The cost of capital

Corporate Finance

Cost of capital

- The cost of capital measures the average expectations of a corporation's capital providers

- A measure of it, for a corporation with n securities $i = 1,2, \ldots$ each with market value MV_i, is:

$$WACC = \frac{\sum_i^n MV_i \times r_i}{\sum_i^n MV_i}$$

- Its most common implementation is

$$WACC = \frac{MV(D)r^D(1-t) + MV(H)r^H + MV(E)r^E}{MV(D) + MV(H) + MV(E)}$$

where r^D, r^H, r^E are the expectation of debt, hybrid, and equity investors and t is the tax rate a corporation pays on its EBIT

Minimizing WACC

- One practical way to think of optimal capital structure management is that, **given an asset mix**, a corporation should seek to minimize its cost of capital

- Quite sensible, as long as one keeps in mind that asset mix and financing cannot be fully dissociated

- A few examples:
  I. Security makes financing cheaper but reduces asset flexibility
  II. Financing mix can mitigate agency frictions hence projected cash-flows
  III. Some stakeholders contribute expertise as well as money

- This chapter discusses the measurement of a corporation's cost of capital

Corporate debt

- Debt contracts issued by corporations fall in three principal categories:
  1. *Bonds*: long-term tradeable securities
  2. *Loans*: long-term debt contracts NOT treated as securities issued by banks (TLAs) and, increasingly, institutions (TLBs)
  3. *Commercial paper*: unsecured, short-term debt instrument issued by corporations typically used to finance short-term liabilities such as payroll, accounts payable, and inventories

Seniority

- *Senior securities* get paid first (within their type) in the event of default, disposition, or reorganization
- *Subordinated/junior securities* get paid after senior securities have been paid
- *Pari Passu*: All creditors at the same level of capital structure are treated as one class

Security

- Secured instruments (mortgages, e.g.) are collateralized by specific tangible assets (real estate, plants, warehouses, machines, airplanes…)
- In theory, secured claims have *absolute priority* over other financial claims, including senior unsecured claims
- To be precise: "*Under the bankruptcy code, secured claims are entitled to receive value equal to the full value of their interest in the collateral before any value is given to holders of unsecured claims, and any priority unsecured claims are entitled to receive the full value of their claims before any general unsecured claims receive any value.*"
- See "A Primer on Second Lien Term Loan Financings", Neil Cummings and Kirk A. Davenport, 2004

Callable/Redeemable securities

- A fixed income security is *callable* if the issuer may *recall/retire* it before maturity
- This is an option which the issuer has the right but not the obligation to exercise
- Terms are specified by the indenture document (aka prospectus):
  1. Lockout period
  2. Call price
  3. Make-whole provisions

Putable securities

- A fixed income security is *putable* if lender(s) may force the issuer to retire it before maturity
- This is an option which the lender has the right but not the obligation to exercise

- Note well: most fixed income contracts feature *acceleration clauses* that are triggered by specific events such as default, but putable securities feature essentially unconditional puts at specific dates

Sinking fund provisions

- A commitment by the issuer to make regular deposits into a trustee-managed fund to be used, eventually, to pay back the issue

Convertible securities

- A fixed income security is *convertible* if the buyer has the right but not the obligation to exchange its debt claims for a fixed number of equity claims
- A traditional bond with a call option

Covenants

- Legally binding agreement between issuer and investors
- *Positive, negative, and financial*
- Typical financial covenants:
  1. Maximum leverage (Debt/EBITDA, say)
  2. Minimum DSCR (EBITDA/Interest)
  3. Minimum rating
- *Incurrence* covenants (as opposed to *maintenance* covenants) are triggered by specific events

Universal principles of debt math

- A debt contract stipulates:
  1. An initial balance/original face value b_0
  2. A frequency of payments and total number T of payments (term to maturity)
  3. An interest rate r_t for each period $t = 1,2, \ldots T$
  4. Payments m_t for each period $t = 1,2, \ldots T$
- Debt algebra:
  1. At a given date, interest due is $b_{t-1}\, r_t$
  2. Balance at the end of date t is $b_t = b_{t-1} + b_{t-1} r_t - m_t$
  3. If $b_T > 0$, balance is due in one *balloon payment*

Some language

- Debt contracts whose balance is zero after T periods ($b_T = 0$) are called *fully amortizing*
- Interest rates can be fixed, vary on a fixed schedule, or according to some other market rate
- Amortization can be negative -- balance can grow -- from one period to the next (if $m_t < b_{t-1}r_t$)

Ex: Bullets (fixed-rate IO bonds)

- For all t:
  1. $r_t = r$
  2. $m_t = b_o r$

- Zero amortization: $b_T = b_0$

Ex: zero-coupon bond

- For all t:
  1. $r_t = 0$
  2. $m_t = 0$

- Zero amortization: $b_T = b_0$

Ex: Floating rate coupon bonds (floaters)

- For all t:
  1. Initial rate: r_0
  2. At reset, $r_t^* = index_t + QM$
  3. QM is the typically fixed quoted margin
  4. r_t adjust towards r_t^* subject to caps and floors, both absolute and on adjustment steps
  5. $m_t = b_o r_t$

- Zero amortization: $b_T = b_0$

- Libor was the most typical index until last year but we have now moved towards alternative reference rates like CMT or SOFR

Ex: Inverse floaters

- For all t:
  1. $r_t = \bar{r} - index_t$ where $\bar{r}$ is some fixed reference
  2. caps and floors

- Those, together with duration/maturity mismatch, killed Orange County finances in 1994

Ex: Fixed rate, fixed payment debt

- For all t:
  1. $r_t = r$
  2. $m_t = m$
- Fully amortizing: $b_T = 0$
- What must m be? (Fixed annuity formulae)
- $m = b_0 r / (1 - (1 + r)^{-T})$

Term loan facilities (*Project Financing*)

Term loan A	Term loan B
Bank ("pro rata") loans issued to typically IG corporate borrowers	Syndicated loans often made to below IG-borrowers, institutional lenders active
Bespoke	Bespoke to some extent
Senior, first lien	Senior, first lien
Short tenor (5-7 years typical)	May have longer tenor
Floating rates	Floating rates
Allow for prepayment with little to no penalties	Allow for prepayment with little to no penalties
Significant amortization	Little to no amortization
Tight covenants, especially w.r.t. operations and leverage	Covenant-lite

Revolving credit facilities (*revolvers*)

- Credit lines in exchange for fees
- "Deal sweeteners," usually combined with TLs

The fundamental equation of debt design

- Full amortization means:

$$b_T = 0, \quad \text{or, equivalently,} \qquad b_0 = \sum_{t=1}^{T} \frac{m_t}{(1+r)^t}$$

- More generally:

$$b_0 = \sum_{t=1}^{T} \frac{m_t}{(1+r)^t} + \frac{b_T}{(1+r)^T}$$

- Absent side payments, r is the loan's IRR <u>if all payments are made</u>, i.e. the YTM on the debt contract

Fixed payment example

- $100K$, monthly payments, 10 years, $r = 7\%/12$

1. With full amortization: $m = \$1{,}161.08$
2. With 30K balloon: $m = \$\,987.76$

Graduated payment example

- A corporation wants to finance 50% of a $\$1M$ investment with some debt
- Cash-flows are expected to increase by 0.5% a month over ten years, so the corporation would like debt payments to match that profile
- A lender is willing to do it with a 7% annual interest rate and a fully amortizing contract
- What is the payment schedule?
- Answer: fundamental equation of debt design

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Market value vs book value

- The book value of a bond is its face value b (we'll often write *FV* too)

- Market value is what the bond would sell for in the market:

$$MV = \sum_{t=1}^{T} \frac{m_t}{(1+y)^t} + \frac{b_T}{(1+y)^T}$$

where y is the *yield-to-maturity* investors are currently requiring from the bond

Market value drivers

- Tautologically:

$$y = r^F + RP$$

- Ergo, two things move market values:
  1. The general level of interest rates (*market conditions*)
  2. Spread changes caused by category-wide or issuer specific reasons

- Trivially, $MV < FV$ if and only if $y > r$, and vice versa

Credit-default swaps, a preview

Protection/CDS buyer **Protection/CDS seller**

$$Premium = Notional(A) \times Swap\ rate\ (\kappa)$$

If "credit event"

$$\begin{aligned} Payment &= Face\,Value\ of\ oustanding\ debt - Market\,Value \\ &= Loss\ given\ default\ (LGD) \end{aligned}$$

CDS rates are risk-premia

- Intuitively (we will prove it under premise of perfect markets), CDS rates should match the spread between the yield on the underlying and the risk-free counterpart
- Indeed, both are compensation for probability of default times loss given default
- Two big, practical applications of this fundamental fact:
    1. Synthetic CDOs *(Who needs to issue securities any more? Just fake it with purely nominal CDS contracts)*
    2. CDS basis convergence trades

Convertible bonds

- A *convertible bond* gives the holder the right to convert her bond investment into equity at an agreed-upon *conversion price* and/or *conversion ratio*

- Example: Consider a 6-month convertible bond with face-value $1,000, S/A coupon rate of 10%, and a conversion price of $25. No issuer call option, no default.

- The conversion ratio is $\frac{1000}{25} = 40$ shares per bond

Convertible bond valuation

- Assume the share price 6 months from now is either \$20 or \$30, each with equal ***risk-neutral probability*** and that the 6-month risk-free (z) rate is 5% (annualized as always)
- What is the value of the bond?
- If the holder does not convert in 6 month, they get \$1,050
- If they do convert they get $40 \times$ share price plus accrued interest
- Obviously the option is exercised when and only when the share price is \$30

Fundamental theorem in Finance

No arbitrage

$$\Longleftrightarrow$$

$$q = \frac{E^*(X)}{1 + r^F}$$

where the expectation* is with respect to a synthetic probability distribution called the risk-neutral probability and r^F is the risk free rate

Most of modern finance prices assets by estimating the RNP first and then pricing assets as if agents were risk neutral

This is especially true in fixed income

Convertibles in WACC

- Decomposing a convertible into straight debt and the value of the warrant is a standard exercise in corp fin, e.g. in the context of measuring a company's WACC

- Process is simple:
  1. Discount coupons in principal at the YTM the market requires on debt of this type absent options
  2. Get the value of the call as a residual
  3. Get the beta of the call as $\beta \times \frac{S}{C} \times \delta$ where β is the stock's beta, *S* is the current stock price, *C* is the value of the call, and δ is the call's delta, estimated using Black-Scholes *(see proof next slide)*
  4. Put it all together to get the overall impact of convertible on cost of capital

- This is all well and good for corp fin but this presumes that the warrants are properly priced by the market

- Fixed income investors look for and look to exploit warrant mispricing

Approximate warrant beta formula

- Let S and C be the current stock and call prices, δ be the call's delta, while β_E and β_C are the stock and the call's betas, respectively
- Then over a given holding period:

$$\beta_C = \frac{COV(r_C, r_M)}{VAR(r_M)} = \frac{COV\left(\frac{dC}{C}, r_M\right)}{VAR(r_M)}$$

$$= \frac{COV\left(\frac{\delta dS}{C}, r_M\right)}{VAR(r_M)}$$

$$= \frac{\delta S}{C} \times \frac{COV\left(\frac{dS}{S}, r_M\right)}{VAR(r_M)}$$

$$= \beta_E \times \delta \times \frac{S}{C}$$

- Notes:
  1. Only first order stock price effect is captured
  2. This presumes a good estimate of δ, usually based on Black-Scholes

Pure convertible bond arbitrage

- Assume that immediately prior to the final coupon payment, the share having already moved up to $30, the bond trades at $1,150
- Then buy the convertible, convert and sell the resulting shares gives you a pure arbitrage
- But that is not what hedge funds mean by convertible arbitrage, a practice made popular by Ed Thorp, the spiritual father of all quants
- *Convertible arbitrage* in the way quants use the expression is not an arbitrage, it is a *convergence trade*

Ed Thorp's version

- If value of the convertible bond is less than value of the non-convertible equivalent plus the value of the call option (the value of the *warrants*):
    1. *Buy the convertible bond*
    2. *Short the stock*

- The number of shares (the *hedge ratio*) is set to immunize the impact of movements in the share price on the bond, this is called *delta hedging*

- Hedge ratio is continuously/dynamically adjusted by buying or selling more stock

No arbitrage

- Tons of risks remain:
  1. *Interest rate risk (can be hedged in standard ways)*
  2. *Default risk (negative gamma/convexity in the large)*
  3. *Risk that mispricing will worsen rather than increase, like it does during financial crises*

Positive gamma?

- When price goes up, bond's delta goes up **<u>locally</u>**, and vice versa
- The rate of change in delta as prices change is called gamma
- When the price goes up the bond's value goes up both because the probability of conversion goes up and the conversion price goes up
- In principle, this makes Ed Thorpe's position **<u>locally</u>** convex

Bond value vs stock price

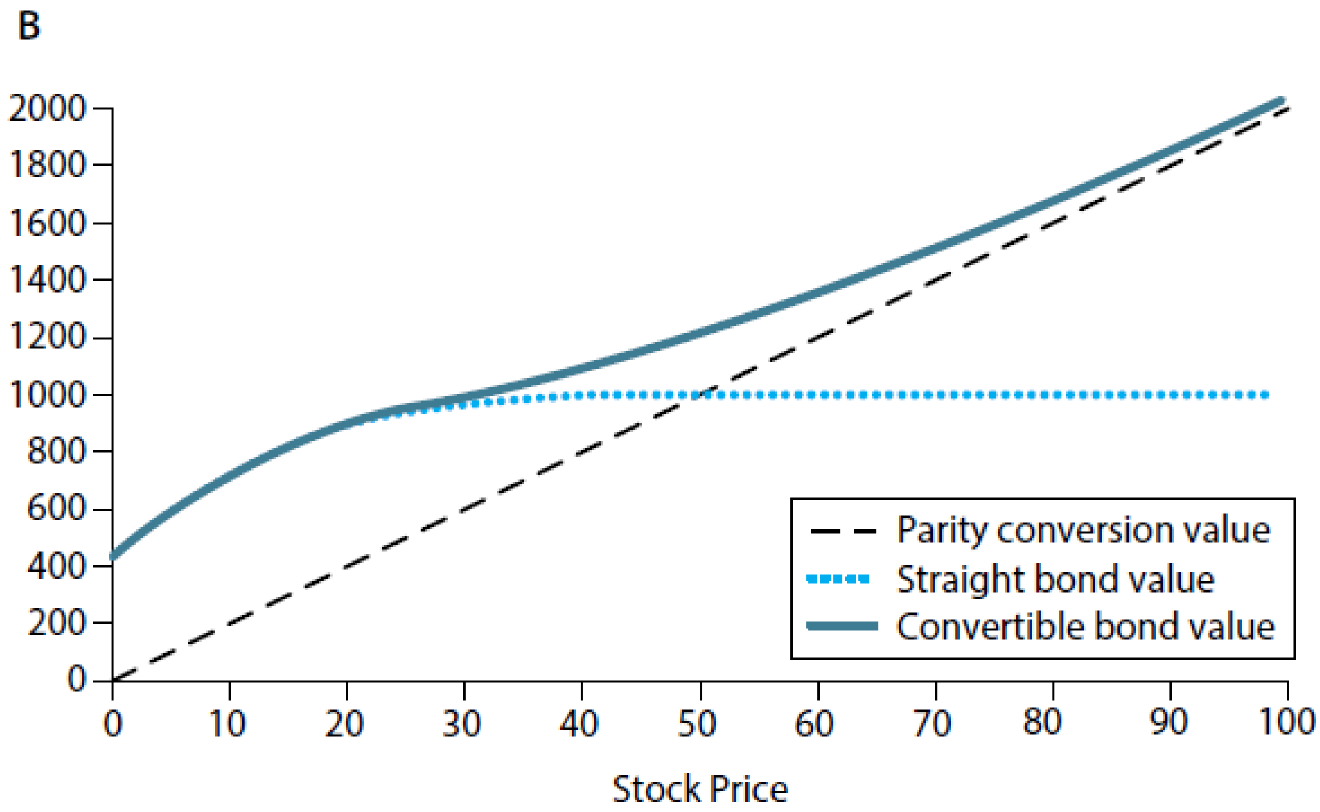


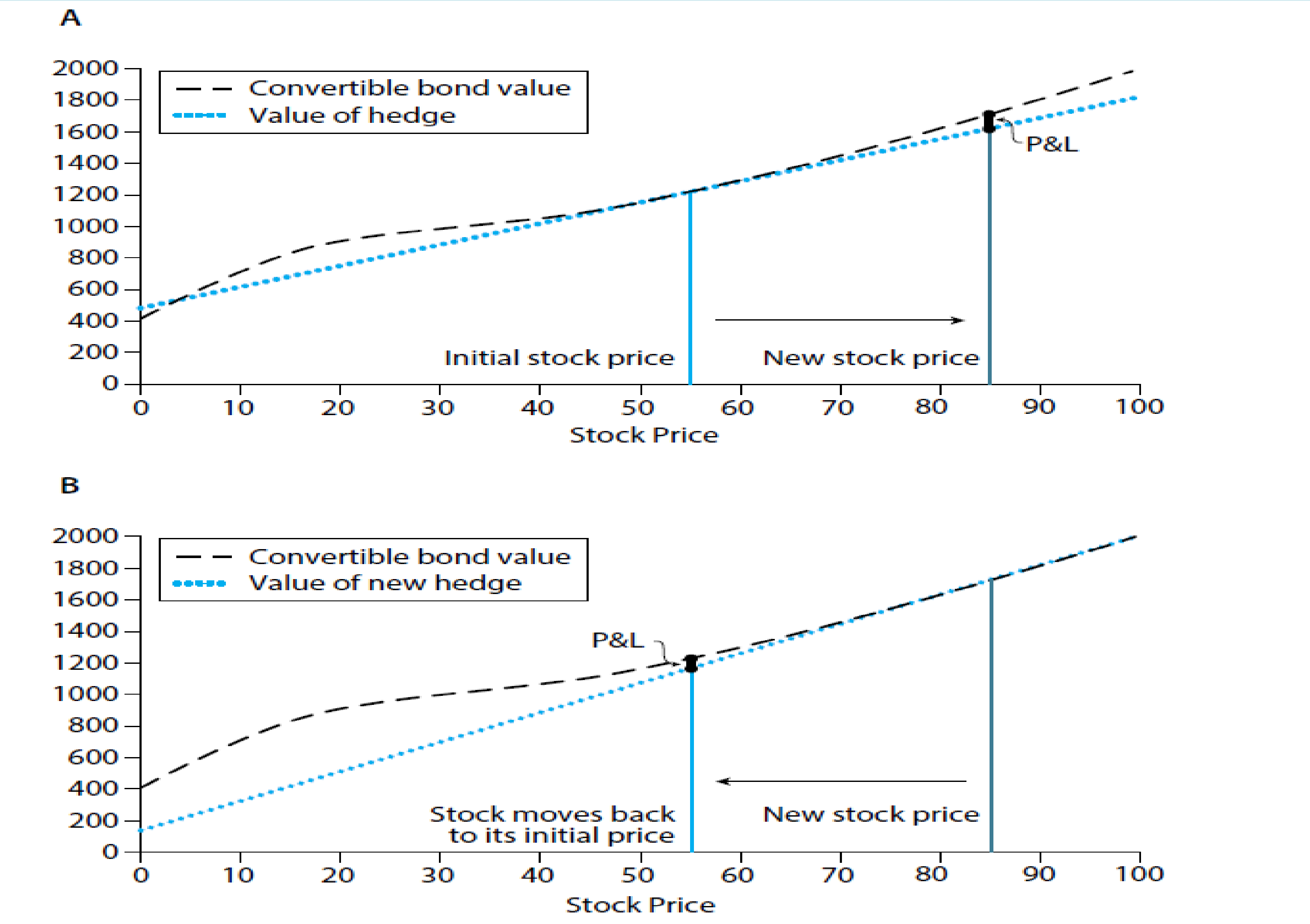
Figure 15.3. How the value of a convertible bond depends on the firm value and stock price.

Panel A. Convertible bond value vs. firm value.

Panel B. Convertible bond value vs. stock price.

- Source: Pedersen's "Inefficiently efficient," a great book

Dynamic hedging



- Source: Pedersen's "Inefficiently efficient," a great book

Preferred equity in WACC

- Preferred equity "promises" a dividend yield calculated on a notional of 100
- Example: assume a corporation's preferred stock trades for $\$80$ per unit and earns a dividend yield of 4%, and that one million units are outstanding
- The market value of preferred stock in that case is is $\$80\text{M}$ while its required return is $\frac{4}{80} = 5\%$