

RE410 - Homework 2
Due : September 29

Presentation will count for 5 points.

Problem 1 (A simulation primer, 95pts, quite difficult, start early)

Consider a real estate project which yields cash flows for up to 20 years. Starting the project costs \$500,000. The project's subsequent cash flows follow what is called a *Markov Process* where the probability distribution of next year's cash flow depends on this year's cash flow. Markov processes try and capture in a parsimonious fashion the possibility that good times or bad times may be persistent.

In the first year, the project's cash flow is \$50,000 with certainty. Each year, there is a 2% probability that next year's cash flow will be zero. Once it becomes zero, it stays there for ever. In this context, zero is called an *absorbing state*.

If this year's cash flow is not zero yet, then there is a 10% probability that next year's cash flow will be higher by 10%, and a 10% probability that it will be lower by 10%. With the remaining probability, namely 78% ($= 100 - 2 - 10 - 10$), the cash flow is unchanged.

Before you begin, a piece of advice. The final question in this problem asks you to perform some scenario analysis. Make sure that your spreadsheet is flexible enough from the get-go to make that last step easy. For instance, if I were you, I'd put a flag variable in the parameter input part of your file that will capture the first two potential mistakes I ask you to consider in the final question.

1. Use Excel's random number generator to simulate 100 possible cash flow paths for the project. To make your life easier below, make sure to include the initial \$500,000 outlay in year 0.
2. Use these 100 simulated histories to estimate the expected value of the cash flow in each of the project's year. (That is, for each year, average cash flow values across histories.)
3. Assuming a discount rate of 10%, calculate the net present value of the project using your estimates of expectations. (Use Excel's NPV function.)

4. Now calculate the present value of each history separately using the same discount rate, and calculate the average of the 100 resulting values. Is the final result the same as in the previous question?
5. Estimate the project's IRR using your simulated estimate of the expected value of the cash flow in each period. (Use Excel's IRR function.)
6. Now calculate the IRR associated with each of the histories you generated.¹ Then calculate the average of the resulting 100 IRRs. Do you get the same value as in the previous question? Any idea why?
7. Your statistics/market analysis team now inform you that they are no longer sure that they gave you the right raw material. They cite three potential mistakes in their analysis:
 - (a) The likelihood of a decrease in cash flow after year 1 may be 5% rather than 10%. The likelihood of an increase is unchanged.
 - (b) The project's useful life may be 15 rather than 20 years, making the final 5 cash flows zero with probability 1.
 - (c) The proper discount rate for this project may be 15% rather than 10%.

Use Excel's scenario analysis macro to display the effects of these potential mistakes on the net present value of the project, one mistake at a time. (Note: no need to redraw random numbers if your parameter input section has flags for mistake 1 and for mistake 2.)

¹Warning: sometimes Excel's IRR function gets stuck, basically because it is not "trained" to look for very negative IRRs. Long story short, if that happens, write, say, `IRR(A1:A20,-0.8)`, rather than simply `IRR(A1:A20)`, which will start the search at -80%.