

The term structure

Fixed income

Bootstrapping

- Needed: rates on zeros at all maturities
- What is the present value of 1\$, risk free, to be delivered 1, 2, 3.5, 10 years from now?
- This is the information we need to discount “risk-free” strings of payments...
- ... and can be inferred from the yield curve
- Only issue is that zero-coupon bonds don't exist for all maturities
- But we can engineer and price zero-coupon portfolios of treasuries
- This gives the *theoretical spot yield curve*



Theories of the term structure

- The *normal shape* of the yield curve is upward sloping and concave
- So much so that we refer to a downward sloping curve as *inverted*...
- ...a situation viewed as temporary, a bad omen for the economy, and destabilizing
- Why does the yield curve typically slope up?
- What drives its steepness?



The expectations hypothesis (EH)

- According to this theory, long-term returns are the expected return of short-term roll-over strategies

- Under EH, with t being today:

$$(1 + r_{t,2})^2 = (1 + r_{t,1})(1 + Er_{t+1,1})$$

$$(1 + r_{t,3})^3 = (1 + r_{t,1})(1 + Er_{t+1,1})(1 + Er_{t+2,1})$$

...

- So if $r_{t,1} = 2\%$ and $Er_{t+1,1} = 3\%$ then $r_{t,2} = 2.5\%$
- Note that the yield curve slopes up if and only if short-term rates are expected to rise, under this hypothesis



Relations to forward rates

- By definition, forward rates solve:

$$(1 + r_{t,2})^2 = (1 + r_{t,1})(1 + f_{t+1,1})$$

- So under EH and under EH only, $Er_{t+1,1} = f_{t+1,1}$
- We would expect $Er_{t+1,1} < f_{t+1,1} \dots$
- ... because long-term investors are exposed to more risk
- In fact, $f_{t+1,1} - Er_{t+1,1}$ is an excellent measure of that premium



General formula for forward rates

- Notation: $f(t, k)$ forward rate at date today + t for tenor k
- Formula:

$$f(t, k) = \left[\frac{(1 + r_{t+k})^{t+k}}{(1 + r_t)^t} \right]^{1/k} - 1$$

where r_t is today's spot yield at tenor t

- Plus standard annualization adjustment as needed
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A very influential variation

- The *5-year, 5-year forward inflation expectation rate*:

$$\left[\frac{(1 + r_{10} - r_{10}^{TIPS})^{10}}{(1 + r_5 - r_5^{TIPS})^5} \right]^{1/5} - 1$$

is the Fed's favorite gauge of medium-term inflation expectations

- This is what they're currently using to tell us that everything is honkey-dorey
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Changing shapes

- *Bear flattening*: short-term rates rise more than long-term rates
- *Bear steepening*: short-term rates rise less than long-term rates
- *Bull flattening*: short-term rates fall less than long-term rates
- *Bull steepening*: short-term rates fall more than long-term rates



Yield-curve strategies

- **If stable:**
 1. Buy and hold
 2. If upward sloping, ride the YC
 3. Sell convexity
 4. Carry trade (borrow short, invest long)
- **For movements in level, slope, or curvature**
 1. Duration management
 2. Buy convexity
 3. Bullet, barbells, butterflies, and condors



Riding the yield curve



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FINANCE QUIZ DU JOUR

You invest in a 2 YR Bond yielding 2.42%. What is your total return after one year assuming all rates stay the same?

Treasury Yields

NAME	COUPON	PRICE	YIELD
GB3:GOV 3 Month	0.00	0.65	0.66%
GB6:GOV 6 Month	0.00	1.08	1.10%
GB12:GOV 12 Month	0.00	1.64	1.68%
GT2:GOV 2 Year	2.25	99.66	2.42%
GT5:GOV 5 Year	2.50	99.38	2.63%
GT10:GOV 10 Year	1.88	94.05	2.56%
GT30:GOV 30 Year	2.25	92.67	2.61%