Interest rate models

Fixed income

Modeling interest rate uncertainty

- Consider an investment horizon with capital T periods
- The path of T one-period interest rates $(r_1, r_2, \dots r_T)$ is uncertain, except for the first one
- Assume that the path lives on a binomial tree (rates can go up or down from one period to the next)
- The tree is recombining: value at a given date only depends on total number of ups and downs

Calibration

- We need:
  1. Size of jumps every period
  2. Probability of up or down, under RNP
- Calibrate both to match 1) estimates of interest rate volatility and 2) match the spot-yield curve
- The result is a model that prices treasuries exactly right
- It can and should also price Treasury derivatives trivially

Example: Black-Derman-Troy tree

- Assume yearly rates of $4\%, 4.5\%$, at horizons 1 and 2
- Assume that option prices imply that the volatility (std of log) of rates a year from now is 8%
- Assume symmetric up and down, this implies $r^u = 5.68\%$ and $r^d = 4.83\%$

Interest rate caps (or floors)

- Interest caps are a derivative contract on an underlying interest rate rate r_t that pays:

$$\max(0, r_t - \bar{r}) \times A$$

where $\bar{r}$ is a fixed value and A is some notional

- It's a call option so its price implies in the standard option theoretic way a volatility of the underlying rate

How about bonds with options?

- Can a tree, so-built, price bonds that feature default and prepayment risks?
- It should
- But it doesn't
- Enters *OAS*, the measure of our ignorance