



Preliminaries



Real estate capital markets

(a) Real Estate Assets

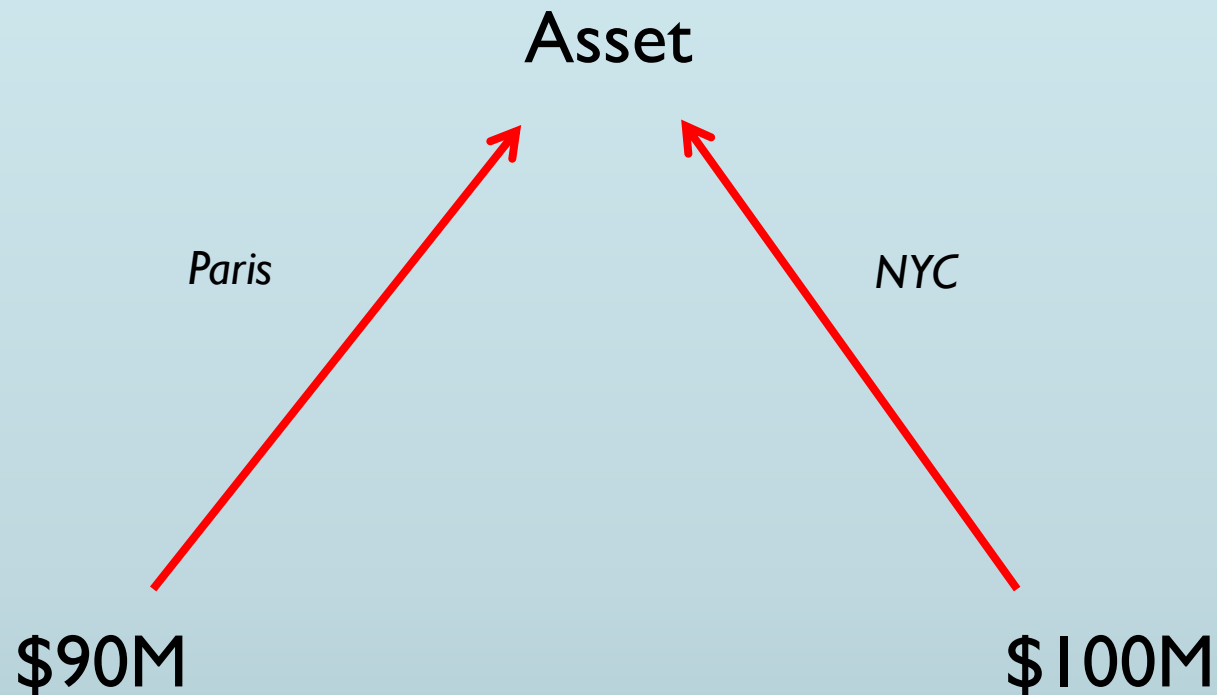


The question

- How should one price real estate assets?
- Asset: store of value with well defined property rights
- A title to a string of cash flows (or payoffs) to be received over time, and subject to some uncertainty
- Two basic tasks:
 1. Describe the distribution of payoffs (i.e. *forecast*)
 2. Price that distribution
- Arbitrage principle: “similar” assets should be priced in such a way that they earn similar returns
- Otherwise...



Arbitrage opportunities



Opportunity cost of capital

- Investing in a given asset is foregoing the opportunity to invest in other assets with similar properties
- Investor should be compensated for foregoing that opportunity
- Asset under consideration, therefore, should yield at least the same return as other similar assets



Main asset pricing recipes

1. **Discounted cash flow approach**
 - a. Write asset as a string of expected cash flows
 - b. Find return similar assets earn
 - c. Discount cash flows using that rate

2. **Ratio/Peer Group/Multiple approach**
 - a. Find a set of similar assets, with known value
 - b. Find average value/key statistic ratio
 - c. Apply that ratio to asset under consideration



The multiple approach in real estate

- Find a group of comparable properties ('Comps') with known value
- Comparable: similar location, purpose, vintage...
- Compute average ratio of value to gross rental income (**Gross Rent Multiplier** approach)
- Compute average ratio of Net Operating Income (NOI) to value, a key ratio known as the **Capitalization Rate**
- Get an estimate of the current Gross Rent and NOI for your target property, and apply ratio



Example

- A target property has a NOI of \$400,000
- You have obtained the following two recent sales data:

	NOI	Selling price
Property 1	\$424,200	\$4,200,000
Property 2	\$387,200	\$3,400,000

- What is the estimated value of your target using the cap rate approach (assign equal weights to the two sales)?
-



Solid comp case:

1. Quality of the comparables
2. Consistency of calculations
3. Good treatment of outliers



Sources for real estate comps/multiples

1. Databases of recent transactions: RCA analytics, Costar...
2. Survey data: PwC, RERC, CBRE, Costar...
3. “Fundamentals”



NOI vs. EBITDA

- $\text{NOI} = \text{operating income} - \text{operating expenses}$
- Like EBITDA, a fuzzy notion
- My preference is to figure cash operating expenses only, making my NOI equivalent to “Normalized EBITDA”
- But not everybody agrees...



NOI vs. PBTCF

- $\text{NOI} = \text{Operating income} - \text{Operating expenses}$
- $\text{BT bottom line} = \text{NOI} - \text{Capital Expenses}$
 $= \text{Property Before Tax Cash Flow}$
 $= \text{PBTCF}$
- Before-tax IRR is the discount rate that makes the PV of all future PBTCF equal to the property's price



The holy trinity of real estate

- Consider a property with current PBTCF cap rate $y\%$
- Assume that PBTCF is expected to grow by $g\%$ for ever
- Then the before-tax IRR associated with buying this property is:

$$r = y + g$$



Cap rate “fundamentals”

- NOI yield \approx required return (r)
 - expected income growth (g)
 - + investment rate (CAPEX/V)

- Required return =
 - + real risk-free rate
 - + expected inflation
 - + risk premium
 - + liquidity premium



Example: Is Manhattan office overvalued?

- Cap rates on Manhattan office have fallen back to pre-crisis levels
- Could spell trouble, but...
- ...PwC survey (Q3-2012) is consistent with current valuations:

Required return (r)	7.44%
- Cap rate (<i>PBTCF</i> or <i>NOI</i> ?)	- 5.25%
- Rent growth (g)	- <u>3.67%</u>
	< 0

- ... and spreads over treasuries have actually risen
 - ... though not as much as in other markets
-



Real estate assets

- Bedrock: real estate properties (land + structures affixed to it)
- Residential (deliver housing services) or Commercial (held for a business purpose)
- Real estate properties are strings of cash flows
- Real estate *assets* are all assets whose payoffs derive -- however remotely -- from some underlying property



Some language

- Debt: financial contract that gives specific claims to asset's payoff, but no ownership rights
- Equity: financial contract that gives only a residual (or subordinated) claim to asset's payoff, but carries ownership rights
- Public Markets: Markets with many buyers and sellers, observable transaction prices and sizes, and stringent disclosure rules
- Private Markets: Markets where transactions involve limited numbers of buyers and sellers, and where transaction information and financials need not be disclosed



(b) Asset Pricing



Asset pricing models

- Stylized worlds in which fundamental asset values can be calculated exactly
- We are going to make a number of heroic assumptions
- These stylized models enable us to:
 1. emphasize and understand fundamental determinants of asset value
 2. derive asset pricing rules that serve as useful benchmarks in practice



Notions of probability

- Asset returns are subject to uncertainty
- Let S be the set of possible states of the world
- Roll of a fair dice: $S=\{1,2,3,4,5,6\}$
- An *event* is a subset of S
- Ex: $A=\{2,4,6\}$ is the event that the roll is even
- A *probability distribution* is a function that assigns probabilities to each possible state of the word
- Ex: If dice is fair, $P(s)=1/6$ for all $s \in \{1,2,3,4,5,6\}$, and, for any event A :



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$$P(A) = \frac{\#A}{\#S}$$



Random variables

- A random variable X on S attaches a value to each possible state of the world
- Assets (risky strings of cash flows) are random variables
- Ex: X pays \$1 if roll of dice is even, nothing otherwise:
$$P(X=1)=P(s \in \{2,4,6\})=0.5$$



Expectations

- The *expected value* of a random variable X is defined as:

$$E(X) = \sum_{s \in S} P(s) X(s)$$

- X pays \$1 if roll of dice is even, nothing otherwise:

$$\begin{aligned} E(X) = & P(s=1) \times 0 + P(s=2) \times 1 + P(s=3) \times 0 \\ & + P(s=4) \times 1 + P(s=5) \times 0 + P(s=6) \times 1 = 0.5 \end{aligned}$$



Variances and standard deviations

- $\text{VAR}(X) = \sum_{s \in S} P(s) (X(s) - E(X))^2$
 $= E[X - E(X)]^2$

- X pays \$1 if roll of dice is even, nothing otherwise:

$$\text{VAR}(X) =$$

$$\begin{aligned} & P(s=1) \times (0-0.5)^2 + P(s=2) \times (1-0.5)^2 + P(s=3) \times (0-0.5)^2 \\ & + P(s=4) \times (1-0.5)^2 + P(s=5) \times (0-0.5)^2 + P(s=6) \times (1-0.5)^2 \\ & = 0.25 \end{aligned}$$

- The standard deviation of X is the square root of its variance:



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- The standard deviation of X is the square root of its variance:

$$\sigma_X = \sqrt{\text{VAR}(X)}$$



Risk

- A random variable X is *risk-free* if $\text{VAR}(X)=0 \Leftrightarrow X(s)=x$ for all $s \in S$
- It is *risky* if $\text{VAR}(X)>0$
- The closest risk we have to risk-free asset in the US (the world?) is a T-bill



Covariance

- We need a notion of how two random variables X and Y are related:

$$\begin{aligned}\text{COV}(X,Y) &= \sum_{s \in S} P(s) (X(s)-E(X))(Y(s)-E(Y)) \\ &= E[(X-E(X))(Y-E(Y))]\end{aligned}$$

- $\text{COV}(X,Y) > 0$ means that X tends to be high when Y tends to be high, and vice-versa
 - Note 1: if X is risk-free, then $\text{COV}(X,Y) = 0$
 - Note 2: $\text{COV}(X,X) = \text{VAR}(X)$
 - Note 3: $\text{COV}(X,Y) = \text{COV}(Y,X)$
-



Example

- X pays \$1 if roll of dice is even, Y pays \$1 if roll of dice is 4 or more
- Then $E(X)=E(Y)=0.5$, and:

$$\begin{aligned} \text{COV}(X,Y) &= P(s=1)(0-0.5)(0-0.5) + P(s=2)(1-0.5)(0-0.5) + \\ &\quad P(s=3)(0-0.5)(0-0.5) + P(s=4)(1-0.5)(1-0.5) + \\ &\quad P(s=5)(0-0.5)(1-0.5) + P(s=6)(1-0.5)(1-0.5) \\ &= 1/12 \end{aligned}$$



Coefficient of correlation

- $\rho_{X,Y} = \text{COV}(X,Y) / (\sigma_X \sigma_Y)$
- Varies from -1 to 1
- $\rho_{X,Y} = 1$ means that $Y = aX + b$, where $a > 0$
- $\rho_{X,Y} = -1$ means that $Y = aX + b$, where $a < 0$



Example

- X pays \$1 if roll of dice is even, Y pays \$1 if roll of dice is 4 or more
- $\rho_{X,Y} = \text{COV}(X,Y) / (\sigma_X \sigma_Y) =$



Example

- X pays \$1 if roll of dice is even, Y pays \$1 if roll of dice is 4 or more

- $\rho_{X,Y} = \text{COV}(X,Y) / (\sigma_X \sigma_Y) = \frac{\frac{1}{12}}{\sqrt{0.25 \times 0.25}} = \frac{1}{3}$



Mixing assets

- Let a and b be numbers, and X and Y be the returns on two assets
- Investing a in X and b in Y returns $aX(s) + bY(s)$ in state s
- (a,b) , in this context, is called a *portfolio*
- We write $aX + bY$ for the resulting random variable



Big facts

- $E(aX+bY) = aE(X) + bE(Y)$
- $VAR(aX)=a^2VAR(X) \Leftrightarrow \sigma_{aX} = a \sigma_X$
- $VAR(aX+bY)= a^2VAR(X) + b^2VAR(Y) +2ab COV(X,Y)$
- $VAR(0.5X+0.5Y) =$
 $0.25VAR(X) + 0.25VAR(Y) +0.5 COV(X,Y)$



Diversification

- Combining **risky** assets reduces risk unless $\rho_{X,Y} = 1$
- Returns on assets that do not covary perfectly tend to offset each other, at least a little bit
- If they co-vary negatively, diversification is even greater
- If you bet the same amount on both red and black at the roulette, you're taking on virtually no risk



More facts

- $\text{COV}(aX+bY,Z) = a\text{COV}(X,Z) + b\text{COV}(Y,Z)$



More facts

- $\text{COV}(aX+bY,Z) = a\text{COV}(X,Z) + b\text{COV}(Y,Z)$
- And the big monster:

$$\text{VAR} \left(\sum_{i=1}^n a_i X_i \right) = \sum_{i=1}^n \sum_{j=1}^n a_i a_j \text{COV}(X_i, X_j)$$



Financial economies

- Two dates: $t=0$, $t=1$
- Time in between is called the holding period
- N assets, available in fixed (given) supply
- Asset $i \in \{1, 2, \dots, N\}$ has random payoff X_i at date $t=1$
- If it costs q_i at date 0, return is $r_i(s) = X_i(s)/q_i - 1$
- Expected return is $E(r_i) = E(X_i)/q_i - 1$



Investors

- J investors, with given wealth to invest at date 0
- Choose a portfolio $(\alpha_1, \alpha_2, \dots, \alpha_n)$ where $\alpha_1 + \alpha_2 + \dots + \alpha_n = 1$
- α_i is the fraction of her wealth the investor spends on asset i
- If investor has wealth w and buys $(\alpha_1, \alpha_2, \dots, \alpha_n)$, she spends $\alpha_i w$ on asset i
- Note: α 's can be negative \Rightarrow *short-selling*



Portfolio risk and return

- Return on portfolio: $\sum_i \alpha_i r_i$
- Expected return: $E(\sum_i \alpha_i r_i) = \sum_i \alpha_i E(r_i)$
- Variance: $\text{VAR}(\sum_i \alpha_i r_i) = \sum_i \sum_j \alpha_i \alpha_j \text{COV}(r_i, r_j)$



Mean-variance preferences

- Investors care about average (or mean) returns and standard-deviations (or variances)
- Holding variance the same, all investors prefer higher returns
- A *risk-neutral* investor only cares about expected returns
- A *risk-averse* investor prefers less risk, holding expected return the same
- A *risk-loving* investor prefers more risk, holding expected return the same



Equilibrium

- An equilibrium is a set (q_1, q_2, \dots, q_n) of asset prices and a set of portfolio choices by all investors such that:
 1. All investors choose the portfolio that maximizes their utility
 2. Total demand for each asset equals supply

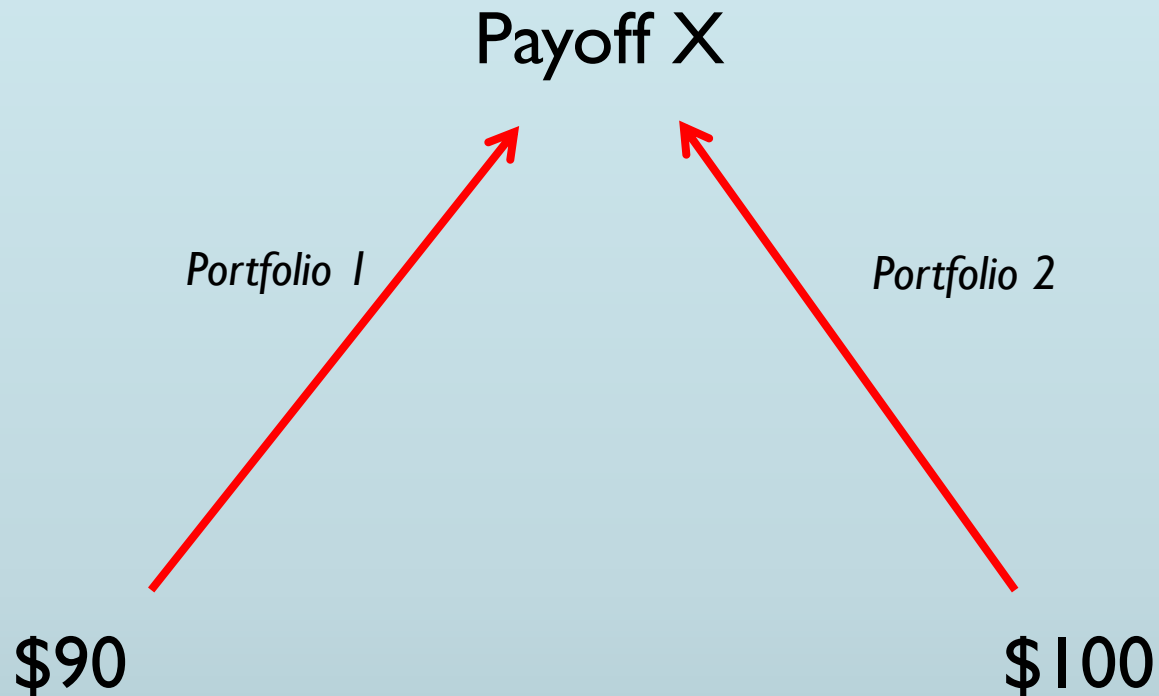


Law of one price

- The law of one price holds if whenever two portfolios yield the exact same payoff in all states, they cost the same.
- Remark: If there are no restriction on short-selling, the law of one price must hold in equilibrium
- Proof: take two portfolio with the same payoff but different prices. Buy the cheap one, sell the expensive one, no payoff implication at date 1, but you are richer at date 0.



A deviation from the Law of One Price



Arbitrage

- A strong arbitrage is a portfolio with a negative price today and a non-negative payoff in all states at date T
- A deviation from the law of one price is a strong arbitrage opportunity
- No strong arbitrage can exist in equilibrium



Fundamental theorem of finance

No arbitrage



$$q_i = E^*(X_i) / (1+r) \text{ for all } i$$

where the expectation* is with respect to a synthetic probability distribution called the risk-neutral probability and r is the risk free rate

Most of modern finance prices assets by estimating the RNP first and then pricing assets as if agents were risk neutral



Classical portfolio theory

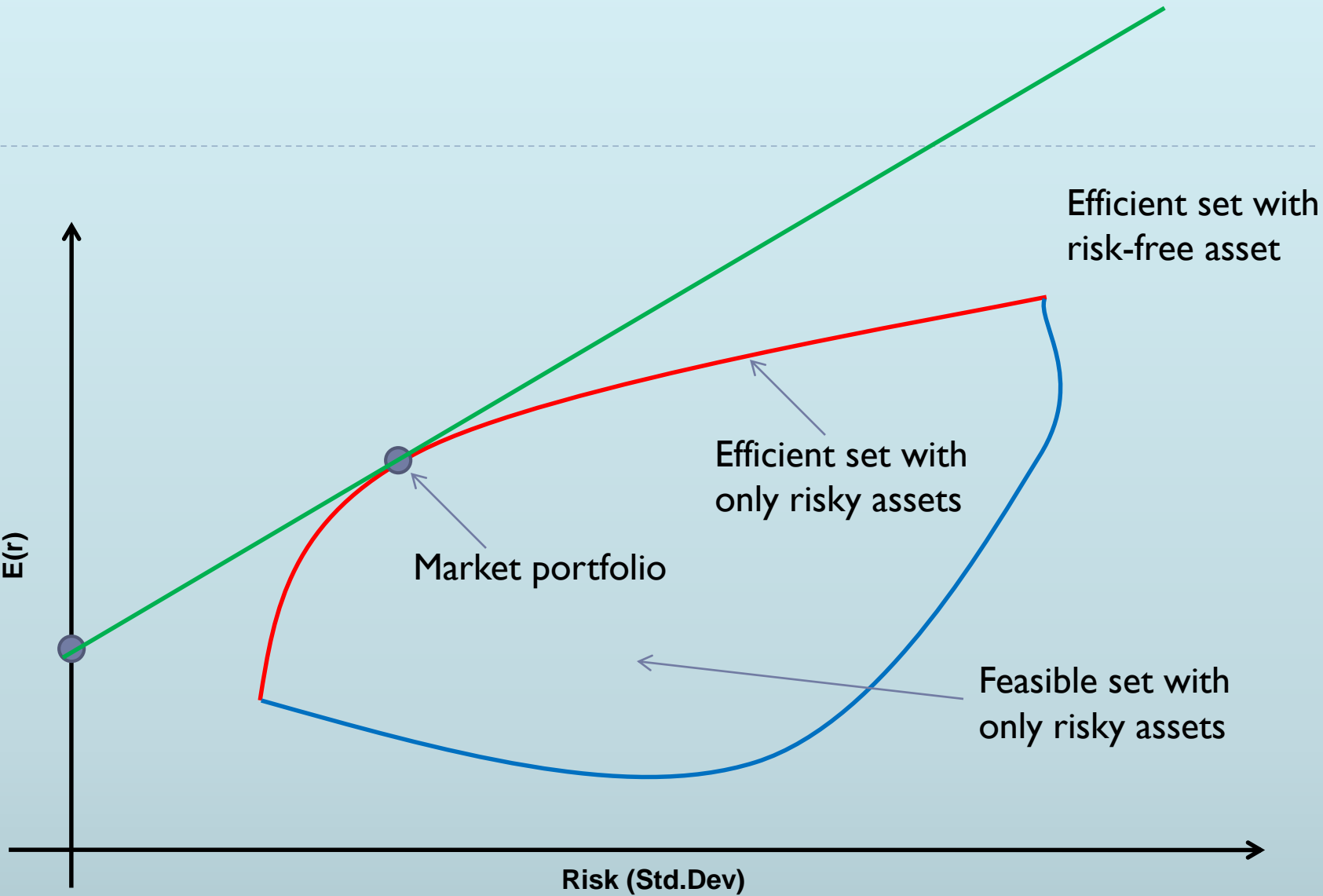
- All investors have mean-variance preferences, and are risk-averse
- Can divide their wealth across assets however they wish
- No taxes or transaction costs
- Investors have all the information they need about assets
- There is a risk-free asset, and investors can borrow and lend at will at the risk-free rate

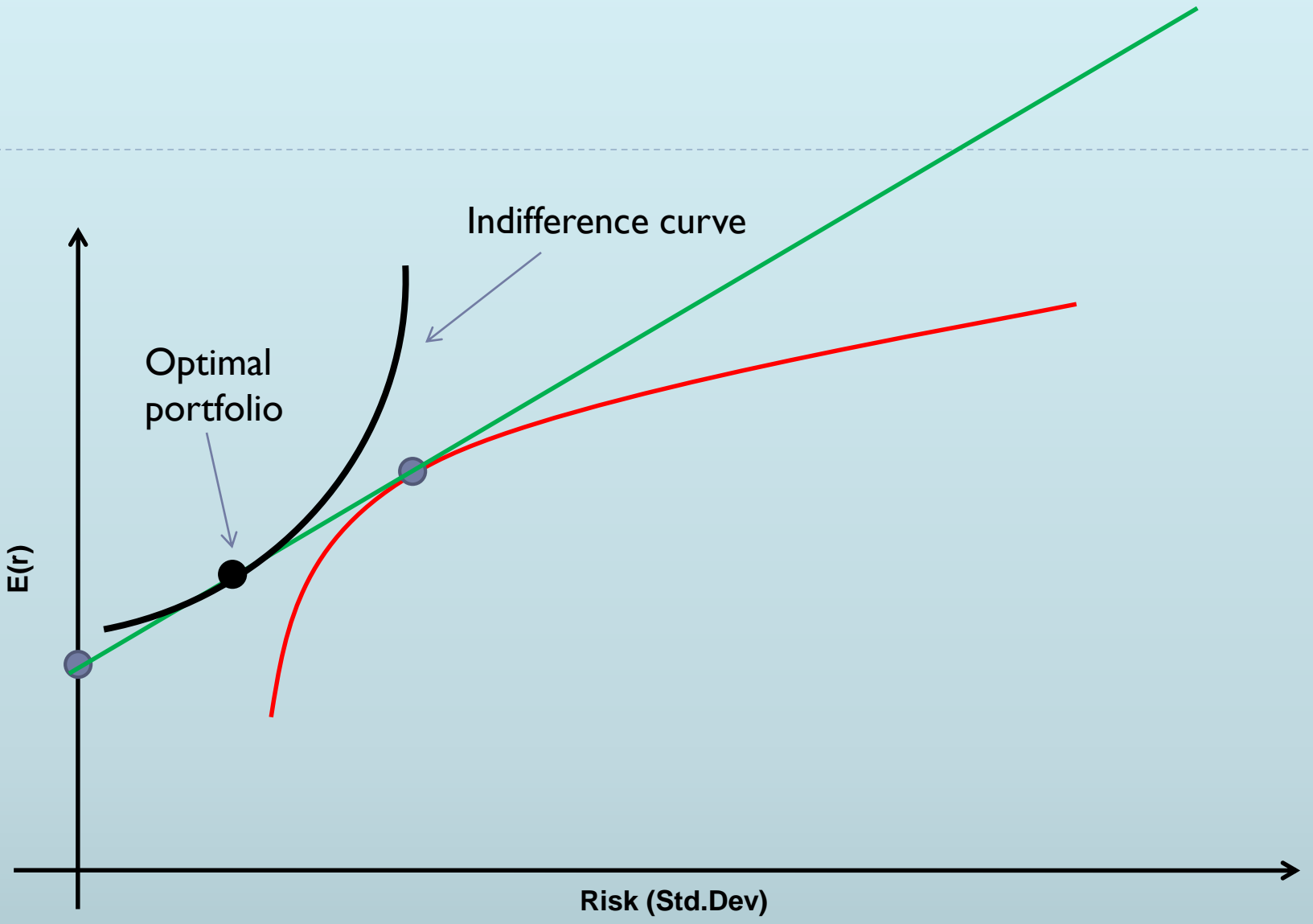


Two-portfolio theorem

- With risk-free asset, efficient set begins at portfolio that puts all wealth in risk-free asset, and touch the risky part of the feasible set in exactly one point
- That point is called the market portfolio
- **Theorem:** In equilibrium, all investors hold a portfolio made of a positive investment in the market portfolio, and a positive or negative investment in the risk-free asset







Market portfolio

- All risky assets have positive weight in it
- The risky-part of all investors portfolios is the same, namely the market portfolio
- It follows that the market portfolio can be computed as the fraction of total risky holdings in a given asset
- Decent practical proxy: capitalization-weighted index, such as the S&P500



Capital Asset Pricing Model (CAPM)

- What should be the average return on asset i in equilibrium? Equivalently, what should be its price?
- Intuitively, riskier assets should command a higher return
- Investors should be compensated for the risk a given asset contributes to their portfolio
- This contribution depends on how it co-varies with all elements of the portfolio, including itself



Capital Asset Pricing Model (CAPM)

- Theorem:

$$E(r_i) = r_f + \beta_i [E(r_m) - r_f]$$

where:



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- Theorem:

$$E(r_i) = r_f + \beta_i [E(r_m) - r_f]$$

where:

$$\beta_i = \frac{\text{COV}(r_i, r_m)}{\text{VAR}(r_m)}$$



CAPM

- Investors want to be compensated for a very specific form of risk: the asset's beta
- Return on a given asset is the risk-free rate plus a risk premium
- Risk premium is the product of beta (the *quantity of risk*) and $E(r_m) - r_f$ (the *market price of risk*)



CAPM in practice

1. Estimate's asset beta using historical data on r_i and r_m
 - ▶ Typical to use 5 years of daily, weekly or monthly data
 - ▶ Use peers or industry data when information is missing or noisy
 - ▶ Regress r_i on r_m directly or, if you have too much time on your hands, $(r_i - r_f)$ on $(r_m - r_f)$
2. Get r_f for expected holding period from yield curve
3. Forecast $E(r_m) - r_f$ for holding period
 - ▶ See Fama-French data library



Diversifiable risk does not matter

- Asset i 's beta is the slope you get if you regress r_i on r_m
- Therefore,
$$r_i = r_f + \beta_i (r_m - r_f) + \varepsilon_i$$
where:
$$\text{COV}(r_m, \varepsilon_i) = 0$$
- It follows that
$$\text{VAR}(r_i) = \beta_i^2 \text{VAR}(r_m) + \text{VAR}(\varepsilon_i)$$
- Asset's risk is the sum of its *systematic risk*, and its *specific (unique, diversifiable) risk*
- Only the first type of risk affects pricing



A key CAPM point

- β 's are linear
- Consider a portfolio made of share α_1 in asset 1 and α_2 in asset 2
- The portfolio's beta is:

$$\begin{aligned}\beta_P &= \text{COV}(\alpha_1 r_1 + \alpha_2 r_2, r_m) / \text{VAR}(r_m) \\ &= [\alpha_1 \text{COV}(r_1, r_m) + \alpha_2 \text{COV}(r_2, r_m)] / \text{VAR}(r_m) \\ &= \alpha_1 \beta_1 + \alpha_2 \beta_2\end{aligned}$$

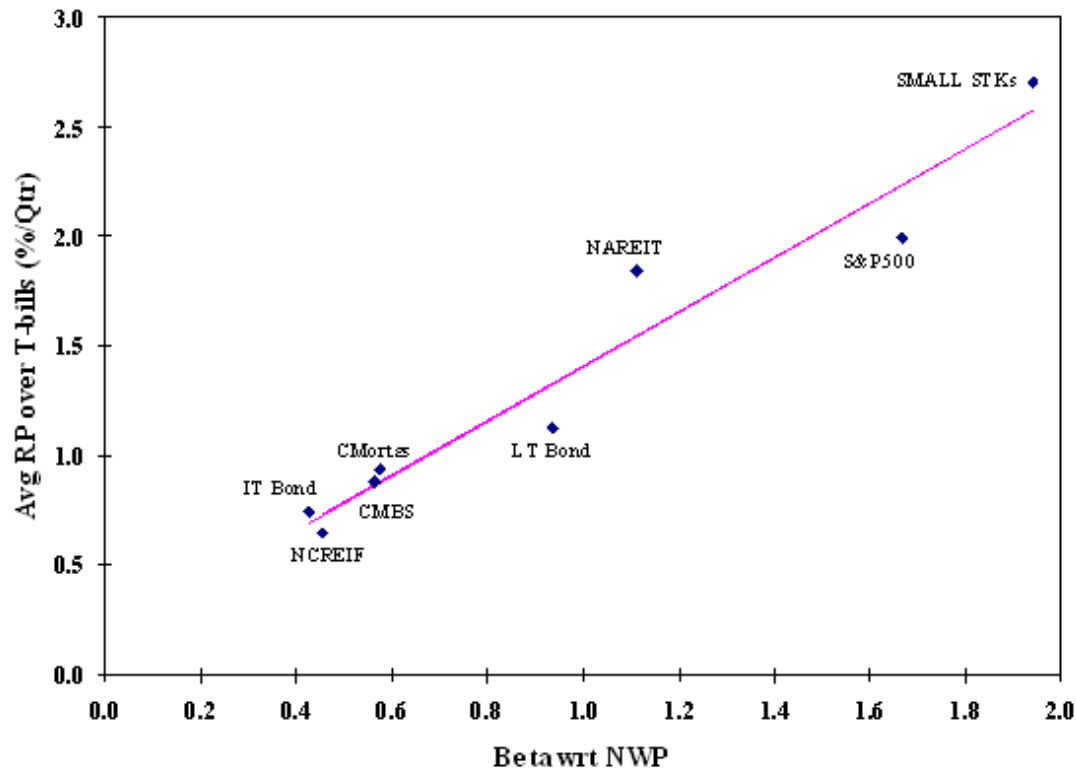


CAPM works OK for broad asset classes

Empirical Security Market Line and Historical Risk & Return on Eight U.S. Domestic Asset Classes

Based on Quarterly Returns 1980-2004 (95 obs)

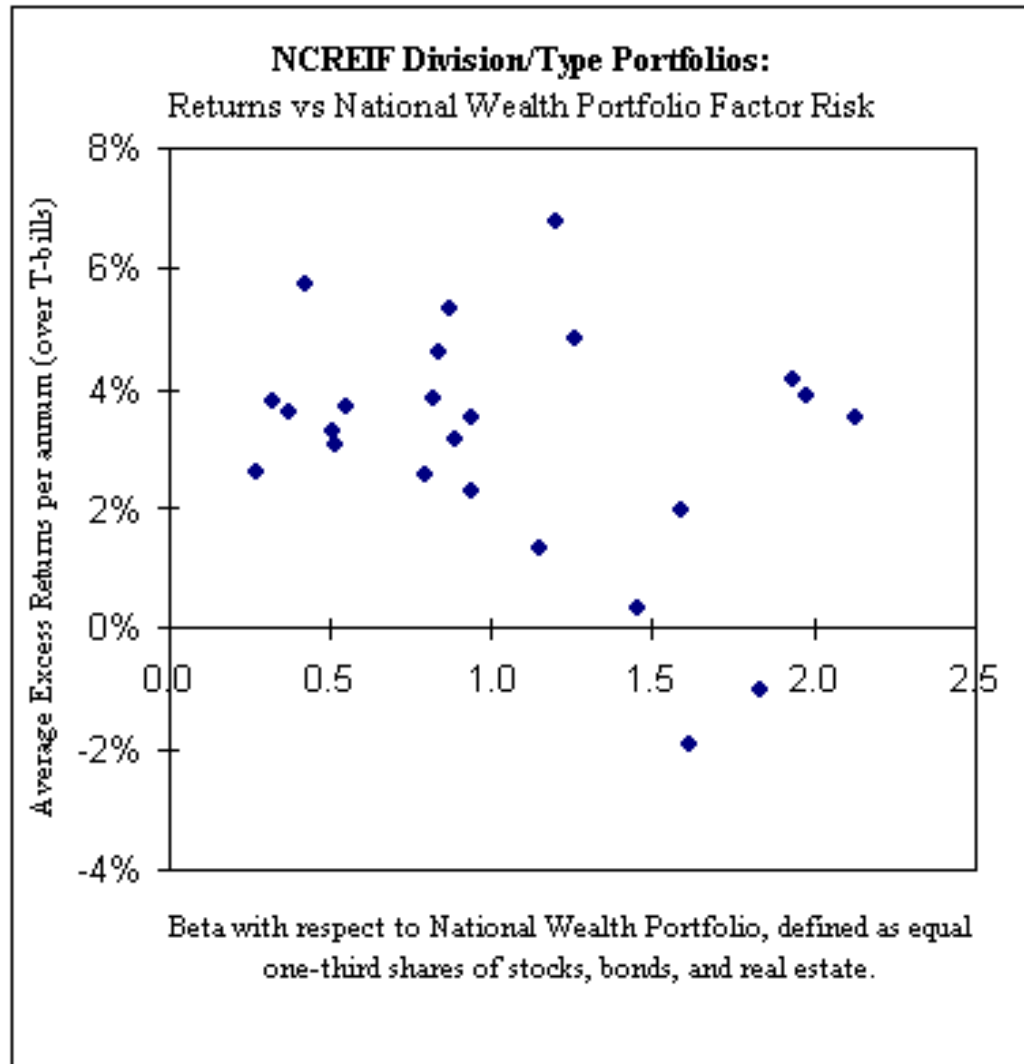
"NWP" = 1/3 Stocks + 1/3 Bonds + 1/3 Real Estate



Regression: $R^2 = 93\%$; Intercept = 0.17% (t-stat = 1.2); Slope = 1.24 (t-stat = 10.0).

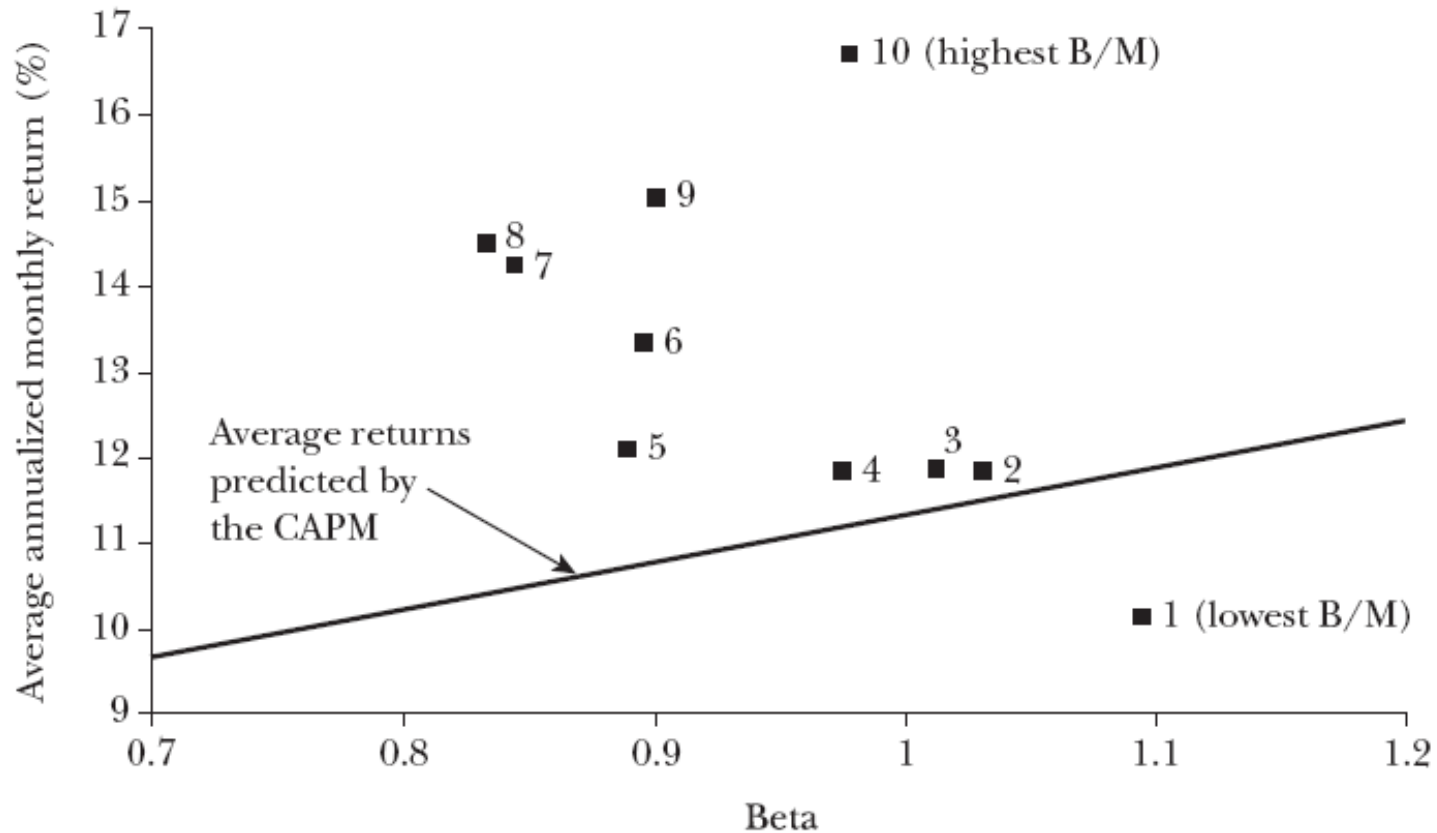
Source: Li & Price (2005) as modified by the authors.

Not so well for narrower classes



True outside of real estate as well

Average Annualized Monthly Return versus Beta for Value Weight Portfolios Formed on B/M, 1963–2003



Fama-French 3-factor model

- Augment CAPM regression to:

$$r_i - r_f = \alpha_i + \beta_i (r_m - r_f) + \beta_{SML} (SML) + \beta_i (HML) + \varepsilon_i$$

Where:

SML = difference between returns on diversified portfolios of small and large capitalization stocks

HML = difference between returns on diversified portfolios of high and low B/M stocks

- The asset/portfolio's empirical intercept (the “alpha”) is the return that can't be explained by exposure to FF factors
 - Estimates of factor loadings (the beta's) together with forecasts for expected factor values over holding period can be used to calibrate required returns
 - Data: Fama-French library
-



(c) Capital structure



Modigliani-Miller (MM)

- Does capital structure matter?
- Does the value of an asset depend on the mix of debt and equity that is used to finance its purchase?
- No, at least absent taxes, transaction costs or limits, and other frictions
- Obvious from CAPM: asset value depends on its payoffs alone



Arbitrage argument

- Consider two properties with the same random payoff X over $t=1,2,3, \dots$
- First property is purchased with equity E and debt D , its value at date 0 is $V^L=E+D$
- We assume that property lives for ever, and keeps structure fixed
- L for *levered* or *leverage*
- Second property is 100% equity financed, and has value V^U
- Can we have $V^L > V^U$?



Two strategies

- Strategy 1: Buy fraction α of levered asset's equity, which costs αE
- Payoff: $\alpha(X - Dr_f)$

- Strategy 2: Borrow αD and buy αV^U of equity in unlevered firm, which costs:

$$\alpha V^U - \alpha D = \alpha(V^U - D) < \alpha(V^L - D) = \alpha E$$

- Payoff: $\alpha X - \alpha D r_f$
 - Violation of the law of one price
-



What does MM tell us?

- Not so much that capital structure does not matter
- It says that if CS matters, it must be because of the frictions MM assume away:
 1. Taxes
 2. Costs associated with financial distress
 3. Agency problems (manager incentives vs. shareholder objectives)
 4. ...



Return on equity

- Unlevered case: $r^U = X / V^U$
- Levered case: $r^E = (X - r^f D) / E = r^U + (D/E) (r^U - r^f)$
- Leverage: more debt means more return on equity as long as $E(r^U) > r^f$
- What's the catch? Risk goes up:
- $\text{VAR}(r^E) = \text{VAR}(r^U) (1 + D/E)^2$



Levered betas

- How does the beta of the levered property's equity compare to the beta of the unlevered property?
- $$\beta^L = \beta(r^E) = \beta(r^U + (D/E)(r^U - r^f))$$
$$= (1 + (D/E)) \beta^U$$
- It is higher, confirming that leverage implies risk
- Some stake-holders (debt-holders) assume “no” risk leaving equity holders to bear more risk



Weighted average cost of capital (WACC)

- $WACC = E/(E+D) E(r^E) + D/(E+D) r^f$
- MM proposition II: $WACC = E(r^U)$ regardless of D
- WACC fact: the asset's value is the expected present value of all future cash flows discounted at the WACC
- Loosely speaking, a positive NPV when discounted at WACC means that cash-flows, in expected terms, are sufficient to meet the expected returns of all stakeholders



When reality strikes: Taxes

- If asset's owner is a corporation, they face taxes, but debt payments are tax deductible

- Net cash flows in each period, are:

$$X - \tau(X - Dr^f) = (1 - \tau)X + \tau Dr^f$$

- The last term is called the tax shield, it adds value to the asset

- One shows: $V^L = V^U + \tau D$

- General principle: $APV = NPV(\text{property}) + NPV(\text{financing})$
-



If debt's so great, why use equity at all?

- MM abstract from issues associated with financial distress
- Distress is costly both for obvious reasons and more subtle ones
- As a result, optimal debt-to-value ratio is less than 100%



Other MM results with taxes

- Unlevered case: $r^U = X(1-\tau) / V^U$
- Levered case: $r^E = r^U + ((1-\tau) D/E) (r^U - r^f)$
- $\beta^L = (1 + (1-\tau) D/E) \beta^U$
- $WACC = E/(E+D) E(r^E) + D/(E+D) (1-\tau) r^f$
- Discounting expected net-of-taxes cash flows at WACC continues to give the right asset value answer



The WACC method

1. Project after-tax cash flows: $X(1-\tau)$
2. Discount at WACC
3. Result: $D + E$



Practical implementation

- Cost of debt is “easy”
- Cost of equity is tough:
 1. Find the beta of “similar” assets
 2. Unlever those betas: $\beta^U = (1 + (1 - \tau) D/E)^{-1} \beta^L$, average
 3. Relever using the actual financing mix used in project under study
 4. Invoke CAPM



Method's advantages

1. Works in some theoretical contexts
2. Has intuitive appeal
3. Time-tested
4. Industry standard
5. What's better out there?



Method's drawbacks

1. Assumptions that make it OK don't hold in practice
2. Levered beta formulae very MM specific
3. Relies on CAPM's approximate validity
4. Often misapplied: one-size WACC don't fit all projects
5. For private projects, what's the market value of debt, what's the market value of equity?



Three levels of cash-flows

- *Before tax cash flows* accrue to:
 1. Taxes (income and capital gains)
 2. Debt holders
 3. Equity holders

- *Free-cash flows to the firm (FCFF)* accrue to:
 1. Debt holders
 2. Equity holders

- *Free-cash flows to equity (FCFE)* accrue to equity holders



Three appropriate discount rates

- *Before tax cash flows* should be discounted at before-tax WACC
- *FCFF* should be discounted at after-tax WACC
- *FCFE* should be discounted at required return on equity
- First two calculations give the value of the firm, the last one gives the value of equity



(d) Review of Real Estate Investment Analysis



Pricing a revenue-generating property

- Consider a property made of a collection of leasable units
- How much should a given investor pay for such a property?
- Two approaches:
 1. DCF method (forecast expected flows, discount them)
 2. Ratio approach (cap and GRM)
- Both approaches require detailed cash flow data



Cash flow pro-forma

- Table of expected cash flows associated with the property over a certain horizon
- Typical horizon: 5 to 10 years, yearly data
- We will first ignore the potential role of debt and taxes, and focus on before tax cash flows



Property before tax cash flow

- $PBTCF = NOI - \text{Capital expenditures}$
- Expected inflows minus expected outflows
- Cash flows to be distributed across three types of stakeholders:
 1. Equity holders
 2. Debt holders
 3. The tax man



Reversion cash flows

- Reversion cash flows are the result of selling all or part of the property
- In most cases, one big reversion cash flow in the last year of the analysis, equal to the expected value of the property at that time, net of transaction costs
- Two methods:
 1. Guess a perpetual rate of growth of PBTFCF and discount the perpetuity at appropriate rate
 2. Use multiple approach (guess year 11 NOI or EGI, and apply standard multiple)



Typical Pro Forma Items

Operating (all years):

Potential Gross Income = (Rent*SF)	=	PGI
- Vacancy Allowance = -(vac.rate)*(PGI)	=	- V
+ Other Income = (eg, parking, laundry)	=	+OI
- Operating Expenses	=	- OE
<hr/>		<hr/>
Net Operating Income	=	NOI
- Capital Expenditures	=	- CE
<hr/>		<hr/>
Property Before-tax Cash Flow	=	PBTCF

Reversion (last year & yrs of partial sales only):

Property Value at time of sale	=	V
- Selling Expenses = -(eg, broker)	=	- SE
<hr/>		<hr/>
Property Before-tax Cash Flow	=	PBTCF



Going in IRR

- Given a proposed property price, and a full pro-forma, a “total” IRR can be calculated
- It is the discount rate that makes the present value of all expected PBTFCF equal to the price
- A sound decision rule: compute typical IRR on similar properties, and take project if property IRR exceeds this typical IRR



Equivalently, use the DCF method

- Estimate required return on similar property (*the opportunity cost of capital, before-tax WACC*)
- Discount PBTCF at rate
- Another sound decision rule: accept project if resulting value exceeds the price



Typical returns: real estate indices

- *NCREIF property index (NPI)*
- Survey
- Cap rate approach (holy trinity)
- CAPM



Multiple/Ratio approach

- Find a group of peer properties on which good value data is available due to recent transaction, or rock-solid appraisal
- Alternatively, collect/purchase info on appropriate multiples
- Apply Cap rate and GRM approach to current property



Debt and taxes

- Many investors are subject to taxes at the property level, which matters greatly for value
- Two ways to deal properly with the effects of debt and taxes:
 1. Discount after tax cash flows at after tax WACC
 2. Discount flows-to-equity (EATCF) at the required rate of equity
- First approach yields the property's total value, the second one yields the value of equity in the property



Calculating taxes and flows-to-equity

- Taxable income = NOI – Depreciation – Interest expenses
- Income taxes = Taxable income x Tax rate
- ATCF= PBTCF – Income taxes
- EATCF=ATCF – Debt service payments



Calculating reversion cash flows

- $\text{Capital gains} = \text{Net sale proceeds} - \text{Adjusted basis}$
- $\text{Adjusted basis} = \text{Original basis} + (\text{Total CAPEX} - \text{Depreciation})$
- $\text{Capital gains tax} = \text{Capital gains} \times \text{relevant tax rate}$
- It is also proper to show a final debt payment in a pro-forma table (even though, in principle, it could be folded into standard debt service line)



Capital Gains Tax

- Capital gains = (Net sale price – (Original basis + Capex))
+ Depreciation
- The two pieces are taxed differently
- *Net sale proceeds – (Original Basis + Capex)* is taxed at the capital gains tax rate
- *Depreciation* is taxed at the “depreciation recapture tax rate”, which is typically higher



Example

- Net sale price=\$1,000,000, Original Basis=\$800,000, CAPEX=\$100,000, Depreciation=\$50,000
- Capital gains tax: 15%, Recapture Tax: 25%
- CGT = $(1,000,000 - 800,000 - 100,000) \times 0.15$
 + $(50,000) \times 0.25$
 = 27,500



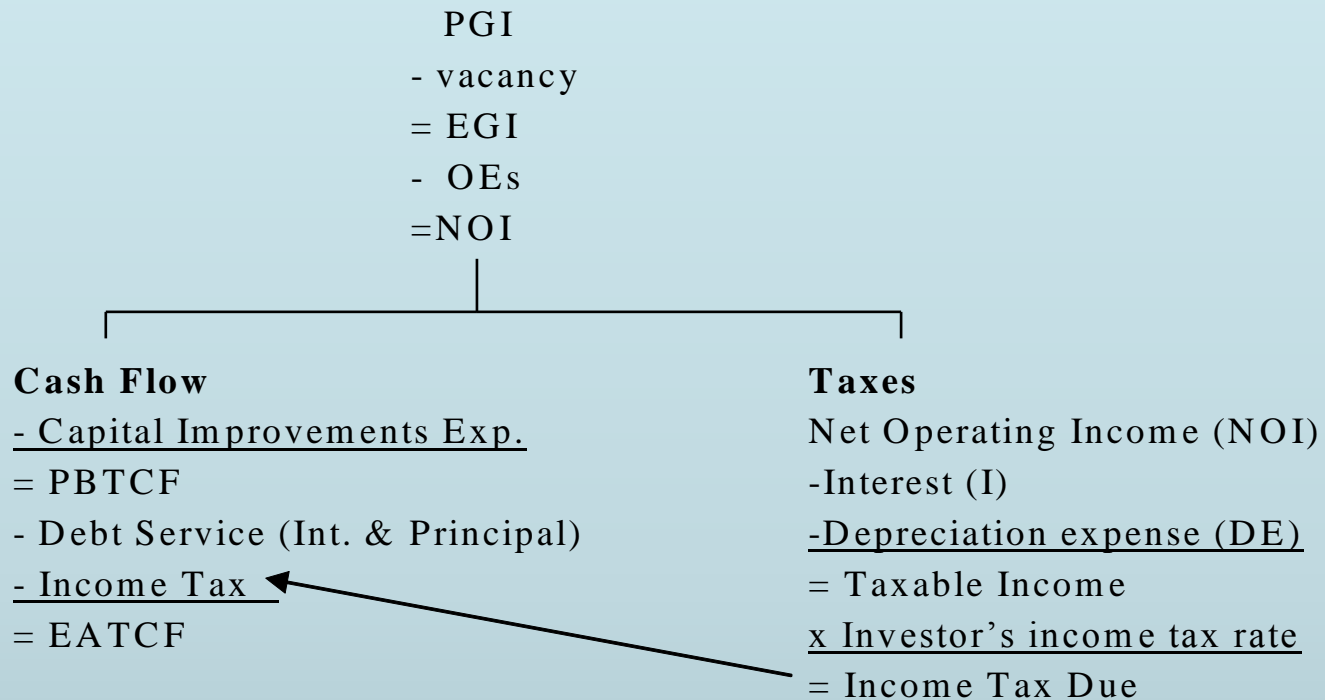
And we're done

- Discount ATCF at WACC, or EATCF at required return on equity
- One should do an obvious set of multiple calculations too, but in practice it is seldom done
- Key point: leverage can make a deal worth it, or kill it, depending on the direct and indirect costs of debt



EATCF from operations

Exhibit 14-1a: Equity After-Tax Cash Flows from Operations



Calculating reversion cash flows

- $\text{EATCF (reversion)} = \text{Net Sale Price} - \text{Loan Balance} - \text{CGT}$



Exhibit 14-2: Example After-Tax Income & Cash Flow Proformas . . .

Property Purchase Price (Year 0):	\$1,000,000	Unlevered:	Levered:
Depreciable Cost Basis:	\$800,000	Before-tax IRR:	6.04%
Ordinary Income Tax Rate:	35.00%	After-tax IRR:	4.34%
Capital Gains Tax Rate:	15.00%	Ratio AT/BT:	0.719
Depreciation Recapture:	25.00%		0.870

	Year:										Oper.	Reversion	Rever.	Total	
	1	2	3	4	5	6	7	8	9		Yr.10	Item:	Yr.10	Yr.10	
Operating:															
Accrual Items:															
NOI	\$60,000	\$60,600	\$61,206	\$61,818	\$62,436	\$63,061	\$63,691	\$64,328	\$64,971	\$65,621		Sale Price	\$1,104,622		
- Depr.Exp.	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091		- Book Val	\$809,091		
- Int.Exp.	\$41,250	\$41,140	\$41,030	\$40,920	\$40,810	\$40,700	\$40,590	\$40,480	\$40,370	\$40,260					
=Net Income (BT)	(\$10,341)	(\$9,631)	(\$8,915)	(\$8,193)	(\$7,465)	(\$6,730)	(\$5,990)	(\$5,243)	(\$4,490)	(\$3,730)	=Book Gain		\$295,531	\$291,801	
- IncTax	(\$3,619)	(\$3,371)	(\$3,120)	(\$2,867)	(\$2,613)	(\$2,356)	(\$2,096)	(\$1,835)	(\$1,571)	(\$1,305)	- CGT		\$73,421		
=Net Income (AT)	(\$6,722)	(\$6,260)	(\$5,795)	(\$5,325)	(\$4,852)	(\$4,375)	(\$3,893)	(\$3,408)	(\$2,918)	(\$2,424)	=Gain (AT)		\$222,111	\$219,686	
Adjusting Accrual to Reflect Cash Flow:															
- Cap. Imprv. Expdtr.	\$0	\$0	\$50,000	\$0	\$0	\$0	\$0	\$50,000	\$0	\$0					
+ Depr.Exp.	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	+ Book Val		\$809,091		
-DebtAmort	\$2,000	\$2,000	\$2,000	\$2,000	\$2,000	\$2,000	\$2,000	\$2,000	\$2,000	\$2,000	-LoanBal		\$730,000		
=EATCF	\$20,369	\$20,831	(\$28,704)	\$21,766	\$22,239	\$22,716	\$23,198	(\$26,317)	\$24,173	\$24,667	=EATCF		\$301,202	\$325,868	
+ IncTax	(\$3,619)	(\$3,371)	(\$3,120)	(\$2,867)	(\$2,613)	(\$2,356)	(\$2,096)	(\$1,835)	(\$1,571)	(\$1,305)	+ CGT		\$73,421		
=EBTCF	\$16,750	\$17,460	(\$31,824)	\$18,898	\$19,626	\$20,361	\$21,101	(\$28,152)	\$22,601	\$23,361	=EBTCF		\$374,622	\$397,983	

CASH FLOW COMPONENTS FORMAT

	Year:										Oper.	Reversion	Rever.	Total	
	1	2	3	4	5	6	7	8	9		Yr.10	Item	Yr.10	Yr.10	
Operating:															
Accrual Items:															
NOI	\$60,000	\$60,600	\$61,206	\$61,818	\$62,436	\$63,061	\$63,691	\$64,328	\$64,971	\$65,621		Sale Price	\$1,104,622		
- Cap. Imprv. Expdtr.	\$0	\$0	\$50,000	\$0	\$0	\$0	\$0	\$50,000	\$0	\$0					
=PBTCF	\$60,000	\$60,600	\$11,206	\$61,818	\$62,436	\$63,061	\$63,691	\$14,328	\$64,971	\$65,621	=PBTCF		\$1,104,622	\$1,170,243	
- Debt Svc	\$43,250	\$43,140	\$43,030	\$42,920	\$42,810	\$42,700	\$42,590	\$42,480	\$42,370	\$42,260	- LoanBal		\$730,000		
=EBTCF	\$16,750	\$17,460	(\$31,824)	\$18,898	\$19,626	\$20,361	\$21,101	(\$28,152)	\$22,601	\$23,361	=EBTCF		\$374,622	\$397,983	
-taxNOI	\$21,000	\$21,210	\$21,422	\$21,636	\$21,853	\$22,071	\$22,292	\$22,515	\$22,740	\$22,967	taxMktGain		\$693	\$23,661	
+ DTS	\$10,182	\$10,182	\$10,182	\$10,182	\$10,182	\$10,182	\$10,182	\$10,182	\$10,182	\$10,182	- AccDTS		(\$72,727)	(\$62,545)	
+ ITS	\$14,438	\$14,399	\$14,361	\$14,322	\$14,284	\$14,245	\$14,207	\$14,168	\$14,130	\$14,091			\$14,091	\$14,091	
=EATCF	\$20,369	\$20,831	(\$28,704)	\$21,766	\$22,239	\$22,716	\$23,198	(\$26,317)	\$24,173	\$24,667	EATCF		\$301,202	\$325,868	

(d) Mortgages



Mortgages

- Mortgage: debt contract secured by a real estate property
- Characteristics:
 1. Initial balance or principal (b_0)
 2. Maturity (T)
 3. Yield (or contract rate) structure (r_t , for all periods t)
 4. Payment structure (m_t , for all periods t)
- Mechanics:
 1. At a given date, interest due is $b_{t-1} r_t$
 2. $b_t = b_{t-1} + b_{t-1} r_t - m_t$
 3. If $b_T > 0$, balance is due in one *balloon payment*



Some language, and notes

- Mortgage whose balance is zero after T periods ($b_T=0$) are called *fully amortizing*
- Yield can be fixed, vary on a fixed schedule, or according to some other market rate
- Amortization can be negative -- balance can grow -- from one period to the next (if $m_t < b_{t-1} r_t$)
- *Interest-only mortgages* (IOMs) have payments equal interest due ($m_t = b_{t-1} r_t$) for part of the contract



FRMs: fixed-rate, fully amortizing mortgages

- For all t :
 1. $r_t = r$
 2. $m_t = m$
- Fully amortizing: $b_T = 0$
- What must m be? (Fixed annuity formulae)
- $m = b_0 r / (1 - (1+r)^{-T})$



The lender's perspective

- Full amortization means:

$$b_T=0, \quad \text{or, equivalently,} \quad b_0 = \sum_{t=1, \dots, T} m_t / (1+r)^t$$

- Absent other payments from the borrower to the lender (e.g. “points”):

$$b_0 = \sum_{t=1, \dots, T} m_t / (1+r)^t + b_T / (1+r)^T$$

- Whether or not amortization is full, r is the loan's IRR if all payments are made, or *yield-to-maturity* (YTM or **APR**)



Yield vs. return

- Yield (YTM/**APR**) is the mortgage's IRR if **and only if** all payments are made as planned
- In practice, borrowers default, fail to make payments on time, refinance or prepay when interest rates are low,...
- Causes transaction costs, and capital losses
- $IRR < YTM$
- Riskier borrowers should pay more
- But paying more makes default more likely...
- Fixed point problem, which may or may not have a solution: market exclusion



GPMs: graduated payment mortgages

- Can we design a fully-amortizing mortgage with contract rate r whose payments grow by $g\%$ each period?
- Easy:
 1. Guess that the first payment is $\$1$
 2. Calculate the corresponding payment schedule, and its PV at discount rate r , call it F
 3. First payment must be $m_1 = b_0/F$
 4. Rest follows trivially
- Logic: all payments are proportional to m_1 , so is PV



GPM example

- 5 years, monthly payments, 2 step ups of 8% (months 13 and 25), $r=10\%$
- Initial payment: \$1918.84
- Loan's half-life: 36 months



Adjustable rate mortgages (ARMs)

- Interest rate adjusts at fixed frequency as a function of a given market interest rate (1 year CMT rates, LIBOR...)
- Payment in a given period is calculated in FRM fashion assuming that the current rate will prevail to maturity
- ARM stipulations:
 1. r_1
 2. Adjustment interval: 1 year, 3 years, 5 years
 3. Index: publicly observable market interest rate index
 4. Margin: $r_t = \text{index}_t + \text{margin}$
 5. Caps and floors (lifetime, or max adjustment)
 6. Full indexation: $r_1 = \text{index}_1 + \text{margin}$
 7. Teaser rate: $r_1 < \text{index}_1 + \text{margin}$



Mortgage schedules for ARMs

- Future rates on ARMs are not known
- One can produce a payment schedule based on index forecasts
- In practice, people use current value of index, assume it will remain where it is, and compute all contract rates
- If the loan is fully indexed, this gives you the same table as a standard FRM
- If the loan features teaser rates, rates and payments rise in full at first adjustment if there are no caps, in several steps if there are binding caps



Annual Percentage Rate (APR)

- YTM from the lender's viewpoint
- Loan's IRR from the point of view of the lender if all payments are made as planned
- On any mortgage with fixed rates (whether or not payments are fixed) and no "points", $YTM = \text{contract rate}$



APRs on ARMs

- In principle, APR depends on expected path of market rates
- In practice, government regulations require that the “official” APR reported for ARMs be based on a flat forecast of market interest rates
- If there is a teaser rate, APR must be calculated under the fastest possible path to fully indexed rate



Example

- 5-year ARM, \$100,000, 2% margin over a market index that can be either 8% or 10%
- Teaser rate of 6%, two resets (Months 13 and 25), no caps
- Index begins at 8%
- 40% chance that it will change value to 10% by first reset, 40% that it will change value again by second reset
- 4 possible histories for the index: high-high (10%-10%), HL, LL, LH
- Hence 4 possible histories for the payments



Points

- Payments from borrower to lender at origination
- 1 point = 1% of initial balance
- Does not reduce initial balance (not a down-payment)
- Effective loan size = $b_0(1-n)$, where n is the number of points at origination
- Raises lender's YTM (APR) above contract rate
- Indeed: $b_0 = PV(\text{payments, contract rate})$
while $b_0(1-n) = PV(\text{payments, APR})$
- $APR > \text{contract rate}$



Why do we see points?

- Points, all else equal, reduce the contract rate (**that the lender is willing to offer**)
- In PV terms, borrower only recovers their initial fees if they stick with the loan until maturity
- Points discourage prepayment
- Borrowers who know they are not going to prepay can use points to convey their type to lender, and secure better terms
- Alternative to prepayment penalty



YTM $>$ lender's IRR (typically)

- APR (=YTM) is the mortgage's IRR if **and only if** all payments are made as planned
- In practice, borrowers default, fail to make payments on time, prepay when interest rates are low,...
- Causes transaction costs, and capital losses



Prepayment risk

- Borrowers prepay loans for a variety of reasons
- If prepayment occurs when market rates are below the contract rate, this causes losses for lender
- In fact, refinancing gains are one of the main reasons for prepaying
- Borrower's refinancing gains = Lender's prepayment loss
- This makes prepayment risk a very bad form of reinvestment risk



Refinancing

- Consider a borrower with $(T-k)$ payments left
- Assume that refinancing carries a fixed cost $c > 0$ for the borrower
- This cost includes transaction costs and penalties
- Assume the borrower's current (fixed) payment is m , and that rates fall in a way that she can make remaining payments $m' < m$
- The gain is the present value of $m - m'$, to maturity
- Discount rate: new market rate on a loan of maturity $T - k$
- Refinancing is potentially beneficial if $PV(m - m') > c$



Refinancing example (part 1)

- Consider a 15-year FRM with initial balance \$100,000 and contract rate 9%
- After 5 years, rates on 10-year FRMs are 8.5%
- Refinancing costs \$1000
- Assuming that it is one-time only option, should you refinance?



The option to delay

- As long as $c > 0$, refinancing now kills the option to refinance a moment (day, month...) later
- What is the value of the option to delay?
- Assume that you can either refinance today ($t=1$) or one period from now ($t=2$)
- Rates at date 2 are either r^h or r^l and the corresponding present values or remaining payments are either $PV(r^h)$ or $PV(r^l)$
- Value of option is $\max(PV(m,r) - B - c, 0)$ given r
- That's a binominal option which can be priced using Cox-Ross-Rubinstein (1979) or, equivalently, Black-Scholes



Option value of refinancing

- A call option's value is high when:
 1. the strike price is low relative to the expected value of the underlying asset
 2. the value of the underlying asset is volatile

- The refinancing option is particularly valuable when:
 1. contract rate is high relative to market rates, mortgage is far from maturity, penalties are low...
 2. interest rates are volatile



Timing

- Refinancing is a call option:
 1. Strike price: loan balance + refinancing costs (c)
 2. Value of underlying asset: PV of remaining payments at the new rate
- Exercising the option kills it
- “Refinance if $PV(m-m') > c$ ” may not be optimal decision
- It may make sense to wait until $PV(m-m')$ rises further



The option to delay

- In previous refi example, assume that the lender has the option to wait another 24 hours
- Tomorrow, rates will be either 8.25% or 8.75%
- The risk free rate during that period is 0.005%
- What is the value of the option to delay? (*Binomial option pricing formula says \$810 or so*)
- Should the borrower wait another 24 hours?



Prepayment from the lender's viewpoint

- Lenders need to forecast, for each period:
 1. Prepayment *hazard rate*
 2. Prepayment losses and/or *yield degradation*
- Date t hazard rate: likelihood of a prepayment at date t , given no prepayment prior to date t
- *Yield degradation*: Loss in IRR for lender if prepayment occurs
- Yield degradation conditional on prepayment at date t = APR- **IRR** conditional on prepayment event at a given date



Refinancing example (part 1)

- Consider a 15-year FRM with initial balance \$100,000 and contract rate 9%
- After 5 years, rates on 10-year FRMs are 8.5%
- Refinancing costs \$1000
- What is yield degradation if the borrower refinances after 5 years?



How lenders deal with prepayment

1. Prepayment penalties
2. Points
3. A contract rate premium (*fixed point problem*)



Lockout/Yield Maintenance clauses

- Lockout clauses prohibit early prepayments regardless of borrower's ability to pay off the loan in its entirety
- A yield maintenance clause requires the borrower to make a lump sum payment to cover the lender's potential loss from reinvesting prepaid sums.
- Typical on CMBS loans, making prepayment essentially a non-issue on those loans



Default

- On commercial loans, default is the primary concern
- Expected cash-flows depend on 1) the likelihood of default and 2) the likely size of losses in the event of default
- Lenders need to forecast both objects



Hazard rates

- h_t = probability that the loan will default in period t conditional on not having defaulted before
- Probability that the loan will default after exactly t periods is $(1-h_1) (1-h_2) (1-h_3) \dots (1-h_{t-1}) h_t$
- This gives $T+1$ mutually exclusive events, with associated probabilities that sum up to 1

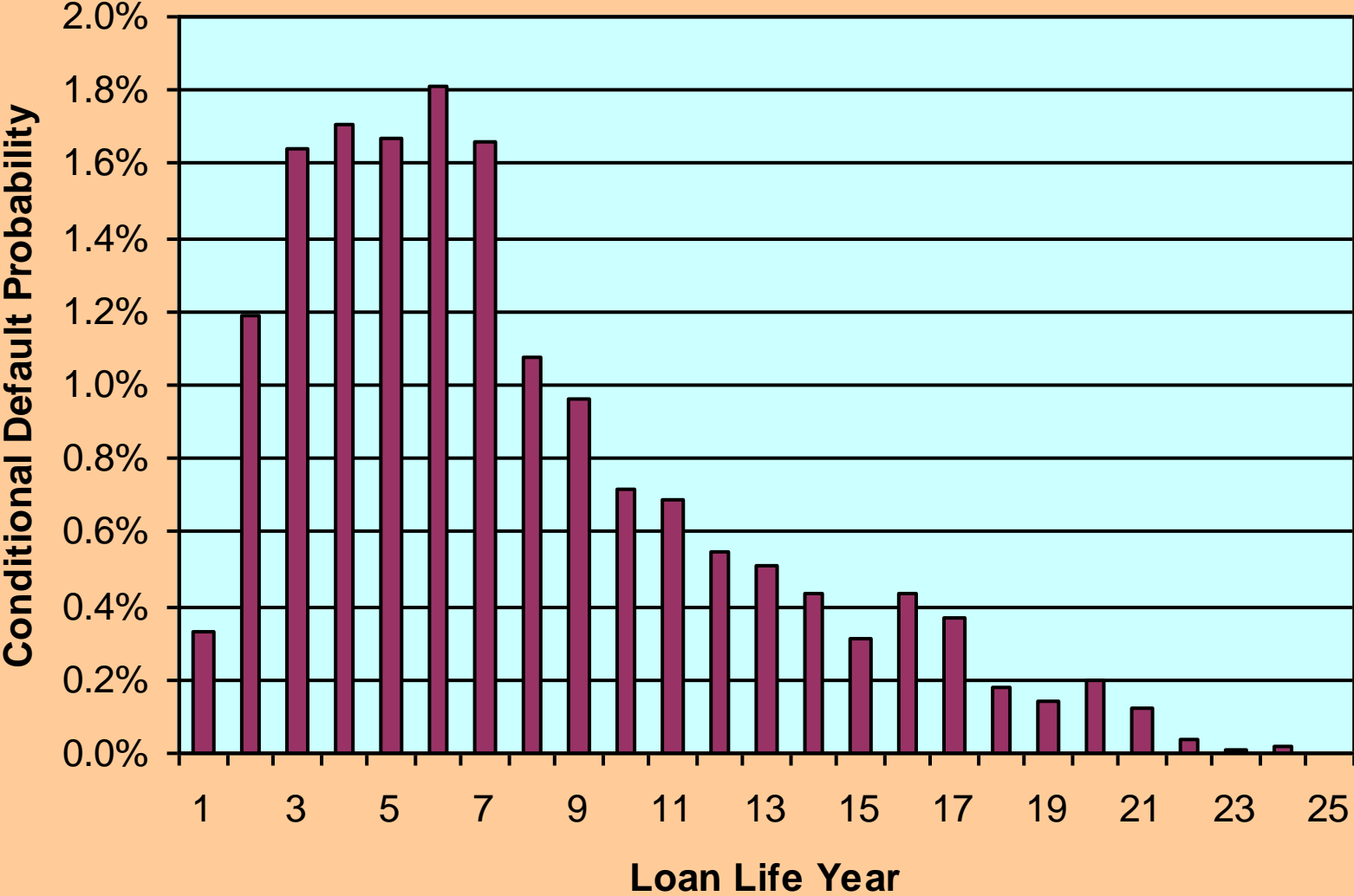


How do lenders forecast hazard rates?

- Use industry standards (SDA: standard default assumptions, scaled up or down)
- Or use econometrics: $h_t = f(\text{loan characteristics, property type, location, borrower characteristics, economic conditions...})$
- Loan characteristics: LTV, DCR (debt-coverage ratio)
- Borrower characteristics: ownership type
- Fit f to historical loan data and hope that past is informative for future



Typical Commercial Mortgage Hazard Rates*



*Source: Esaki et al (2002)

Loss severity rates / Recovery rates

- Date t *loss severity rates* are expected losses if default takes place at date t , as a fraction of outstanding balance
- *Recovery rates* are the opposite: the fraction of the balance the lender expects to recover if default takes place at date t
- Forecast using the same two methods as hazard rates



Why is default so costly?

- Loss severity rates can exceed 50%, and typically range from 30 to 40% on commercial loans
- Many causes:
 1. Transaction costs
 2. Payment delays
 3. Low foreclosure proceeds
- It is estimated that residential properties sell at a 25% discount on average when foreclosed relative to observably similar properties that have not foreclosed



Conditional yield degradation

- Yield degradation if default occurs at date $t =$
 $YTM - IRR \text{ if default at date } t$
- Consider a 3-year IOM loan with initial balance \$100,000 and contract rate 10%
- Year 3 loss severity is 30%, so that the lender only expects to recover $\$77,000 = \$110,000 \times (1 - 0.3)$ in year 3
- IRR in that case is -1.12%
- Yield degradation = $10\% - (-1.12\%) = 11.12\%$



Expected return

- Expected return = $\sum_t P(\text{default at } t) \times (\text{YTM} - (\text{Yield Degradation})_t)$
+ $P(\text{no default}) \times \text{YTM}$
- $E(\text{IRR}(\text{CFs}))$
- In IOM example, assume that default occurs with probability 10% in year 2 and year 3, with loss severity 30% in either case
- Expected return = $.10 \times (-17.11\%) + .10 \times (-1.12\%) + .80 \times 10\%$
= 7.18%
- Average Yield Degradation = $\text{YTM} - \text{Expected Return}$
= $E(\text{Yield Degradation})$



A better measure

- True IRR is IRR(Expected Cash Flows) which can differ greatly from expected return
- In IOM example, assume again that default occurs with probability 10% in year 2 and year 3, with loss severity 30% in either case



A better measure

- True IRR is IRR(Expected Cash Flows) which can differ greatly from expected return
- In IOM example, assume again that default occurs with probability 10% in year 2 and year 3, with loss severity 30% in either case

Year	0	1	2	3	IRR
Default at date 2 (10%)	-100000	10000	77000	0	-7.11%
Default at date 3 (10%)	-100000	10000	10000	77000	-1.12%
No default (80%)	-100000	10000	10000	110000	10.00%
Expected CF	-100000	10000	16700	95700	7.82%



Pricing mortgages with default

- Assume that lender wants to hit a given IRR on a loan
- Contract rate must exceed this IRR target because of default
- Problem: when contract rate increases, so do default probabilities
- There may be many solutions to this problem (which do we choose?) or no solution (exclusion)



Example

- 3-year FRM, yearly payments, initial balance of \$100,000
- Default hazard rate on the mortgage in each year is:
 $[3 + m/40,000] \%$
- Loss severity: 25%
- Target **IRR**: 10%
- Is there a contract rate that delivers the right **IRR**?
- Can the right **IRR** be delivered with a contract rate of 10% and positive points?

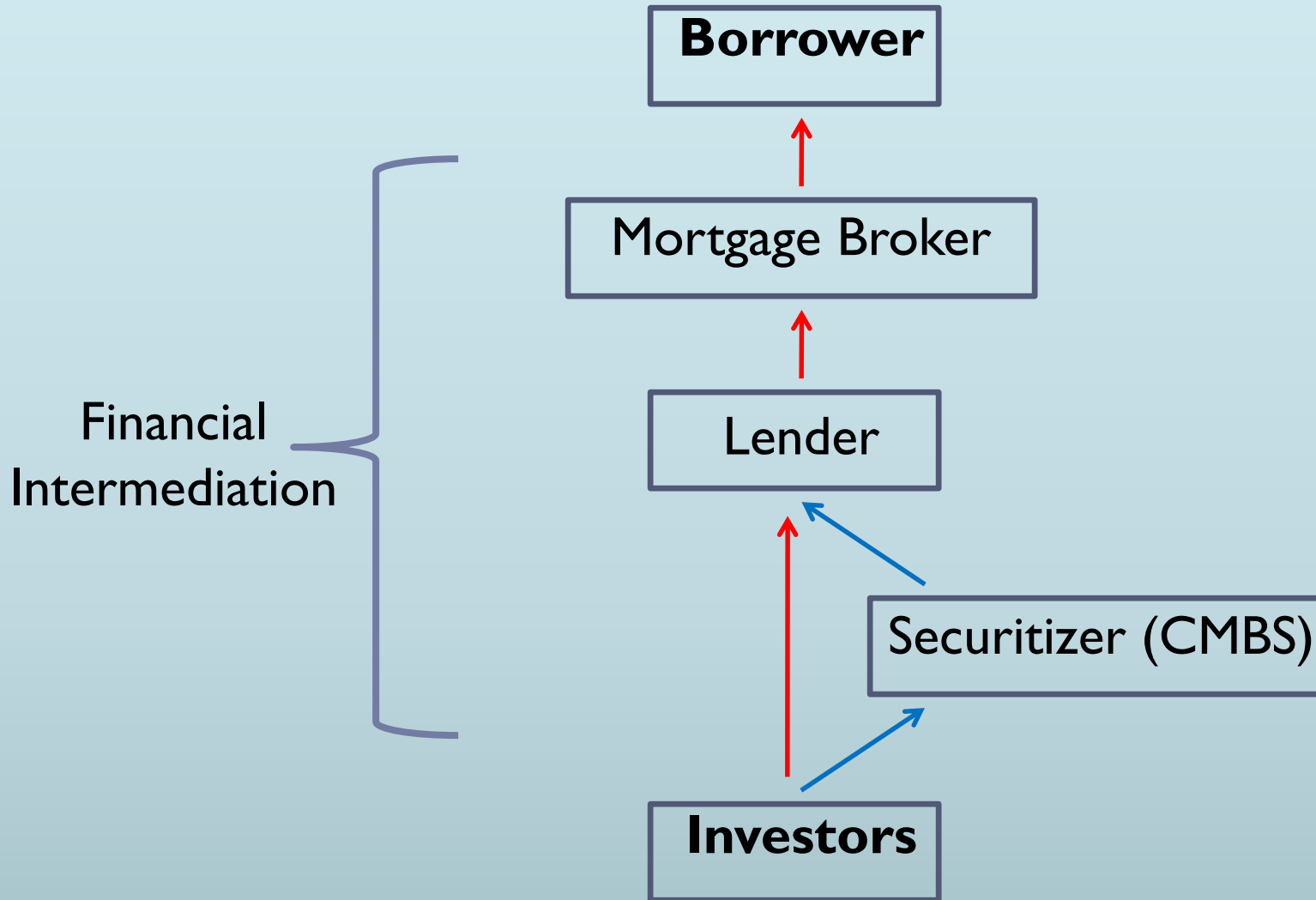


Example with no solution (exclusion)

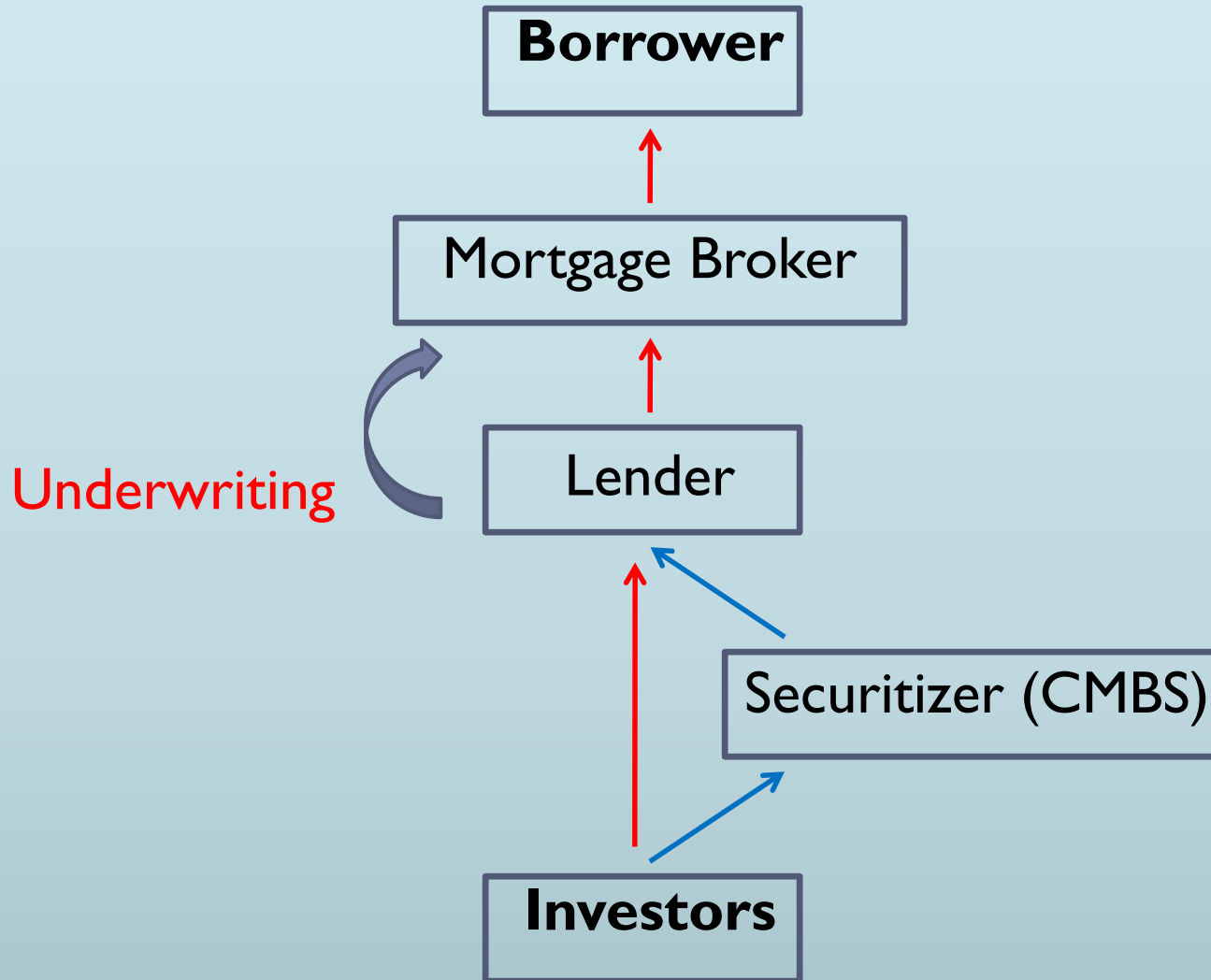
- 3-year FRM, yearly payments, initial balance of \$100,000
- Loss severity is 80%
- Default hazard rate on the mortgage in each year is:
 $[2 + (m/10,000)^2] \%$
- Then, it is not possible to hit a target of 10%
- Hazard rates rise too fast as we try to raise the payment
- This borrower is too risky
- At lower targets, a different problem may arise: multiple solutions
- This second problem is an easy one to deal with



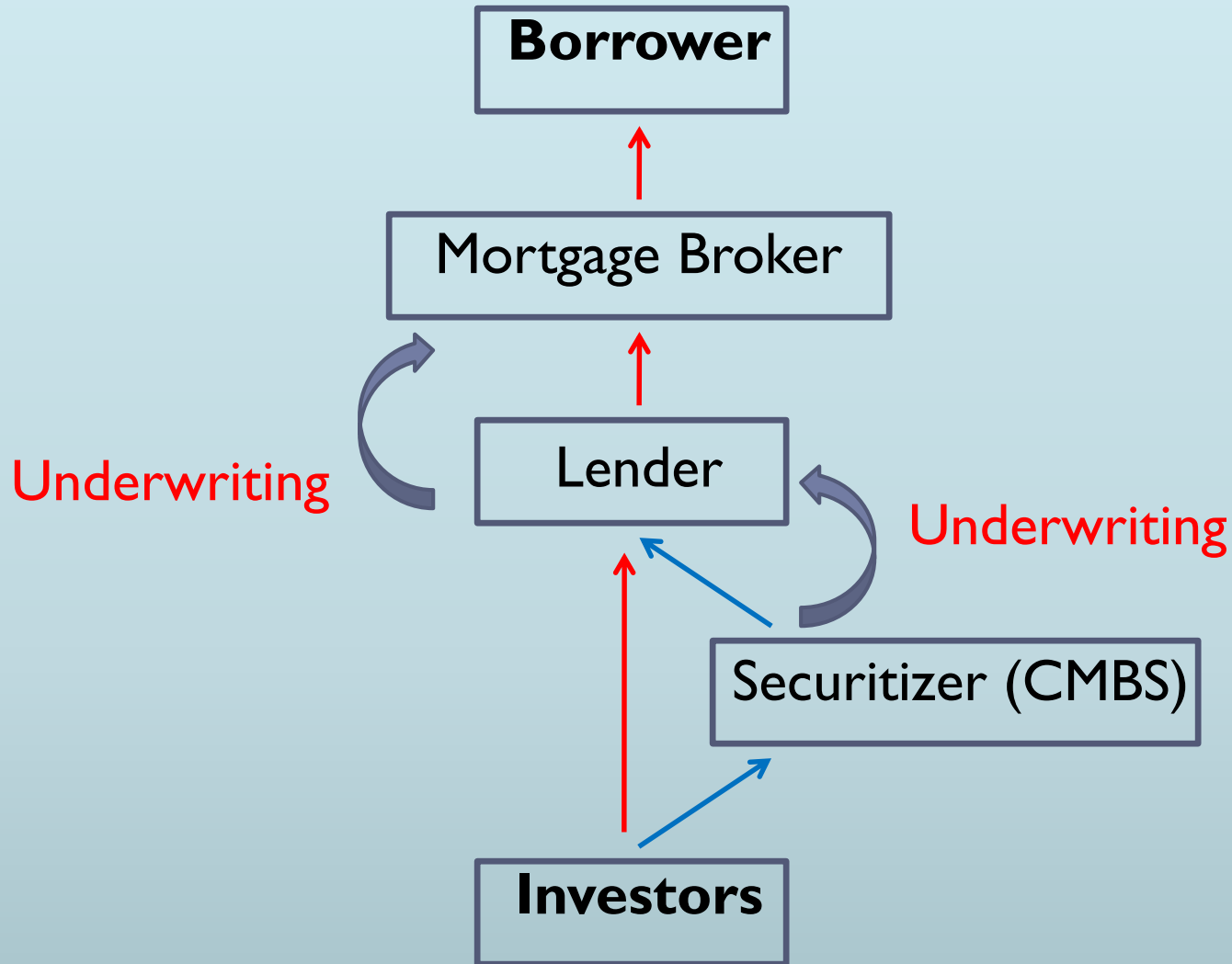
The mortgage process



Underwriting criteria



Underwriting criteria



Underwriting criteria

- Lenders tell brokers what they'll fund:
 1. Leverage (loan-to-value ratio)
 2. Credit worthiness of borrowers
 3. Proper documentation
 4. Ratio of projected cash-flows to debt-service
 5. ...
- Likewise, securitizers tell lenders what they'll buy
- When secondary markets are involved, lenders pass underwriting standards on to brokers

