The Cyclical Volatility of Labor Markets under Frictional Financial Markets

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Abstract

In an economy with search on credit and labor markets, a financial multiplier raises the elasticity of labor market tightness to productivity shocks. This multiplier increases with total financial costs. We show that the financial costs are minimized under a credit market Hosios-Pissarides rule. Using a flexible calibration method based on small perturbations (a trembling-hand calibration method), we find the parameter values to get the US share of the financial sector. Those values are far away from Hosios and lead to a financial accelerator of about 3 (exogenous wages) to 5.5 (endogenous wages). Both match Shimer (2005)’s elasticity of labor market tightness to productivity shocks. Financial frictions are thus an alternative to the “small labor surplus” assumption in Hagedorn and Manovskii (2008): we keep the value of wages over productivity below 0.77. We conclude that financial frictions are a good candidate to solve the volatility puzzle and rejoin Pissarides (2009) in arguing that hiring costs must be partly non-proportional to congestion in the labor market, which is the case of financial costs.

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1 Introduction

Cole and Rogerson (1999) and Shimer (2005) have investigated the cyclical properties of the search matching models following Pissarides (1985) and Mortensen and Pissarides (1994). The celebrated Shimer’s puzzle is the demonstration of the inability of the conventional matching model to replicate the US statistics regarding the volatility of job vacancies, unemployment and their ratio (called labor market tightness), in response to productivity shocks. Shimer’s main finding is that the elasticity of labor market tightness to productivity shocks is around 20 in the data, and around 1 in a calibration of the Mortensen-Pissarides model. Several calibration improvements have been proposed. One of them, called the “small labor surplus” assumption, implies that the calibrated value of non-employment utility (Hagedorn and Manovskii 2008) becomes closer to market productivity, with only a few percentage points differences and very low values for the bargaining power of workers. This leads firms to also face a small surplus, by a few percents, after bargaining over the surplus. Firms are therefore more fragile to productivity shocks, leading the market to be overall more volatile. Other promising roads have been proposed, such as wage rigidity (Hall 2005) and on-the-job search (Mortensen and Nagypáł 2007).

One line of research that has so far been ignored but seems promising is the existence of credit market imperfections. In this paper we pursue this logic, following two previous papers. On the one hand, Petrosky-Nadeau (2009) shows that introducing credit market imperfections, with in particular costly state verification, in a search model can lead to a large amplification of the volatility of labor market tightness. The standard deviation in his model of the vacancy-unemployment ratio approaches 12.5 relative to that of output, while it is 15.4 in US data and merely 3.7 in the standard search model. Credit market imperfections induce an amplification factor of 3.5. On the other hand, Wasmer and Weil (2004), who develop financial imperfections in a Mortensen-Pissarides economy with two matching functions (one in the labor market, one in the credit market), show that the steady-state volatility of labor market tightness to profit shocks is augmented by a factor 1.7 by the existence of moderate credit market imperfections. They call this a financial accelerator, in line with an earlier literature.

Despite recent papers attempting to bring together credit market imperfections and the search-matching approach, the macro-labor literature has been slow to incorporate the well-known message of an earlier literature. Indeed, it has been known for a while that credit market imperfections generate additional volatility of the business cycle. Early papers such as Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and subsequent papers (such as Bernanke and Gertler 1995, Bernanke Gertler and
Gilchrist 1996, and several others), have emphasized the amplification role of credit markets and the existence of a *financial accelerator*. Although part of this literature is centered on the role of credit shocks and the credit channel of monetary policy, the ingredients generating the amplification of credit shocks can very well be adapted to the amplification of business cycle shocks to labor markets.

Firms in our model arise from the result of the meeting of an entrepreneur and a banker on a frictional credit market. The average cost of creating a firm is the sum of all prospecting costs on the credit market which, compared to the world with perfect credit markets in Mortensen and Pissarides (1994), imposes a lower limit on the value of a job vacancy to a firm.

Our results regarding the amplification of productivity shocks in this double matching economy can be summarized as follows. Consistent with Wasmer and Weil (2004), financial imperfections raise the calibrated elasticity of labor market tightness to productivity shocks by a factor $M_f$ called the financial multiplier, which is an increasing function of total financial costs in the economy. This paper brings in addition several new results.

**First**, a Hosios-Pissarides rule exists in the credit market: the bargaining power of firms vis-à-vis banks is equal, at the social optimum, to the elasticity of the finding rate of banks with respect to credit market tightness. Under the Hosios rule, the search costs in the credit market are minimized, and so is $M_f$. Relaxing that condition leads to a larger financial accelerator, which can match or even overshoot the elasticity of market tightness in the data.

**Second**, using a flexible calibration method based on small perturbations (a trembling-hand calibration method), we find the parameter values to get the US share of the financial sector (3.3%) and the right elasticity of labor market tightness to productivity shocks (22 in the data). Those values are generally far away from Hosios. With exogenous wages, we find that the share of banks is 0.86 and an elasticity of the finding rate of banks with respect to credit market tightness of 0.51 and lead to a financial accelerator of about 3.64.

**Third**, with endogenous wages, the same remains true: we find that the share of banks is 0.92 and an elasticity of the finding rate of banks with respect to credit market tightness of 0.58 and lead to a financial accelerator of about 5.6.

**Fourth**, this result is obtained in keeping the share of wages to be around 0.76 of productivity, thus quite far away from the “small labor surplus” assumption in Hagedorn and Manovskii (2008). Financial frictions are thus an alternative to be taken seriously. However, as Mortensen and Nagypal (2007) point out, this assumption implies that there is very little utility gain to accepting a job, nor does it fit well
with estimates of the value of non-employment. Financial imperfections in our model enable us to relax the "small labor surplus" assumption in order to match the elasticity of market tightness to productivity found in the data.

This paper is organized as follows. In Section 2, we summarize the main equations in Wasmer and Weil (2004), calculate the volatility of labor market tightness to productivity shocks and show how the Hosios rule in the credit market affects the volatility of the labor market. In Section 3, we describe the stochastic extension of the model, with both endogenous and exogenous wages. In Section 4, we describe our calibration method. In Section 5, we derive our main results: deviating away from Hosios substantially raises the elasticity with exogenous wages. The calibration with endogenous wages show similar results. In addition, the “small labor surplus” assumption does not need to be maintained. In Section 6, we conclude that financial frictions are a good candidate to solve the volatility puzzle and rejoin Pissarides (2009) in arguing that hiring costs must be partly non-proportional to congestion in the labor market, which is the case of financial costs.

2 Hosios-Pissarides in a continuous time economy with credit and labor market frictions and the elasticity of labor market tightness to shocks

2.1 Model

Time is continuous and there are three types of agents: entrepreneurs with no capital; banks with no ability to produce; and workers with no capital and no ability to start a business. The timing of events for entrepreneurs is as that they initially need to find a "banker" in order to start a business. This search process costs $e$ units of effort per unit of time. Search is successful with probability $p$. The newly formed firm, from the successful meeting of entrepreneur and banker, then goes to the labor market. The bank finances the vacancy posting cost $\gamma$ to attract workers (the so-called recruitment costs) for the firm. This search process succeeds with probability $q$. The firm is then able to produce and sell in the good market, which generates a flow profit $y - w - \rho$ where $y$ is the marginal product, $w$ is the wage, $r$ is the flow rate of discount, and $\rho$ is the flow repayment to the bank (determined through bargaining). Jobs are subject to destruction shocks with Poisson parameter $s$. The steady-state asset values of the entrepreneurs are denoted by $E_j$ with $j = c, l$ or $g$ the market in which the entrepreneur is operating, standing respectively for the credit, labor and good markets. We also assume free entry at the first stage,
that is $E_c \equiv 0$. We therefore have the following Bellman equations:

\[
\begin{align*}
    rE_c &= 0 = -e + pE_l & (1) \\
    rE_l &= 0 + q(E_g - E_l) & (2) \\
    rE_g &= y - w - \rho + s(0 - E). & (3)
\end{align*}
\]

In the last line, it was assumed that job destruction also leads to the destruction of the firm and the lending relation with the bank.

Symmetrically, the bank’s asset values are denoted by $B_j$, $j = c, l$ or $g$ for each of the stages. We also assume free entry of the banking relationship: $B_c = 0$. We denote by $k$ the screening cost per unit of time of banks in the first stage, and by $\hat{p}$ the Poisson rate at which a bank finds a firm to be financed. We have:

\[
\begin{align*}
    rB_c &= 0 = -k + \hat{p}B_l & (4) \\
    rB_l &= -\gamma + q(B_g - B_l) & (5) \\
    rB_g &= \rho + s(0 - B_g). & (6)
\end{align*}
\]

The matching rates $p$ and $\hat{p}$ are made mutually consistent by the existence of a matching function $M_c(\mathcal{B}, \mathcal{E})$, where $\mathcal{B}$ and $\mathcal{E}$ are respectively the number of bankers and of entrepreneurs in stage $c$. This function is assumed to have constant returns to scale. Hence, denoting by $\phi$ the ratio $\mathcal{E}/\mathcal{B}$, which is a reflection of the tension in the credit market and that we shall call credit market tightness from the point of view of entrepreneurs, we have

\[
\begin{align*}
    p &= \frac{M_c(\mathcal{B}, \mathcal{E})}{\mathcal{E}} = p(\phi) \text{ with } p'(\phi) < 0. \\
    \hat{p} &= \phi p(\phi) \text{ with } \hat{p}'(\phi) > 0.
\end{align*}
\]

After the contact, the bank and the entrepreneur engage in bargaining about $\rho$ which is such that

\[
(1 - \beta)B_l = \beta E_l \quad (7)
\]

where $\beta$ is the bargaining power of the bank relative to the entrepreneur. With $\beta = 0$ the bank leaves all the surplus to the entrepreneur.

Combining (1), (4) and (7), we obtain the equilibrium value of $\phi$ denoted by $\phi^*$ with

\[
\phi^* = \frac{k}{e} \frac{1 - \beta}{\beta}.
\]
Matching in the labor market is denoted by $M_l(\mathcal{V}, u)$ where $u$ is the rate of unemployment and the total number of unemployed workers since the labor force is normalized to 1. $\mathcal{V}$ is the number of "vacancies", that is the number of firms in stage $l$. The function is also assumed to be constant return to scale, hence the rate at which firms fill vacancies is a function of the ratio $\mathcal{V}/u$, that is tightness of the labor market.

We have

$$q(\theta) = \frac{M_l(\mathcal{V}, u)}{\mathcal{V}} \text{ with } q'(\theta) < 0.$$  

Further using (2), (3) and (5), (6), we finally simultaneously solve for $\rho$:

$$\frac{\rho}{r + s} = \beta \frac{y - w}{r + s} + (1 - \beta) \frac{\gamma}{q(\theta)}$$

and obtain the two main equations of the model:

$$(EE) : \frac{e}{p(\phi^*)} = \frac{q(\theta)}{r + q(\theta)} \left( \frac{y - w}{r + s} - \frac{\gamma}{q(\theta)} \right) (1 - \beta)$$

$$(BB) : \frac{k}{\phi^* p(\phi^*)} = \frac{q(\theta)}{r + q(\theta)} \left( \frac{y - w}{r + s} - \frac{\gamma}{q(\theta)} \right) \beta$$

Each equation provides a link between $\theta$ and $\phi$ that is of opposite sign. There is therefore at most one equilibrium set of $(\theta^*, \phi^*)$.\(^1\) Finally, summing up (EE) and (BB), one obtains a single market equation denoted by (CC) for $\theta^*$ describing a job creation condition for this double matching economy:

$$(CC): \frac{e}{p(\phi^*)} + \frac{k}{\phi^* p(\phi^*)} = \frac{q(\theta)}{r + q(\theta)} \left( \frac{y - w}{r + s} - \frac{\gamma}{q(\theta)} \right)$$

where the left-hand side is a measure of the total amount of search costs in financial markets. These are the total financial costs associated with the creation of a firm and that we shall denote by

$$K \equiv \frac{e}{p(\phi^*)} + \frac{k}{\phi^* p(\phi^*)} \quad (11)$$

\[2.2\] Steady-state volatility of $\theta$ to shocks

For the moment, to keep the analysis simple, we fix wages at some exogenous value. Endogenous wages are introduced only in the stochastic extensions, in next Section. We now want to calculate the elasticity of $\theta$ to profit shocks, denoted by $\Lambda_{\theta/\pi}$. Let $\theta^p$ be the value of tightness solving for

$$\frac{y - w}{r + s} = \frac{\gamma}{q(\theta^p)}$$

\(^1\)Wasmer and Weil (2004) provide a condition for existence.
The value of $\theta^P$ defined here is the credit frictionless world in Pissarides (1985), which one would obtain from (10) when $K = 0$. In using (CC), one has:

$$\frac{\gamma}{q(\theta^P)} - \frac{\gamma}{q(\theta^*)} = K \left( \frac{r + q(\theta^*)}{q(\theta^*)} \right) > 0$$

Hence, given that $q'$ is downward sloping, we have that $\theta^* < \theta^P$, as was shown in Wasmer and Weil (2004) and arises in Petrosky-Nadeau (2009), and the difference is precisely due to the existence of search costs in the credit market. Posing $r = 0$ to marginally simplify the analysis, we have an equilibrium job creation condition under frictional credit markets which states that the profit flows from a job net of the total financial costs to creating a firm must equal the average cost of filling a job vacancy:

$$\frac{y-w}{r+s} = K + \frac{\gamma}{q(\theta^*)}.$$  (13)

What the model does is to add a new component in entry costs for firm that, in the special case $r = 0$, is independent of labor market tightness. As we will see, this will rise the volatility of the economy, an insight already brought by Pissarides (2009).²

Let $\pi = (y-w)/(r+s)$ be the present discounted value of profits. Taking logs and differentiating, we have

$$-q'(\theta^*)/q(\theta^*) d\theta = d\ln \pi = \frac{\pi}{\pi - K}$$

or, reusing (12) and (13) and where $\eta = -q'(\theta)/q(\theta)$ is the (non-necessarily constant) elasticity of $q$ to $\theta$, we have

$$\Lambda_{\theta/\pi} = \frac{1}{\eta} \frac{\gamma}{q(\theta^*)} = \frac{1}{\eta} \frac{q(\theta^*)}{q(\theta^P)}.$$

Two remarks are in order. First, in the (credit) frictionless world in Pissarides, the elasticity is simply the inverse of the elasticity of $q$ to $\theta$, that is $1/\eta$. Second, the existence of credit market imperfections reduces $\theta^*$ relative to $\theta^P$, and therefore raise the volatility $\Lambda_{\theta/\pi}$ by a factor due to the financial accelerator identified in Wasmer and Weil (2004): higher profits raise the entry of firms, hence banks make faster profits, which in turn benefits firms, and so on. Denote by

$$M_f = \frac{q(\theta^*)}{q(\theta^P)}$$

²A stated in Pissarides (2009, page 1341) : “(...) a simple remodeling of the [matching] costs from proportional to partly fixed and partly proportional can increase the volatility of tightness and job finding, virtually matching the observed magnitudes, without violating wage flexibility.” We thank Etienne Lalé for bringing attention to this point. What we will do here is to give an interpretation to this fixed part.
the value of the financial accelerator, which can more generically be defined as the ratio of the elasticity in a world with credit frictions and the elasticity in a world where credit frictions disappear.

Under the assumption of an exogenous wage, the response of this economy to productivity shocks on \( y \) is therefore:

\[
\Lambda_{\theta/y} = \frac{d \ln \theta}{d \ln y} = \frac{d \ln \theta}{d \ln \pi} \frac{d \ln \pi}{d \ln y} = \frac{1}{\eta} \frac{y}{y - w} M_f
\]

The first component of this elasticity is the amplification due to the existence of search frictions on the labor market. The second component is the gap between wages and marginal product - the smaller the gap, the more responsive job creation is to productivity shocks; and finally, the third is the financial accelerator.

With the parameters values in Wasmer and Weil (2004), \( \eta = 0.5 \), \( y = 1 \), \( w = 2/3 \), and \( M_f = 1.74 \), such that

\[
\Lambda_{\theta/y} = 2 \times 3 \times 1.74 = 10.44.
\]

This is a large factor compared to the conventional Pissarides model elasticity as in Shimer (2005), for example, who found a much smaller number value of 1.13. This difference is due to three factors:

1. the choice of the matching elasticity in Shimer (1/0.72) ; assuming \( \eta = 0.5 \) instead raises the elasticity with respect to Shimer by a factor \( 2 \times 0.72 = 1.44 \).

2. the assumption of wage rigidity in our model (see Hall 2005): in the absence of rigidity in wages, the factor \( \frac{y}{y - w} = 3 \) would have to be replaced by a more complex term, derived and discussed later on in the part devoted to endogenous wages. In short, wage rigidity raise volatility by a factor of 4 to 5.

3. The last part of the difference is due to the existence of a financial accelerator \( M_f = 1.74 \), consistent with the literature initiated by Bernanke and Gertler (1989).

The labor literature has attempted to raise the elasticity of market tightness to productivity with either wage rigidities (Hall 2005) or by making what we will call hereafter the "small labor surplus" assumption by choosing higher values of non-employment utility and lower values for the bargaining power of workers (Hagedorn and Manovskii 2008) and reducing the gap between wages and marginal product. While acknowledging the interest of these approaches, we pursue another avenue here and attempt to understand the determinants of \( M_f \).
2.3 Hosios-Pissarides in the credit market and the entry costs for firms

We start by noting that frictions in the credit market may lead to a second best efficiency condition similar to that in Hosios (1990) and Pissarides (1990).

2.3.1 The efficiency of financial markets in a search-economy

To see this, we can calculate the social welfare function as output net of all search costs. We have:

\[ \Omega = y(1-u) + zu - \gamma \theta u - k \mathcal{B} - e \mathcal{E} \]

where \( z \) is the value of non-employment utility and \( \theta u = \mathcal{V} \) is the number of firms prospecting in the labor market. To obtain a simpler expression for \( \Omega \), we can note that in a steady-state, we have \( \mathcal{E} p(\phi) = q(\theta) \mathcal{V} \) which states that inflows into the financing stage are compensated by outflows out of that stage. It follows that

\[ \mathcal{E} = \frac{q(\theta) \theta u}{p(\phi)} \quad \text{and} \quad \mathcal{B} = \frac{\mathcal{E}}{\phi} = \frac{q(\theta) \theta u}{\phi p(\phi)} \]

Therefore, the social planner’s program can be rewritten as

\[
\begin{align*}
\max_{u, \theta, \phi} \Omega &= y(1-u) + zu - \gamma \theta u - \left( \frac{k}{\phi p(\phi)} + \frac{e}{p(\phi)} \right) q(\theta) \theta u \\
\text{s.t.} \quad u &= s/(s + \theta q(\theta))
\end{align*}
\]

Relative to the choice of the optimal \( \phi \) denoted by \( \phi^{opt} \), the problem is simple and block-recursive in \( \phi \) and then in \( u \) and \( \theta \). For the first block that we only consider here, the optimal choice of \( \phi \) amounts to minimizing total search costs \( K(\phi) = \frac{k}{\phi p(\phi)} + \frac{e}{p(\phi)} \):

\[
\frac{\partial \Omega}{\partial \phi} = q(\theta) \theta u \frac{\partial}{\partial \phi} K(\phi) = 0
\]

\[ \iff \phi^{opt} = \frac{1 - \varepsilon k}{\varepsilon} \quad \text{where} \quad \varepsilon = -\frac{\phi p'(\phi)}{p(\phi)} \]

Hence, since \( \frac{\partial^2}{\partial \phi^2} K(\phi) > 0 \), the socially optimal value of credit market tightness is the one that minimizes search costs on credit markets. The Hosios-Pissarides rule, which states that there is a value of the

\[
\frac{\partial \Omega}{\partial \phi} = 0 \iff \frac{k}{\phi p(\phi)} \frac{\phi p'(\phi) + p(\phi)}{\phi p(\phi)} + \frac{e}{p(\phi)} p'(\phi) = 0
\]

\[ \iff \frac{k}{\phi p(\phi)} (1 - \varepsilon) = \frac{e}{p(\phi)} \varepsilon \]

\[ \text{Intermediate steps are:} \]

\[
\frac{\partial \Omega}{\partial \phi} = 0 \iff \frac{k}{\phi p(\phi)} \frac{\phi p'(\phi) + p(\phi)}{\phi p(\phi)} + \frac{e}{p(\phi)} p'(\phi) = 0
\]

\[ \iff \frac{k}{\phi p(\phi)} (1 - \varepsilon) = \frac{e}{p(\phi)} \varepsilon \]

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bargaining parameter over $\rho$ that internalizes the matching externalities due to the search frictions, applies here:

$$\phi^* = \phi^{opt}$$

$\iff \beta = \varepsilon$ : Hosios condition in the credit market

### 2.3.2 Minimizing the financial costs and the gap between $\theta^*$ and $\theta^P$

One may think that the Hosios condition is the one that minimizes entry costs in the credit market. One can check this formally. The left-hand side of job creation condition (CC) is a function of $\beta$ and $\varepsilon$ denoted by $K(\beta, \varepsilon)$; the right-hand side is increasing in $\theta$. It is therefore enough to show that $K(\beta, \varepsilon)$ is minimized in $\beta = \varepsilon$. Before doing so, we can use two intermediate steps. First, note that $K(\beta, \varepsilon) = \frac{\pi \phi^*}{1-\beta}$ from equation (EE) divided by $(1-\beta)$. Second, we have $\frac{\partial \phi^*}{\partial \beta} = -\frac{1}{\beta^2}$ hence

$$\frac{\partial K}{\partial \beta} = \frac{-e' \phi^*}{1-\beta} + \frac{e}{\rho^2(\phi^*)} \frac{\partial \phi^*}{\partial \beta} = 0$$

$\iff \varepsilon = \beta$

Given that $M_f$, and hence $\Lambda_{\theta/y}$, is increasing in the gap between $\theta^*$ and $\theta^P$, at any $\phi^*$, the Hosios condition in the credit market is the one minimizing the volatility induced by financial imperfections. Away from this equation, one has a larger financial accelerator.

### 3 A stochastic extension

#### 3.1 Dynamic setup

In this Section, we study a dynamic stochastic model with first exogenous and then endogenous wages and offer a flexible – that is, easy-to-implement– calibration method to obtain both a set of first order moments (unemployment and financial sector’s share of GDP) and then second moments (the volatility of labor market tightness to productivity shocks).

We make the following assumptions for convenience. First, time is discrete and labor productivity is assumed to follow a stationary AR(1) process $y_t = \rho_y y_{t-1} + \nu_t$, where $0 < \rho_y < 1$ and $\nu_t$ is white noise. Second, we relax the assumption that $r = 0$. Third, an entrepreneur meeting a banker begins the recruiting process within the period. A successful meeting between a firm and worker begins production the following period. Maintaining our assumption of free entry on both sides of the credit market and
bargaining over $\rho$, we find that the equilibrium credit market tightness $\phi^*$ is time invariant and of the same form as earlier.\footnote{Time invariance follows from the sharing rule $(1-\beta)B_{l,t} = \beta E_{l,t}$ which implies a constant ratio $\frac{E_{l,t}}{B_{l,t}} = \frac{1-\beta}{\beta}$.} Moreover, $\rho$ is assumed to be determined when a banker and an entrepreneur meet and is solved as

$$E_t [\rho_{t+1}] = \beta E_t [y_{t+1} - w_t] + (1-\beta)E_t \left[ \frac{(1+r)\gamma}{q(\theta_t)} - \frac{(1-s)\gamma}{q(\theta_{t+1})} \right]$$

(14)

where $E_t$ is an expectations operator over productivity and $w_t$ is a wage determined later on.

From the constant values of being in the recruiting stage, $B_{l,t} = \kappa \phi^* p(\phi^*)$ and $E_{l,t} = e_p(\phi^*)$, we can combine the (EE) and (BB) curves in this stochastic environment,

$$\frac{e}{p(\phi)} = \frac{q(\theta_t)}{1+r} E_t [E_{g,t+1}] + \frac{(1-q(\theta_t))}{1+r} \frac{e}{p(\phi)}$$

and

$$\frac{\kappa}{\phi p(\phi)} = -\gamma + \frac{q(\theta_t)}{1+r} E_t [B_{g,t+1}] + \frac{(1-q(\theta_t))}{1+r} \frac{\kappa}{\phi p(\phi)}$$

to obtain a job creation condition in the presence of frictional credit markets

$$\frac{\Gamma_t}{q(\theta_t)} = \frac{1}{1+r} E_t \left[ y_{t+1} - w_t + (1-s) \frac{\Gamma_{t+1}}{q(\theta_{t+1})} \right]$$

(15)

where

$$\Gamma_t \equiv \gamma + K \left( 1 - \frac{1}{1+r} (1-q(\theta_t^*)) \right)$$

(16)

are vacancy costs augmented for frictional credit markets and $K = \frac{e}{p(\phi^*)} + \frac{\kappa}{\phi^* p(\phi^*)}$ is once again total search costs on the credit market.

It is worth noting two special cases. First, when $r = 0$, $\Gamma_t$ is simply the sum of all prospection costs in credit and labor markets, unadjusted for discounting. Second, when credit markets are perfect, $\Gamma_t$ boils down to $\gamma$, and the job creation condition reduces to

$$\frac{\gamma}{q(\theta_t^*{\gamma})} = \frac{1}{1+r} E_t \left[ y_{t+1} - w_t + (1-s) \frac{\gamma}{q(\theta_{t+1}^*)} \right]$$

(17)

### 3.2 Elasticity of $\theta_t$ to productivity shocks, fix wage

Define period profits from labor as $\Pi_t = y_t - w$ where $w_t = \overline{w}$. We can compute two elasticities of tightness to productivity innovations, first in the absence of financial imperfections, second with financial imperfections, and compare them.
Taking log-linear deviations around a steady state of equation (17), deviations in market tightness in the credit frictionless world can be expressed as a discounted sum of deviations in future expected profits

\[ \hat{\theta}_t^P = \frac{q(\theta^P) \Pi}{\eta \gamma (1 + r)} \sum_{i=0}^{\infty} \left( \frac{1 - s}{1 + r} \right)^i \hat{\Pi}_{t+i+1} \]

Given a fixed wage and the assumption on productivity, this is simply \( \hat{\theta}_t^P = \frac{q(\theta^P)}{\eta \gamma (1 + r)} \sum_{i=0}^{\infty} \rho^i \hat{\Pi}_t + 1 + i \) such that the elasticity of market tightness to a productivity shock in the Pissarides world with a fixed wage is denoted by \( \Lambda^P \) with

\[ \Lambda^P = \frac{\partial \hat{\theta}_t^P}{\partial \nu_t} = \frac{q(\theta^P) \rho_{\nu}}{\eta \gamma [(1 + r) - (1 - s) \rho_{\nu}]} \] (18)

By the same steps, the elasticity in the presence of credit frictions is given by \( \Lambda \) with

\[ \Lambda = \frac{\partial \hat{\theta}_t^*}{\partial \nu_t} = \frac{q(\theta^*) \rho_{\nu}}{\eta \gamma^T [(1 + r) - (1 - s) \rho_{\nu}]} \] (19)

where \( \gamma^T \equiv [\gamma + K (\frac{r}{1+r})] > \gamma \) is a measure of total frictional costs in both credit and labor markets.

The financial multiplier in this dynamic setting is thus:

\[ M_f \equiv \frac{\Lambda}{\Lambda^P} = \frac{q(\theta^*) \gamma}{q(\theta^P) \gamma^T} \]

which is exactly identical to the accelerator derived in Section 2 when \( r = 0 \). Finally, the elasticity of the job finding rate is simply:

\[ \frac{\partial f(\theta)}{\partial \nu} = \frac{\partial f(\theta)}{\partial \theta} \frac{\partial \theta}{\partial \nu} = (1 - \eta) \frac{\partial \theta}{\partial \nu} \cdot \frac{\partial f(\theta)}{\partial \theta} \]

### 3.3 Elasticity of \( \theta_t \) to productivity shocks, endogenous wages

Endogenous wages seriously reduce the elasticity of labor market tightness to productivity shocks. We thus expect that the financial multiplier will need to be higher to generate an as high value of volatility of the economy as in the fix wage section.

The wage determination we select is as follows. We assume that the worker bargains the wage with a firm, defined as the entrepreneur-banker block, at the time of meeting, instead of a bilateral bargaining between the worker and the entrepreneur (leaving the bank aside).\(^5\)

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\(^5\)There are two related reasons for this choice. The first one is that the natural alternative, bargaining between the entrepreneur and the worker, leads to complex strategic interactions illustrated in Wasmer and Weil (2004, Section IV-A): the entrepreneur and the bank wish to raise the debt of the firm above what is needed in order to reduce the size of total surplus to be shared between the firm and the worker at a later time. Hence, wages are driven down to the reservation wage of workers and do not vary with the firm’s productivity, which is counterfactual. This leads to the second reason, which is that we want our endogenous wage extension to be comparable to the classical wage solution in the labor search literature in order to compare the volatility in the model to other elasticities found in the literature.
Define the values of employment and unemployment in a discrete time stochastic setting as

\[ U_t = z + f(\theta_t)\beta E_t W_{t+1} + (1 - f(\theta_t))\beta E_t U_{t+1} \]

\[ W_t = w_t + \beta E_t [(1 - s)W_{t+1} + sU_{t+1}] \]

where \( z \) is the value of non-employment activities and \( f(\theta) = \theta q(\theta) \) the job finding rate. The Pissarides wage is \( w_t^P = \alpha (y_t + \gamma \theta_t^P) + (1 - \alpha)z \) where \( \alpha \) is the bargaining power of workers vis-à-vis the firm. Taking log-deviations, movements in labor market tightness to future productivity are given by

\[ \hat{\theta}_t^P = q(\theta_t^P)(1 - \alpha) \frac{\rho}{\eta \gamma (1 + r)} \sum_{i=0}^{\infty} \Psi_t^i \gamma_{t+i} \]

where the second term in \( \Psi = \frac{(1 - s)}{1 + r} - \frac{a(\theta_t^P)}{\eta (1 + r)} \) reflects the share of the change in productivity accruing to the worker through the wage. The latter strongly reduces the elasticity of labor market tightness to productivity shocks which, with our specification, is

\[ \Lambda^P = \frac{\partial \hat{\theta}_t^P}{\partial v_t} = \frac{q(\theta_t^P)(1 - \alpha)\rho}{\eta \gamma (1 + r) - \gamma \eta (1 - s) - \alpha f(\theta_t^P)\rho + \gamma} \]

Compared to the elasticity when wages are fixed, only a share \((1 - \alpha)\) of the rise in productivity accrues to the firm. In addition, the equilibrium rise in labor market tightness following a positive productivity shock improves the outside option of the worker and his bargaining position in the wage determination. This appears in the denominator as the term \( \alpha f(\theta_t^P) \), further reducing the elasticity of labor market tightness to productivity shocks.

Turning now to the responsiveness of labor market tightness under frictional credit markets, we begin by detailing the determination of the wage. As discussed earlier, we assume that the wage negotiated in a worker-firm pair, and in the presence of credit market frictions, it must satisfy a sharing rule

\[ \alpha F_{g,t} = (1 - \alpha) (W_t - U_t) \]

where \( F_{g,t} = E_{g,t} + B_{g,t} \) is the joint value of the firm to the entrepreneur-banker pair. Under this assumption the wage is

\[ w_t = \alpha [y_t + \Gamma_t \theta_t^P] + (1 - \alpha)z \]

and differs from the Pissarides wage by the coefficient \( \Gamma_t \) on market tightness. To the extent that this term is negatively correlated with productivity, credit market frictions induce a certain degree of wage

---

6To check the result, note that if \( \rho_y = 1 \) this is the elasticity obtained when comparing steady states, or to a permanent productivity shock, as in Shimer (2005), i.e. \( \epsilon_{\theta,y} = \frac{(1 - \alpha)}{\gamma \frac{a(\theta_t^P)}{\eta (1 + r)} + \alpha \theta_t^P} \). The details for deriving the elasticities can be found in the appendix.
rigidity by limiting the effect of a rise in market tightness on wages, a feature also present in Petrosky-Nadeau (2009). To see why this is the case, recall that \( \Gamma_t \equiv \gamma + K \left( 1 - \frac{1}{1+\tau}(1 - q(\theta^*)) \right) \) are vacancy costs augmented for frictional credit markets. Since \( q \) is decreasing in market tightness, so is \( \Gamma \).

Finally, the elasticity of labor market tightness under frictional credit markets and an endogenous wage is

\[
\Lambda = \frac{\partial \hat{\theta}^*_t}{\partial \nu} = \frac{q(\theta^*) (1 - \alpha) \rho_y}{\eta \gamma^T (1 + r) - [\eta \gamma^T (1 - \delta) - \alpha f(\theta^*) (\gamma^T + (1 - \eta) \tilde{\kappa})] \rho_y}
\]

(21)

where \( \tilde{\kappa} \equiv K \frac{g(\theta^*)}{1+r} \).

4 Calibration

4.1 Targets: first and second moments

Our first objective is to find a precise measure of the share of the financial sector in GDP and try to reproduce it in model in the steady-state of the stochastic extension. Theoretically, the share of the financial sector in the value added is

\[
\Sigma = \frac{(1 - u) \rho - \gamma \gamma - B \kappa}{1 - u}
\]

(22)

where in the numerator, the first term represents total banks gross profits \( \rho \) times the number of banks in the profit state, which is equal to the number of firms \( 1 - u \); the second term represent the negative cash flows of banks financing vacancies times the number of job vacancies \( \gamma^T \) where \( \gamma^T = \Gamma^T \); and the last term represents the financial intermediation costs paid by banks. Note that we assumed the effort costs \( e \) paid by entrepreneurs don’t enter GDP as they are effort costs. The denominator is total production at \( y = 1 \).

US national account data (Table1-14 in NIPA\(^7\)) allow us to calculate the gross value added of financial services. On average over years 2000-2009, this value was 3.3% of GDP. This will be the target for \( \Sigma \).

Our second target for first moments will be the rate of unemployment that we try to keep in the neighborhood of 6%. Our third target is to find an elasticity of \( \theta \) to productivity shocks around 22.

Our approach can be summarized in three stages, described in the following sub-sections.

\(^7\)http://www.econstats.com/nipa/NIPA1_1_14_.htm
4.2 Initial values of parameters under Hosios-Pissarides

We first find, both for the fixed wage model and the endogenous wage model, a set of parameters that reaches the target unemployment level. The calibration of the credit market requires choosing parameters of the credit matching function, assumed to be of the form \( M_c(\mathcal{P}, e) = \xi e^{\varepsilon - \varepsilon \mathcal{P}} \), the costs of prospecting on credit markets and the bargaining weight \( \beta \). We start agnostically from a Hosios-Pissarides rule in the credit market and proceed as follows.

We start from an initial, informed guess on parameters, where we choose in particular a symmetric set of parameters regarding prospecting costs and matching functions in the credit market. We include them in the vector \( X \) using as a starting point a “balanced” credit matching function and the credit market Hosios condition; i.e., \( \beta = \varepsilon = 0.5 \). We also begin with symmetry in prospecting costs \( \kappa = e = 0.05 \) and set the remaining parameter, \( \varsigma \), to 0.05. On the labor market, we value include the unit recruitment costs \( \gamma \) and the level parameter \( \chi \) in the vector of parameters \( X \) to achieve a desired level of unemployment. In this section the wage is assumed exogenous and equal to three quarters of labor productivity. The steady state rate of job separation is set to \( s = 0.06 \). We assume an the elasticity of the labor matching function with respect to unemployment of \( \eta = 0.5 \).\(^8\) The labor matching function is assumed to be a Cobb-Douglas \( M_l(\mathcal{V}, u) = \chi \mathcal{V}^{1-\eta} u^\eta \). Finally, the risk free rate is set to 4% annually, corresponding to a 3-month treasury bill, and the persistence coefficient in the process for productivity is set to 0.975, a commonly used value in the real business cycle literature.

In this initial calibration, the value of \( \Sigma \) (share of financial sector in GDP in equation 22) happens to be too low and relatively stable to parameters. Since the parameter space is large, we cannot find the “right” parameter values easily and therefore had to undertake a second step, described below, to raise the calibrated value of \( \Sigma \).

4.3 A “trembling hand” calibration method

The procedure consists of perturbing each element of an initial vector of parameters \( X \) by a random shock drawn for a normal distribution in a 7-dimensional parameter space for exogenous wages : \( \gamma, \beta, \varepsilon, e, \kappa \) and the two scale parameters in the matching functions denoted by \( \chi \) in the labor market and \( \varsigma \) in the credit market. With endogenous wages, the parameter space is 8-dimensional, as we also let \( \alpha \) be shaken too.

\(^8\)See Petrongolo and Pissarides (2001) for a survey of estimates of the labor matching function.
We run perturbations of the set of parameters $X$ in a neighborhood of the starting values of parameters, where perturbations are small: each parameter receives a multiplicative normal shock of variance $1/60$ (exogenous wages) or $1/80$ (endogenous wages). We obtain a corresponding value of the equilibrium variables $\theta$, $u$ and $\phi$ as well as a value of the credit market share in GDP $\Sigma$. We only retain values of the parameters for which $u$ is between 6 and 7% and for which $q(\theta)$ is between 0 and 1. After we reach 100 “acceptable” draws, we pick up the set of parameters where $\Sigma$ is maximal. Denote the new vector of parameters $X'$.

We then iterate on the same procedure, where the initial value of parameters is $X'$ of the previous iteration. We stop the iterations when the value of $\Sigma$ exceeds 3.3%, generally slightly above this threshold. The convergence occurs relatively fast, in about 10 to 15 steps. We call this first procedure Step 1, and it aims at matching the credit market share in GDP $\Sigma$.

Next, we replicate this procedure to progressively raise the elasticity of tightness of the labor market to productivity shocks. With exogenous wages, we perturbate the same parameters. With endogenous wages, we also shock the value of leisure and the bargaining power of workers. We stop when we obtain an elasticity $\Lambda$ of labor market tightness to productivity shocks larger than 22.

5 Results of the calibration

5.1 Exogenous wages

Table 1 summarizes the baseline parameter values, both the starting point and the results of the numerical search procedure. It also presents the steady state values of a series of endogenous quantities that are part of the constraint set. The process is stopped after 14 iterations. The values of the credit matching function’s elasticity $\varepsilon$ and the bargaining weight $\beta$ evolve quite smoothly at each iteration. Both parameters start at 0.5 and diverge away from Hosios-Pissarides: the matching elasticity remains around 0.5 while the bank bargaining weight $\beta$ increases to 0.85.

The value of $\Sigma$ is matched, but the elasticity of tightness to productivity innovations is still a bit low, with a value of 19.5 and a credit multiplier of 3.07. We thus launch the second step calibration procedure that aims at raising the value the elasticity, keeping the value of $\Sigma$ below 3.4%. We end up fast to the required value of 22: we overshoot a bit at 23. The financial multiplier reaches 3.64. We deviate marginally more away from Hosios: $\beta$ reaches 0.86 and $\varepsilon$ is lower at 0.51. The appendix plots the evolution of all the parameters in the vector $X$ and show that they are all converge quickly to their
<table>
<thead>
<tr>
<th>Labor market</th>
<th>Initial</th>
<th>Step1</th>
<th>Final</th>
<th>Steady state values</th>
<th>Initial</th>
<th>Step1</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td>matching elasticity</td>
<td>$\eta$</td>
<td>0.5</td>
<td>0.5 (*)</td>
<td>0.5 (*)</td>
<td>$q(\theta)$</td>
<td>0.17</td>
<td>0.21</td>
</tr>
<tr>
<td>fixed wage</td>
<td>$\tilde{w}$</td>
<td>0.77</td>
<td>0.77 (*)</td>
<td>0.77 (*)</td>
<td>$f(\theta)$</td>
<td>0.99</td>
<td>0.86</td>
</tr>
<tr>
<td>$w/y$</td>
<td>0.77 (*)</td>
<td></td>
<td></td>
<td></td>
<td>$w/y = 0.77 (*)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>job separation rate</td>
<td>$s$</td>
<td>0.06</td>
<td>0.06 (*)</td>
<td>0.06 (*)</td>
<td>Unmp. rate (6%;7%) (*)</td>
<td>5.4%</td>
<td>6.5%</td>
</tr>
<tr>
<td>vacancy cost</td>
<td>$\gamma$</td>
<td>0.2</td>
<td>0.198</td>
<td>0.215</td>
<td>Average recruiting cost</td>
<td>3.28</td>
<td>3.28</td>
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<td>matching level param.</td>
<td>$\chi$</td>
<td>0.42</td>
<td>0.422</td>
<td>0.460</td>
<td>Credit market share $\Sigma$</td>
<td>2.55%</td>
<td>3.39%</td>
</tr>
<tr>
<td>Credit market</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>bank’s barg. weight</td>
<td>$\beta$</td>
<td>0.5</td>
<td>0.85</td>
<td>0.86</td>
<td>Financial multiplier $M_f$</td>
<td>2.51</td>
<td>3.07</td>
</tr>
<tr>
<td>matching elasticity</td>
<td>$\varepsilon$</td>
<td>0.5</td>
<td>0.53</td>
<td>0.51</td>
<td>Duration, search for credit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>matching level param.</td>
<td>$\zeta$</td>
<td>0.05</td>
<td>0.056</td>
<td>0.055</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bank search costs</td>
<td>$\kappa$</td>
<td>0.05</td>
<td>0.045</td>
<td>0.047</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firm search costs</td>
<td>$e$</td>
<td>0.05</td>
<td>0.049</td>
<td>0.047</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Match $\Sigma$ after 14 iterations, match $\Lambda$) after 10 iterations (*): constrained

5.2 Endogenous wages

In the endogenous wage case, we can replicate the same calibration strategy for Step 1. Again the parameters deviate away from Hosios: $\beta$ is at 0.79 while $\varepsilon$ remains close to 0.5. We match the share of the financial sector in GDP but here, contrary to the previous case, obtain a quite low elasticity of $\theta$ to shocks. The value remains around 4.38 with a financial accelerator of 2.26.

We thus proceed to the final stage, in adding one more shock on the value of leisure $z$, hence in a 9-dimensional parameter space. We constrain the equilibria to remain in an area where $\Sigma$ remains between 3.2 and 3.4%, and the rate of unemployment remains between 6 and 7%. Finally, for reasons we develop in the next sub-section, we restrict the ratio of wages to productivity to remain below their initial value of 0.76 plus 1% (that is less than 0.77). We obtain the following values: the elasticity jumps to 23.9 and overshoot a bit in the last iteration. We obtain this in letting the gap from Hosios $\beta - \varepsilon$ go from $0.79 - 0.49 = 0.3$ to $0.92 - 0.58 = 0.34$, thus slightly higher.
Table 2: Baseline parameter and steady state values, endogenous wage

<table>
<thead>
<tr>
<th>Labor market</th>
<th>Initial</th>
<th>Step 1</th>
<th>Step 2</th>
<th>Steady state values</th>
<th>Initial</th>
<th>Step 1</th>
<th>Step 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>matching elasticity</td>
<td>η</td>
<td>0.5</td>
<td>0.5 (*)</td>
<td>0.5 (*)</td>
<td>0.27</td>
<td>0.21</td>
<td>0.81</td>
</tr>
<tr>
<td>worker’s share</td>
<td>α</td>
<td>0.15</td>
<td>0.1078</td>
<td>0.025</td>
<td>0.93</td>
<td>0.80</td>
<td>0.83</td>
</tr>
<tr>
<td>value of leisure</td>
<td>z</td>
<td>0.4</td>
<td>0.4 (*)</td>
<td>0.69</td>
<td>0.83</td>
<td>0.76</td>
<td>0.76 (*)</td>
</tr>
<tr>
<td>job separation rate</td>
<td>s</td>
<td>0.06</td>
<td>0.06 (*)</td>
<td>0.06 (*)</td>
<td>Unmp. rate (6%;7%) (*)</td>
<td>6.0%</td>
<td>6.9%</td>
</tr>
<tr>
<td>vacancy cost</td>
<td>γ</td>
<td>0.1</td>
<td>0.102</td>
<td>0.095</td>
<td>Average recruiting cost</td>
<td>2.43</td>
<td>3.41</td>
</tr>
<tr>
<td>matching level param.</td>
<td>χ</td>
<td>0.5</td>
<td>0.409</td>
<td>0.822</td>
<td>Credit market share Σ</td>
<td>1.58%</td>
<td>3.37%</td>
</tr>
</tbody>
</table>

Credit market

<table>
<thead>
<tr>
<th></th>
<th>Initial</th>
<th>Step 1</th>
<th>Step 2</th>
<th>Steady state values</th>
<th>Initial</th>
<th>Step 1</th>
<th>Step 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>bank’s barg. weight</td>
<td>β</td>
<td>0.5</td>
<td>0.81</td>
<td>0.94</td>
<td>Financial multiplier M_f</td>
<td>2.05</td>
<td>2.24</td>
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<tr>
<td>matching elasticity</td>
<td>ε</td>
<td>0.5</td>
<td>0.47</td>
<td>0.50</td>
<td>Duration, search for credit</td>
<td>20.0</td>
<td>10.6</td>
</tr>
<tr>
<td>matching level param.</td>
<td>ζ</td>
<td>0.05</td>
<td>0.047</td>
<td>0.057</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bank search costs</td>
<td>κ</td>
<td>0.05</td>
<td>0.049</td>
<td>0.042</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firm search costs</td>
<td>e</td>
<td>0.05</td>
<td>0.050</td>
<td>0.042</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Match Σ after 39 iterations, match Λ after 58 iterations (*): constrained

Most other parameter values remain stable. In addition, we manage to keep the duration of search by entrepreneurs of a financier quite low, around 6 quarters which is slightly more than a year (Wasmer and Weil 2004). This duration is an important target, and we will see that it results from a trade-off between having a small labor surplus and large costs of financial friction (next sub-section).

6 Conclusion

Financial imperfections raise the calibrated elasticity of labor market tightness to productivity shocks by a factor $M_f$ called the financial multiplier. With exogenous wages, it is easy to generate a plausible large elasticity of labor market tightness to productivity shocks, if one relaxes the Hosios-Pissarides rule in the credit market. Under the assumption of a large enough difference between the bargaining power of banks vis-à-vis entrepreneurs ($β$) with the elasticity of the rate at which entrepreneurs meet bankers with respect to credit market tightness ($ε$), one can obtain an elasticity around 20 or even larger.

Under endogenous wages with bargaining power $α$ of workers relative to firms, defined as the joint
bank-entrepreneur entity, all elasticities are divided by a factor 4 to 5, as was established by Shimer (2005) and Hall (2005). Hence, the model requires a higher financial multiplier. However we manage to keep the wage/productivity ratio below 0.77, thus relaxing the “small labor surplus assumption”.

We perceive our results to be in fact readily a generalization of the “small labor surplus” assumption: when the credit market is either very tight or very slack for firms, one side of the market has a very small total surplus to entering the relationship. Consequently, the entry of that side of the credit market is restricted and even small productivity shocks can generate large relative increases in the number of agents on the restricted side of the market: the small surplus is on firms in the credit-prospection stage.

In addition, we can go back to the intuitions of equation (13) or its discrete time equivalents combined in (15) and (16). These entry equations for firms have a common denominator: they introduce a new element to hiring costs, which is not strictly proportional to the duration of a vacancy \(1/q(\theta)\). As pointed out in Pissarides (2009), this leads to a greater volatility. An interpretation of our paper, fully consistent with Pissarides (2009), is that this non-proportional part is a financial costs arising from frictions in the credit market.

In our calibration with endogenous wages, those results are obtained with a duration of search in the credit market of approximately 6 quarters, that is on average 1.5 years, while keeping intermediation costs at 3.3% of GDP as in national accounts. Future research should attempt at reducing further this calibrated duration. One way of doing so is to introduce financial frictions on existing firms, and not only on starting firms.
References


Appendix: The Cyclical Volatility of Labor Markets under Frictional Financial Markets

A Introduction

This appendix details the derivation of the various equations and elasticities presented in the main text. We begin by fully describing the stochastic model in discrete time.

A.1 Asset values of an entrepreneur

\begin{align*}
E_{c,t} &= -e + p_t E_{l,t} + (1 - p_t) \frac{1}{1+r} \mathbb{E}_t E_{c,t+1} \\
E_{l,t} &= -\gamma + \gamma + \frac{1}{1+r} \mathbb{E}_t [q_t E_{g,t+1} + (1 - q_t) E_{l,t+1}] \\
E_{g,t} &= y_t - w_t - \rho_t + \frac{1}{1+r} \mathbb{E}_t [sE_{c,t+1} + (1 - s)E_{g,t+1}]
\end{align*}

The cost of convincing a bank to fund future negative cash flows is \( e \), and with probability \( 0 < p_t < 1 \) this results in a successful match within the period. During the second stage, the bank covers the cost of recruiting a worker, \( \gamma \), who is met with probability \( 0 < q_t < 1 \). During the production stage, \( y \) goods are produced which must cover both the wage rate \( w \) and interest payments \( \rho \). During the last stage, firms are subject to death shocks with probability \( s \).

An assumption of free entry for entrepreneurs leads \( \frac{E_l}{E} = E_{l,t} \) such that the final stage may be simplified to

\[ E_{g,t} = y_t - w_t - \rho_t + \frac{1}{1+r} \mathbb{E}_t E_{g,t+1} \]

A.2 Matching on credit markets

We follow the matching literature and assume that the total number of matches is governed by a matching technology associating the total number of banks in stage 0, denoted by \( B \), and the total number of entrepreneurs in stage 0, denoted by \( E \). Let \( M_C(E, B) \) be the matching process in the credit market. We have that \( p = M_C(E, B) / E \). Symmetrically, the rate at which banks find a project they are willing to finance is \( M_C(E, B) / B = \phi p \) where \( \phi = E / B \). Under the assumption of constant returns to scale of \( M_C(E, B) \), we have that \( p = p(\phi) \) with \( p'(\phi) < 0 \), elasticity \( e(\phi) = -\phi p'(\phi) / p(\phi) \), and it follows that
φ is a natural measure of the tightness of the credit market. We also make the assumptions

\[
\lim_{\phi \to 0} p(\phi) = 1 \\
\lim_{\phi \to +\infty} p(\phi) = 0
\]

The first line states that in the relative scarcity of competing firms relative to banks, matching with a banker is instantaneous, and the second line states that in the relative abundance of competing firms relative to banks, matching with a banker is infinitely slow.

A.3 Asset values for a banker

\[
B_{c,t} = -\kappa + \phi_t p(\phi_t) B_{l,t} + (1 - \phi_t p(\phi_t)) \frac{1}{1 + r} \mathbb{E}_t B_{c,t+1}
\]

\[
B_{l,t} = -\gamma + \frac{1}{1 + r} \mathbb{E}_t [q_t B_{g,t+1} + (1 - q_t) B_{l,t+1}]
\]

\[
B_{g,t} = \rho_t + \frac{1}{1 + r} \mathbb{E}_t [s B_{c,t+1} + (1 - s) B_{g,t+1}]
\]

Bankers search for a suitable investment at a cost of \(\kappa\) and enter the recruiting stage with probability \(\phi_t p(\phi_t)\) during which the vacancy cost \(\gamma\) must be disbursed. Meeting a worker occurs at the rate \(q_t\), at which point a banker enters the production stage and the remuneration \(\rho\) is received. An assumption of free entry for bankers leads \(\frac{\kappa}{\phi_t p(\phi_t)} = B_{l,t}\).

A.4 Time invariant credit market tightness

Free entry on both sides of the credit market, along with Nash bargaining over the surplus of a credit relationship, results in a time invariant tightness. To show this, note first that we had under free entry

\[
B_{l,t} = \frac{\kappa}{\phi_t p(\phi_t)}; \quad \text{and} \quad E_{l,t} = \frac{e}{p(\phi_t)}
\]

Denoting the banker’s bargaining weight by \(\beta\), and defining the credit relationship surplus as \(S_{c,t} = (E_{l,t} - E_{c,t}) + (B_{l,t} - B_{c,t})\), results in \(E_{l,t} B_{c,t} = \frac{1-\beta}{\beta} \) and

\[
\phi^* = \frac{1-\beta}{\beta} \frac{\kappa}{e}
\]
A.5 A calibration strategy for the credit matching function

The first figure plots the values of the parameters in the vector $X$ over each iteration $j$ for the model with a fixed wage. The second Figure plot the corresponding information for the model with an endogenous wage.

A.6 Deriving a job creation condition:

It will be convenient at this stage to express the joint value of recruiting a worker to banker and entrepreneur as $F_{g,t} = E_{g,t} + B_{g,t}$, which corresponds to the surplus from the credit relationhsip, as

$$
\frac{e}{p(\phi)} + \frac{\kappa}{\phi p(\phi)} = -\gamma + q_t \frac{1}{1+r} \mathbb{E}_t [E_{g,t+1} + B_{g,t+1}] + (1-q_t) \frac{1}{1+r} \left[ \frac{e}{p(\phi)} + \frac{\kappa}{\phi p(\phi)} \right]
$$

Define total costs on the credit market as $K(\phi) = \frac{e}{p(\phi)} + \frac{\kappa}{\phi p(\phi)}$ and $\Gamma_t \equiv \gamma + K(\phi) \left( 1 - \frac{1}{1+r} (1-q_t) \right)$, then

$$
\frac{\Gamma_t}{q_t} = \frac{1}{1+r} \mathbb{E}_t [E_{g,t+1} + B_{g,t+1}]
$$

Using the Bellman equations for entrepreneur and banker during production to define $[E_{g,t} + B_{g,t}] = F_{g,t} = y_t - w_t + (1-s) \frac{1}{1+r} \mathbb{E}_t [F_{g,t+1}]$, we obtain a job creation condition in the presence of frictional credit and labor markets

$$
\frac{\Gamma_t}{q_t} = \frac{1}{1+r} \mathbb{E}_t \left[ y_{t+1} - w_{t+1} + (1-s) \frac{\Gamma_{t+1}}{q_{t+1}} \right]
$$
Note that when the credit market is perfect $K(\phi) = 0$ and $\Gamma_t = \gamma$ such that the job creation condition collapses to the familiar

$$\frac{\gamma}{q_t} = \frac{1}{1 + r} \mathbb{E}_t \left[ y_{t+1} - w_{t+1} + (1 - s) \frac{\gamma}{q_{t+1}} \right]$$

### A.7 Rental rate

This section provides the details in deriving the rental rate

$$\mathbb{E}_t [\rho_{t+1}] = \beta \mathbb{E}_t [y_{t+1} - w_{t+1}] + (1 - \beta) \mathbb{E}_t \left[ \frac{(1 + r) \gamma}{q_t} - \frac{(1 - s) \gamma}{q_t} \right]$$

Define the surplus to the credit relationship as $S_{C,t} = E_{t,t} + B_{t,t}$. The sharing rule under Nash bargaining implies $B_{t,t} = \beta S_{C,t}$ and $E_{t,t} = (1 - \beta) S_{C,t}$. Expanding on the former,

$$-\gamma + \frac{1}{1 + r} \mathbb{E}_t [q_t B_{g,t+1} + (1 - q_t) B_{l,t+1}] = -\beta \gamma + \beta q_t \frac{1}{1 + r} \mathbb{E}_t [E_{t,t+1} + B_{g,t+1}]$$

$$+ \beta (1 - q_t) \frac{1}{1 + r} \mathbb{E}_t [E_{t,t+1} + B_{l,t+1}]$$

Rearranging terms,

$$\mathbb{E}_t B_{g,t+1} + \frac{(1 - q_t)}{q_t} \mathbb{E}_t B_{l,t+1} = (1 - \beta) \frac{\gamma (1 + r)}{q_t}$$

$$+ \beta \mathbb{E}_t \left[ E_{t,t+1} + B_{g,t+1} \right] + \frac{(1 - q_t)}{q_t} \mathbb{E}_t \left[ E_{t,t+1} + B_{l,t+1} \right]$$

$$\mathbb{E}_t \left[ \rho_{t+1} + \frac{1}{1 + r} (1 - s) B_{g,t+2} \right] = (1 - \beta) \frac{\gamma (1 + r)}{q_t}$$

$$+ \beta \mathbb{E}_t \left[ y_{t+1} - w_{t+1} + (1 - s) \frac{1}{1 + r} \left( B_{g,t+2} + E_{g,t+2} \right) \right]$$

$$+ \beta \frac{(1 - q_t)}{q_t} \mathbb{E}_t \left[ E_{t,t+1} + B_{l,t+1} \right] - \frac{(1 - q_t)}{q_t} \mathbb{E}_t B_{l,t+1}$$

Since $B_{l,t} = \beta \left[ E_{t,t} + B_{t,t} \right]$, $\mathbb{E}_t B_{g,t+1} = (1 - \beta) \frac{\gamma (1 + r)}{q_t} + \beta \mathbb{E}_t \left[ E_{g,t+1} + B_{g,t+1} \right]$, or

$$\mathbb{E}_t [(1 - \beta) B_{g,t+1} - \beta E_{g,t+1}] = (1 - \beta) \frac{\gamma (1 + r)}{q_t}$$

then

$$\mathbb{E}_t \left[ \rho_{t+1} + \frac{1}{1 + r} (1 - s) B_{g,t+2} \right] = (1 - \beta) \frac{\gamma (1 + r)}{q_t} + \beta \mathbb{E}_t \left[ y_{t+1} - w_{t+1} + (1 - s) \frac{1}{1 + r} \left( B_{g,t+2} + E_{g,t+2} \right) \right]$$

and

$$\mathbb{E}_t [\rho_{t+1}] = \beta \mathbb{E}_t [y_{t+1} - w_{t+1}] + (1 - \beta) \mathbb{E}_t \left[ \frac{(1 + r) \gamma}{q_t} - \frac{(1 - s) \gamma}{q_t} \right]$$
A.8 Workers and wages

An individual may be unemployed and earning income \( z < y \). The unemployed meet job offers at rate \( f(\theta) = \theta q \). Once employed, workers earn wage \( w \) until separation, which occurs with probability \( s \) per unit of time. The Bellman equations describing each of these stages are

\[
U_t = z + f(\theta_t) \frac{1}{1+r} \mathbb{E}_t W_{t+1} + (1 - f(\theta_t)) \frac{1}{1+r} \mathbb{E}_t U_{t+1}
\]

\[
W_t = w_t + \frac{1}{1+r} \mathbb{E}_t [(1-s)W_{t+1} + sU_{t+1}]
\]

We assume that the wage negotiated in a worker-firm pair in the presence of credit market frictions, with surplus \( S_{L,t} = F_{g,t} + W_t - U_t \), satisfies \( \alpha F_{g,t} = (1 - \alpha) (W_t - U_t) \), where \( F_{g,t} = E_{g,t} + B_{g,t} \) is the joint value of the firm to the entrepreneur-banker pair. Applying this sharing rule to the worker-firm surplus, first we have

\[
S_{L,t} = y_t - w_t + (1-s) \frac{1}{1+r} \mathbb{E}_t F_{g,t+1} + \frac{1}{1+r} \mathbb{E}_t [(1-s)W_{t+1} + sU_{t+1}] - z - \frac{1}{1+r} \mathbb{E}_t [\theta_t q_t W_{t+1} - (1 - \theta_t q_t) U_{t+1}]
\]

\[
S_{L,t} = y_t - z + (1-s) \frac{1}{1+r} \mathbb{E}_t [F_{g,t+1} + W_{t+1} - U_{t+1}] - \theta_t q_t \frac{1}{1+r} \mathbb{E}_t [W_{t+1} - U_{t+1}]
\]

\[
S_{L,t} = w_t - z + (1-s) \frac{1}{1+r} \mathbb{E}_t S_{L,t+1} + \alpha \theta_t q_t \frac{1}{1+r} \mathbb{E}_t S_{L,t+1}
\]

and second, using \( F_{g,t} = (1 - \alpha) S_{L,t} \) and \( \Gamma_t = \frac{1}{1+r} \mathbb{E}_t (1 - \alpha) F_{g,t+1} \),

\[
y_t - w_t + (1-s) \frac{1}{1+r} \mathbb{E}_t F_{g,t+1} = (1 - \alpha) \left( y_t - z + (1-s) \frac{1}{1+r} \mathbb{E}_t S_{L,t+1} \right) - \alpha \theta_t \Gamma_t
\]

Rearranging terms yield the wage rule under frictional labor and credit markets:

\[
w_t = \alpha (y_t + \theta_t \Gamma_t) + (1 - \alpha) z
\]

B Deriving the elasticity of market tightness to a productivity shock

B.1 Canonical framework

Assume the matching function is Cobb-Douglas such that \( q(\theta_t) = \chi \theta_t^{-\eta} \). Define period profit flows
as \( \Pi = y - w \). Taking log linear deviations around a stationary steady state, \( \eta \frac{\gamma(1+r)}{q(\theta^p)} \theta_t^p = \Pi \varepsilon_t \hat{\Pi}_{t+1} \) and \( \eta \frac{\gamma(1+r)}{q(\theta^p \eta)} \hat{\theta}_{t+1}^p \), and using the forward operator we have that \( [1 - \frac{1}{1+r} \varepsilon_t L^{-1}] \hat{\theta}_t^p = \frac{q(\theta^p \Pi)}{E \eta \gamma(1+r)} \varepsilon_t \hat{\Pi}_{t+1} \) such that

\[
\hat{\theta}_t^p = \frac{q(\theta^p \Pi)}{\eta \gamma(1+r)} \varepsilon_t \sum_{i=0}^{\infty} \left( 1 - \frac{s}{1+r} \right)^i \hat{\Pi}_{t+1+i}
\]

Deviations of market tightness are forward looking, discounting future deviations of profits. Using the definition of the wage \( w_t = \alpha (y_t + \gamma \theta_t) + (1 - \alpha) z_t \):

\[
\hat{\theta}_t^p = \frac{q(\theta^p \Pi)}{\eta \gamma(1+r)} \varepsilon_t \sum_{i=0}^{\infty} \psi^i \hat{\Pi}_{t+1+i}
\]

where \( \psi = \left( \frac{1-s}{1+r} \right) - \alpha \theta^p q(\theta^p) \). Assuming that productivity follows an AR(1) with persistence parameter \( 0 < \rho_s < 1 \) and innovation \( \nu_t \) as white noise, then

\[
\hat{\theta}_t^p = \frac{q(\theta^p \Pi)}{\eta \gamma(1+r)} \varepsilon_t \sum_{i=0}^{\infty} \psi^i \theta_t^{i+1} \nu_t
\]

so that \( \hat{\theta}_t^p = \frac{q(\theta^p \Pi)}{\eta \gamma(1+r)} \left( \frac{1}{1 - \psi_r} \right) \rho_s \nu_t \), and

\[
\frac{\partial \hat{\theta}_t^p}{\partial \nu_t} = \frac{(1 - \alpha) q(\theta^p) \rho_s}{\eta \gamma(1+r) - \gamma(1 - s) - \alpha f(\theta)} \frac{(1 - \alpha) q(\theta^p) \rho_s}{\eta \gamma(1+r) - \gamma(1 - s) - \alpha f(\theta)} \frac{(1 - \alpha) q(\theta^p) \rho_s}{\eta \gamma(1+r) - \gamma(1 - s) - \alpha f(\theta)} \frac{(1 - \alpha) q(\theta^p) \rho_s}{\eta \gamma(1+r) - \gamma(1 - s) - \alpha f(\theta)}
\]

To check the result, note that if \( \rho_s = 1 \) this is the elasticity obtained when comparing steady states, or to a permanent productivity shock, as in Shimer (2005), i.e. \( \frac{(1 - \alpha) q(\theta^p) \rho_s}{\eta \gamma(1+r) - \gamma(1 - s) - \alpha f(\theta)} \).

### B.2 Frictional credit markets - fixed wage

Recall the job creation condition \( \frac{y_t}{q(\theta)} = \frac{1}{1+r} \varepsilon_t \left[ y_{t+1} - w + (1-s) \frac{\Gamma_{t+1}}{q_{t+1}} \right] \), with \( \gamma_t \equiv \gamma + K \left( 1 - \frac{1}{1+r} (1 - q_t) \right) \) and \( q(\theta_t) = \chi \theta_t^{-\eta} \). Taking log linear deviations around a stationary steady state: \( \frac{y_t}{q(\theta)} \left[ \gamma + K \left( \frac{r}{1+r} \right) \right] \hat{\theta}_t = \frac{\eta}{q(\theta)} \left[ \gamma + K \left( \frac{r}{1+r} \right) \right] \hat{\theta}_t = \varepsilon_t \hat{\Pi}_{t+1} \), Call \( \gamma_T \equiv \gamma + K \left( \frac{r}{1+r} \right) \), then \( [1 - \frac{1}{1+r} \varepsilon_t L^{-1}] \hat{\theta}_t = \frac{q(\theta)}{\eta \gamma(1+r)} \varepsilon_t \hat{\Pi}_{t+1} \), and

\[
\hat{\theta}_t = \frac{q(\theta)}{\eta \gamma(1+r)} \varepsilon_t \sum_{i=0}^{\infty} \left( 1 - \frac{s}{1+r} \right)^i \hat{\Pi}_{t+1+i}
\]

If productivity follows the same AR(1) process, then \( \hat{\theta}_t = \frac{q(\theta)}{\eta \gamma^p (1+r)} \varepsilon_t \sum_{i=0}^{\infty} \left( 1 - \frac{s}{1+r} \right)^i \rho_s^{i+1} \nu_t \),

\[
\hat{\theta}_t = \frac{q(\theta)}{\eta \gamma^p (1+r)} \left( \frac{\rho_s}{1 - \frac{s}{1+r} \rho_s} \right) \nu_t
\]

and

\[
\frac{\partial \hat{\theta}_t}{\partial \nu_t} = \frac{q(\theta) \rho_s}{\eta \gamma^p [(1 + r) - (1 - s)] \rho_s}
\]
B.3 Frictional credit markets - flexible wage

If the wage outcome is \( w_t = \alpha [y_t + \Gamma_t \theta_t] + (1 - \alpha) z \), we can write the job creation condition as

\[
\frac{\Gamma_t}{q_t} = \frac{1}{1 + r} \mathbb{E}_t \left[ (1 - \alpha) \left( y_{t+1} - z \right) - \alpha \Gamma_{t+1} \theta_{t+1} + (1 - \alpha) \frac{\Gamma_{t+1}}{q_{t+1}} \right].
\]

The following preparatory steps are useful. First, write \( \Gamma_t = \gamma T + K_1 + r \theta_t q_t(\theta_t) \). Then take log-linear deviation around a stationary steady state of the job creation condition:

\[
\eta (1 + r) \gamma^T \theta_t = (1 - \alpha) \mathbb{E}_t \left[ y_{t+1} - \alpha \left( y^T + q(\theta) \right) \right] E_t \theta_{t+1} + \eta (1 + s) r \eta \gamma^T \theta_t q(\theta) \theta_t q(\theta)
\]

Calling \( \Phi \equiv \left[ (1 + s) - \frac{\alpha q(\theta)}{\eta \gamma^T (1 + r)} \right] E_t \theta_{t+1} \), we then follow similar steps by obtaining:

\[
\hat{\theta}_t = \frac{(1 - \alpha) q(\theta)}{\eta \gamma^T (1 + r)} \mathbb{E}_t \sum_{i=0}^{\infty} \Phi^i \gamma_{t+1+i}
\]

and making use of the specification for labor productivity, \( \hat{\theta}_t = \frac{(1 - \alpha) q(\theta)}{\eta \gamma^T (1 + r)} \mathbb{E}_t \sum_{i=0}^{\infty} \Phi^i \rho_{t+1}^i \). Finally \( \hat{\theta}_t = \frac{(1 - \alpha) q(\theta)}{\eta \gamma^T (1 + r)} \left( \frac{\rho_t}{1 - \Phi \rho_t} \right) \nu_t \) and

\[
\frac{\partial \hat{\theta}_t}{\partial \nu_t} = \frac{(1 - \alpha) q(\theta) \rho_y}{\eta \gamma^T (1 + r) - \left[ \eta \gamma^T (1 + s) - \alpha f(\theta) \gamma^T + (1 - \eta) \kappa \right] \rho_y}
\]

where \( \kappa \equiv K_{1+\Gamma_t} \).

**Online Appendix : not for publication**

C Online Technical Appendix 1 : additional numerical results

The baseline results kept the unemployment rate constant by adjusting the labor matching function’s level parameter \( \chi \). We show in the following table the this level parameter in the labor matching function has no incidence on the propagation of productivity shocks when wages are fixed.
Figure 2: Elasticity of labor market labor tightness to productivity shocks.

Table A1: Keeping $\chi$ fixed

<table>
<thead>
<tr>
<th>$q(\theta)$</th>
<th>Wage</th>
<th>U rate</th>
<th>Elasticity of $\theta$</th>
<th>Financial accelerator $M_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pissarides</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- fixed wage</td>
<td>0.11</td>
<td>0.75</td>
<td>0.08</td>
<td>6.48</td>
</tr>
<tr>
<td>Credit friction - fixed wage</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta = 0.5, \varepsilon = 0.5$</td>
<td>0.20</td>
<td>0.75</td>
<td>0.08</td>
<td>11.57</td>
</tr>
<tr>
<td>$\beta = 0.2, \varepsilon = 0.8$</td>
<td>0.72</td>
<td>0.75</td>
<td>0.24</td>
<td>39.11</td>
</tr>
<tr>
<td>$\beta = 0.8, \varepsilon = 0.2$</td>
<td>0.72</td>
<td>0.75</td>
<td>0.24</td>
<td>39.11</td>
</tr>
</tbody>
</table>

D Online Technical Appendix 2: additional results with endogenous wages

Table 3 presents the results for several scenarios. The first row shows that the Pissarides model with a fixed wage yields an elasticity of labor market tightness of $XX\text{?}6.48$, which is 4.7 times greater than when wages are flexible (see Table 2). The next three rows present the results for a multi-frictional economy. In the baseline calibration, the matching function is "balanced" and the economy is at a Hosios condition on the credit market. The resulting minimized financial accelerator has a value of 1.79, and the elasticity of labor market tightness to productivity shocks is 11.57, values that are close to those obtained from our steady state calculations.
Table 3: Baseline results

<table>
<thead>
<tr>
<th></th>
<th>Elasticity of $\theta$</th>
<th>Elasticity of $f(\theta)$</th>
<th>Financial accelerator $M_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pissarides - fixed wage</td>
<td>7.30</td>
<td>3.65</td>
<td>1</td>
</tr>
<tr>
<td>Credit friction - fixed wage</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta = 0.45, \varepsilon = 0.96$</td>
<td>75.67</td>
<td>37.74</td>
<td>10.36</td>
</tr>
<tr>
<td>Hosios: $\beta = \varepsilon = 0.45$</td>
<td>11.17</td>
<td>5.58</td>
<td>1.53</td>
</tr>
</tbody>
</table>

When we move away from the credit market Hosios condition, the value of the financial accelerator can become very large and the elasticity of market tightness to productivity shocks overshoots the value in the data. Figure 3 presents the values of the elasticity of labor market tightness to productivity shocks keeping a "balanced" credit matching function, i.e., $\varepsilon = 0.5$, and varying the bargaining weight $\beta$ between 0.1 and 0.9. As was known from the analysis in Section 3, Figure 3 illustrates that the financial accelerator is minimized at $\beta = \varepsilon = 0.5$ and increases away from this point.

At one extreme, e.g., when $\beta$ is small, firms in the entry stage get a very low share of the surplus. Hence, any small positive productivity shock leads to a large impact on the entry of firms on the credit market. This is essentially a generalization of the small surplus idea in the literature, and in particular of Hagedorn and Manovskii (2008), although applied to firms in a different stage, that is before the production of the final good. Conversely, when $\beta$ is close to 1 it is banks who get a small surplus. In that case, any small profit shock would lead to a large relative increase in the number of prospecting banks and hence to a large amplification of the same shock. The small surplus here corresponds to that of banks, and their optimal response to shocks is therefore large.

We explore this further with two cases in which there are large departures in the degree of matching externalities and creditor’s bargaining weight. In both cases, there is near linearity in the matching function; first in the supply of creditors, second in the supply of entrepreneurs. The results are shown in the last two rows of Table 1, and in the first ($\varepsilon = 0.8$) and second ($\varepsilon = 0.2$) panels of Figure 4. Case 1 shows that if matching is near linear in supply of creditors and we depart from the credit market Hosios condition by reducing the bargaining weight of bankers, the elasticity of labor market tightness to productivity shocks becomes potentially very large. Intuitively, when entrepreneurs extract a larger
Figure 3: Elasticity of job finding rate to productivity shocks.

Figure 4: Two cases for the credit market and the financial multiplier.

share of the surplus generated by a firm they respond more strongly to the change in productivity by wanting to enter the credit market. On the other side of the market, the average search cost for creditors is relatively unresponsive to the bankers entering the market from the near linearity in the matching function. As a result, when $\varepsilon = 0.8$ and $\beta = 0.2$ the elasticity of labor market tightness to productivity shocks nears 40, and the financial accelerator contributes a factor $M_f = 6.04$. A step further, at $\beta = 0.15$, the elasticity exceeds 80. The exact inverse is observed when the matching function is near linear in the supply of entrepreneurs (see the second panel of Figure 4).\(^9\)

\(^9\)The two regimes imply very different annualized excess return on the loan. In the first case, when the firm has most of the weight in sharing the surplus, the excess return is 3.5%. In the second case, the excess return is 26%.
Table 2 presents the results when wages are endogenous following a similar calibration exercise as in the previous section, along with a Hosios condition on the labor market to determine the bargaining weight $\alpha$. We find an elasticity of labor market tightness when credit markets are perfect and wages endogenous of 1.37. For the economy with a frictional credit market, we focus on two sets of results: 1) the results for our baseline calibration and, 2) calibrating to a small labor surplus. Table 2 reveals, first, that size of the financial accelerator is greater in the presence of endogenous wages, rising from 1.79 to 2.14, the additional amplification arising from the slight degree of wage rigidity outlined above. Second, amplification is less sensitive to departures from the credit market Hosios condition. For $\beta = 0.2$ and $\epsilon = 0.8$ the financial accelerator reaches a factor of 2.54. Note that the magnitude of the multiplier in this scenario also corroborates the results in Petrosky-Nadeau (2009). More extreme departures are necessary for a large financial accelerator: at $\beta = 0.1$ and $\epsilon = 0.9$ we obtain $M_f = 8.6$.

E Online Technical Appendix 3: Convergence through the trembling hand calibration method

E.1 Convergence with endogenous wages, step 1 (getting the right value for the share of financial sector $\Sigma = 3.3\%$)

The next figures present the progression of the target in step one, the share of credit markets in aggregate value added, at each iteration $j$ until the target of $\Sigma = 3.3\%$ is reached, along with the progression of the parameter values.
E.2 Convergence with endogenous wages, step 2 (getting the right value for the elasticity of tightness with respect to productivity shocks)