## Real Estate Finance - Homework 3

Due : Monday, October 20, in class.

Presentation will count for 5 points.

## Problem 1 ( 15 pts)

1. What main advantage can S-corporations and Limited Liability Corporations (LLCs) give investors over traditional (C-) corporations?
2. From the point of view of real estate investors, list 3 key differences between LLCs and S-corporations.
3. Explain how the 1986 tax reform changed the treatment of "passive income" (one or two sentences), and why this change probably helped the popularity of equity REITs (one or two sentences.)

## Problem 2 (20 pts)

Consider the following probability space and random variable.

| S | $s_{1}$ | $s_{2}$ | $s_{3}$ |
| :--- | :---: | :---: | :---: |
| p | 0.1 | 0.4 | 0.5 |
| X | 0.5 | 0.2 | 0.1 |

1. What are the expectation, the variance and the standard deviation of X ?
2. Find a random variable $Y$ such that $\rho_{X Y}=1$.
3. Find a random variable Z such that $\rho_{X Z}=0$.
4. Find a random variable $W$ such that $\rho_{X W}=0.5$
5. Find a random variable H such that $\rho_{X H}=-1$.

## Problem 3 (20 pts)

A bank is funding a 2-year mortgage to be paid back in full in two yearly payments. The loan size is $\$ 100,000$. The contract rate is $10 \%$. The mortgage is a Constant Amortization Mortgage, or CAM. In other words, the payment towards principal is the same in the two years.

1. What two payments will the borrower make if all goes well?
2. What is the loan's YTM?
3. There is a possibility of default on the two payments. When the borrower defaults, the lender recovers half of the loan's balance and unpaid interest. The likelihood of default on the first payment is $p>0$. If no default occurs on the first payment, then the likelihood of default in year 2 is, once again, $p$. The lender requires an IRR of $8 \%$ on this sort of loan. What is the highest value of $p$ compatible with that objective?

## Problem 4 (20 pts)

Consider a financial economy with three securities. The (percentage terms) expected returns of securities 1,2 , and 3 are $(20,30,13)$, respectively. The variance-covariance matrix of these returns is:

$$
\left[\begin{array}{ccc}
198 & 200 & 145 \\
200 & 1000 & 75 \\
145 & 75 & 310
\end{array}\right]
$$

Assuming that investors cannot shortsell any of these securities, use Excel to show the corresponding feasible set.

## Problem 5 (10 pts)

Below are the (fictional) returns on a REIT and the S\&P500 over the past 13 quarters.

| REIT | S\&P500 |
| ---: | ---: |
| 10 | 5 |
| 4 | 6 |
| 5 | 3 |
| 0 | -2 |
| 3 | -2 |
| 8 | 4 |
| 15 | 5 |
| 10 | 2 |
| 13 | 4 |
| 8 | 3 |
| 0 | -2 |
| 3 | -2 |
| 8 | 4 |

Use this historical information to estimate:

1. the REIT's beta,
2. its unique risk.

Please use Excel's regression function under data analysis to answer both questions in one fell swoop. On the regression's output table, show where beta and an estimate of the unique risk are.

## Problem 6 (10 pts)

Re-run the CAPM regression you ran on HW1 but augment the model to include Fama-French's two factors. Use an excess-return specification so that the intercept has a traditional alpha interpretation. Do those factors enter the regression significantly? How about alpha? What, if anything, does all this say about CAPM?

## Problem 6 - Midterm-style questions, for preparation only, no need to turn them in

1. If you want to buy a house asking $\$ 1,000,000$ and are looking for an $80 \%$ loan-to-value, how much principal will you have paid on the loan after 24 months if it is a 30 -year (full amortization) FRM with an interest rate of $5.675 \%$ ?
2. You wish to borrow $\$ 200,000$ for 20 years at $7 \%$ interest rate and amortize the loan by making fixed monthly payments. You also agree to make a balloon payment of $\$ 30,000$ at the end of your last month (240th month). What will be your monthly payment?
3. An investors can split his wealth across 3 assets, but cannot shortsell any of those assets. All three assets have the same expected return, namely 0.1 , and the same variance, namely 0.05 . The return on asset 1 has zero correlation with the returns on both asset 2 and asset 3 . The returns on asset 2 and asset 3 are perfectly correlated. What is the lowest variance the investor can achieve? Show your work.
4. Consider the following probability space and random variables.

| S | $s_{1}$ | $s_{2}$ | $s_{3}$ |
| :--- | :---: | :---: | :---: |
| p | 0.3 | 0.2 | 0.5 |
| $r_{1}$ | 0.5 | 0.0 | 0.3 |

Assume that CAPM holds exactly. Assume further that the market portfolio has variance 0.01 , and expected return 0.2 . The risk-free rate is 0.1 . What must be the covariance of $r_{1}$ with the market portfolio? Show your work.
5. An investor is considering buying a property and holding it for 3 years. She expects the property to generate a constant NOI of $\$ 500,000$ over the next 4 years. Capital expenses are expected to be 0 in the first year but $50 \%$ of NOI in year 2 and 3. Finally, the investor expects to sell the property at a NOI-cap rate of $8 \%$ at the end of year 3. The investor wants a PBTCF return of $10 \%$ from this investment. How much is she willing to pay for the property today?

