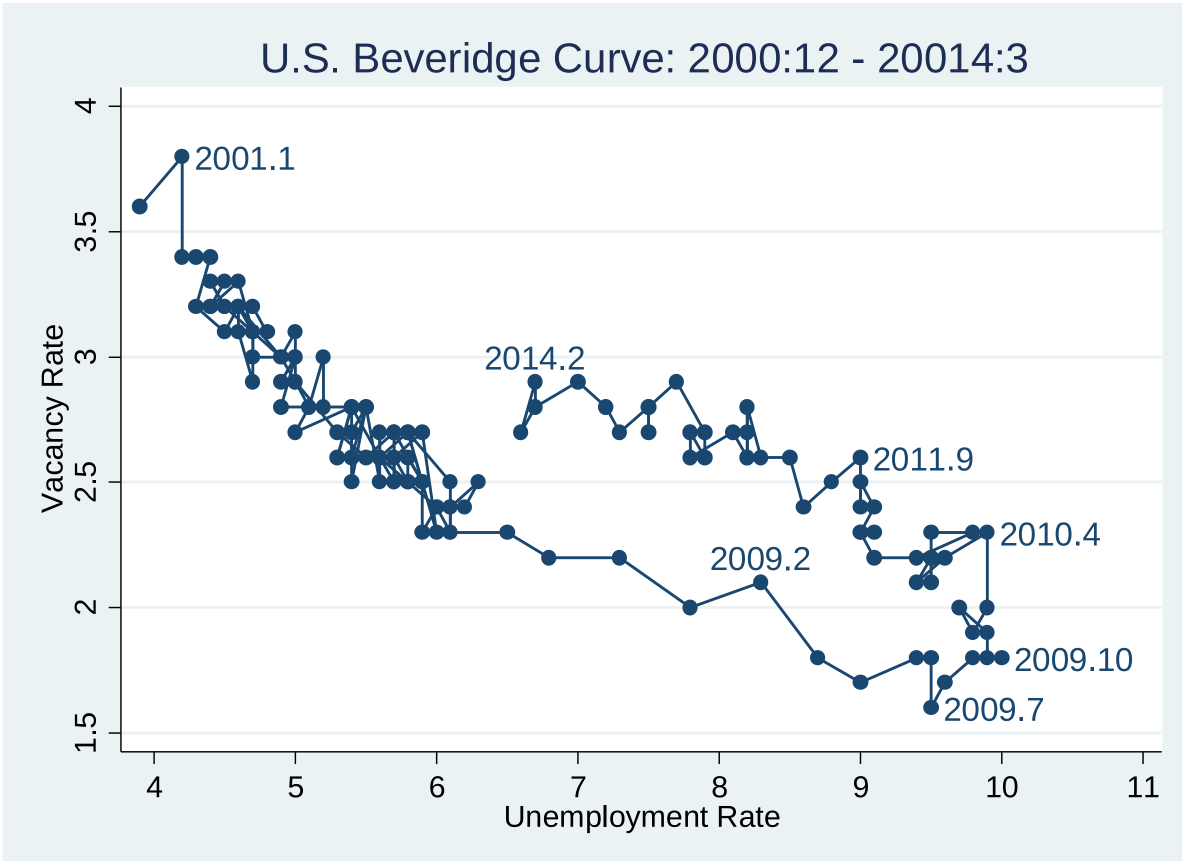
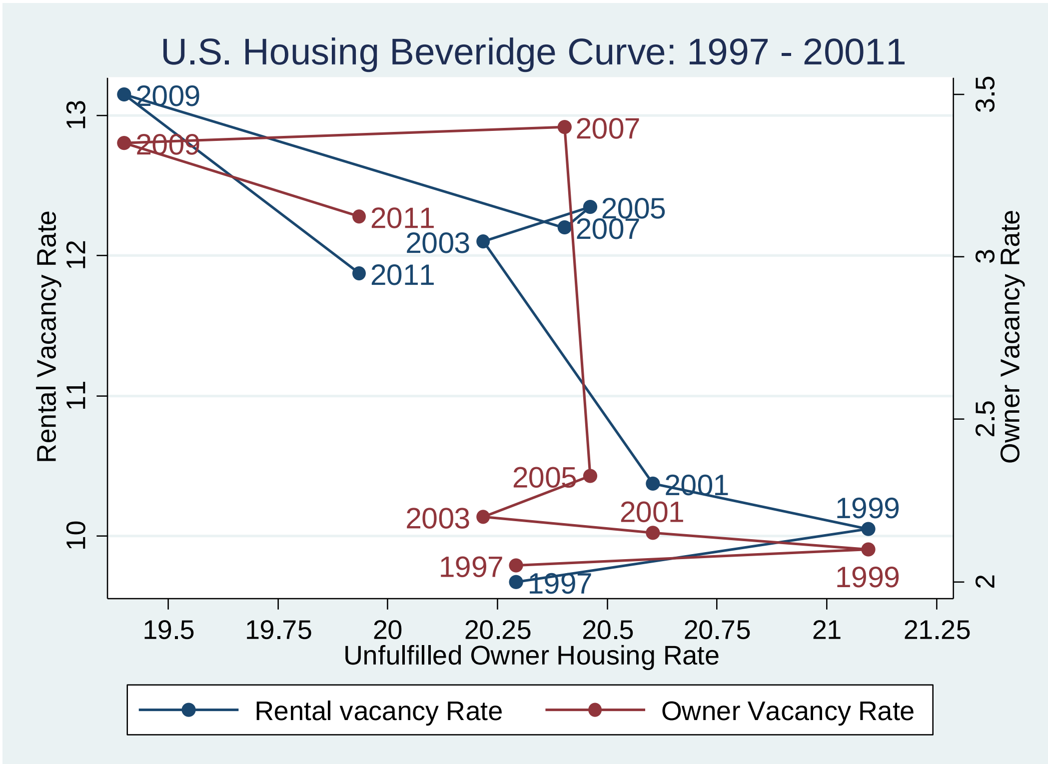
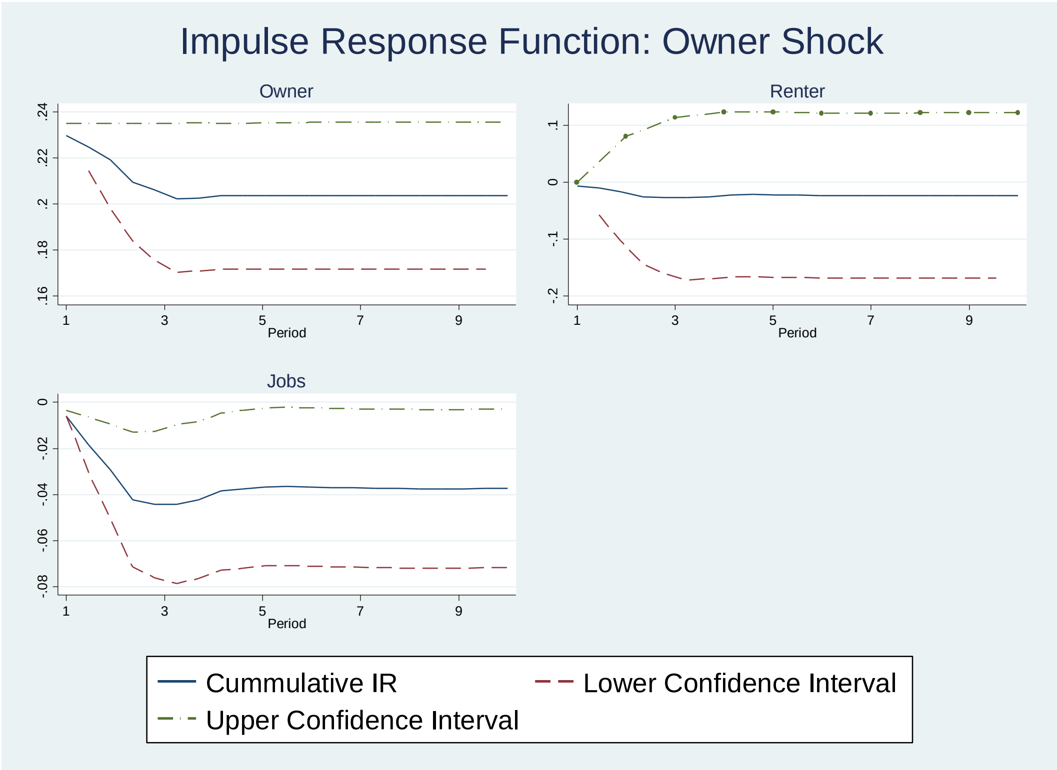

U.S. Beveridge Curve: 2000:12 - 20014:3
Vacancy Rate
Unemployment Rate
1.5
2
2.5
3
3.5
4
4
5
6
7
8
9
10
11
2001.1
2014.2
2011.9
2010.4
2009.2
2009.10
2009.7

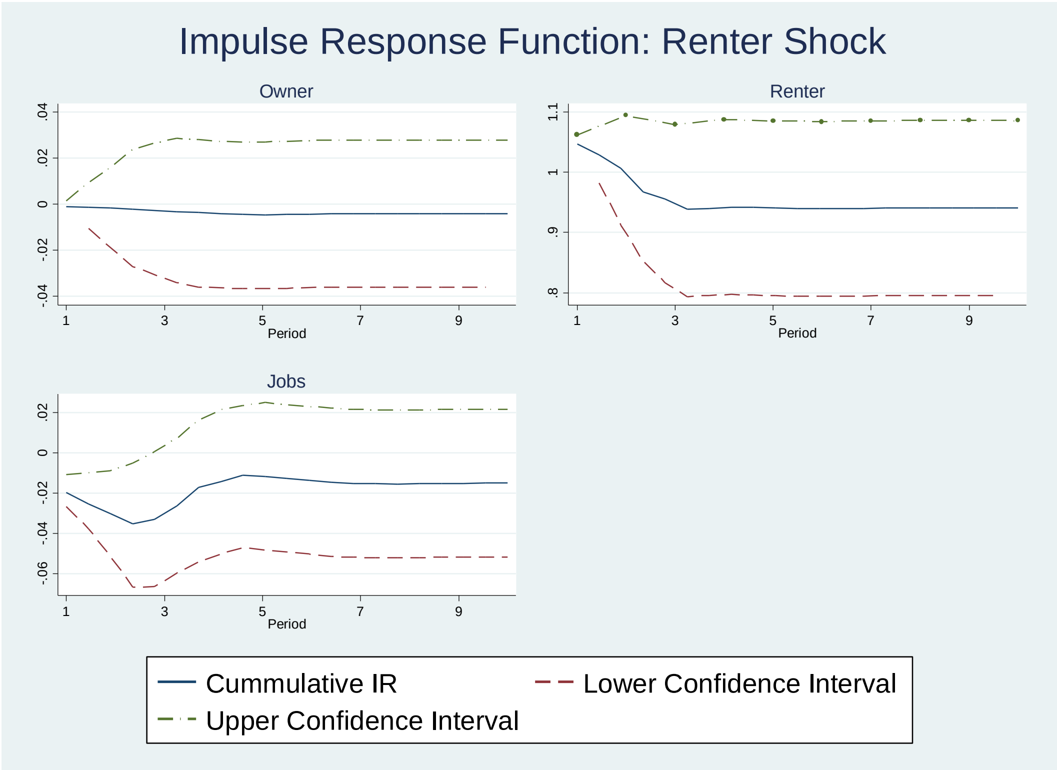
**Figure 6**

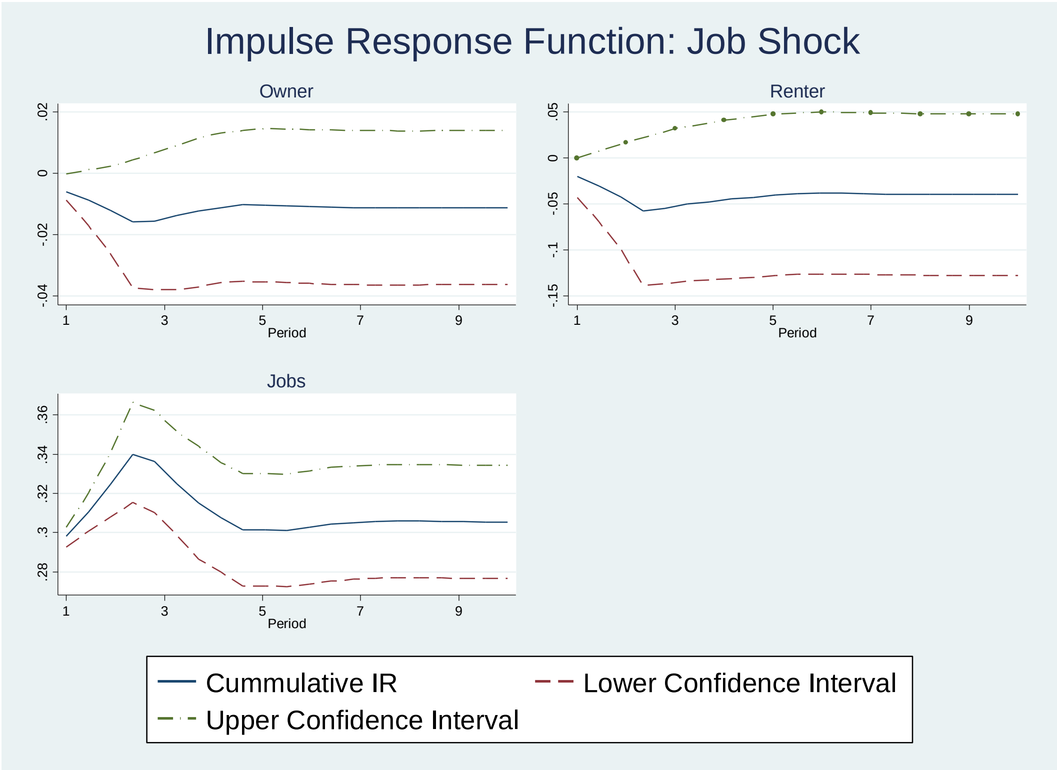


U.S. Housing Beveridge Curve: 1997 - 20011
Rental Vacancy Rate
Owner Vacancy Rate
Unfulfilled Owner Housing Rate
13
12
11
10
3.5
3
2.5
2
19.5
19.75
20
20.25
20.5
20.75
21
21.25
2009
2007
2009
2005
2011
2007
2003
2011
2005
2001
1999
2003
2001
1997
1997
1999
Rental vacancy Rate
Owner Vacancy Rate

## Figure 7: Impulse Response Functions




Impulse Response Function: Job Shock
Owner
Renter
Jobs
Period
Cummulative IR
Lower Confidence Interval
Upper Confidence Interval

# 10 Tables

**Table 1: Summary Statistics**

| Variable | Nobs | Mean | Std. Dev. | Minimum | Maximum |
|---|---|---|---|---|---|
| | | HVS Sample | | | |
| Single Family House Price Index | 1423 | 144.44 | 48.26 | 67.31 | 333.53 |
| Single Family Housing Permits (in hundreds) | 1423 | 85.76 | 90.35 | 0.52 | 615.58 |
| Owner Occupied Vacancy Rate (HVS) | 1492 | 1.67 | 1.00 | 0.10 | 6.30 |
| Rental Vacancy Rate (HVS) | 1492 | 8.56 | 4.21 | 2.50 | 21.10 |
| Employment (1,000s) | 1351 | 1006.96 | 1174.33 | 16.79 | 7737.40 |
| Fair Market Rent | 1423 | 715.03 | 231.19 | 370.16 | 1791.65 |
| Population (1,000s) | 1423 | 2439.26 | 2847.47 | 105.18 | 19069.80 |
| Per Capita Income (1,000s) | 1150 | 34.31 | 8.65 | 15.59 | 80.14 |
| Age Adjusted Ownership Rate | 1423 | 56.32 | 6.97 | 37.75 | 78.30 |
| Unemployment Rate | 1421 | 5.62 | 2.21 | 1.56 | 15.87 |
| Unemployment Compensation (millions) | 1423 | 396.46 | 793.45 | 2.31 | 11456.67 |
| Wages (1,000s) | 1150 | 39.08 | 9.75 | 19.73 | 94.75 |
| | | ACS Sample | | | |
| Single Family House Price Index | 2465 | 181.32 | 37.07 | 105.03 | 362.87 |
| Single Family Housing Permits (in hundreds) | 2465 | 20.19 | 45.64 | 0.10 | 615.58 |
| Owner Occupied Vacancy Rate | 2465 | 2.34 | 1.31 | 0.10 | 11.90 |
| Employment (1,000s) | 2112 | 283.90 | 653.85 | 14.79 | 7737.40 |
| Fair Market Rent | 2465 | 726.51 | 191.21 | 356.17 | 1730.00 |
| Population (1,000s) | 2465 | 716.26 | 1596.95 | 70.26 | 19069.80 |
| Per Capita Income (1,000s) | 2458 | 35.29 | 6.79 | 17.29 | 80.14 |
| Age Adjusted Ownership Rate | 2465 | 61.05 | 6.22 | 38.90 | 78.30 |
| Unemployment Rate | 2463 | 6.82 | 2.99 | 2.07 | 29.67 |
| Unemployment Compensation (millions) | 2458 | 180.48 | 567.83 | 0.59 | 11456.67 |
| Wages (1,000s) | 2458 | 38.91 | 6.81 | 24.44 | 94.75 |
| Wages – Construction (1,000s) | 2443 | 34.81 | 8.79 | 10.48 | 67.48 |
| | | Monthly Composite Help Wanted Index | | | |
| HWI | 13680 | 98.40 | 67.27 | 11.35 | 628.13 |
| | | JOLTS Data | | | |
| Jobs Vacancy Rate – National | 160 | 2.65 | 0.41 | 1.6 | 3.8 |
| Jobs Vacancy Rate – Northeast | 160 | 2.52 | 0.36 | 1.7 | 4 |
| Jobs Vacancy Rate – Midwest | 160 | 2.39 | 0.39 | 1.4 | 3.8 |
| Jobs Vacancy Rate – South | 160 | 2.82 | 0.49 | 1.7 | 3.9 |
| Jobs Vacancy Rate – West | 160 | 2.77 | 0.5 | 1.6 | 4.3 |

**Table 2: Beveridge Curve Results: 1950-2014**
**Dependent Variable is Job Vacancy Rate**

| Variables | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| Unempl Rate$^{-1}$ | 12.864*** | 20.388*** | 24.618*** | 25.900*** |
| | (3.630) | (3.193) | (2.332) | (2.316) |
| Owner Vacancy Rate | | 1.221** | | 0.566 |
| | | (0.587) | | (0.459) |
| Rental Vacancy Rate | | -0.921*** | | -0.379*** |
| | | (0.162) | | (0.140) |
| 1 if 1959-1971 | | | 2.056*** | 1.789*** |
| | | | (0.363) | (0.489) |
| 1 if 1972-1989 | | | 4.352*** | 3.673*** |
| | | | (0.406) | (0.493) |
| 1 if 1990-2000 | | | 2.131*** | 1.891*** |
| | | | (0.393) | (0.525) |
| 1 if 2001-2014 | | | 1.060** | 1.214* |
| | | | (0.403) | (0.650) |
| 1 if 1951 to 1959 | | -7.269*** | | -2.686*** |
| | | (0.914) | | (0.873) |
| Constant | 4.590*** | 8.282*** | 0.188 | 2.173** |
| | (0.696) | (0.889) | (0.647) | (0.952) |
| | Elasticities | | | |
| Unemployment Rate | -0.31 | -0.50 | -0.60 | -0.63 |
| Owner Vacancy Rate | | 0.28 | | 0.13 |
| Rental Vacancy Rate | | -0.98 | | -0.40 |
| Observations | 64 | 64 | 64 | 64 |
| R-squared | 0.168 | 0.610 | 0.805 | 0.836 |

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

**Table 3: Housing Beveridge Curve Results: 1997-2011**

| | Dependent Variable | | | |
|---|---|---|---|---|
| | Owner Vacancy Rate | | Rental Vacancy Rate | |
| Variables | (1) | (2) | (3) | (4) |
| Unfulfilled Ownership | 241.03* | 215.07 | 570.72* | -18.40 |
| | (120.110) | (233.871) | (247.348) | (362.501) |
| Job Vacancy Index | | -0.03 | | -0.58 |
| | | (0.190) | | (0.294) |
| Constant | -9.82 | -8.39 | -18.90 | 13.50 |
| | (5.921) | (12.458) | (12.194) | (19.310) |
| | Elasticities | | | |
| Unfulfilled Ownership | -5.76 | -5.14 | -3.05 | 0.10 |
| Job Vacancy Index | | -0.9 | | -0.44 |
| Observations | 8 | 8 | 8 | 8 |
| R-squared | 0.402 | 0.404 | 0.470 | 0.701 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 4 - VAR Regression Results

| | Dependent Variable: Vacancy Rates (if First-Differences) | | |
|---|---|---|---|
| | Ownership | Rental | Jobs |
| Vacancies in First Diff | | | |
| $\text{Ownership}_{t-1}$ | -0.077 | -0.090 | -0.167 |
| | (0.048) | (0.219) | (0.055)** |
| $\text{Ownership}_{t-2}$ | -0.077 | -0.093 | 0.002 |
| | (0.048) | (0.220) | (0.056) |
| $\text{Rental}_{t-1}$ | -0.002 | -0.066 | -0.049 |
| | (0.010) | (0.048) | (0.012)** |
| $\text{Rental}_{t-2}$ | -0.004 | -0.069 | 0.034 |
| | (0.011) | (0.048) | (0.012)** |
| $\text{Jobs}_{t-1}$ | -0.067 | -0.236 | 0.323 |
| | (0.033)* | (0.149) | (0.038)** |
| $\text{Jobs}_{t-2}$ | 0.026 | 0.103 | -0.210 |
| | (0.031) | (0.143) | (0.036)** |
| Other Variables in First-Diff | | | |
| $\text{Unpl}^{-1}_{t-1}$ | -0.661 | -2.080 | 0.351 |
| | (0.459) | (2.091) | (0.528) |
| $\text{Unpl}^{-1}_{t-2}$ | 0.019 | 0.140 | -0.244 |
| | (0.443) | (2.019) | (0.510) |
| $\text{House Price Index}_{t-1}$ | -0.002 | -0.012 | -0.006 |
| | (0.001) | (0.006)* | (0.002)** |
| $\text{House Price Index}_{t-2}$ | 0.000 | 0.001 | 0.004 |
| | (0.001) | (0.006) | (0.001)** |
| _cons | 0.002 | 0.026 | 0.053 |
| | (0.044) | (0.198) | (0.050) |
| $\hat{\sigma}_{\varepsilon}$ | 0.2332 | 1.0622 | 0.2685 |
| $R^2$ | 0.16 | 0.37 | 0.41 |
| $N$ | 740 | 740 | 740 |

Standard Errors are in Parentheses. Regressions include time dummies and CBSA fixed effects

* $p$<0.05; ** $p$<0.01

# 11 Appendix: Generating the Composite Help Wanted Index

The method we use to construct the full National help wanted index up through 2014 is similar to Barnichon (2010) but not as complicated. It consists of the following 4 steps.

Step 1. 1951-1994: online help-wanted (HWOL) index, $O_t$ does not exist, $O_t = 0$. As in Barnichon, we use the HWI print index through 1994; $H_t = P_t$ where $H_t$ and $P_t$ are the composite and print Help-Wanted advertising indices, respectively.

Step 2. 1995-2005:5: $O_t > 0$, but not observed. Step 2 is also the same as in Barnichon. To get the composite index, we inflate the print index by the estimated print share: $H_t = P_t/\hat{s}_t^p$. But the procedure wee use to estimate is the simpler version in Barnichon. That is, we fit a quartic polynomial to $P_t$ over 1951-2010:10 (the last month that we have the National HWI print index), and estimate as the ratio of the polynomial's value at time $t$ to the polynomial's value in 1994:12. Figure A1 below reproduces Figure 2 in Barnichon. One can see that the print share based on the polynomial trend fits the Sp-JOLTs print share very well. What is key here is that, unlike in Barnichon, the Sp-JOLTs print share DOES exhibit a constant rate of decline and does NOT appear to follow an $S-$curve. Hence, we use the polynomial trend in the above calculation to estimate and not the more complicated method used by Barnichon.

Step 3. 2005:6-2010:10: both $O_t$ and $P_t$ are observed. Same as in Barnichon: $H_t$ is constructed from

$$\frac{dH_t}{H_{t-1}} = s_{t-1}^p \frac{dH_t}{P_{t-1}} + (1 - s_{t-1}^p)\frac{dO_t}{O_{t-1}}$$

where $O_t$ is the online help-wanted advertising index.

What we have from the online data is the total number of ads (seasonally adjusted and not seasonally adjusted) and the total number of new ads (seasonally adjusted and not seasonally adjusted). We use the seasonally adjusted total number of ads to construct $O_t$.

Step 4. 2010:11-2014:6: Only $O_t$ is observed. We construct $H_t$ from from $d\ln H_t = d\ln O_t$. That is, we assume that $s_t^p = 0$ starting in 2010:11 (the estimated value from the polynomial trend is 0.008). The composite index, the print index and the rescaled JOLTS index are plotted on Figure 5.

We can use the same procedure at the MSA level. The one complication is that the last date that the print index is observed varies across MSAs and can be less than 2010.10; the earliest date for this is June 2005. Between this date and 2010:11 (call this Step 3.1), we use the inflated value of $O_t$ to construct $H_t$ from $d\ln H_t = \ln \frac{O_t}{1-\hat{s}_t^p} - \ln \frac{O_{t-1}}{1-\hat{s}_{t-1}^p}$.