

Fixed income fundamentals

Real estate finance

Fixed income securities

- Debt: contractually specified cash flows
- If CFs are risk-free, market value only depends on interest rate path
- Two main sources of CF risks: prepayment and default
- Building blocks needed:
 1. Interest rate model (discount factors)
 2. Prepayment model
 3. Default model
- All (heroically) under the risk-neutral probability kernel



(a) Spot yield curve



Theoretical spot yield curve

- What is the present value of 1\$, risk free, to be delivered 1, 2, 3.5, 10 years from now?
- This is the information we need to discount “risk-free” strings of payments...
- ... and can be inferred from the yield curve
- Only issue is that zero-coupon bonds don't exist for all maturities
- But we can engineer and price zero-coupon portfolios of treasuries
- This gives the theoretical spot yield curve



(a) Interest rate models



Interest rate trees (Black-Derman-Troy)

- Consider an investment horizon with capital T periods
- The path of T one-period interest rates (r_1, r_2, \dots, r_T) is uncertain, except for the first one
- Assume that the path lives on a binomial tree (rates can go up or down from one period to the next)
- The tree is recombining: value at a given date only depends on total number of ups and downs
- We need:
 1. Size of moves in each period
 2. Probability of up or down, under RNP
- Calibrate both to 1) match estimates of interest rate volatility and 2) match spot yield curve
- Note: the model prices treasuries exactly right by design
- It can/should also price treasury derivatives trivially
- Can it price MBSs at the same time?
- Absolutely not. Yet...



(a) Prepayment and default



Prepayment model

- Assume that prepayment rates are a random variable that lives on the same tree as interest rates (!)
- Example 1: deterministic CPR (PSA, say, or constant)
- Example 2: (Bjorn Eraker): $x_t = (\underline{x} + k (r_t - \Theta)) \min(t/30, 1)$
- What about factors other than interest rates?
- Typical assumption is that these other factors are orthogonal to (independent of) interest rates hence need not be modeled on pathwise basis
- Standard practice is to level-shift interest rate dependent model as a function of characteristics at origination



Default model

- Assume that default rates are a random variable that lives on the same tree as interest rates (!)
- Example 1: deterministic CDR (SDA, say, or flat)
- What about factors other than interest rates?
- Again, typically treated as level shift



(a) Yield spreads (*YAS*)



Plain-vanilla spreads

- Compute a bond's YTM, or its IRR under a specific prepayment/default scenario
- Report spread vs. benchmark: 10-year treasury rate or swap rates
- Compare to competition



OAS

- If our IR/P/D model were correct, simulated price ought to equal market price
- It never does, expect for treasuries (why?)
- Most instruments price at a spread over model
- Question: what constant shift of the interest rate model yields the correct price?
- The answer is called the Option-Adjusted-Spread or OAS



Z-spread

- Same except the calculation is made under the assumption that all underlying sources of cash-flows make it to maturity (no prepayment, no default) or under PSA (say) but not interest rate uncertainty
- Discount rates are spot rates + constant
- That constant is the Z-spread



(a) Pricing derivatives



Derivatives

- Derivatives are assets whose payoffs derive from some other asset or set of assets
- Example: swaps
- A swap contract stipulates an exchange of payoffs between two assets



Interest rate swaps

- Two parties exchange (risky) return from some real estate asset for a fixed return
- At origination, fixed rate is set so that the value of the swap is zero
- As time goes by, swap value rises or falls (symmetrically for the two counterparties)
- Swaps are traded in secondary markets, where investors can buy or sell exposure to real estate payoffs...
- ...without the underlying asset being much involved



Pricing with forwards

- Future rates can be locked-in today using forward contracts
- Result is a risk-free set of cash flows, so that the appropriate discount rate at date t is the spot rate
- Trivial calculations



Pricing without forwards

- Cash-flows associated with swap can be replicated by investing notional amount in index and reinvesting all returns until maturity
- Result is a quick way to value the swap, and proof that swap positions should exactly earn the risk-free rate
- Practical issues make this magic trick difficult to apply to RE swaps
- RE index return is estimated, not known, for one



Real estate swap

- In practice, RE swaps involve returns on large indices such as NCREIF, for various subtypes of institutional properties
 - Institutional Properties: large, safe, premium quality properties in which institutional investors invest
 - Say you own lots of properties; to offset the risk associated with your investment, you sell the NCREIF return to Credit Suisse for a safe return
 - Hedge vs. systematic real estate risk
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Market has yet to take off

- Four possible explanations:
 1. No NCREIF forwards
 2. A redundant asset
 3. “Liquidity begets liquidity”
 4. Tough to price

- More success in Europe with IPD instruments?



Credit-default swap (CDS)

- Protection buyer owns asset subject to default (a MBS, say)
- Pays protection seller (AIG, say) fixed premia
- Seller covers default risk
- Perfect way to eliminate diversifiable risk
- Systematic risk remains, however
- Real-estate related CDS played a big role in the recent financial mess



Pricing CDS' (a la Hull-White)

- Write/calibrate a tree of credit events for underlying asset, under RNP
- How?
 1. Compare bonds issued by target (or proxy) to T-bond of similar maturity
 2. Differences “must” reflect default risk
 3. Given severity rate scenario, RNP can be fit to these data
- Simulate tree forward, discount using spot yield curve, done



Summary

- Fixed income pricing requires *only* three ingredients:
IR/P/D
- Many alternative ways to specify these objects however, which lead to disagreements among traders hence to trading opportunities

