Immunization

Fixed income

Bullet immunization

- Say you need to finance a payment of $100k in 8 years
- You could invest in an 8-year zero, and you are done
- If not feasible, you could invest in a bond or portfolio of bonds with an 8 year duration
- *Bullet immunization*:
  1. duration=investment horizon
  2. PV of investment= PV of liability
- *Fisher-Weil classical immunization theorem*: Assume a flat yield curve and level-shifts only, interest rate risk and reinvestment risk exactly balance one another

Balance-sheet immunization (stolen from Ivan)

Example 3: Hedging interest rate risk

- Consider the balance sheet of the financial institution:

Table 3.5 Asset and Liabilities of a Financial Institution

Assets				Liabilities			
Item	Amount	Duration	Dollar Duration	Item	Amount	Duration	Dollar Duration
Cash	100	0	0	Deposits	600	0	0
S.T. Loans	300	0.8	240	S.T. Debt	400	0.5	200
M.T. Loans	500	3	1500	M.T. Debt	400	4	1600
L.T. Loans	1500	12	18000	L.T. Debt	400	8	3200
Total	2400		19740	Total	1800		5000
				Equity	600		14740

- How much would the equity change due to a 1% shift in interest rates?

The math

- Letting *E* be equity while *A* and *L* are assets and liability:

$$
\begin{aligned}
dE &= dA - dL \\
&= A\frac{dA}{A} - L\frac{dL}{L}
\end{aligned}
$$

- Dollar duration of equity equals dollar duration of assets minus dollar duration of liabilities

Insurance companies

- The liabilities of insurance and pension companies are or very much look like a predictable, growing annuity
- They can cash-flow match (expensive) or duration match (cheaper)
- But growing annuities become super convex at low interest rates (why?), so:
  1. Frequent readjustments are needed as rates fall
  2. Demand for safe assets from insurance companies rise as rates fall
- Appears borne out by the data (see BIS paper) and may create downward spiral for rates

The general idea

- A portfolio that features fixed-rate bonds is subject to interest risk
- Duration (first order) and convexity (second order) are measures of that risk
- This risk can be mitigated:
  1. Forward/futures (set the delivery price today)
  2. Buy bond put options, sell call options
  3. Cash-flow matching
  4. Duration matching

Option example

- A risk-free bond has duration of 5 and both a face value and market value of $10M

- A T-bond put option on an underlying bond worth $9.5M and of duration 10 has a delta of -0.5

- First order dollar risk (in $M): $10 \times 5 \times \frac{1}{100}$

- Put option dollar risk (in $M): $9.5 \times 10 \times \frac{1}{100} \times 0.5$

- Buy $\frac{10\times5}{9.5\times10\times0.5}$ puts to eliminate all first-order risk

Using plain vanilla swaps

- Swapping fixed payments for floating payments reduces an intermediary's duration risk
- Indeed, floating instrument have very low duration (no higher than the time to the next reset)
- Intermediaries can measure their duration exposure and control it at will via swaps
- In fact they have to, by law, and swaps are the quickest way to get compliant

Using interest caps and floors

- A bank can buy full or partial protection against interest rates movements by buying caps or floors

- Interest caps are a derivative contract on an underlying interest rate rate r_t that pays:

$$\max(0, r_t - \bar{r}) \times A$$

where $\bar{r}$ is a fixed value and A is some notional

- It's a call option so its price implies in the standard option theoretic way a volatility of the underlying rate

Cap and floor menu example

SOFR Cap Pricing (% of Notional)						
Strike	2Y	3Y	4Y	5Y	7Y	10Y
3.00%	2.90%	3.94%	4.92%	5.97%	8.37%	12.11%
3.25%	2.54%	3.48%	4.35%	5.30%	7.52%	11.00%
3.50%	2.06%	2.78%	3.57%	4.45%	6.36%	9.39%
3.75%	1.73%	2.37%	3.08%	3.87%	5.62%	8.43%
4.00%	1.42%	1.98%	2.62%	3.33%	4.94%	7.55%
4.25%	1.14%	1.63%	2.20%	2.86%	4.33%	6.76%
4.50%	0.86%	1.27%	1.75%	2.31%	3.60%	5.74%
4.75%	0.64%	1.00%	1.43%	1.94%	3.12%	5.10%
5.00%	0.45%	0.76%	1.15%	1.61%	2.70%	4.54%
5.25%	0.30%	0.58%	0.92%	1.35%	2.35%	4.06%
5.50%	0.21%	0.45%	0.76%	1.14%	2.07%	3.66%
5.75%	0.15%	0.37%	0.65%	1.00%	1.85%	3.34%
6.00%	0.12%	0.31%	0.56%	0.88%	1.67%	3.06%

SOFR Floor Pricing (% of Notional)						
Strike	2Y	3Y	4Y	5Y	7Y	10Y
0.00%	0.04%	0.10%	0.17%	0.26%	0.46%	0.83%
1.00%	0.10%	0.23%	0.39%	0.57%	0.98%	1.66%
2.00%	0.22%	0.52%	0.87%	1.25%	2.05%	3.30%
2.50%	0.33%	0.78%	1.28%	1.81%	2.90%	4.56%
2.75%	0.41%	0.95%	1.54%	2.16%	3.44%	5.35%
3.00%	0.50%	1.14%	1.83%	2.56%	4.03%	6.19%
3.25%	0.61%	1.37%	2.18%	3.02%	4.70%	7.16%
3.50%	0.76%	1.68%	2.64%	3.63%	5.61%	8.49%
4.00%	1.09%	2.30%	3.55%	4.81%	7.31%	10.88%
4.50%	1.68%	3.33%	4.87%	6.43%	9.62%	14.19%
5.00%	2.22%	4.23%	6.12%	8.00%	11.80%	17.19%
5.50%	2.90%	5.28%	7.55%	9.79%	14.23%	20.48%
6.00%	3.74%	6.51%	9.17%	11.78%	16.90%	24.04%

Why consider derivatives?

Insure, Not Speculate

- ✓ Objective: protect against worst case scenario, not speculate.

Cost Savings & Flexibility

- ✓ Pricing favorable for term funding.
- ✓ Accounting flexibility

Off-Balance Sheet

- ✓ No inflating the balance sheet.

FASB Easy

- ✓ Portfolio layer hedge method for swaps
- ✓ Straight Line Amortization for caps and floors

Very Common

- ✓ Many financial institutions getting macro-hedging in place.

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The Orange County bankruptcy, again

- In addition to investing in inverse floaters…
- … they also engaged in massive duration mismatch (2-5 year treasuries on the asset side, 24h repos on the liability side)
- Both are bets on falling rates
- In 1994, rates rose, OC became insolvent, fire sale ensued, game over

ASWs: the ideal duration management tool

- *Asset Swaps:*
  1. Buy a fixed rate bond
  2. Swap the fixed payment for LIBOR/SFOR + spread

- The swap continues (or is unwound at market value) even if the underlying bond defaults so its pricing depends on the counterparty risk inherent to the swap traders, not the underlying bond*

- This is an ideal way to mitigate interest rate risk because this is based on the very fixed payment whose duration impact you're trying to mitigate/eliminate

*Terms may vary. Default may constitute "breakage" in which case the contract is subject to breakage costs and fees. Bloomberg pricing assumes no breakage so that ASWs are in fact a bond purchase plus a swap agreement between default-free and prepayment-free entities.

Par asset swaps

- When bond trades away from par (say at $P \neq 100$), asset buyer "pays" and additional *100-P* to seller at inception

- Letting swap spread be s^A while c is the coupon rate, fair pricing means:

$$PV(100(LIBOR + s^A - c)) = 100 - P$$

- So the swap spread reflects both c and where the bond trades vis-à-vis par

- Bloomberg calculates all that for us under <ASW>

Leverage mechanics: the case of M-Reits

- REITs are corporations that are exempt from corporate taxation as long as:
  1. The invest mostly in real estate assets
  2. They distribute most of their net income each quarter
  3. They have a diffuse shareholder base
  4. …
- Mortgage REITs invest in mortgages and mortgage-backed securities
- Their dividend yield oscillates between 10 and 20 percent a year (!)
- How? Massive leverage