



Mortgages



HEC Paris, 2012

Basics

- Mortgages are loans secured by a real estate asset:
 1. Commercial vs. Residential
 2. Permanent vs. construction
 3. CMBS loans vs. portfolio loans
- Two parties: *mortgagor* (borrower), *mortgagee* (lender)
- Two basic components:
 1. Promissory note: stipulates payment obligations
 2. Mortgage deed: stipulates claims to collateral



The question

- Mortgage pricing and approval would be trivial absent default and prepayment risk:
 1. Measure opportunity cost of capital
 2. Make that the contract rate or more generally the loan's APR (YTM)
- How should mortgages be priced when default and prepayment are a possibility?



Annual Percentage Rate (APR)

- YTM from the lender's viewpoint
- Loan's IRR from the point of view of the lender if all payments are made as planned
- On any mortgage with fixed rates (whether or not payments are fixed) and no "points", $YTM = \text{contract rate}$



Mortgages

- Mortgage: debt contract secured by a real estate property
- Characteristics:
 1. Initial balance or principal (b_0)
 2. Maturity (T)
 3. Yield (or contract rate) structure (r_t , for all periods t)
 4. Payment structure (m_t , for all periods t)
- Mechanics:
 1. At a given date, interest due is $b_{t-1} r_t$
 2. $b_t = b_{t-1} + b_{t-1} r_t - m_t$
 3. If $b_T > 0$, balance is due in one *balloon payment*



FRMs: fixed-rate, fully amortizing mortgages

- For all t :
 1. $r_t = r$
 2. $m_t = m$
- Fully amortizing: $b_T = 0$
- What must m be? (Fixed annuity formulae)
- $m = b_0 r / (1 - (1+r)^{-T})$



The lender's perspective

- Full amortization means:

$$b_T=0, \quad \text{or, equivalently,} \quad b_0 = \sum_{t=1, \dots, T} m_t / (1+r)^t$$

- More generally:

$$b_0 = \sum_{t=1, \dots, T} m_t / (1+r)^t + b_T / (1+r)^T$$

- Absent points and whether or not amortization is full, r is the loan's IRR if all payments are made, i.e. the APR or YTM



Fixed payment example

- 100K, monthly payments, 10 years, $r=7\%$
 1. With full amortization: $m=\$1,161.08$
 2. With 30K balloon: $m=\$987.76$



Constant-amortization mortgages (CAMs)

- Each period, principal payment is b_0/T :
 - $b_t = b_{t-1} - b_0/T = b_0 - (t/T) b_0$
 - $m_t = b_0/T + b_{t-1} r_t$
- Popular for a while after great depression, rare today
- Possible advantages over traditional FRMs:
 1. Less default risk
 2. Less prepayment risk



Graduated-payment mortgages (GPMs)

- Low initial payment
- Fixed number of steps, fixed size of increment (“step-ups”)
- Example: at the end of each of the first 4 years, payment goes up by 7.5%, fixed contract rate
- Can feature negative amortization initially



GPM math

- Guess initial payment m_1 and update guess until $b_T=0$
- Adjust m_1 until present value at contract rate is b_0
- Simple trick:
 1. Calculate PV of payments if first payment is \$1
 2. Divide b_0 by resulting factor to get m_1
- Example: 30-yr GPM with 4 annual step-ups of 7.5% each, 12% contract rate, monthly payments
- If first payment is \$1, PV of loan at 12% annual discount rate is \$121.12
- Therefore, $m_1 = b_0 / 121.12$, for any b_0



2-period GPM example

- A client wants a 100K mortgage with 2 yearly payments ($T=2$), a 30K balloon, a rate of 7%, and payments that increase by 50% in year 2. What are the two payments?
- One unknown: m_1
- Indeed, $m_2 = 1.5 m_1$
- One equation:
$$100K = m_1/1.07 + 1.5m_1/1.07^2 + 30K/1.07^2$$
- $m_1 = \$32,875.49$, $m_2 = \$49,313.24$



Another 2-period example

- ▶ 2-year, \$100,000 mortgage, yearly payments
- ▶ Payments increase by 10% each year
- ▶ The first payment will be made for sure
- ▶ There is a 20% chance that the second payment will not be made. In that case, the bank expects to recover 50% of the remaining balance plus unpaid interest.
- ▶ What rate must the bank charge to generate a 10% IRR on this loan?



Adjustable rate mortgages (ARMs)

- Interest rate adjusts at fixed frequency as a function of a given market interest rate (1 year CMT rates, LIBOR...)
- Payment in a given period is calculated in FRM fashion assuming that the current rate will prevail to maturity
- ARM stipulations:
 1. r_1
 2. Adjustment interval: 1 year, 3 years, 5 years
 3. Index: publicly observable market interest rate index
 4. Margin: $r_t = \text{index}_t + \text{margin}$
 5. Caps and floors (lifetime, or max adjustment)
 6. Full indexation: $r_1 = \text{index}_1 + \text{margin}$
 7. Teaser rate: $r_1 < \text{index}_1 + \text{margin}$

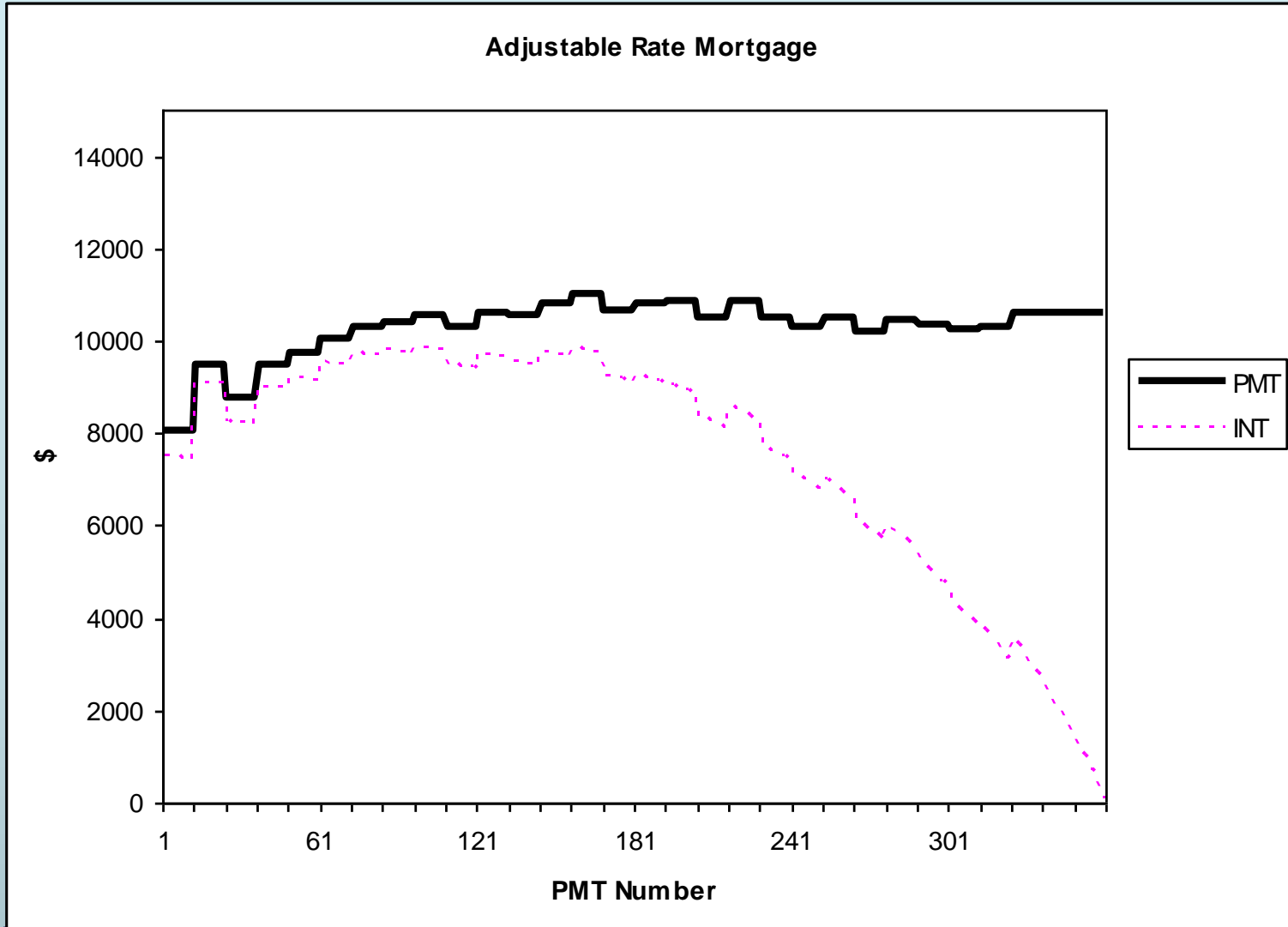


Mortgage schedules for ARMs

- Future rates on ARMs are not known
- **One** can produce a payment schedule based on index forecasts
- In practice, people use current value of index, assume it will remain where it is, and compute all contract rates
- If the loan is fully indexed, this gives you the same table as a standard FRM
- If the loan features teaser rates, rates and payments rise in full at first adjustment if there are no caps, in several steps if there are binding caps



Typical path for ARM payments



Annual Percentage Rate (APR)

- YTM from the lender's viewpoint
- Loan's IRR from the point of view of the lender if all payments are made as planned
- On any mortgage with fixed rates (whether or not payments are fixed) and no "points", $YTM = \text{contract rate}$



APRs on ARMs

- In principle, APR depends on expected path of market rates
- In practice, US government regulations require that the “official” APR reported for ARMs be based on a flat forecast of market interest rates
- If there is a teaser rate, APR must be calculated under the fastest possible path to fully indexed rate



Example

- 5-year ARM, \$100,000, 2% margin over a market index that can be either 8% or 10%
- Teaser rate of 6%, two resets (Months 13 and 25), no caps
- Index begins at 8%
- 40% chance that it will change value to 10% by first reset, 40% that it will change value again by second reset
- 4 possible histories for the index: high-high (10%-10%), HL, LL, LH
- Hence 4 possible histories for the payments



Points

- Payments from borrower to lender at origination
- 1 point = 1% of initial balance
- Does not reduce initial balance (not a down-payment)
- Effective loan size = $b_0(1-n)$, where n is the number of points at origination
- Raises lender's YTM (APR) above contract rate
- Indeed: $b_0 = PV(\text{payments, contract rate})$
while $b_0(1-n) = PV(\text{payments, APR})$
- $APR > \text{contract rate}$



Why do we see points?

- Points, all else equal, reduce the contract rate (**that the lender is willing to offer**)
- In PV terms, borrower only recovers their initial fees if they stick with the loan until maturity
- Points discourage prepayment
- Borrowers who know they are not going to prepay can use points to convey their type to lender, and secure better terms
- Alternative to prepayment penalty



Prepayment risk

- Borrowers prepay loans for a variety of reasons
- If prepayment occurs when market rates are below the contract rate, this causes losses for lender
- In fact, refinancing gains are one of the main reasons for prepaying
- Borrower's refinancing gains = Lender's prepayment loss
- This makes prepayment risk a very bad form of reinvestment risk



Refinancing

- Consider a borrower with $(T-k)$ payments left
- Assume that refinancing carries a fixed cost $c > 0$ for the borrower
- This cost includes transaction costs and penalties
- Assume the borrower's current (fixed) payment is m , and that rates fall in a way that she can make remaining payments $m' < m$
- The gain is the present value of $m - m'$, to maturity
- Discount rate: new market rate on a loan of maturity $T - k$
- Refinancing is potentially beneficial if $PV(m - m') > c$



Refinancing example (part 1)

- Consider a 15-year FRM with initial balance \$100,000 and contract rate 9%
- After 5 years, rates on 10-year FRMs are 8.5%
- Refinancing costs \$1000
- Assuming that refinancing is a one-time only option in this case, should the borrower refinance?



Refinancing example (part 2)

- Assume that at origination the borrower could have picked a loan with two points and the same APR
- Would it make sense to refinance had the borrower taken that loan?



Timing

- Refinancing is a call option:
 1. Strike price: loan balance + refinancing costs (c)
 2. Value of underlying asset: PV of remaining payments at the new rate
- Exercising the option kills it
- “Refinance if $PV(m-m') > c$ ” may not be optimal decision
- It may make sense to wait until $PV(m-m')$ rises further



Option value of refinancing

- A call option's value is high when:
 1. the strike price is low relative to the expected value of the underlying asset
 2. the value of the underlying asset is volatile
- The refinancing option is particularly valuable when:
 1. contract rate is high relative to market rates, mortgage is far from maturity, penalties are low...
 2. interest rates are volatile



The option to delay

- In previous refi example, assume that the lender has the option to wait another 24 hours
- Tomorrow, rates will be either 8.25% or 8.75%
- The risk free rate during that period is 0.005%
- What is the value of the option to delay? (*Binomial option pricing formula says \$810 or so*)
- Should the borrower wait another 24 hours?



Prepayment from the lender's viewpoint

- Lenders need to forecast, for each period:
 1. Prepayment *hazard rate*
 2. Prepayment losses and/or *yield degradation*
- Date t hazard rate: likelihood of a prepayment at date t , given no prepayment prior to date t
- *Yield degradation*: Loss in IRR for lender if prepayment occurs
- Yield degradation conditional on prepayment at date $t =$
APR- IRR conditional on prepayment event at a given date



Refinancing example (part 1)

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- After 5 years, rates on 10-year FRMs are 8.5%
- Refinancing costs \$1000
- What is yield degradation if the borrower refinances after 5 years?



How lenders deal with prepayment

1. Prepayment penalties
2. Points
3. A contract rate premium (*fixed point problem*)



Lockout/Yield Maintenance clauses

- Lockout clauses prohibit early prepayments regardless of borrower's ability to pay off the loan in its entirety
- A yield maintenance clause requires the borrower to make a lump sum payment to cover the lender's potential loss from reinvesting prepaid sums.
- Typical on CMBS loans, making prepayment essentially a non-issue on those loans



Default

- On commercial loans, default is the primary concern
- Expected cash-flows depend on 1) the likelihood of default and 2) the likely size of losses in the event of default
- Lenders need to forecast both objects



Hazard rates

- h_t = probability that the loan will default in period t conditional on not having defaulted before
- Probability that the loan will default after exactly t periods is $(1-h_1) (1-h_2) (1-h_3) \dots (1-h_{t-1}) h_t$
- This gives $T+1$ mutually exclusive events, with associated probabilities that sum up to 1

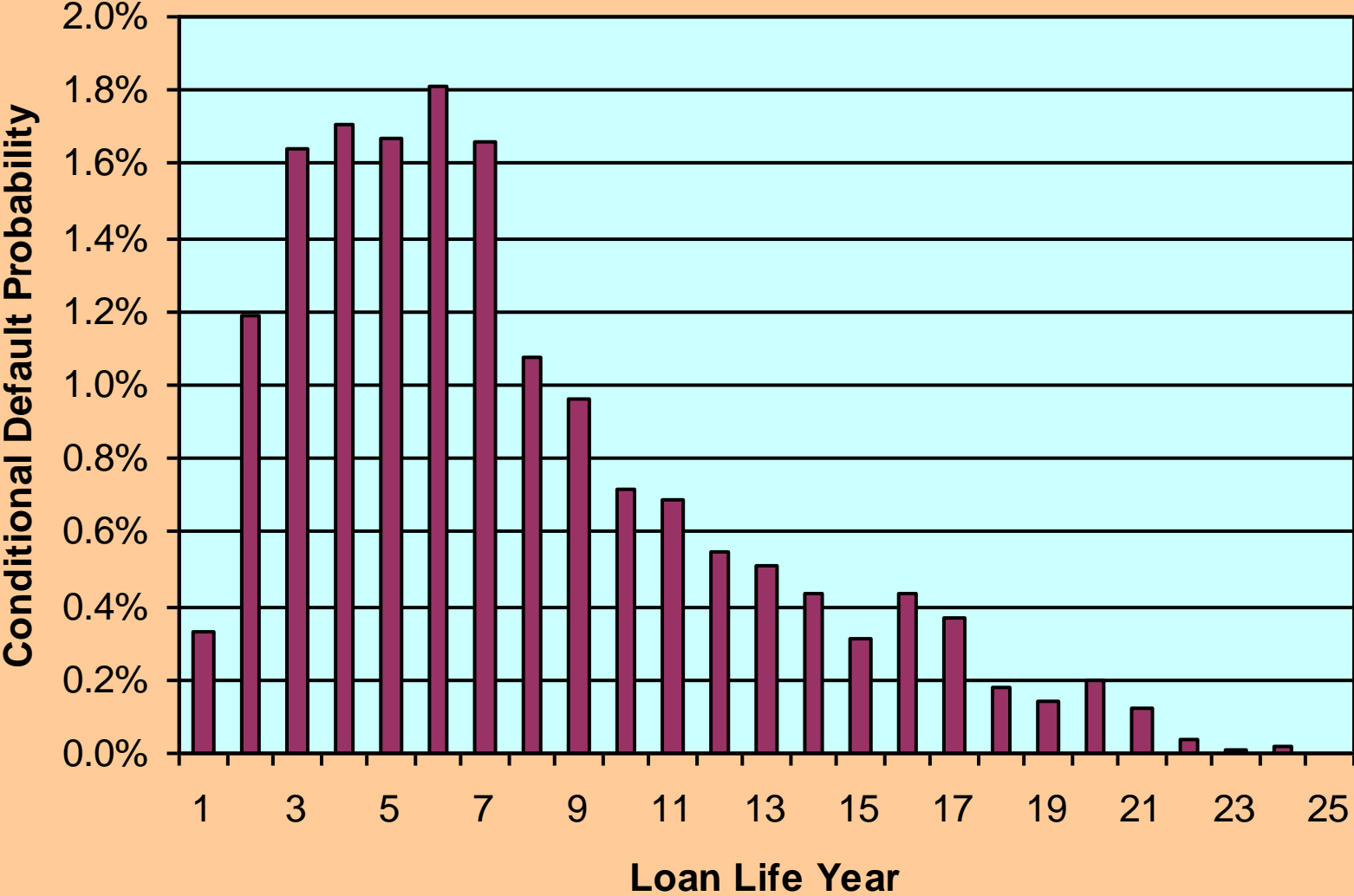


How do lenders forecast hazard rates?

- Use industry standards (SDA: standard default assumptions, scaled up or down)
- Or use econometrics: $h_t = f(\text{loan characteristics, property type, location, borrower characteristics, economic conditions...})$
- Loan characteristics: LTV, DCR (debt-coverage ratio)
- Borrower characteristics: ownership type
- Fit f to historical loan data and hope that past is informative for future



Typical Commercial Mortgage Hazard Rates*



*Source: Esaki et al (2002)

Loss severity rates / Recovery rates

- Date t *loss severity rates* are expected losses if default takes place at date t , as a fraction of outstanding balance
- *Recovery rates* are the opposite: the fraction of the balance the lender expects to recover if default takes place at date t
- Forecast using the same two methods as hazard rates



Why is default so costly?

- Loss severity rates can exceed 50%, and typically range from 30 to 40% on commercial loans
- Many causes:
 1. Transaction costs
 2. Payment delays
 3. Low foreclosure proceeds
- It is estimated that residential properties sell at a 25% discount on average when foreclosed relative to observably similar properties that have not foreclosed



Conditional yield degradation

- Yield degradation if default occurs at date t =
$$\text{YTM} - \text{IRR if default at date } t$$
- Consider a 3-year IOM loan with initial balance \$100,000 and contract rate 10%
- Year 3 loss severity is 30%, so that the lender only expects to recover \$77,000 = \$110,000 \times (1-0.3) in year 3
- IRR in that case is -1.12%
- Yield degradation = 10% - (-1.12%) = 11.12%



Expected return

- Expected return = $\sum_t P(\text{default at } t) \times (\text{YTM} - (\text{Yield Degradation})_t)$
+ $P(\text{no default}) \times \text{YTM}$
- $E(\text{IRR}(\text{CFs}))$
- In IOM example, assume that default occurs with probability 10% in year 2 and year 3, with loss severity 30% in either case
- Expected return = $.10 \times (-17.11\%) + .10 \times (-1.12\%) + .80 \times 10\%$
= 7.18%
- Average Yield Degradation = $\text{YTM} - \text{Expected Return}$
= $E(\text{Yield Degradation})$



A better measure

- True IRR is IRR(Expected Cash Flows) which can differ greatly from expected return
- In IOM example, assume again that default occurs with probability 10% in year 2 and year 3, with loss severity 30% in either case



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Year	0	1	2	3	IRR
Default at date 2 (10%)	-100000	10000	77000	0	-7.11%
Default at date 3 (10%)	-100000	10000	10000	77000	-1.12%
No default (80%)	-100000	10000	10000	110000	10.00%
Expected CF	-100000	10000	16700	95700	7.82%



Pricing mortgages with default

- Assume that lender wants to hit a given IRR on a loan
- Contract rate must exceed this IRR target because of default
- Problem: when contract rate increases, so do default probabilities
- There may be many solutions to this problem (which do we choose?) or no solution (exclusion)



Example

- 3-year FRM, yearly payments, initial balance of \$100,000
- Default hazard rate on the mortgage in each year is:
 $[3 + m/40,000] \%$
- Loss severity: 25%
- Target **IRR**: 10%
- Is there a contract rate that delivers the right **IRR**?
- Can the right **IRR** be delivered with a contract rate of 10% and positive points?

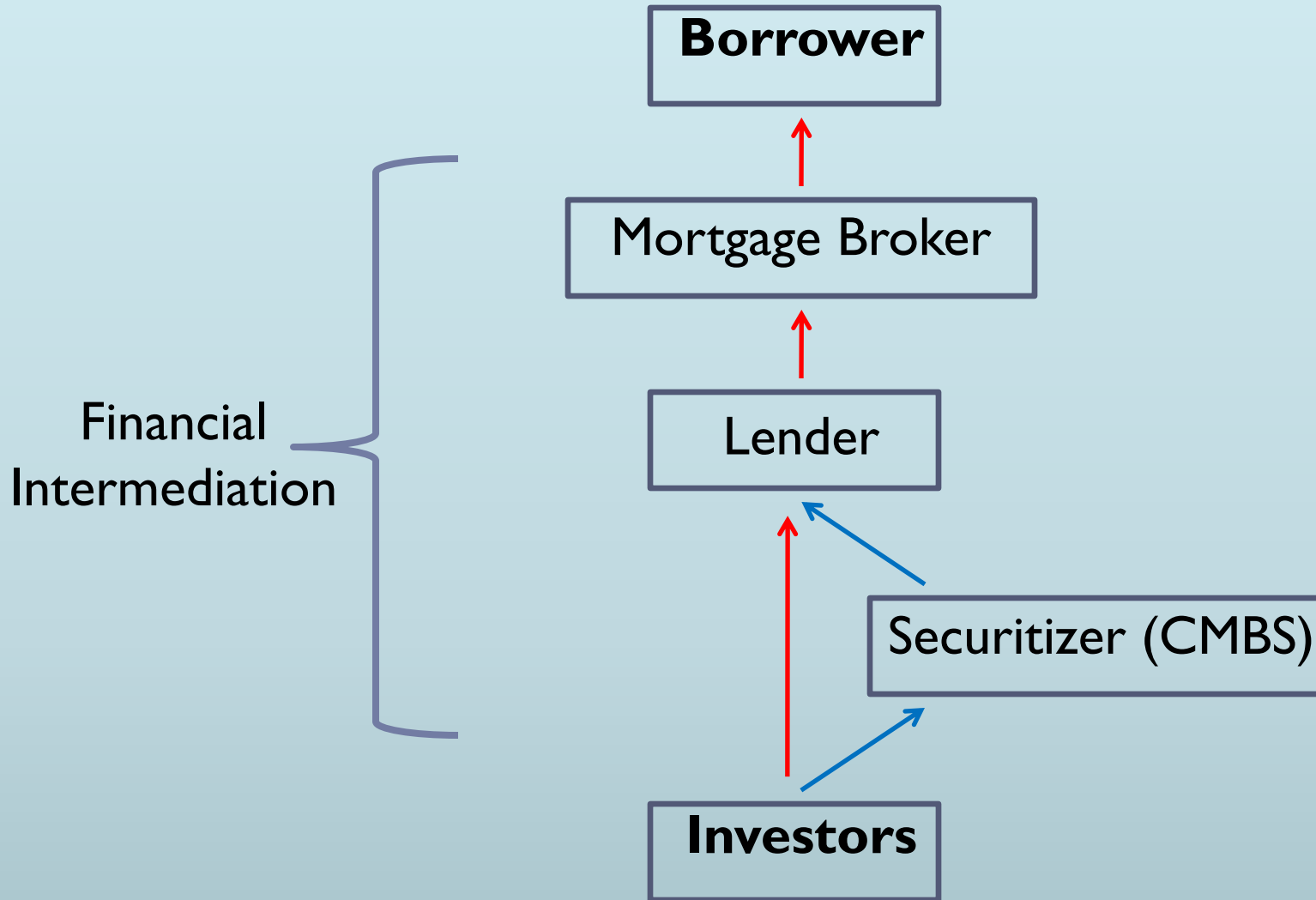


Example with no solution (exclusion)

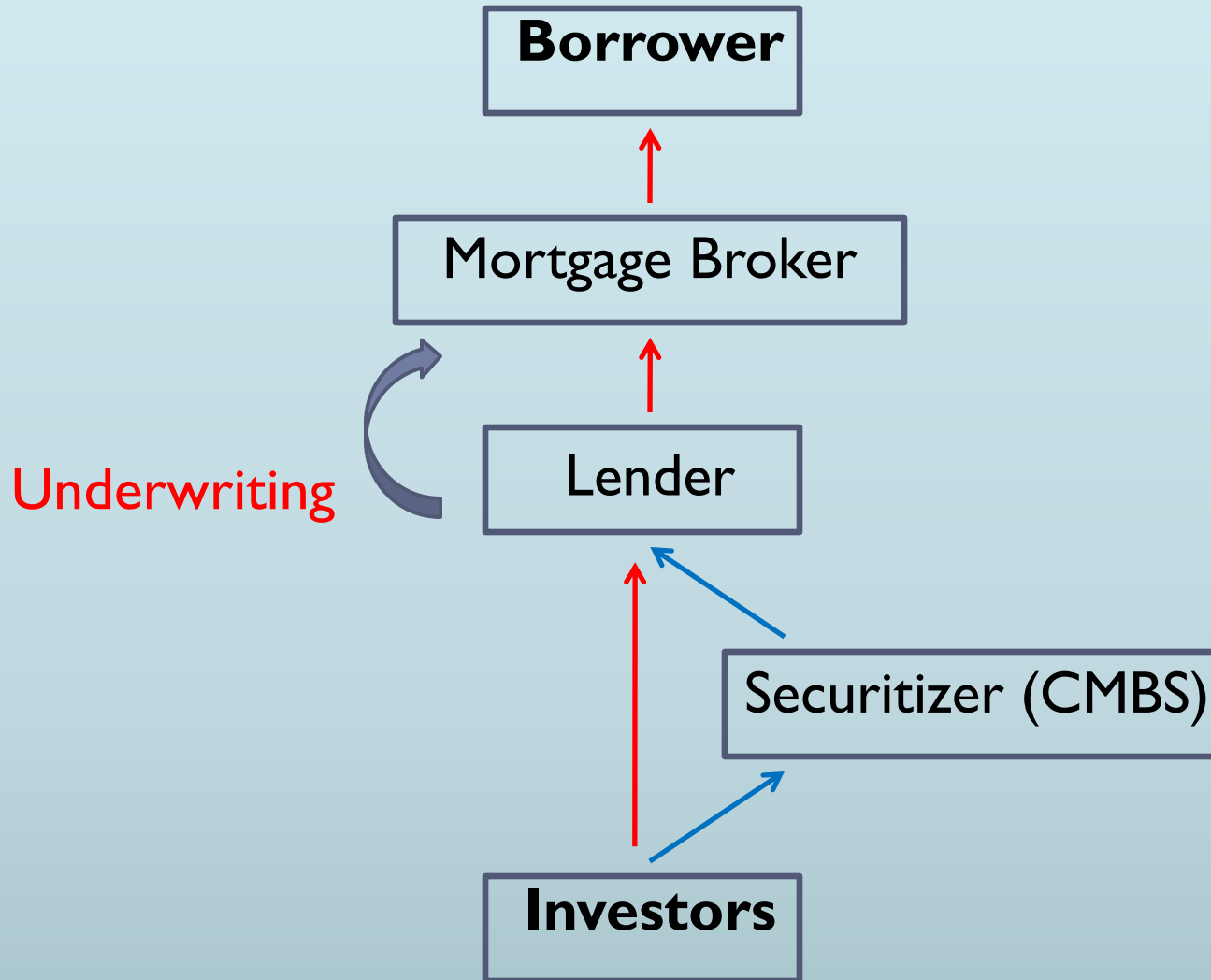
- 3-year FRM, yearly payments, initial balance of \$100,000
- Loss severity is 40%
- Default hazard rate on the mortgage in each year is:
 $[2 + (m/10,000)^2] \%$
- Then, it is not possible to hit a target of 10%
- Hazard rates rise too fast as we try to raise the payment
- This borrower is too risky
- At lower targets, a different problem may arise: multiple solutions
- This second problem is an easy one to deal with



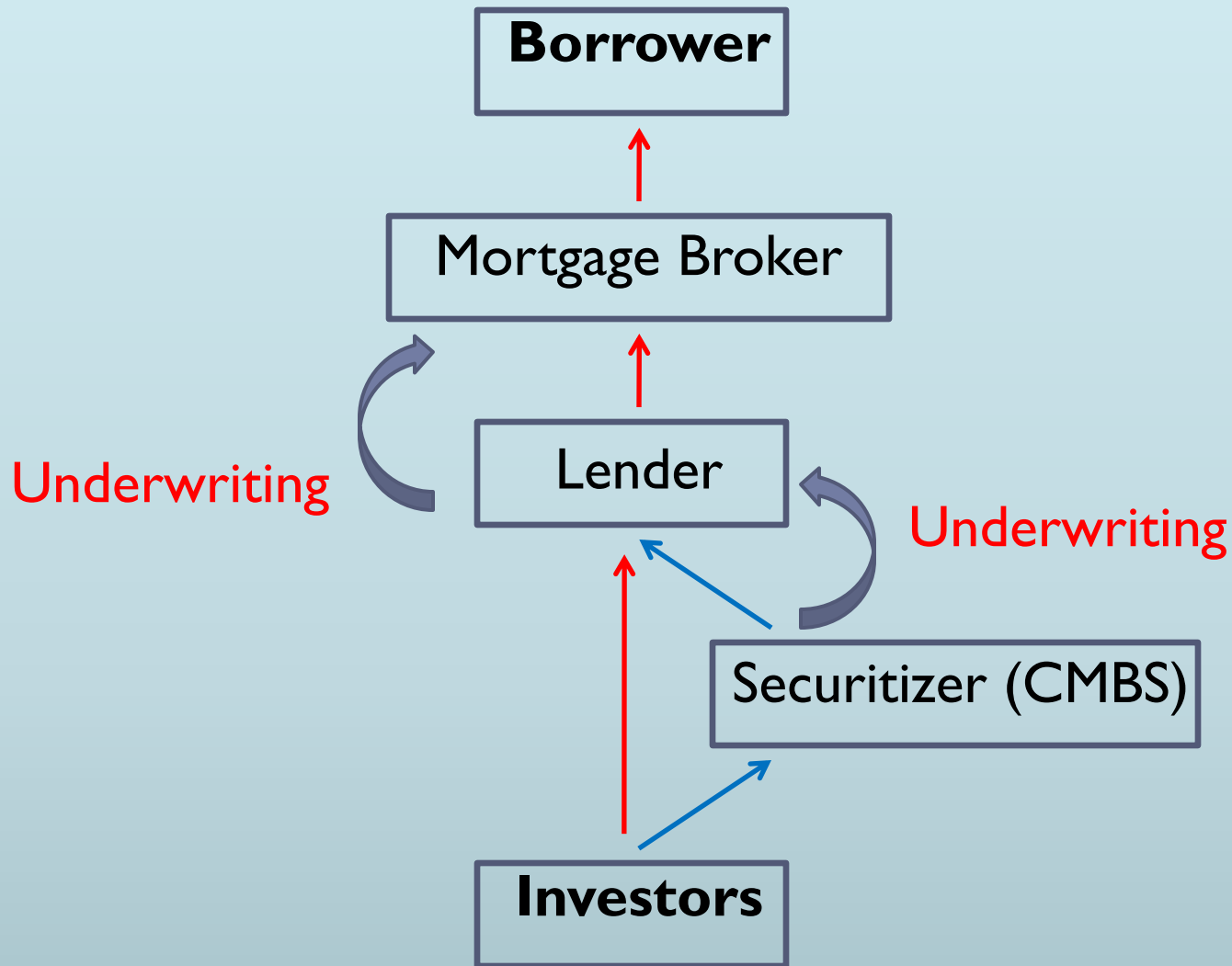
The mortgage process



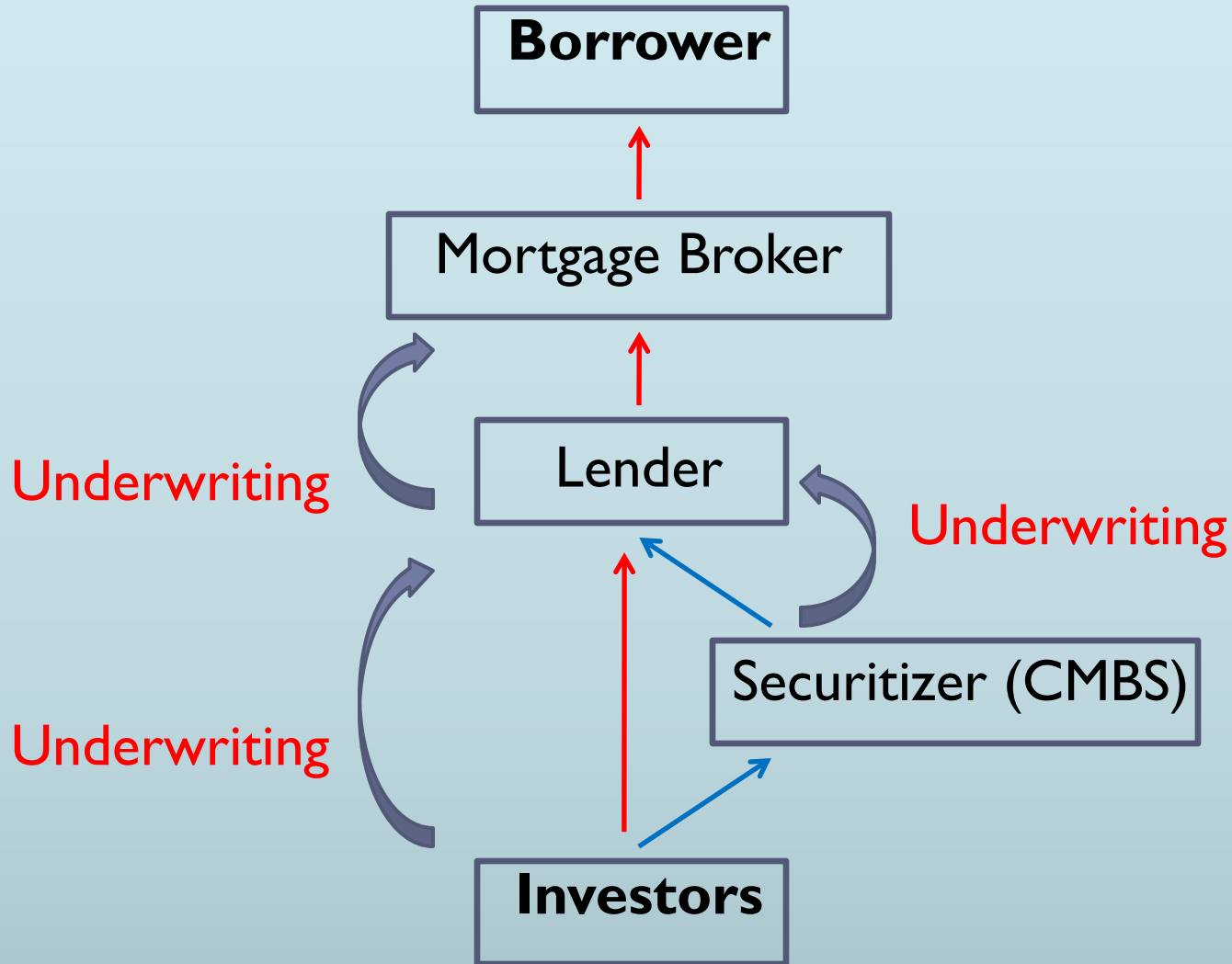
Underwriting criteria



Underwriting criteria



Underwriting criteria



Underwriting criteria

- Lenders tell brokers what they'll fund:
 1. Leverage (loan-to-value ratio)
 2. Credit-worthiness of borrowers
 3. Proper documentation
 4. Ratio of projected cash-flows to debt-service
 5. ...
- Likewise, securitizers tell lenders what they'll buy
- When secondary markets are involved, lenders pass underwriting standards on to brokers



Standard numerical criteria

- *Loan-to-value ratio (LTV)* = Loan size / Market value, at origination, and at termination
- *Debt Service Coverage Ratio (DCR)* = NOI / Debt service
- *Break-even ratio (BER)* = (DS + OE) / PGI
- Lower bound on EBTCF



Underwriting standards

- Besides specific numerical criteria, underwriting standards also specify:
 1. What information must be provided (type of financial statements horizon, borrower information...)
 2. Appraisal source and method
 3. How financial ratios need to be calculated



Example (GM, section 18.2.3)

- Borrower & seller claim property is worth \$12,222,000
- Buyer wants to borrow 75% (\$9.167 Million, or \$91.67/SF)
- Wants non-recourse, 10-yr interest-only loan, monthly pmts
- Willing to accept “lock-out” (No prepayment)



Opportunity cost of capital

- Mortgages of this broad type are going for a “spread” of 200 basis points over 10-year treasury yields, currently at 6%
- This mortgage should yield 6% + 2%, in *bond-equivalent yield (BEY) terms* (**see GM, chapter 8**)
- This means that the effective yield on loans of this sort is
$$(1 + 8\%/2)^2 - 1 \approx 8.17\%$$
- If i is contract rate on a monthly mortgage, effective annual rate is: $\text{EAR} = (1 + i/12)^{12} - 1$
- Contract rate must solve $i = 12 \times [(1.0817)^{(1/12)} - 1] \approx 7.87\%$



Question

- Premium is a reward for liquidity differential vis-à-vis treasuries, and for default risk, for typical loan of this sort
- Why don't we stop here? Why aren't we done?



Underwriting criteria

- Max initial LTV at origination = 75%
- Max projected terminal LTV = 65%
- In computing LTV, use:
 - ✓ going-in NOI cap rate of 9% or more
 - ✓ terminal NOI cap rate of 10% or more
 - ✓ multi-yr DCF with Disc. Rate of 10% or more applied to PBTDCF
 - ✓ Lowest resulting value
- Min DCR = 120%.
- Max BER = 85%, or, if less, 5% less than market vacancy
- “Avoid” EBTDCF < 0



Property information

- 100,000SF, fully occupied, single-tenant, office building.
- 10-yr lease signed 3 yrs ago.
- \$11/SF net (NOI=EGI)
- "Step-ups" of \$0.50 in lease yr.5 & 8
- Current mkt rents on new 10-yr leases are \$12/SF net
- Expect mkt rents to grow @ 3%/yr. (same age)



Underwriting steps

1. Build a 10-year pro-forma
2. Calculate DCR, BER, EBTCF and ask if income criteria are met
3. Estimate initial and final property value and ask if value (LTV) criteria are met
4. If compliance fails, ask if loan modifications can be found that make loan acceptable



Pro-forma

- **Assume:**
 1. 1% rent growth
 2. Lease renewal probability is 75%, \$2/SF commission if renew, 5\$/SF otherwise, \$10/SF in TI if renew, \$20/SF otherwise
 3. 10% year 10 cap rate
- **Then:**



Pro-forma

- Assume:
 1. 1% rent growth
 2. Lease renewal probability is 75%, \$2/SF commission if renew, 5\$/SF otherwise, \$10/SF in TI if renew, \$20/SF otherwise
 3. 10% year 10 cap rate
- Then:

Year:	1	2	3	4	5	6	7	8	9	10	Year 11
Mkt Rent (net) /SF	\$12.12	\$12.24	\$12.36	\$12.49	\$12.61	\$12.74	\$12.87	\$12.99	\$13.12	\$13.26	\$13.39
Property Rent(net)	\$11.00	\$11.50	\$11.50	\$11.50	\$12.00	\$12.00	\$12.00	\$12.99	\$12.99	\$12.99	\$12.99
Vacancy Allow	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.00	\$0.81	\$0.00	\$0.00	\$0.00
NOI/SF	\$11.00	\$11.50	\$11.50	\$11.50	\$12.00	\$12.00	\$12.00	\$12.18	\$12.99	\$12.99	\$12.99
NOI	\$1,100,000	\$1,150,000	\$1,150,000	\$1,150,000	\$1,200,000	\$1,200,000	\$1,200,000	\$1,218,214	\$1,299,428	\$1,299,428	\$1,299,428
Lease Comm	\$0	\$0	\$0	\$0	\$0	\$0	\$0	-\$275,000	\$0	\$0	\$0
Ten.Imprv	\$0	\$0	\$0	\$0	\$0	\$0	\$0	-\$1,250,000	\$0	\$0	\$0
Reversion@10%Cap										\$12,994,280	
Less OLB										\$9,167,000	
PBTCF	\$1,100,000	\$1,150,000	\$1,150,000	\$1,150,000	\$1,200,000	\$1,200,000	\$1,200,000	-\$306,786	\$1,299,428	\$14,293,709	
Debt Svc	-\$721,443	-\$721,443	-\$721,443	-\$721,443	-\$721,443	-\$721,443	-\$721,443	-\$721,443	-\$721,443	-\$9,888,443	
EBTCF	\$378,557	\$428,557	\$428,557	\$428,557	\$478,557	\$478,557	\$478,557	(\$1,028,229)	\$577,985	\$4,405,266	
DCR	152%	159%	159%	159%	166%	166%	166%	169%	180%	180%	
BER @ Mkt	60%	59%	58%	58%	57%	57%	56%	56%	55%	54%	

Issues

1. Negative EBTCF in year 8 (minor)
2. At 9% cap rate, loans meets the ILTV criterion, but PV(PBTCF) @ 10% is \$11,557,000, which makes ILTV 79%
3. Terminal LTV @ 10% NOI cap rate is too low



Solutions

1. Include a covenant in contract under which borrower commits to set funds aside for large CAPEX in year 8 (*sinking fund covenant*)
2. Lower loan size
3. Use a loan with some amortization, and debt service characteristics that remain compatible with income criteria



But seriously...

- On the residential side where loans are more or less standardized, standardized underwriting works fine (?)
- In fact, automated underwriting has come to dominate in the residential market
- On the commercial side, ok for boiler plate deals and when the seas are calm
- Big, unique deals require more serious calculations
- Those serious calculations are where underwriting criteria come from in the first place

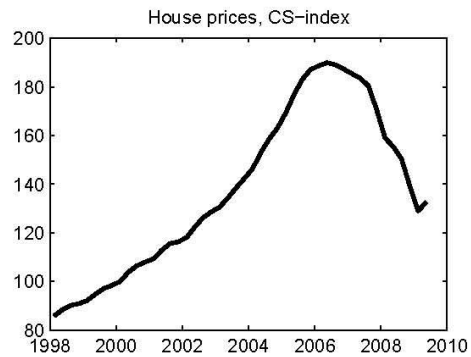
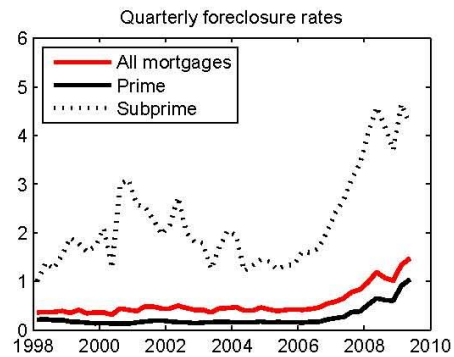
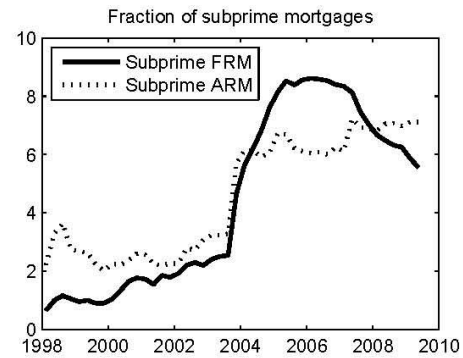
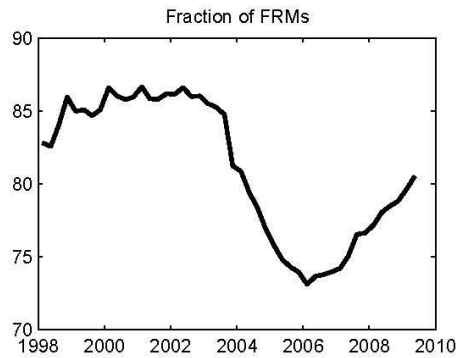


The foreclosure crisis

- Between 2004 and 2006, underwriting standards were greatly loosened in the residential market (why?)
- Low-down payment, delayed amortization products gained ground
- High-risk borrowers entered market, and products with slow build-up of equity proliferated
- When house prices collapsed in mid-2006, foreclosure rates skyrocketed like never before



Recent trends in US housing



Source: National Delinquency Survey, Mortgage Bankers Association

Solving the foreclosure crisis

- Almost all foreclosure involve negative equity...
- ... but most households with negative equity do NOT foreclose absent something else
- Second trigger: income difficulties (e.g. job loss)
- Obama plan subsidizes loan modification. It won't help much.
- Instead, plan should offer mortgage payment vouchers to households with verifiable income difficulties
- This is the Wi-Fur/Boston Fed plan

