The term structure

Fixed income

Bootstrapping risk-free spot yields

- Needed: rates on safe zeros at all maturities
- What is the present value of 1$, risk free, to be delivered 1, 2, 3.5, 10 years from now?
- This is the information we need to discount "risk-free" strings of payments…

- As we discussed in our first chapter, this can be inferred via bootstrapping from:
  1. Outstanding treasuries
  2. Strip markets
  3. Swap and Euro-Dollar rates
- The last option dominates practice because it relies on the most complete and liquid markets

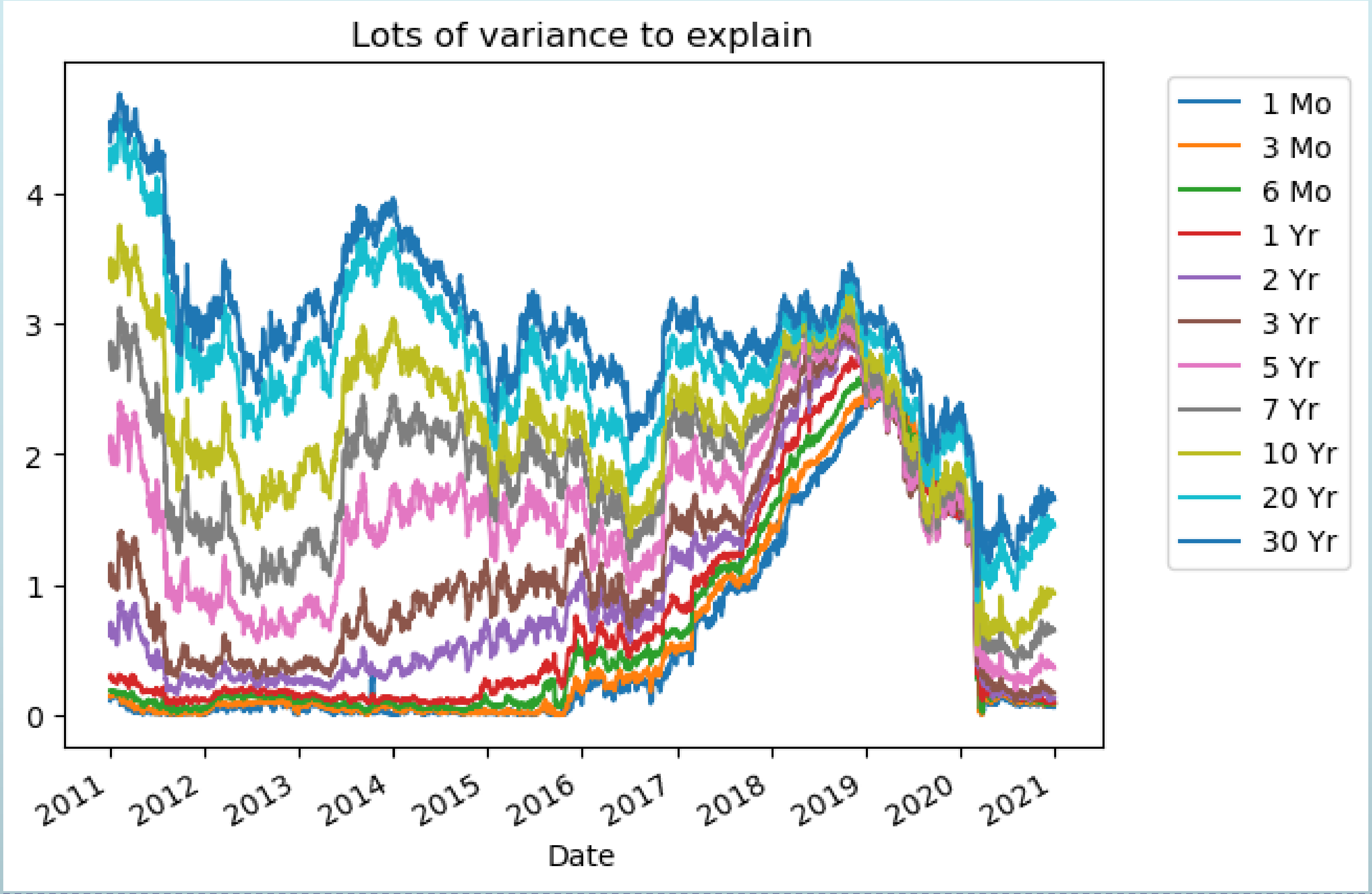
Theories of the term structure

- The *normal shape* of the yield curve is upward sloping and concave
- So much so that we refer to a downward sloping curve as *inverted*…
- …a situation viewed as temporary, a bad omen for the economy, and destabilizing
- Why does the yield curve typically slope up?
- What drives its steepness?

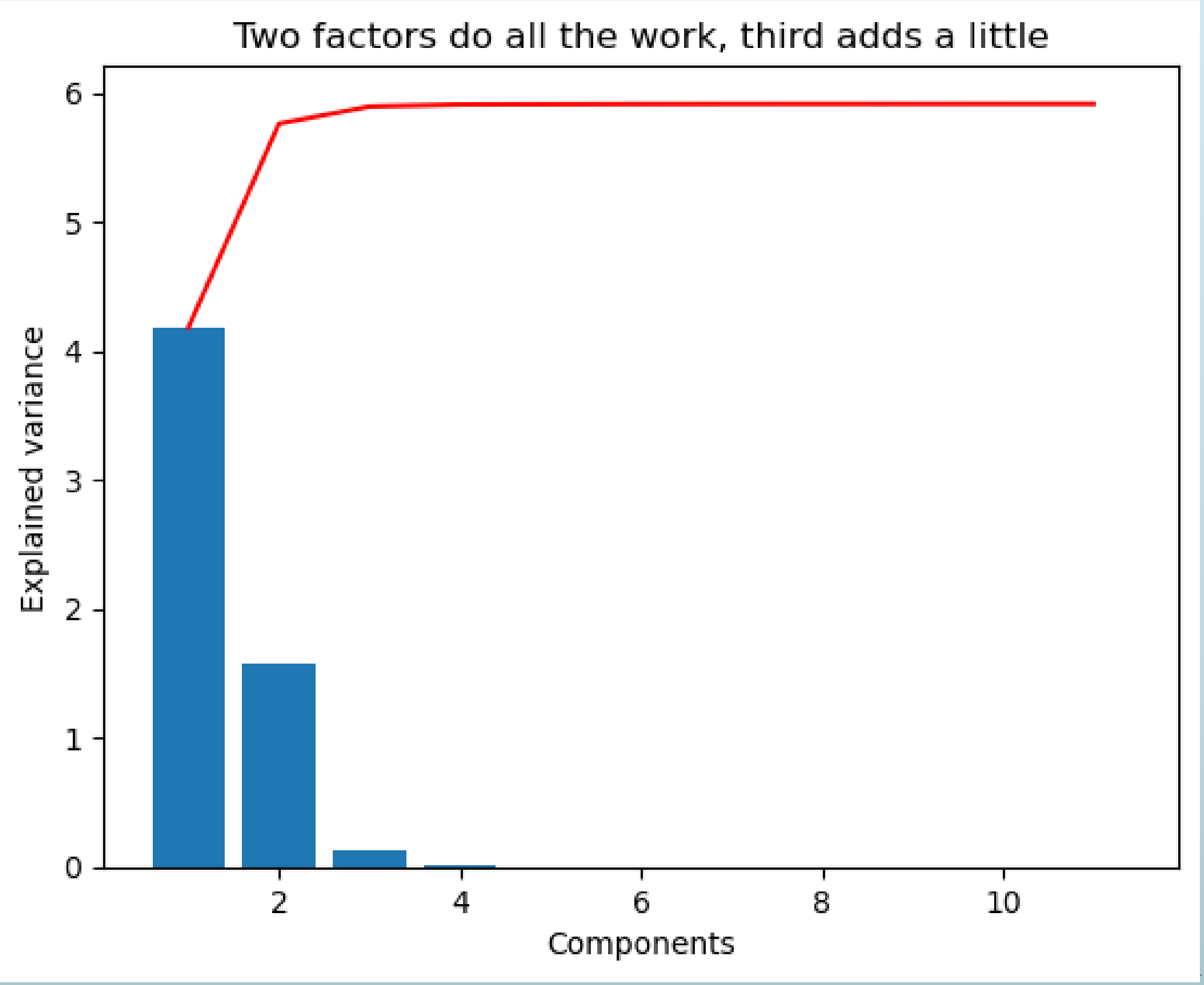
Shape talk

- In common industry parlance, we speak of the curve's *level*, *slope*, and *curvature*
- This practice can be rationalized via low-rank approximation theory encapsulated by the glorious Eckart–Young–Mirsky theorem
- In other words, via *principal component analysis* or equivalently *singular value decomposition* of historical yield curve data
- See Python notebook

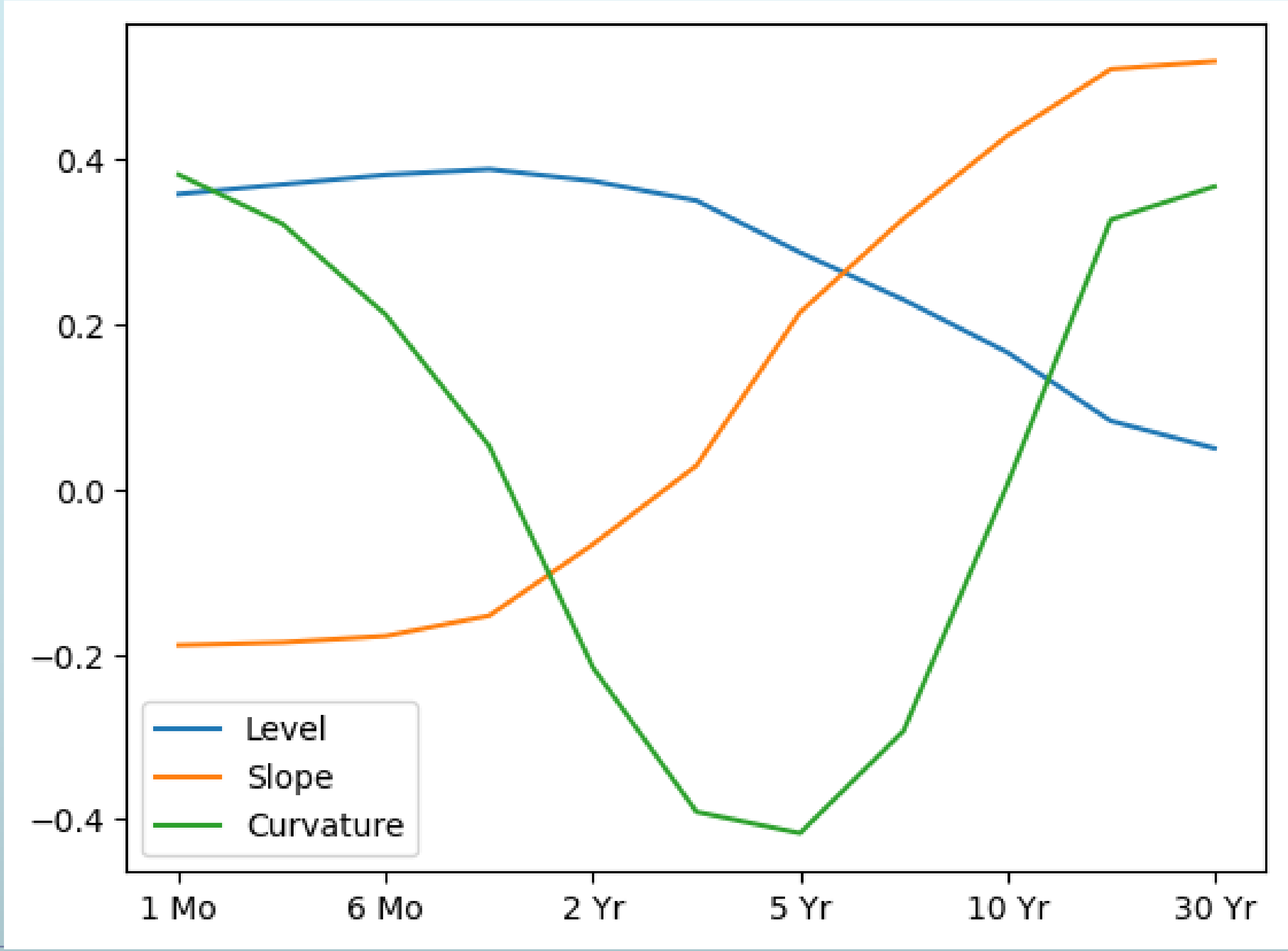
The variation to explain



Two, maybe three factors



Level, slope, and curvature



Three main theories

1. *Expectation hypothesis*: long-term rates are geometric averages of expected short-term rates

2. *Market segmentation theory*: different terms are different markets with different investor preferences; supply and demand pin down rates one tenor at a time

3. *Term/liquidity premium theory*: long rates are geometric averages of expected short-term rates plus a risk premium

The expectations hypothesis (EH)

- According to this theory, long-term returns are the expected return of short-term roll-over strategies

- Under EH, with t being today:

$$(1+r_{t,2})^2 = (1+r_{t,1})(1+Er_{t+1,1})$$
$$(1+r_{t,3})^3 = (1+r_{t,1})(1+Er_{t+1,1})(1+Er_{t+2,1})$$
$$\ldots$$

- So if $r_{t,1} = 2\%$ and $Er_{t+1,1} = 3\%$ then $r_{t,2} = 2.5\%$

- Note that the yield curve slopes up if and only if short-term rates are expected to rise, under this hypothesis

Relations to forward rates

- By definition, forward rates solve:

$$(1+r_{t,2})^2 = (1+r_{t,1})(1+f_{t+1,1})$$

- So under EH and under EH only, $Er_{t+1,1}$=$f_{t+1,1}$

Forward contracts must pay f

- Assume, by way of contradiction, that in forward markets:

$$(1 + r_{t,2})^2 > (1 + r_{t,1})(1 + f_{t+1,1})$$

- Then:
  1. Buy 1$ par-worth of 2-period bond at date t for DF(2), since it's cheap
  2. Finance by selling $\frac{DF(2)}{DF(1)}$ par worth of 1-period bond since it seems expensive
  3. Agree to borrow $\frac{DF(2)}{DF(1)}$ at $f_{t+1,1}$ at date 1 to pay the par you owe on the bond you just shorted
  4. Pay back loan at date 2

- No cash flows at date 0 or 1 by construction

- Cash flow at date 2:

$$1 - \frac{DF(2)}{DF(1)}(1 + f_{t+1,1}) = 1 - \frac{(1 + r_{t,1})(1 + f_{t+1,1})}{(1 + r_{t,2})^2} > 0$$

- Arbitrage!!!!!!!!!!!!!

General formula for forward rates

- Notation: $f(t, s)$ forward rate at date today $+ t$ for tenor s

- Formula:

$$f(t, s) = k\left[\left(\frac{\left(1 + \frac{r_{t+s}}{k}\right)^{k(t+s)}}{\left(1 + \frac{r_t}{k}\right)^{kt}}\right)^{1/ks} - 1\right]$$

where r_t is today's spot yield at tenor t (annualized bien sur) and k is the payment frequency

A very influential variation

- The *5-year, 5-year forward inflation expectation rate:*

$$\left[\frac{\left(1 + r_{10} - r_{10}^{TIPS}\right)^{10}}{\left(1 + r_5 - r_5^{TIPS}\right)^5}\right]^{1/5} - 1$$

is the Fed's favorite gauge of medium-term inflation expectations

- This is what they're currently using to tell us that everything is honkey-dorey

The term premium view

- We would expect $Er_{t+1,1} < f_{t+1,1}\ldots$
- … because long-term investors are exposed to more risk
- In fact, $f_{t+1,1} - Er_{t+1,1}$ is an excellent "measure" of that premium
- But $Er_{t+1,1}$ is unobservable so the vast term premium literature does two things:
  1. Find ways to measure the term premium
  2. Attempt to explain (and ideally forecast) pattern in the term premium

Changing shapes

- *Bear flattening*: short-term rates rise more than long-term rates
- *Bear steepening*: short-term rates rise less than long-term rates
- *Bull flattening*: short-term rates fall less than long-term rates
- *Bull steepening*: short-term rates fall more than long-term rates

Yield-curve strategies

- If stable:
  - Long only:
    1. If flat, buy and hold
    2. If upward sloping, ride the YC
    3. If downward sloping, rollover
  - Long-short:
    1. Sell convexity
    2. Carry trade (borrow short, invest long)
- For movements in level, slope, or curvature
  1. Duration management
  2. Buy convexity
  3. Bullet, barbells, butterflies, and condors

Riding the yield curve

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FINANCE QUIZ DU JOUR
You invest in a 2 YR Bond yielding 2.42%. What is your total return after one year assuming all rates stay the same?

Treasury Yields

NAME	COUPON	PRICE	YIELD
GB3:GOV **3 Month**	0.00	0.65	0.66%
GB6:GOV **6 Month**	0.00	1.08	1.10%
GB12:GOV **12 Month**	0.00	1.64	1.68%
GT2:GOV **2 Year**	2.25	99.66	2.42%
GT5:GOV **5 Year**	2.50	99.38	2.63%
GT10:GOV **10 Year**	1.88	94.05	2.56%
GT30:GOV **30 Year**	2.25	92.67	2.61%