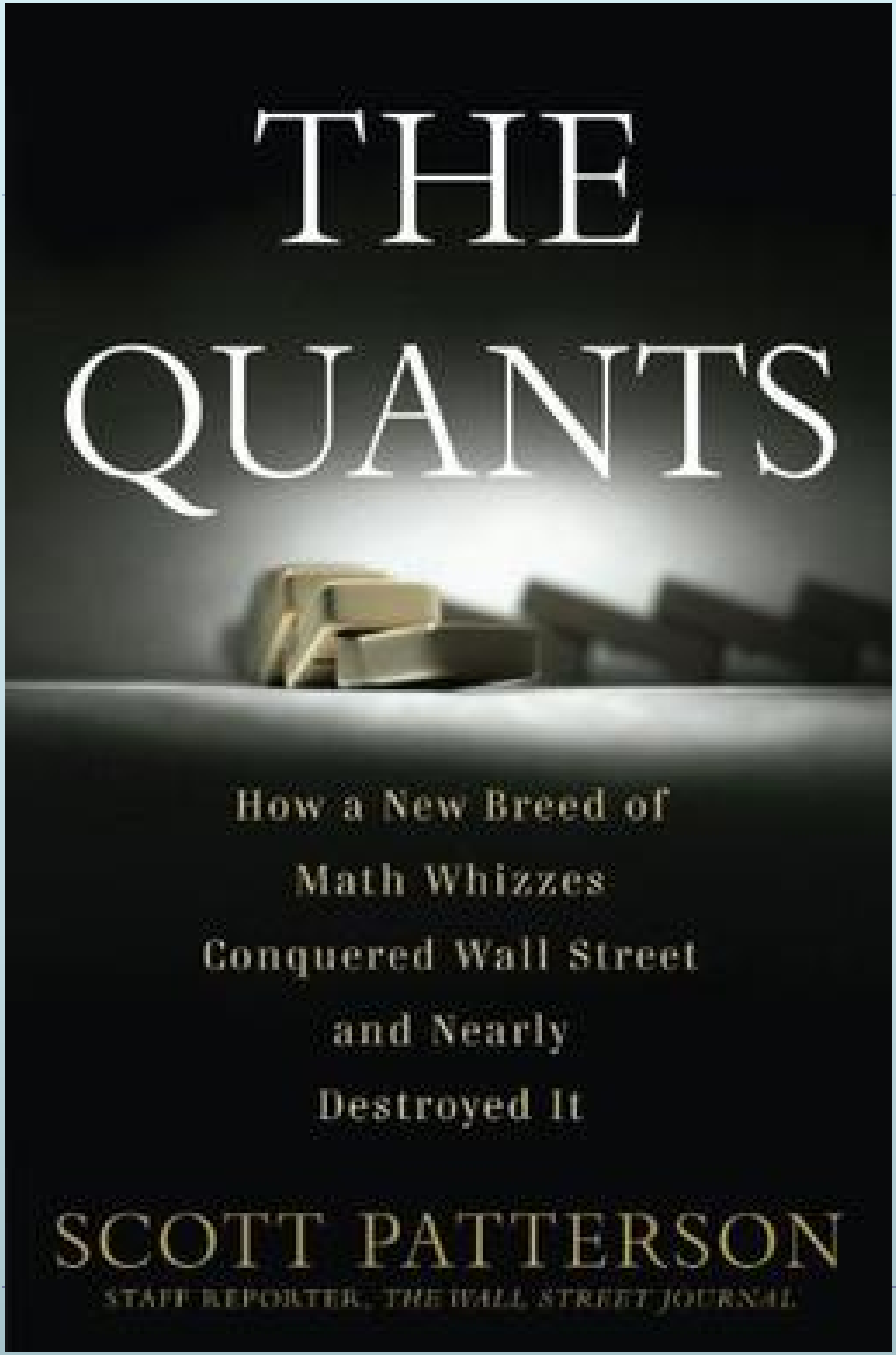
Convergence trades

Corporate Finance

Three “arbitrage” opportunities

- CDS rates that are cheap compared to bond spreads
- Convertible bonds that undervalue warrants
- Pair trades and more broadly statistical arbitrage


THE
QUANTS
How a New Breed of
Math Whizzes
Conquered Wall Street
and Nearly
Destroyed It
SCOTT PATTERSON
STAFF REPORTER, THE WALL STREET JOURNAL

Three "arbitrage" opportunities

- CDS rates that are cheap compared to bond spreads
- Convertible bonds that undervalue warrants
- Pair trades and more broadly statistical arbitrage

These are all **convergence trades…**

… but are sold as arbitrage

Credit-default swaps, a preview

Protection/CDS buyer

Protection/CDS seller

$$Premium = Notional(A) \times Swap\ rate\ (\kappa)$$

If "credit event"

$$\begin{aligned} Payment &= Face\ Value\ of\ oustanding\ debt - Market\ Value \\ &= Loss\ given\ default\ (LGD) \end{aligned}$$

CDS rates are risk-premia

- Intuitively (we will prove it under premise of perfect markets), CDS rates should match the spread between the yield on the underlying and the risk-free counterpart
- Indeed, both are compensation for probability of default times loss given default
- Two big, practical applications of this fundamental fact:
  1. Synthetic CDOs *(Who needs to issue securities any more? Just fake it with purely nominal CDS contracts)*
  2. CDS basis convergence trades

CDS basis trade

- Take a bond whose current yield is $y = r^F + RP$ and whose CDS rate is κ

- The CDS basis is $\kappa - RP$

- Assume the basis is negative, i.e $\kappa < RP$

Then:

1. Buy (long) the bond
2. Buy (long) the CDS with notional=purchase price of bond
3. Borrow notional at r^F

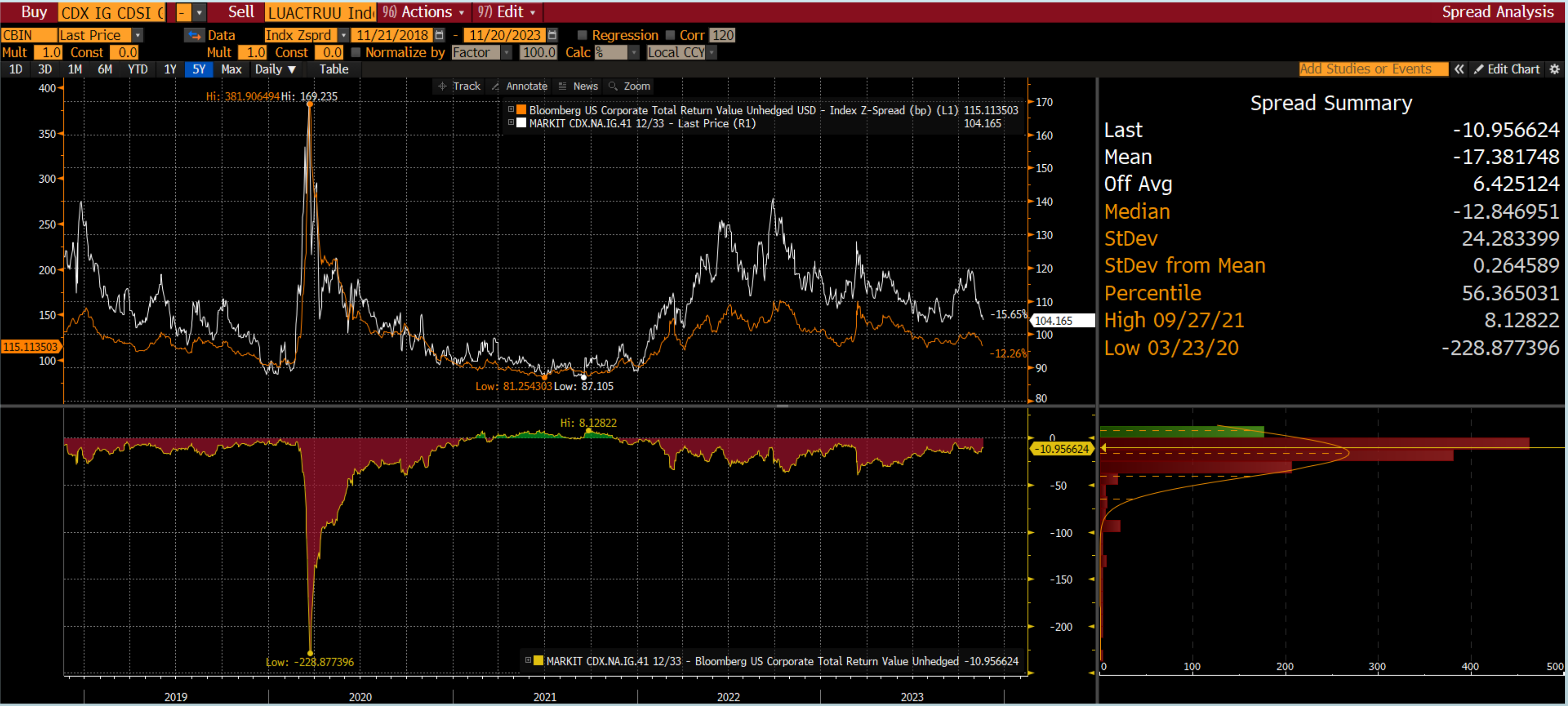
CDS basis trade (continued)

- Cost is zero at onset
- Income per $ invested:

$$(r^F + RP - \kappa) - r^F > 0$$

- Risk free (?)
- And any convergence of the basis generates capital gains

CDS basis: dimes before the steamroller



Convertible bonds

- A *convertible bond* gives the holder the right to convert her bond investment into equity at an agreed-upon *conversion price* and/or *conversion ratio*

- Example: Consider a 6-month convertible bond with face-value $1,000, S/A coupon rate of 10%, and a conversion price of $25. No issuer call option, no default.

- The conversion ratio is $\frac{1000}{25} = 40$ shares per bond

Convertible bond valuation

- Assume the share price 6 months from now is either \$20 or \$30, each with equal ***risk-neutral probability*** and that the 6-month risk-free (z) rate is 5% (annualized as always)
- What is the value of the bond?
- If the holder does not convert in 6 month, they get \$1,050
- If they do convert they get $40 \times$ share price plus accrued interest
- Obviously the option is exercised when and only when the share price is \$30

Fundamental theorem in Finance

No arbitrage

$$\Longleftrightarrow$$

$$q = \frac{E^*(X)}{1 + r^F}$$

where the expectation* is with respect to a synthetic probability distribution called the risk-neutral probability and r^F is the risk free rate

Most of modern finance prices assets by estimating the RNP first and then pricing assets as if agents were risk neutral

This is especially true in fixed income

Convertible bond valuation (2)

- The value of the bond is (should be, to be precise)

$$\frac{\frac{1}{2}\times 1050+\frac{1}{2}\times(40\times 30+50)}{1+\frac{5\%}{2}} = 1{,}121.95$$

- Another way to write this is:

$$\frac{1{,}050}{1+\frac{5\%}{2}} + \frac{\frac{1}{2}\max(40\times 20-1000{,}0)+\frac{1}{2}\max(40\times 30-1000{,}0)}{1+\frac{5\%}{2}}$$

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Value of the bond without the conversion feature
(*investment value*)

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Value of the call option to convert, a.k.a warrant

$$\frac{1{,}050}{1+\frac{5\%}{2}} + \frac{\frac{1}{2}\max(40\times 20-1000{,}0)+\frac{1}{2}\max(40\times 30-1000{,}0)}{1+\frac{5\%}{2}}$$

Value of the bond without the conversion feature (*investment value*)

Pure convertible bond arbitrage

- Assume that immediately prior to the final coupon payment, the share having already moved up to $30, the bond trades at $1,150
- Then buy the convertible, convert and sell the resulting shares gives you a pure arbitrage
- But that is not what hedge funds mean by convertible arbitrage, a practice made popular by Ed Thorp, the spiritual father of all quants
- *Convertible arbitrage* in the way quants use the expression is not an arbitrage, it is a *convergence trade*

Ed Thorp's version

- If value of the convertible bond is less than value of the non-convertible equivalent plus the value of the call option (the value of the *warrants*):
  1. *Buy the convertible bond*
  2. *Short the stock*

- The number of shares (the *hedge ratio*) is set to immunize the impact of movements in the share price on the bond, this is called *delta hedging*

- Hedge ratio is continuously/dynamically adjusted by buying or selling more stock

No arbitrage

- Tons of risks remain:
  1. *Interest rate risk (can be hedged in standard ways)*
  2. *Default risk (negative gamma/convexity in the large)*
  3. *Risk that mispricing will worsen rather than increase, like it does during financial crises*

Positive gamma?

- When price goes up, bond's delta goes up **locally**, and vice versa
- The rate of change in delta as prices change is called gamma
- When the price goes up the bond's value goes up both because the probability of conversion goes up and the conversion price goes up
- In principle, this makes Ed Thorpe's position **locally** convex

Bond value vs stock price

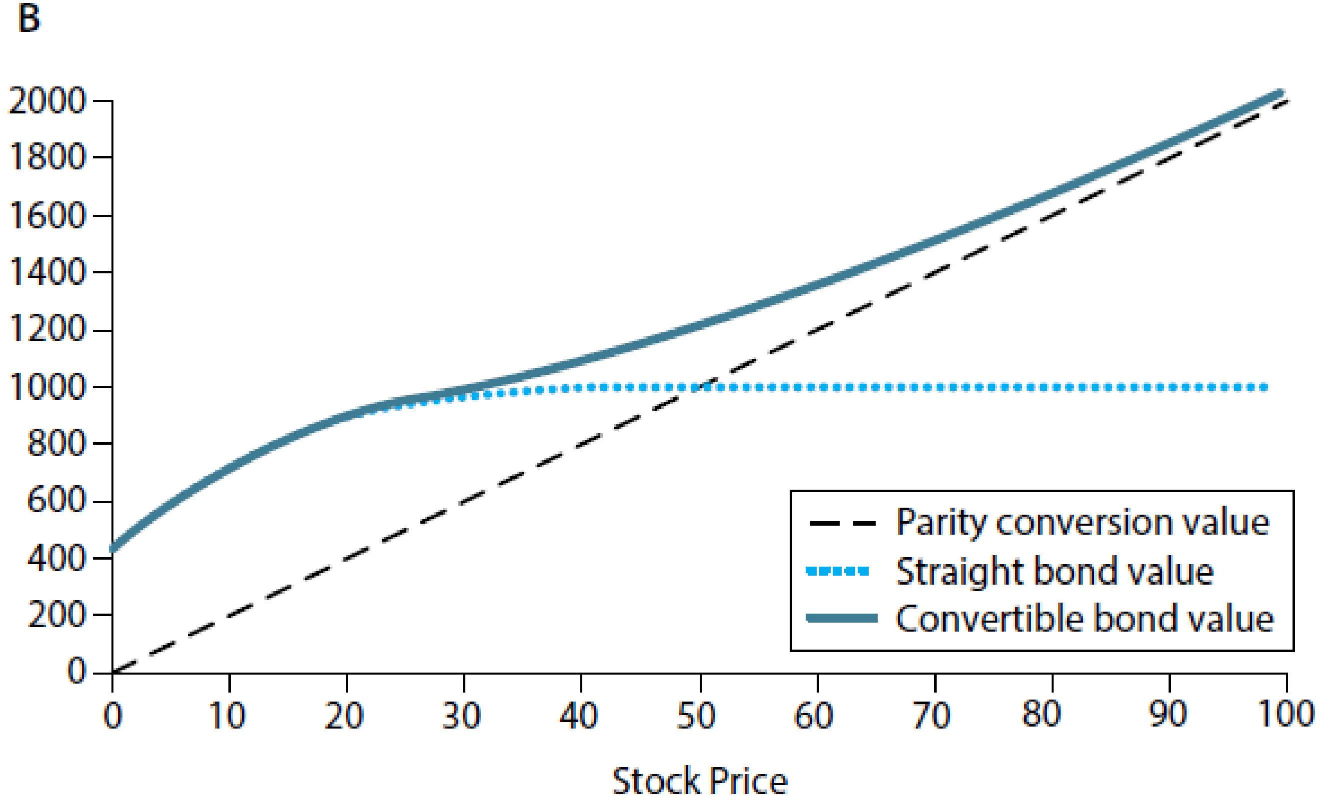


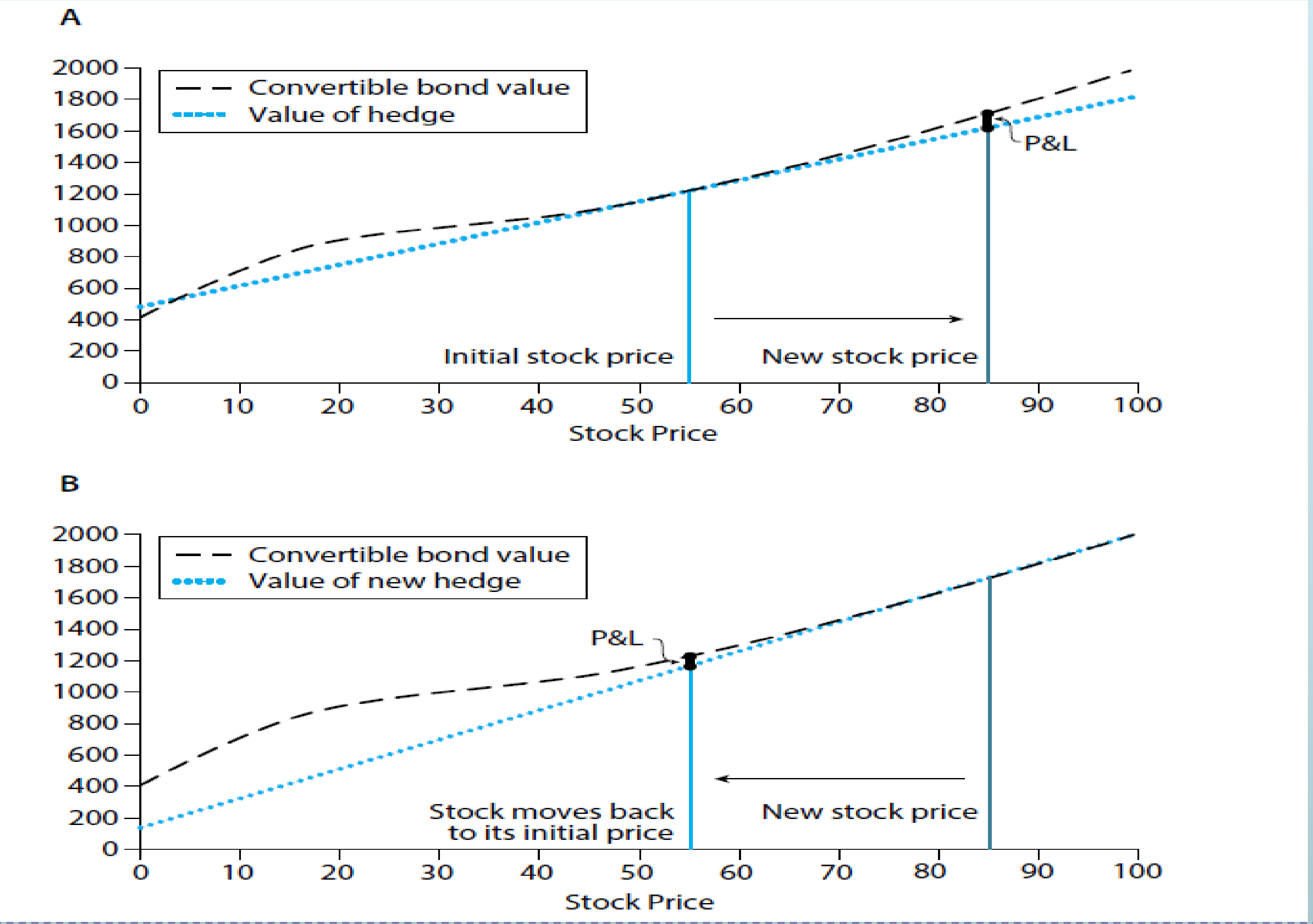
Figure 15.3. How the value of a convertible bond depends on the firm value and stock price.

Panel A. Convertible bond value vs. firm value.

Panel B. Convertible bond value vs. stock price.

- Source: Pedersen's "Inefficiently efficient," a great book

Dynamic hedging



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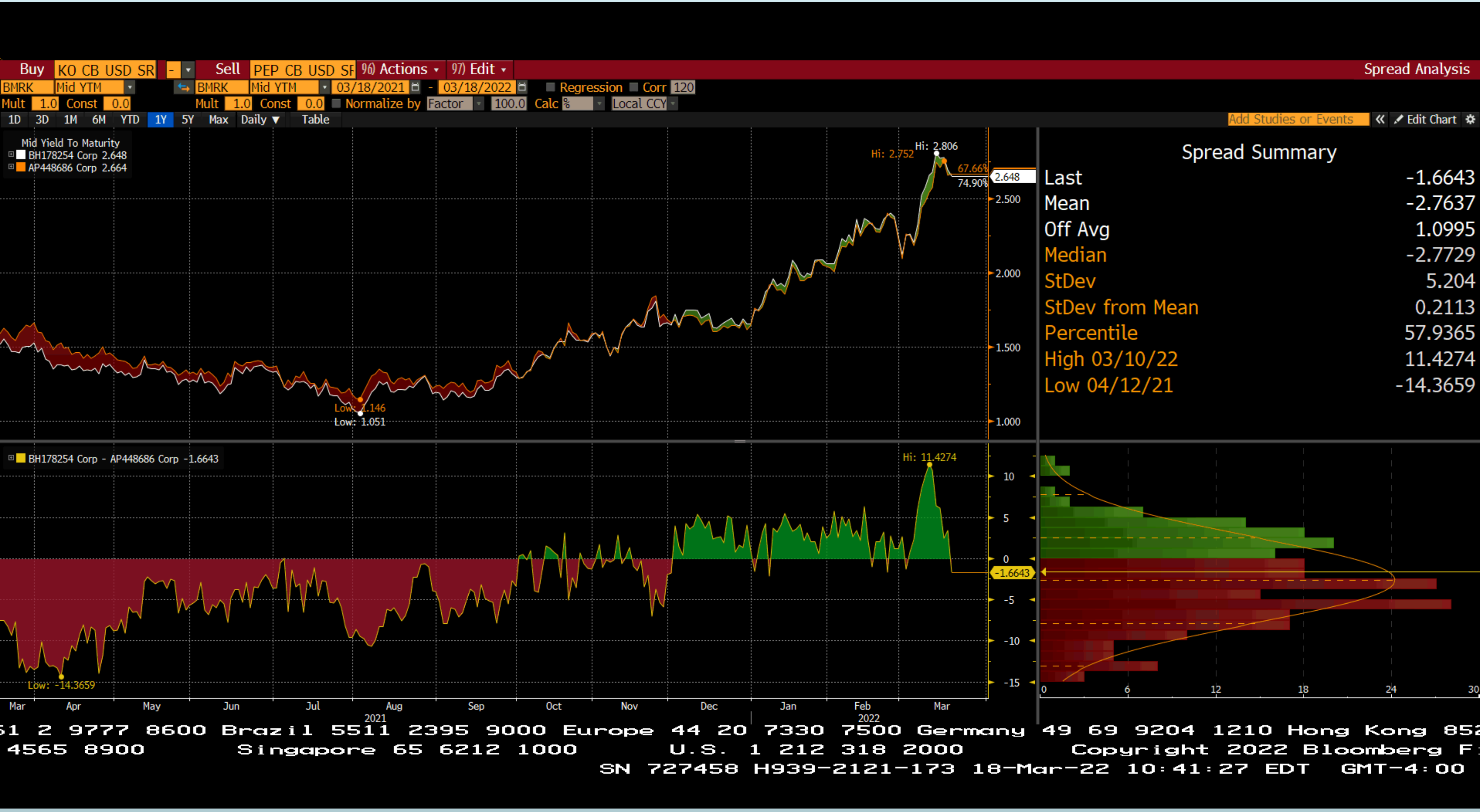
Stat arb

- Mine for and exploit statistical relationships, usually centers around some reversion to the mean argument

- Example: we expect yields on similar instruments for KO and PEP to be *co-integrated*

- If an exogenous event (large trade, say) nudges the relationship, enter a *pair trade* until shock is dissipated



Buy KO CB USD SR
Sell PEP CB USD SR
96) Actions
97) Edit
Spread Analysis
BMRK Mid YTM
BMRK Mid YTM 03/18/2021 - 03/18/2022
Regression Corr 120
Mult 1.0 Const 0.0
Mult 1.0 Const 0.0 Normalize by Factor 100.0 Calc % Local CCY
1D 3D 1M 6M YTD 1Y 5Y Max Daily Table
Add Studies or Events
Edit Chart
Mid Yield To Maturity
BH178254 Corp 2.648
AP448686 Corp 2.664
Hi: 2.752
Hi: 2.806
Low: 1.146
Low: 1.051
Spread Summary
Last -1.6643
Mean -2.7637
Off Avg 1.0995
Median -2.7729
StDev 5.204
StDev from Mean 0.2113
Percentile 57.9365
High 03/10/22 11.4274
Low 04/12/21 -14.3659
BH178254 Corp - AP448686 Corp -1.6643
Hi: 11.4274
Low: -14.3659
Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan Feb Mar
2021 2022
51 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852
4565 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2022 Bloomberg F
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Speed of convergence

- Consider the following model of the spread:

$$S_{t+1} = \gamma S^* + (1-\gamma)S_t + \epsilon_{t+1}$$

where ϵ is noise and γ is the speed of adjustment

- If the spread has a tendency to return to some mean then γ should estimate to a number between 0 and 1
- The closer to 1 the faster the speed of adjustment
- A common stationarity test (Dickey-Fuller) involves testing the hypothesis that $\gamma = 0$