

Fixed income fundamentals

Real estate finance

Fixed income securities

- Debt: contractually specified cash flows
- If CFs are risk-free, market value only depends on interest rate path
- Two main sources of CF risks: prepayment and default
- Building blocks needed:
 1. Interest rate model (discount factors)
 2. Prepayment model
 3. Default model
- All (heroically) under the risk-neutral probability kernel



Fundamental theorem of finance

No arbitrage



$$q_i = E^*(X_i) / (1+r) \text{ for all } i$$

where the expectation* is with respect to a synthetic probability distribution called the risk-neutral probability and r is the risk free rate

Most of modern finance prices assets by estimating the RNP first and then pricing assets as if agents were risk neutral



(a) Spot yield curve



Theoretical spot yield curve

- What is the present value of 1\$, risk free, to be delivered 1, 2, 3.5, 10 years from now?
- This is the information we need to discount “risk-free” strings of payments...
- ... and can be inferred from the yield curve
- Only issue is that zero-coupon bonds don't exist for all maturities
- But we can engineer and price zero-coupon portfolios of treasuries
- This gives the theoretical spot yield curve



(a) Interest rate models



Interest rate trees (Black-Derman-Troy)

- Consider an investment horizon with capital T periods
- The path of T one-period interest rates (r_1, r_2, \dots, r_T) is uncertain, except for the first one
- Assume that the path lives on a binomial tree (rates can go up or down from one period to the next)
- The tree is recombining: value at a given date only depends on total number of ups and downs
- We need:
 1. Size of moves in each period
 2. Probability of up or down, under RNP
- Calibrate both to 1) match estimates of interest rate volatility and 2) match spot yield curve
- Note: the model prices treasuries exactly right by design
- It can/should also price treasury derivatives trivially
- Can it price MBSs at the same time?
- Absolutely not. Yet...



(a) Prepayment and default



Prepayment model

- Assume that prepayment rates are a random variable that lives on the same tree as interest rates (!)
- Example 1: deterministic CPR (PSA, say, or constant)
- Example 2: (Bjorn Eraker): $x_t = (\underline{x} + k (r_t - \Theta)) \min(t/30, 1)$
- What about factors other than interest rates?
- Typical assumption is that these other factors are orthogonal to (independent of) interest rates hence need not be modeled on pathwise basis
- Standard practice is to level-shift interest rate dependent model as a function of characteristics at origination



Default model

- Assume that default rates are a random variable that lives on the same tree as interest rates (!)
- Example 1: deterministic CDR (SDA, say, or flat)
- What about factors other than interest rates?
- Again, typically treated as level shift



(a) Yield spreads (*YAS*)



Plain-vanilla (I) spreads

- Compute a bond's YTM, or its IRR under a specific prepayment/default scenario
- Report spread vs. benchmark: treasury rate or swap rates at similar WAL
- $WAL = \frac{P}{YTM - \text{benchmark}}$ — where P is the total face value of the pool
- This is the I-spread



Z-spread

- Write cash flows under a specific prepayment scenario (PSA, typically) but no interest rate uncertainty
- Discount rates are spot rates + constant where the constant is such that $PV \text{ of CFs} = MV \text{ of bond}$
- That constant is the Z-spread



S-spread

- Same as Z-spread except that STRIP rates are used instead of spot rates



OAS

- If our IR/P/D model were correct, simulated price ought to equal market price
- It never does, expect for treasuries (why?)
- Most instruments price at a spread over model
- Question: what constant shift of the interest rate model yields the correct price?
- The answer is called the Option-Adjusted-Spread or OAS



Fixed income macro plays

- Assume that you believe/expect/forecast that:
 1. Consensus view on interest rate model, macro prepayment or macro default factors are off
 2. Spreads will rise or fall
- Then market values will/should move in a predictable direction
- Trade accordingly, pray that you are right and that the market will adjust sufficiently fast



Fixed income micro plays

- Assume that you believe/expect/forecast that, for a particular (type of) security:
 1. Consensus prepayment or default factors are off
 2. Spreads will rise or fall
- Then market value will/should move in a predictable direction
- Trade accordingly, pray that you are right and that the market will adjust sufficiently fast



(a) Pricing derivatives



Derivatives

- Derivatives are assets whose payoffs derive from some other asset or set of assets
- Example: swaps
- A swap contract stipulates an exchange of payoffs between two assets



Interest rate swaps

- Two parties exchange (risky) return from some real estate asset for a fixed return
- At origination, fixed rate is set so that the value of the swap is zero
- As time goes by, swap value rises or falls (symmetrically for the two counterparties)
- Swaps are traded in secondary markets, where investors can buy or sell exposure to real estate payoffs...
- ...without the underlying asset being much involved



Pricing with forwards

- Future rates can be locked-in today using forward contracts
- Result is a risk-free set of cash flows, so that the appropriate discount rate at date t is the spot rate
- Trivial calculations



Pricing without forwards

- Cash-flows associated with swap can be replicated by investing notional amount in index and reinvesting all returns until maturity
- Result is a quick way to value the swap, and proof that swap positions should exactly earn the risk-free rate
- Practical issues make this magic trick difficult to apply to RE swaps
- RE index return is estimated, not known, for one



Real estate swap

- In practice, RE swaps involve returns on large indices such as NCREIF, for various subtypes of institutional properties
 - Institutional Properties: large, safe, premium quality properties in which institutional investors invest
 - Say you own lots of properties; to offset the risk associated with your investment, you sell the NCREIF return to Credit Suisse for a safe return
 - Hedge vs. systematic real estate risk
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Market has yet to take off

- Four possible explanations:
 1. No NCREIF forwards
 2. A redundant asset
 3. “Liquidity begets liquidity”
 4. Tough to price

- More success in Europe with IPD instruments?



Credit-default swap (CDS)

- Protection buyer owns asset subject to default (a MBS, say)
- Pays protection seller (AIG, say) fixed premia
- Seller covers default risk
- Perfect way to eliminate diversifiable risk
- Systematic risk remains, however
- Real-estate related CDS played a big role in the recent financial mess



Pricing CDS' (a la Hull-White)

- Write/calibrate a tree of credit events for underlying asset, under RNP
- How?
 1. Compare bonds issued by target (or proxy) to T-bond of similar maturity
 2. Differences “must” reflect default risk
 3. Given severity rate scenario, RNP can be fit to these data
- Simulate tree forward, discount using spot yield curve, done



Summary

- Fixed income pricing requires *only* three ingredients:
IR/P/D
- Many alternative ways to specify these objects however,
which lead to disagreements among traders hence to
trading opportunities

