Prepayment and default

Fixed income

Prepayment

- Prepayment (calls) occur when the borrower pays back their debt early

- Triggered by:
  1. End of business need
  2. De-leveraging policy
  3. A covenant
  4. Refinancing gains:

$$
\begin{aligned}
NPV\ of\ refi = {} & Market\ value\ of\ Bond \\
& - Book\ value\ of\ Bond \\
& - prepayment\ costs
\end{aligned}
$$

Make-whole, yield maintenance provisions

- Those compensate lenders for reinvestment risks and kill refi gains
- Very common on IG corporate bonds
- Less so on HY
- Even less so on mortgages, hence prepayment is a big deal for MBS

Default

- On defaultable bonds, expected payoffs depend on the likelihood/probability of default (*PD*) and on the likely size of losses (*loss severity rate or loss given default, aka LGD*)

- Default probabilities usually measured as hazard rates

- h_t= probability that the loan will default in period t conditional on not having defaulted before

- Probability that the loan will default after exactly t periods is:
$$(1 - h_1)(1 - h_2) \ldots (1 - h_{t-1})h_t$$

- We expect hazard rates to be humped-shaped with a jump at maturity

Expected loss (EL)

- By definition:

$$EL = \sum_{t=1}^{T} PD_t \times PV(LGD_t) =$$

$$= PD \times \sum_{t=1}^{T} \frac{PD_t}{PD} \times PV(LGD_t)$$

$$= PD \times LGD$$

where $PD = \sum_{t=1}^{T} PD_t$ so that $\left\{\frac{PD_t}{PD}\right\}$ is a bona fide conditional distribution…

…. while $\mathrm{LGD} = \sum_{t=1}^{T} \frac{PD_t}{PD} \times PV(LGD_t)$ is the expected value of LGD as of today conditional on default happening during the holding period

Spreads compensate for EL

- Assuming default is the only risk, one gets, for a zero,

$$spread = E(r) + \frac{EL}{P} - r^F$$

- Let's do that math, courtesy of Ivan Shaliastovich

Implied default probabilities (1)

- More generally, spreads, given LGDs, imply default rates and vice-versa

- Example: Consider a defaultable bond with face value 1,000 that trades for 1,025. The SA bond has a yearly coupon rate of 5% and a remaining maturity of exactly two years. Spot rates are 1, 1.5, 2, 2.5 over the next six-month. Assume constant hazard rates and a constant recovery rate of 60%. (LGD are measured with respect to projected risk-free value at forward rates.) What is the 6-month rate of hazard into default for this bond?

Default probabilities implied by CDS spreads (1)

- Assume constant hazard rates λ, constant recovery rate R, and flat CDS spreads κ
- Then CDS spreads κ should solve:

$$\lambda = \frac{\kappa}{1-R}$$

- Further, letting $P(0,t)$ be probability of default between now and time t:

$$P(0,t) = \int_0^t \big(1 - P(0,s)\big)\lambda ds$$

Default probabilities implied by CDS spreads (2)

- It follows that $\frac{dP(0,t)}{dt} = \left(1 - P(0,t)\right)\lambda$
- This is a first-order differential equation whose solution, given $P(0,0) = 0$ is:

$$P(0,t) = 1 - e^{-\lambda t}$$

- So, finally,

$$P(0,t) = 1 - e^{-\frac{\kappa t}{1-R}}$$

Default probabilities implied by CDS spreads (3)

- If there is only one payment (one maturity) we can just do

$$P(0,T) = \frac{\kappa}{1-R}$$

- That's the CFA default

Prepayment vectors

- Assume that prepayment rates are a random variable that lives on the same tree as interest rates (!)
- Example 1: deterministic CPR (PSA, say, or constant)
- Example 2: (Bjorn Eraker):

$$x_t = \left(x + k\left(r_t - \Theta\right)\right)\min\left(\frac{t}{T}, 1\right)$$

- What about factors other than interest rates?
- Typical assumption is that these other factors are orthogonal to (independent of) interest rates hence need not be modeled on pathwise basis
- Standard practice is to level-shift interest rate dependent model as a function of characteristics at origination

Merton’s distance to default model

- Merton models equity as a call option on the firm’s assets (which it is given limited liability)

- Then (under strong assumptions) one shows “distance to default” to be

$$DD = \frac{\ln\left(\frac{V}{D}\right) + \left(r - \frac{\sigma^2}{2}\right) \times T}{\sigma \times \sqrt{T}}$$

where V is asset value, σ its volatility, T is debt maturity, r is the short-term interest rate

- Under even stronger assumptions, the probability of default is $N(-DD)$

Default vectors

- Assume that default rates are a random variable that lives on the same tree as interest rates (!)
- Example 1: deterministic CDR (SDA, say, or flat)
- What about factors other than interest rates?
- Again, typically treated as level shift

Putting it all together we get a I/P/D tree that is ready to price anything, in principle…