

## Fixed income problem bank

### Problem 1

Complete the Bloomberg Market Concept (< *BMC* >) Fixed Income module with a quiz score of 75% or above. If you choose to do the full Bloomberg Certification already, you can attach your certificate to your assignment. (You will eventually need to produce that certificate anyway.) If for now you choose to do fixed income only, paste an image of your screen showing the module as completed and your quiz score onto your homework.

### Problem 2

Consider a zero-coupon bond due in exactly six months with a face value of 100 and currently selling for 98. Assume you buy now and hold the investment to maturity.

1. What is the simple quarterly compounded rate of interest per annum on this investment?
2. What is the continuously compounded rate of interest on this investment?
3. What is the EAR on this investment?

### Problem 3

Consider a fixed rate semi-annual coupon bond with redemption value \$1,000, whose most recent coupon date was 8/15/2022, clean price on 9/15/2022 was \$990, yield was 6% on 9/15/2022, and maturity is 8/15/2025. Interest accrues according to 30/360 US.

1. What is the bond's coupon rate?
2. What was the bond's dirty price on 9/15/2022?

### Problem 4

These are all quoting convention questions.

1. A treasury bond is quoted at a price of 105.282, in standard form. What is the decimal value of this price?
2. A treasury bond is trading at a decimal price of 101.3359375. What is the quote in standard form?
3. A treasury bill is quoted at a discount of 7 and has 78 days left to maturity. What is the price of the bill per \$100 of par?
4. A treasury bill has 188 days left to maturity. The price of the bill per \$100 of par is \$99. What is the discount quote?

### Problem 5

These are all day-count convention questions. Assume S/A bonds throughout with a principal of \$1,000 and a rate per annum of 10%.

1. Assume 30/360 US. What is the DCF between 1/1/2020 and 3/13/2020?
2. Assume Actual/Actual ICMA. What is the DCF between 1/1/2020 and 3/13/2020?
3. Assume now that 1/1/2020 is a coupon reset date. What is the accrued interest on the bond under 30/360 US as of 3/13/2020?
4. Assume now that 1/1/2020 is a coupon reset date. What is the accrued interest on the bond under Actual/Actual ICMA as of 3/13/2020?

### Problem 6

A corporation has the option to prepay (call) a bond with 4 years to maturity, \$50M in remaining principal, a 10% yearly rate, fixed and monthly payments. It can replace this bond with a 4 year bond with the same remaining payment structure. Prepayment penalties are  $c = \$500,000$ .

How low must the yearly rate on the new bond be to justify calling the old bond (ignoring the option value of waiting to refi)?

### Problem 7

Consider a fully amortizing bond with face value \$100M, twenty yearly payments, and an interest rate of 10%. Payments will grow by  $g\%$  a year for 10 years. After

year 10, payments are flat. (So year 11 payment is the same as year 10, as are all subsequent payments)

1. If  $g = 1\%$ , what is the bond's outstanding principal at the start of year 5?
2. Above what growth rate does the bond begin to feature negative amortization, holding the interest rate the same?

### **Problem 8**

A company just issued a bond with  $100K$  in remaining principal to be fully amortized in full in 20 yearly payments. The first 5 payments are interest only. Then payments grow by a rate of  $g\%$  every year. The interest rate is  $10\%$ . What is  $g$ ?

### **Problem 9a**

A floating rate bond with annual payments has exactly 5 years and 5 payments left to maturity and features a quoted margin of 100 basis points over an index whose current value is  $5\%$ . It has a redemption value of  $\$1,000$  but currently trades at  $\$1,020$ . What is the floater's discount margin? (*In this problem as well as any other QM/DM problem in this class, make the standard assumption that the index will stay at its current value all the way to maturity.*)

### **Problem 9b**

A floating rate bond with annual payments has exactly 5 years and 5 payments left to maturity and features a quoted margin of 100 basis points over an index whose current value is  $r\%$ . It has a redemption value of  $\$1,000$ . The floater's discount margin is 90 basis points. The floater currently trades at  $\$1,004$ . What is  $r$ ?

### **Problem 10**

Use Bloomberg to download total returns on the S&P500 (use SPXT) and on Investment Grade corporate bonds (use LUACTRUU) on a weekly basis since 1990.

1. Compute ttm correlations between the two return series and plot the resulting line chart. Has that correlation been stable over the past thirty years?
2. Repeat the analysis for high-yield bond rather than investment-grade bonds by

using LF98TRUU rather than LUACTRUU. Use weekly data since 1998 for this purpose since LF98TRUU (effectively) starts at that date.

### **Problem 11**

A five-year callable bullet with exactly 5 years to maturity, a coupon rate of 10% and 10 S/A payment left is callable in 2 years (right after payment 4 is made) at a call price of  $100 + x$  per 100 of principal. The redemption value is 1,000 and the current market price is 1,100. Find  $x$  so that  $YTW = YTC = YTM$ .

### **Problem 12a**

In a given economy, spot yields are 3, 4, 5, and 6 percent (per annum, with S/A compounding) at tenors 6, 12, 18, and 24 months.

1. What are discount factors at those same tenors?
2. What is the market value of a risk-free S/A bullet with a coupon rate of 10% per annum, a redemption value of \$1,000, and that matures in 24 months?

### **Problem 12b**

In this exercise we practice going from spot to par and from swap to spot.

1. In one economy, spot yields are 3, 4, 5, and 6 percent (per annum, with S/A compounding) at tenors 6, 12, 18, and 24 months. What are par yields at those same tenors?
2. In a different economy, swap rates are 3, 4, 5, and 6 percent (per annum, with S/A compounding) at tenors 6, 12, 18, and 24 months. What are spot yields at those same tenors?

### **Problem 12c**

Assume that S/A annualized spot rates are 1, 2, and 3 percent, respectively over the next 18 months. An 18 month S/A bullet with coupon rate of 10% and a redemption value of 1,000 is issued at a price of \$1,100. What is the bond's z-spread at issue?

### **Problem 13**

A 10-year fixed-payment, fixed-interest bond with yearly payments that currently trades at par has the same modified duration as a 5-year zero coupon bond with a S/A yield of 10%. What is the interest rate on the fixed payment bond?

*Warning: Since the bond has yearly payments, modified duration is  $\frac{1}{1+y}$  times  $McD$  for that bond. For the zero, since the yield is given in SA terms, modified duration is  $\frac{1}{1+\frac{y}{2}}$  times  $McD$ .*

**Problem 14**

Consider a bond to be paid back in 5 yearly payments plus a balloon payment. Payments grow by 10% a year. The sum of all yearly bond payments equals the size of the balloon. The face value of the bond is \$500M, the interest rate is 5%.

What is the bond's outstanding balance at the end of year 3, i.e. after year-3 payment is made?

**Problem 15**

A corporation has the following portfolio of debt liabilities on its books. All are fixed rate coupon bonds with yearly payments. What is the current market value of the corporation's total debt? What is the modified duration of the corporation's portfolio of liabilities?

Maturity (years)	Face (M)	Rate (contract)	Rate (market, current yield)
1	100	1.90%	2.50%
2	100	2.25%	2.60%
3	0	NA	2.65%
4	230	2.50%	2.70%
5	120	4.25%	3.25%

**Problem 16**

Consider a bond to be paid back in 5 yearly payments plus a balloon payment of size \$50,000. Payment 2 (as in, the payment in year 2) is 10% higher than payment 1, payment 3 is the same as payment 2, payment 4 is half of payment 3, payment 5 is the same as payment 4. The face value of the bond is \$200,000. The size of the balloon payment equals the sum of all interest payments over the life of the bond.

What is the bond payment in year 4?

### Problem 17

A constant amortization bond is a bond such that payments towards principal (i.e. reductions in outstanding principal) are the same each period. So, for instance, a constant amortization bond with \$1M of outstanding principal, monthly payments, and 100 months to maturity features a principal payment/reduction of \$10,000 every month. You also have to pay interest each month so that, for instance, assuming an interest rate of 0.5% a month, the total payment to the bond just described would be \$15,000 in the first remaining month to maturity.

Consider a corporation that holds a constant amortization bond with \$1M of outstanding principal,  $T$  months of remaining maturity, a current rate of 0.5% per month. (**Note: the interest is already quoted in monthly terms, do NOT divide it by 12.**) For a one-time prepayment penalty \$25,000, it can replace (refinance) this bond with a new constant amortization bond with the exact same terms except that the new monthly interest rate is 0.45% per month. Refinancing is a one-time only option.

What is the lowest maturity (the lowest integer value of  $T$ ) such that refinancing has positive NPV?

### Problem 18

A corporation has a bond on its books with \$500K left in principal, 5 years to maturity, yearly payments that grow by 10% each year, a balloon payment of \$50K, and an interest rate of 10%. It can replace this bond with a new bond with the same terms (\$500K left in principal, 5 years to maturity, yearly payments that grow by 10% each year, a balloon payment of \$50K) except that the interest rate is lower. The cost of refinancing is 2% of the outstanding principal.

Assuming that the corporation will have no other opportunity to replace its bond, below what interest rate is refinancing a positive NPV decision? (Assume that the right discount rate in this calculation is the rate on the new bond.)

### Problem 19

Consider a bond to be paid back in 7 yearly payments. In year 2 and in year 3, the payment goes up by 10%. In year 4 and in year 5, the payment goes up by 5%. In year 6 the payment falls by 20%. In year 7 the payment is the same as in year 6. The bond is fully amortizing. The interest rate on the bond is 10%. The **interest**

payment due in year 6 is \$17,945.63.

What is the face value (i.e. the initial balance) of the bond?

### **Problem 20**

A corporation holds a bond with an outstanding principal of \$1M to be paid back in fifteen yearly payments plus a balloon of size \$100,000. Payments are interest-only for 5 years (payment equals interest for 5 years). The payment changes in year 6 and is constant at its year 6 value thereafter. The current interest rate is 15%. The corporation has the opportunity to replace this bond with a new bond with exactly the same payment structure and balloon size but with a lower interest rate of 14%. The prepayment penalty is such that if the corporation were untaxed, the NPV of refi would be \$10,000.

But the corporation is taxed at a rate  $\tau$ . Below what tax rate does refinancing remain a positive NPV decision?

*(Hint: With taxes, the NPV of refi is the same as it normally is except that you have to subtract the PV of the increase in taxes, which gets discounted at the rate on the new bond.)*

### **Problem 21**

A S/A bond with market value \$1M has a modified duration of 5 and a convexity of 300. Yields rise from 7 to 7.5 percent.

1. By what percent, approximately, does the value of the bond fall? By what amount?
2. What is the bond's bloomberg risk before the yield change?
3. What is the bond's DV01 before the yield change?
4. What is the bond's Macaulay duration before the yield change?
5. Following the yield change, does the bond's duration rise or fall?
6. Following the yield change, does the bond's convexity rise or fall?

### **Problem 21b**

Your portfolio consists of two bonds: a 5 year zero with redemption value \$100k whose current S/A yield is 5% and a 10-year S/A bullet with the same redemption value, a 10% coupon rate, and a market value of \$110k. What is the modified duration of your bond portfolio? What is the convexity of your bond portfolio?

### **Problem 22**

A S/A bullet was just originated with face value of \$1M, a market value of \$1.05M, a coupon rate of 10%, and a maturity of 10 years. What are the bond's Macaulay duration, modified duration, convexity, bloomberg risk, and DV01?

### **Problem 22b**

A S/A bullet was just originated with YTM origination of 12%, a coupon rate of 10%, and a maturity of 10 years. The bond's bloomberg risk at origination is \$1M.

1. What is the bond's redemption value?
2. What is the bond's convexity?

### **Problem 23**

Consider a bond with yearly payments, 5 years to maturity, a yield of 10%, a z-spread of 5%. A CDS on this bond trades at 400 basis points.

1. What is the CDS basis on this bond?
2. Assuming constant hazard and recovery rates ( $R = 60\%$ ), what is the implied probability of default on this bond?

### **Problem 24**

Consider a convertible bond with a redemption value of 1,000, a conversion ratio of 13 and a stock price of \$80. The bond currently trades at par value. Accrued interest is zero. Assuming no transaction costs, is this a pure convertible bond arbitrage?

### **Problem 25a**

Consider a default-free convertible bond with one year (=one payment) to maturity, a redemption value of 1,000, a coupon rate of 10%, and a conversion price of \$100. The



stock price may be \$80 or \$120 a period from now with equal risk-neutral probability. The one-year S/A spot yield is 5%. The bond currently trades at \$1,125. Is this a convertible bond arbitrage in the sense of Ed Thorp?

### **Problem 25b**

Consider a convertible S/A bullet with exactly 10 years to maturity, an annualized coupon rate of 5%, and a redemption value of \$1,000. The bond currently trades at a price (invoice=clean, we just paid a coupon) of \$1,100. The market YTM on a bond with exactly the same terms but with no conversion features is 4%. What is the warrant value implied by the current market price?

### **Problem 26**

Get one year of ttm yield data for 30-year generics for AAPL and for MSFT.

1. Estimate the speed of convergence to the mean of the associated spread
2. (Python) Test the hypothesis of no convergence

### **Problem 27**

A bank has portfolio of fixed interest rate investments with market value \$100M and modified duration 10 years. Its liabilities have market value \$20M and a modified duration of 4. It wants to reduce its bloomberg risk to \$2M. To that end, it will add IRS' contracts each with a DV01 of \$ - 5,000. How many such IRS contracts does the bank need to enter into to achieve its goal?

### **Problem 28**

Same set up as the previous problem but this time the Bank wants to buy puts on a T-bond worth \$10M with a delta of -0.8 and a duration of 10. How many put options does the bank need to buy?

### **Problem 29**

An insurance company wants to bullet immunize a a payment of 100k in 8 years. It plans to use a 12-year S/A bullet with an 8-year Macaulay duration, a face value of 50k and a current yield of 10%. How many bonds (fractional investments ok) does the company need to invest in today?

**Problem 30**

Assume spot rates at tenors 6, 12, 18, and 24 months are 1, 3, 7, and 10 percent respectively, all in S/A simple terms. Compute and plot the three associated S/A forward curves.

**Problem 30b**

Assume zero-coupon treasury (STRIP) annually (not S/A!) compounded rates are 5% and 6% at maturity 1 and 2 years, respectively. Assume that one-year ahead, one year zero-coupon treasury forward contracts are available at a 9% annually compounded rate. Build an arbitrage portfolio.

**Problem 31**

The six month spot yield is 5 percent. The 6-month ahead forward curve is 4, 8, and 9 percent respectively, all in S/A simple terms, at tenors 6, 12, and 18 month. Compute the current spot yield curve at tenors 6, 12, 18, and 24 months.

**Problem 32**

Consider the following treasury bond menu:

Maturity	Coupon (S/A)	Price	YTM (S/A)
0.5	5	101.405	Y
1	5	102.653	2.301%
1.5	5	X	2.437%
2	5	104.713	2.567%

Coupons are shown in S/A annualized terms and in percent.

1. Find X and Y in the table.
2. Find the spot yield curve up to maturity 2 implied by the data above.
3. A 10% coupon bond with 6 months to maturity is trading a semi-annual YTM of 3%. Is this an arbitrage opportunity? Explain why or why not.
4. We now want to calibrate an interest rate model over the next year that replicates these bond prices. The interest rate over the first six months is known,

it is the spot rate you calculated above, label it  $z(0.5)$ . In the second period, interest rates can rise to  $z_H$  or fall to  $z_L$ . The risk-neutral probability of moves up or down at any node is 50%. The conditional volatility of interest rates is assumed to be such that  $0.5 \ln \frac{z_H}{z_L} = 10\%$ . Find  $z_H$  and  $z_L$ .

### Problem 33

Consider a mortgage pass-through security backed by a pool of mortgages with remaining face value \$M100. All those mortgages are FRMs of equal size with semi-annual payments (payments every 6 months), a yearly rate of 10% and mature in 12 months. Fraction  $0.02 + 2(0.03 - r)$  of the mortgages prepay after 6 month where  $r$  is the 6-month yield at the start of period 2. Then, at the end of one year, 2.5% of all remaining mortgages default. The average loss severity rate on these defaulting mortgages is 40%. The appropriate interest rate model for this environment is the model you derived in the previous question.

1. The security is currently trading at a Z-spread of 100 basis points under the assumption that 5% of mortgages will be prepaid in period 2. At what price is the security trading today?
2. What is the OAS implied by this price?

### Problem 34

Take the pool of IOM mortgages we used as an example in class (100 \$1M IOMs, all at rate 10%, half mature in year 1, half mature in year 2.) The issuer is considering writing a mortgage-backed bond against this pool with face value \$90M, a yearly coupon payment of 10%, and all principal due at maturity. MBB investors require a YTM of 9% from this kind of instrument, and underwriting costs are 2.5%. There are no other fees or source of income associated with this deal. The issuer wants a positive NPV at a discount rate of 15% from this transaction. Should they do this deal? Show your work.

### Problem 35

Take the same pool of mortgages as in the previous problem. The issuer is now considering issuing two CMBS tranches. The first tranche is an A tranche with face value \$60M. The second tranche is a Z tranche with face value \$30M. The A tranche

receives all principal payments from the pool plus all accrued interest payments from the Z tranche until it is fully retired. It also receives interest payments of 8% of its outstanding principal. The interest rate on the Z tranche is 10%. Assume that both tranches sell at par value.

1. Show the issuer's cash flows from this transaction.
2. There is a probability  $p$  that 50% of the pool's payments will be lost to default in the second year. The issuer wants an IRR of 10% from this transaction. What is the maximum likelihood of default compatible with this target IRR?

### **Problem 36**

Consider the first CMO example we discussed in class with three tranches: A, B and Z. Assume that 50% of all payments are lost to default each year. Principal available for the amortization of the top two tranche is the minimum of two numbers: 1) Principal from the pool + accrued interest on Z and, 2), Total pool payments minus interest on A and B.

Under this default scenario, how many years does it take to retire the A tranche? What IRR do B tranche holders earn?

### **Problem 37**

A benchmark consists of two sectors (1 and 2) with funds equally split between the two. Both sectors generated a return of 10% over a recent holding period. A portfolio manager constrained to the same holdings as the index invested 25% of her funds in sector 1, generated a return of 15% in that sector and an overall return of 13% during the same period. Measure the manager's excess return for the period and attribute it to allocation, selection, and interaction.

### **Problem 38**

In the same set-up as in the previous problem, the manager's portfolio had a duration of 4 in sector 1 and 6 in sector 2 at the start of the holding period. Her carry return was 3% in sector 1 and 7% in sector 2. The reference treasury rate (5-year) dropped 1% during the holding period while the DMT yield was unchanged during the period in sector 1 while it gained 0.5% in sector 2. Calculate and report the shift, twist and residual returns in each sector for this manager.

### Problem 39

Consider the Nassim Taleb screen shot at the end of chapter 3. You invest in a 12 month treasury today and sell it after 6 months. Assuming yields are unchanged during the period, what is your roll-down return? What is your total return? (Assume the 12-month bill has 360 days to maturity while the and will have 180 days to maturity left in 6 months.)

*Note: Measure roll-down return as total return minus the return you would get assuming yields are flat at the 12-month yield today (hence assuming no ride-down.) Bloomberg, recall, measures roll-down as total return minus carry return.*

### Problem 40 (Python)

Test the hypothesis of no-cointegration between the generic 30-year yields of S/A, fixed interest rate bonds issued by AAPL and MSFT using 12-month ttm daily data.

### Problem 41

Consider an asset swap involving a three-year bullet with yearly payments, a coupon rate of 5%, and that trades at a 10% discount. The reference rate is SOFR. SOFR-implied discount rates are 0.99, 0.98, and 0.97 for the next three years.

1. Compute 1-year forward SOFR rates for year 2 and year 3
2. Using those forward rates are your forecast for year 2 and year 3 SOFR rates, compute what the swap spread should be in the way Bloomberg would given those assumptions.

### Problem 42

In the previous problem, compute the z-spread associated with the underlying bond and compare it to the ASW spread you obtained.

*Hint: z-spreads are over current spot yields, so the first thing you need to do is convert the forward rates above into spot yields today. Here, since the yield curve is very flat, there is little difference between forward rates and current spot yields, but in general those two curves can differ a lot.*

### Problem 43

A CDS contract on an underlying bond expires on 12/15/2027. Today is 4/11/2023. The CDS spread is 100 basis points. The expected recovery rate in the event of default is 50%. What is the implied probability of default over the life of this contract as Bloomberg would compute it?

#### **Problem 44**

Consider a bond with six months left and one payment left to maturity. The six-month spot yield is 10%, in annualized S/A terms. If and only if interest rates go up in 6 months, the bond defaults in which case 5% of the one remaining payment is lost. The z-spread is 600 basis points (computed assuming all payments are made as planned). What is the OAS in annualized S/A terms?

#### **Problem 45**

A corporation has the option to prepay (call) a bond with 4 years to maturity, \$50M in remaining principal, a 10% yearly rate, fixed and monthly payments. It can replace this bond with a 4 year bond with the same remaining payment structure. Prepayment penalties are  $c = \$750,000$ .

1. How low must the yearly rate on the new bond be to justify calling the old bond (ignoring the option value of waiting to refi)?
2. What yield-maintenance (a.k.a makewhole) rate  $r_{ym}$  is equivalent to the above prepayment penalty?

#### **Problem 46**

A S/A BDT tree has been calibrated with  $z(0.5) = 3\%$ ,  $z^L = 2\%$ , and  $z^H = 5\%$ , all in annualized terms. What is  $z(1)$ , the one-year spot yield?