Bailouts and Financial Fragility

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Abstract

How does the belief that policy makers will bail out investors in the event of a crisis affect the allocation of resources and the stability of the financial system? I study this question in a model of financial intermediation with limited commitment. When a crisis occurs, the efficient policy response is to use public resources to augment the private consumption of those investors facing losses. The anticipation of such a “bailout” distorts ex ante incentives, leading intermediaries to choose arrangements with excessive illiquidity and thereby increasing financial fragility. Prohibiting bailouts is not necessarily desirable, however: it induces intermediaries to become too liquid from a social point of view and may in addition leave the economy more susceptible to a crisis. A policy of taxing short-term liabilities, in contrast, can correct the incentive problem while improving financial stability.

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1 Introduction

The recent financial crisis has generated a heated debate about the economic effects of public-sector bailouts of private financial institutions. A wide range of policy interventions undertaken in various countries over the past three years can be thought of as “bailouts,” including loans to individual institutions, guarantees of private debt, and direct purchases of certain types of assets. Most observers agree that the anticipation of such bailouts in the event of a crisis distorts the incentives faced by financial institutions and other investors. By insulating these agents from the full consequences of a negative outcome, an anticipated bailout results in a misallocation of resources and encourages risky behavior that may leave the economy more susceptible to a future crisis.

Opinions differ widely, however, on the best way for policy makers to deal with this problem. Some observers argue that policy makers should focus on making credible commitments to not bail out financial institutions in the event of a future crisis. Such a commitment would encourage investors to provision for bad outcomes and, it is claimed, these actions would collectively make the financial system more stable. Others argue that policy makers should focus instead on improving the regulation and supervision of financial institutions and markets. Proponents of this second view believe that it is either infeasible or perhaps even undesirable to limit future policy makers’ ability to deal with possibly unforeseen circumstances. They view the distortions caused by the anticipation of future bailouts as inevitable and argue that policy makers must aim to correct the distortions and promote financial stability through improved regulation in normal times.

Given these widely differing views, it is important to investigate the effects of bailouts in formal economic models and to use these models to ask how policy makers can best address the issue. Would it be desirable for policy makers to commit to never bail out financial institutions? Would doing so be an effective way to promote financial stability? Or is it better to allow bailouts to occur and attempt to offset their distortionary effects through regulation? I address these questions in a model of financial intermediation and fragility based on the classic paper of Diamond and Dybvig [7]. In particular, I study an environment with idiosyncratic liquidity risk and with limited commitment, as in Ennis and Keister [8]. Individuals deposit resources with financial intermediaries, and these resources are invested in a nonstochastic production technology.
form maturity transformation and thereby insure investors against their individual liquidity risk. This maturity transformation makes intermediaries illiquid and may leave them susceptible to a self-fulfilling run by investors.

Fiscal policy is introduced into this framework by adding a public good that is financed by taxing households’ endowments. In the event of a crisis, some of this tax revenue may be diverted from production of the public good and instead given as private consumption to investors facing losses in the financial system. The size of this “bailout” payment is chosen to achieve an ex post efficient allocation of the remaining resources in the economy. Intermediaries and investors anticipate this reaction when making ex ante decisions. Intermediaries are assumed to always act in the best interest of their investors and a benevolent policy maker chooses the tax rate and the size of the bailout payment, if any. Neither intermediaries nor the policy maker can commit to future actions; both will react optimally to whatever situation they face.

I begin the analysis by characterizing a benchmark allocation that represents the efficient distribution of resources in this environment conditional on investors running on the financial system in some states of the world. I show that this allocation always involves a transfer of public resources to private investors in those states. In other words, a bailout is part of the efficient allocation of resources in this environment whenever a crisis is possible. The logic behind this result is straightforward. In normal times, the policy maker chooses the tax rate and the level of public good provision to equate the marginal social values of public and private consumption. A crisis results in a misallocation of resources, which raises the marginal value of private consumption for some investors. The optimal response to this situation is to decrease public consumption in order to transfer resources to these investors – a “bailout.” The efficient bailout policy thus provides investors with partial insurance against the losses associated with a financial crisis.

In a decentralized setting, the anticipation of this type of bailout distorts the ex ante incentives of investors and their intermediaries. As a result, intermediaries choose to perform more maturity transformation, and hence become more illiquid, than in the benchmark allocation. This excessive illiquidity, in turn, implies that the financial system is more fragile in the sense that a self-fulfilling run can occur in equilibrium for a strictly larger set of parameter values. The incentive problem created by the anticipated bailout thus has two negative effects in this environment: it both distorts the allocation of resources in normal times and increases the financial system’s susceptibility to a crisis.
A policy of committing to no bailouts, even if it were feasible, is not necessarily desirable in this setting. Such a policy would *over-correct* the incentive problem described above. By requiring intermediaries to completely self-insure against the possibility of a crisis, a no-bailouts policy would lead them to become more liquid (by performing less maturity transformation) than in the benchmark efficient allocation. Despite this fact, the economy remains more fragile than in the benchmark allocation. While the increase in liquidity tends to make the financial system more stable, this effect is offset by the loss of the insurance investors receive from the efficient bailout policy. In fact, for some economies that are not fragile in a discretionary regime, a no-bailouts policy actually *introduces* the possibility of a self-fulfilling run.

The idea that a credible no-bailout commitment can increase the fragility of the financial system may seem surprising at first, but the mechanism behind this result is straightforward and quite general. A bailout provides insurance – it lessens the potential loss an investor faces if she does not withdraw her funds and a crisis occurs. As such, the anticipation of a bailout decreases the incentive for individuals to withdraw, which, in turn, makes the financial system less susceptible to a self-fulfilling run. This argument is familiar in the context of retail banking: government-sponsored deposit insurance programs can be thought of as a type of “bailout” policy that is explicitly designed to play this stabilizing role. Despite this similarity, discussion of the insurance role of bailouts has been largely absent in the current policy debate.

An optimal policy arrangement in the environment studied here requires permitting bailouts to occur, so that investors benefit from the efficient level of insurance, while offsetting the negative effects on ex ante incentives. One way this can be accomplished is by placing a Pigouvian tax on intermediaries’ short-term liabilities, which can also be interpreted as a tax on the activity of maturity transformation. Specifically, suppose each intermediary must pay a fee that is proportional to the total value of its short-term liabilities. No restrictions are placed on the policy maker and, in the event of a crisis, the ex post optimal bailout policy is followed. In the simple environment studied here, the appropriate choice of tax rate will implement the benchmark efficient allocation. As a result, the scope for financial fragility is strictly smaller under this policy than in either the discretionary or the no-bailouts regime.

There is a large literature in which versions of the Diamond-Dybvig model are used to address issues related to banking policy and financial fragility. This paper follows Green and Lin [14], Peck and Shell [18], Ennis and Keister [9] and other recent work in specifying an explicit sequen-
tial service constraint and allowing intermediaries to offer any contract that is consistent with the information flow generated by that constraint. In particular, intermediaries and the policy maker are able to react as soon as they infer that a run is under way, rather than continuing to follow a simple rule such as allowing withdrawals until all funds are depleted. The paper also focuses on the implications of a lack of commitment power on the part of the banking authorities, as in Mailath and Mester [16], Acharya and Yorulmazer [1], Ennis and Keister [8] and others.

There is a small but growing literature on the incentive effects of financial-sector bailouts and optimal regulatory policy in the presence of limited commitment. Chari and Kehoe [4] study an environment in which committing to a no-bailout policy would generate the first-best allocation of resources if it were feasible. They assume such commitment is infeasible and show how, given that bailout payments are inevitable in some states of nature, ex ante regulation of private contracts can be welfare improving. In the environment studied here, in contrast, committing to a no-bailout policy is never first-best optimal because bailout payments provide socially-valuable insurance. This aspect of the model is similar in some respects to Green [13], which also highlights the fact that policies resembling a bailout can be part of a desirable ex ante insurance arrangement. Other related work includes Fahri and Tirole [11], who focus on the strategic complementarities generated by indiscriminate bailouts, Cooper and Kempf [5], who study the redistributive effects of deposit insurance when agents are ex ante heterogeneous, and Niepmann and Schmidt-Eisenlohr [17], who examine the strategic interaction between governments when bailouts have international spillover effects. In contrast to these papers, a primary focus here is on financial fragility, that is, the conditions under which an economy becomes susceptible to a crisis driven by the self-fulfilling beliefs of investors.

2 The Model

I begin with a fairly standard version of the Diamond and Dybvig [7] model and augment this basic framework by introducing a public good. This section describes the physical environment and the model of the decentralized economy.

2.1 The environment

There are three time periods, $t = 0, 1, 2$, and a continuum of investors, indexed by $i \in [0, 1]$. Each
investor has preferences given by

\[ U(c_1, c_2, g; \theta_i) = u(c_1 + \theta_i c_2) + v(g), \]

where \( c_t \) is consumption of the private good in period \( t \) and \( g \) is the level of public good, which is provided in period 1. The functions \( u \) and \( v \) are assumed to be strictly increasing, strictly concave, and to satisfy the usual Inada conditions. The parameter \( \theta_i \) is a binomial random variable with support \( \Theta = \{0, 1\} \). If the realized value of \( \theta_i \) is zero, investor \( i \) is *impatient* and only cares about early consumption. An investor’s type \( \theta_i \) is revealed to her in period 1 and remains private information. Let \( \omega \) denote a profile of preference types for each investor and let \( \Omega \) denote the set of all such profiles. Let \( \pi \) denote the probability with which each individual investor will be impatient. By a law of large numbers, \( \pi \) is also the fraction of investors in the population who will be impatient.

In some of what follows, it will be useful to assume that preferences are of the constant-relative-risk-aversion (CRRA) form,

\[ u(c) = \frac{(c)^{1-\gamma}}{1-\gamma} \quad \text{and} \quad v(g) = \delta \left(\frac{g}{1-\gamma}\right)^{1-\gamma}, \quad \text{with} \ \gamma > 1. \]

In this specification, the parameter \( \delta \) measures the relative importance of the public good and is assumed to be common to all investors.

Each investor is endowed with one unit of the private good in period 0. As in Diamond and Dybvig [7], there is a constant-returns-to-scale technology for transforming this endowment into private consumption in the later periods. A unit of the good invested in period 0 yields \( R > 1 \) units in period 2, but only one unit in period 1. This investment technology is operated in a central location, where investors can pool resources in an intermediation technology to insure against individual liquidity risk. Investors are isolated from each other in periods 1 and 2 and no trade can occur among them. Upon learning her preference type, each investor chooses either to contact the intermediation technology in period 1 to withdraw funds or to wait and withdraw in period 2. There is also a technology for transforming units of the private good one-for-one into units of the public good. This technology is operated in period 1, using goods that were placed into the investment technology in period 0.

An (ex post) allocation in this environment is a pair \((c, g)\), where \( c : [0, 1] \rightarrow \mathbb{R}_+^2 \) is an assign-
ment of a private consumption level to each investor in each period and \( g \in \mathbb{R}_+ \) is a level of public good provision. An allocation is \textit{feasible} if it can be produced from the period-0 endowments using the technologies described above, that is, if
\[
\int_0^1 c_1(i) \, di + \frac{1}{R} \int_0^1 c_2(i) \, di \leq 1 - g.
\]
Let \( A \) denote the set of feasible allocations. A \textit{state-contingent allocation} is a mapping \( c : \Omega \rightarrow A \) from the set of realized preference types to the set of feasible allocations.

Investors who choose to withdraw in period 1 arrive one at a time in a randomly-determined order. As in Wallace [20]-[21], these investors must consume immediately upon arrival. This sequential-service constraint implies that the payment made to such an investor can only depend on the information received by the intermediation technology up to that point. In particular, this payment can be contingent on the number of early withdrawals that have taken place \textit{so far}, but not on the \textit{total} number of early withdrawals that will occur because this latter number will not be known until the end of the period.

Since all investors are identical in period 0, it is natural to measure ex ante welfare in this economy as the period-0 expected utility of each investor. For ex post measures of welfare, after preference types (and potentially some consumption levels) have been realized, I use an equal-weighted sum of individual utilities to measure welfare. The expression
\[
W = \int_0^1 E[u(c_1,i, c_2,i, g; \theta_i)] \, di
\]
captures both of these notions and is, therefore, used to measure welfare throughout the analysis.

\subsection*{2.2 The decentralized economy}

In the decentralized economy, the intermediation technology is operated by a large number of competitive intermediaries, each of which aims to maximize the expected utility of its investors. Each intermediary serves a large number of investors and, hence, knows that a fraction \( \pi \) of its investors will be impatient. Let \( \sigma_j \) denote the fraction of investors in the population whose endowments are deposited with intermediary \( j \). Because investors’ types are private information, the payment an investor receives from her intermediary cannot depend directly on her realized type. Instead, the intermediary allows each investor to choose the period in which she will withdraw. This arrangement, which resembles a demand-deposit contract, is well known to be a useful tool for implementing
desirable allocations in economies with private information. However, such arrangements can also create the possibility of a “run” on the financial system in which all investors attempt to withdraw early, regardless of their realized preference type.

Intermediaries act to maximize the expected utility of their investors at all times. In reality, there are important agency problems that cause the incentives of financial intermediaries to differ from those of their investors and creditors. I abstract from these agency problems here in order to focus more directly on the distortions in investors’ incentives that are created by the anticipation of a bailout in the event of a crisis. As in Ennis and Keister [8], [10], intermediaries cannot commit to future actions. This inability to commit implies that they are unable to use the type of suspension of convertibility plans discussed in Diamond and Dybvig [7] or the type of run-proof contracts studied in Cooper and Ross [6]. Instead, the payment given to each investor who withdraws in period 1 will be a best response given the intermediary’s current beliefs.

Since there is a continuum of investors, no individual will voluntarily provide any of the public good from her own resources. The public good is instead provided by a benevolent policy maker who has the ability to tax endowments in period 0. The revenue from this tax is placed into the investment technology and transformed into period 1 private goods. In period 1, the policy maker can use these private goods to produce units of the public good or can transfer some of these private goods to the financial intermediaries. I refer to this latter option as a “bailout” payment to the financial system.

Notice that the type of bailout policy I consider here is entirely consistent with the sequential service constraint in the model. The only opportunity to tax agents comes in the first period, before funds are deposited with intermediaries. To keep the notation simple, I assume that the policy maker does not transfer funds to the financial system in the event that there is no run.

2.3 Financial crises

In order to allow a run on the financial system to occur with nontrivial probability, I introduce an extrinsic “sunspot” signal on which investors can potentially condition their actions. Let \( S = \{ s_1, s_2 \} \) be the set of possible sunspot states, with \( \text{prob}[s = s_2] = q \in (0, 1) \). Investor \( i \) chooses a strategy

\footnote{If the environment allowed the government to tax depositors after it has observed the total level of withdrawal demand, then it should also be possible for intermediaries to require advance notice of an investors’ intention to withdraw. The assumption here is that the sequential service constraint applies to the policy maker as well as to the intermediaries, which is consistent with the Wallace [20] critique of Diamond and Dybvig’s [7] analysis.}
that assigns a decision to withdraw in either period 1 or period 2 to each possible realization of her preference type $\theta_i$ and of the sunspot variable

$$y_i : \Theta \times S \rightarrow \{1, 2\}.$$ 

Neither the intermediaries nor the policy maker observe the realization of the sunspot variable. Instead, they must try to infer the state from the flow of withdrawals. This approach is standard\(^2\) and, combined with the sequential service constraint, implies that some payments must be made to withdrawing investors before the intermediaries or policy maker know whether or not a run is underway. Suppose that all investors attempt to withdraw early in, say, state $s_2$. Intermediaries and the policy maker know that at least $\pi$ investors will withdraw in both states and, therefore, as the first $\pi$ withdrawals take place they are unable to infer anything about the realized sunspot state. If the fraction of early withdrawals goes past $\pi$, however, they can immediately to infer that the state is $s_2$ and that a run is underway.

An important element of the model is specifying how intermediaries and the policy maker respond once they discover that a run is underway and how those investors who have not yet been able to withdraw react to this response. In general, this interaction may be quite complex and different patterns of behavior are possible (see Ennis and Keister [10]). To simplify matters, I assume here that once it has discovered a run is underway, an intermediary is able to implement the efficient allocation of its remaining resources among the remaining investors. As part of this allocation, only those remaining investors who are impatient withdraw early; the remaining patient investors wait until period 2 to withdraw.

There are several different ways in which this allocation could come about. It could, for example, be the result of a screening technology that can be used in the event of a run, as in Ennis and Keister [8]. Alternatively, it could be the result of equilibrium behavior in a game played by the intermediary and those investors who anticipate they will be late to arrive at their intermediary in period 1, as in Ennis and Keister [10]. Whatever the mechanism, this approach ensures that none of the results below are driven by some assumed inefficiency in the distribution of resources following a run.\(^3\)

\(^2\) See, for example, Diamond and Dybvig [7, Section 4], Cooper and Ross [6], and Peck and Shell [18].

\(^3\) The results would not change if, for example, a fraction of the intermediary’s remaining assets were lost in the event of a run. Such an inefficiency would only serve to increase the scope for financial fragility under all of the policy regimes studied here.
3 Efficient Allocations and Bailouts

In this section, I study the efficient allocation of resources under the assumption that only impatient investors withdraw early in state $s_1$ but all investors attempt to withdraw early in $s_2$, so that a financial crisis occurs with probability $q \geq 0$. When $q = 0$, this allocation corresponds to the first-best allocation of resources in this environment. For positive values of $q$, this allocation will be a useful benchmark for comparing the equilibrium outcomes under different policy regimes below.

3.1 The $q$-efficient allocation

Using the structure of the model, particularly the absence of any intrinsic aggregate uncertainty, the problem of finding the efficient allocation of resources in this scenario can be simplified considerably. First, note that the form of the utility function (1) implies that the planner would prefer to give consumption to impatient investors only in period 1 and to patient investors only in period 2. Moreover, because investors are risk averse, the planner would like to give the same amount of consumption to all investors of a given type. However, I assume the planner faces the same informational constraints that intermediaries and the policy maker face in the decentralized economy. In particular, the planner correctly anticipates investors’ withdrawal strategies as a function of the sunspot state, but is unable to observe the realized state. Instead, the state must be inferred from the observed withdrawal behavior of investors.

In the scenario considered here, the fraction of investors who attempt to withdraw early will be $\pi$ in state $s_1$ and 1 in state $s_2$. As the first $\pi$ withdrawals are taking place, therefore, no information about the state is revealed to the planner. The efficient policy must give the same consumption level to all of these investors; any feasible allocation in which these investors consume different amounts is strictly dominated by another feasible allocation in which their consumption levels are equalized. Let $c_E$ denote the payment given to these investors who withdraw “early.” If withdrawals cease after a fraction $\pi$ of investors has withdrawn, the planner can infer that the remaining investors are all patient and will withdraw in period 2. The planner will then divide the remaining resources between a common payment $c_L$ for those investors who withdraw “late” and an amount $g$ of the public good.

If, on the other hand, the fraction of investors withdrawing in period 1 goes past $\pi$, the planner is immediately able to infer that state $s_2$ has occurred. At this point, the planner is able to implement
the efficient continuation allocation among the remaining investors. This allocation gives a common amount of consumption, denoted $c_E$, to each remaining impatient investor in period 1. Note that $c_E$ will, in general, be different from the consumption level of the first $\pi$ investors to withdraw, $c_E$. Similarly, the planner will give a common amount $c_L$ to each remaining patient investor in period 2. Let $\hat{g}$ denote the amount of public good provided in this case. Notice the importance of the sequential service constraint here. The planner would like to give different consumption levels to investors in the different states. However, a fraction $\pi$ of investors must be served, and will consume, before the planner is able to infer the state and thus determine the appropriate consumption level.

The problem of finding the efficient allocation of resources given that a run will occur in state $s_2$ can, therefore, be reduced to choosing the consumption levels $(c_E, c_L, \hat{c}_E, \hat{c}_L)$ and the levels of public good provision $(g, \hat{g})$ to solve

$$
\max (1 - q) \left[ \pi u(c_E) + (1 - \pi) u(c_L) + v(g) \right] + 
q \left[ \pi u(c_E) + (1 - \pi) \left( \pi u(\hat{c}_E) + (1 - \pi) u(\hat{c}_L) \right) + v(\hat{g}) \right]
$$

subject to

$$
\pi c_E + (1 - \pi) \frac{c_L}{R} + g \leq 1,
$$

$$
(1 - \pi) \left( \pi \hat{c}_E + (1 - \pi) \frac{\hat{c}_L}{R} \right) + \hat{g} \leq 1 - \pi c_E,
$$

and

$$
c_L \geq c_E, \quad \hat{c}_L \geq \hat{c}_E
$$

Expression (4) is the resource constraint that applies in state $s_1$, while (5) applies in $s_2$. The final two constraints are incentive compatibility conditions that, in a decentralized economy, will ensure that withdrawing early is not a dominant strategy. It is straightforward to show that these latter constraints never bind at the solution. The solution to this problem is called the $q$-efficient allocation.

Letting $(1 - q) \mu$ and $q\hat{\mu}$ denote the multipliers on constraints (4) and (5), respectively, the
solution to this problem is characterized by the conditions

\[ u'(c_E) = (1 - q) Ru' (c_L) + qRu' (\hat{c}_L) \]
\[ Ru'(c_L) = v'(g) = \mu, \quad \text{and} \]
\[ u'(\hat{c}_E) = Ru' (\hat{c}_L) = v' (\hat{g}) = \hat{\mu}. \]

The first condition says that the marginal value assigned to resources paid out before the planner knows whether a run is underway should be equal to the expected marginal value of resources once the state is revealed. The other equations can be interpreted as the standard Samuelson condition for the efficient provision of a public good, which equates the sum of individuals’ marginal rates of substitution to the marginal rate of transformation, in each of the two states.\(^4\)

Let \( c^* = (c^*_E, c^*_L, g^*, \hat{c}^*_E, \hat{c}^*_L, \hat{g}^*) \) denote the solution to this problem and let \( (\mu^*, \hat{\mu}^*) \) denote the corresponding values of the (normalized) multipliers. It is straightforward to show that each element of this solution varies continuously with the probability of a crisis \( q \), and that evaluating \( c^* \) in the limit as \( q \to 0 \) yields the first-best allocation of resources in this environment. For the CRRA utility function in (2), the \( q \)-efficient allocation can be derived in closed form; this solution is presented in Appendix B.

### 3.2 Illiquidity

The degree of illiquidity in the economy will play an important role in the analysis that follows. For any given allocation, define

\[ \rho \equiv \frac{c_E}{1 - g}, \]

so that \( \rho \) represents the private consumption of an investor who withdraws early relative to the per-capita period-1 value of the resources designated for private consumption. Since each investor has, in principle, the option of withdrawing early, \( c_E \) represents the per-capita short term liabilities of the financial system. The short-run value of intermediaries’ assets will equal the fraction of endowments that are invested to provide private consumption, \( 1 - g \). Hence \( \rho \) represents the ratio of the short-term liabilities of the financial system to the short-run value of its assets. I will say that the financial system is illiquid whenever \( \rho > 1 \) holds.

For the CRRA utility function (2), direct calculations show that the degree of illiquidity in

\(^4\) Note that because the \( q \)-efficient allocation is symmetric and there is a measure 1 of depositors, the sum of all investors’ marginal rates of substitution is equal to each individual’s marginal rate of substitution.
the $q$-efficient allocation satisfies $\rho^* > 1$ if and only if $\gamma > 1$ holds, regardless of the value of $q$. This result reflects a standard property of Diamond-Dybvig models: when the coefficient of relative risk aversion is larger than unity, the efficient allocation of resources involves illiquidity. This illiquidity is what potentially opens the door to self-fulfilling financial crises. In addition, it is straightforward to show that the efficient response to an increase in the probability of a crisis is to decrease the degree of illiquidity. In other words, when a crisis is more likely, the planner would like to shift resources toward the bad state of nature by lowering the return offered on early withdrawals. This shift has the effect of making the financial system more liquid. These results are summarized in the following proposition.

**Proposition 1** Under (2), $\rho^* > 1$ holds for all $q$. In addition, $\rho^*$ is strictly decreasing in $q$.

### 3.3 Bailouts

One key feature of the $q$-efficient allocation is that whenever it involves illiquidity, the marginal social value of resources will be higher in the event of a crisis than in normal times.

**Proposition 2** $\bar{\mu}^* > \mu^*$ holds if and only if $\rho^* > 1$.

A proof of this proposition is given in Appendix A. When the financial system is illiquid, the value of its assets per remaining investor falls as more withdrawals take place. If, after $\pi$ withdrawals have taken place, the planner discovers that a run is underway, it realizes that the remaining investors are a mix of patient and impatient types. Consumption must be provided to these investors from a smaller level of assets per capita than the planner initially had available and, as a result, the marginal value of resources is higher.

The next result follows immediately from Proposition 2 and the first-order conditions in (6).

**Corollary 1** $\tilde{g}^* < g^*$ holds if and only if $\rho^* > 1$.

Recall that $g$ is the quantity of resources set aside to provide the public good in the event that there is no crisis. If a crisis occurs, some of these resources are instead used to provide private consumption to those investors who have not yet been able to withdraw. The property $\tilde{g}^* < g^*$ can, therefore, be interpreted as a “bailout” of the financial system. In the event of a run, all
investors pay a cost in terms of a lower level of the public good in order to benefit that subset of agents who are facing losses on their financial investments.

The corollary shows that this bailout is part of the efficient allocation of resources. The logic behind the result is fairly general and seems likely to appear in a wide range of settings. The efficient fiscal plan is designed so that the marginal social value of public consumption will equal the marginal value of the private consumption for the patient investors in normal times. When a crisis occurs, it leads to a misallocation of resources that lowers private consumption for some investors, which raises their marginal value of consumption. The efficient response must, therefore, be to shift some resources away from public consumption and into the private consumption of these investors. Notice that this “bailout” is efficient even from an ex ante point of view; it provides investors with insurance against the losses they may suffer in the event of a crisis.

3.4 Financial fragility

One of the primary goals of the paper is to understand how policies related to bailouts affect the stability of the financial system. The concept of financial fragility – the susceptibility of the financial system to a crisis – has been defined in a variety of different ways, depending on the setting. In the environment studied here, it is natural to say that the financial system is fragile if a crisis can occur with positive probability in the decentralized economy.

**Definition:** The financial system of an economy is fragile under a given policy regime if, for some \( q > 0 \), there exists an equilibrium in which all investors attempt to withdraw early in state \( s_2 \).

For making comparisons across different policy regimes, I examine the set of economies that fit this definition of fragility under each regime. An economy is characterized by a set of parameter values for technologies and preferences \((R, \pi, u, v)\). For each policy regime, I ask what subset of economies have an equilibrium in which investors run on the financial system in state \( s_2 \). If this set is strictly larger under some policy regime A than under B, I say that policy A increases financial fragility relative to B.

Other approaches to defining financial fragility are possible and would likely lead to similar results. One could, for example, impose an equilibrium selection rule such as risk dominance to resolve the multiplicity of equilibrium in the coordination game played by investors. Aggregate uncertainty about, say, the long-run return \( R \) could be introduced so that agents are initially uncertain...
whether or not a run will occur. Financial fragility could then be measured by the probability of the set of realizations of $R$ for which the equilibrium selected by this rule involves a run. While this approach would alter the way financial fragility is measured, the comparative statics of fragility would be qualitatively unchanged. A policy change that makes it more attractive for an individual investor to withdraw early if she believes that others are doing so will tend to increase financial fragility in this setting under any reasonable definition.\footnote{Another alternative would be to attempt to resolve the multiplicity of equilibrium by introducing private information as in the literature on global games pioneered by Carlsson and van Damme\cite{carlsson1994}. However, this approach places rather strict requirements on the information structure of the model. Papers that have used the global games methodology in Diamond-Dybvig type models have done so by placing arbitrary restrictions on contracts between intermediaries and their investors (see, for example, Rochet and Vives\cite{rochet1997} and Goldstein and Pauzner\cite{goldstein1997}). These restrictions themselves are potential sources of financial fragility, quite separate from the issues related to bailouts under consideration here. The approach taken here captures the effects of changes in the incentives faced by investors in a reasonably clear and transparent way, and does not place any additional restrictions on agents other than those imposed by the physical environment.}

The definition of fragility can be extended in a natural way to the benchmark allocation studied above. In the decentralized economy, a patient investor who runs when all other investors are running and is served before the planner discovers that a run is underway receives $c_E \bar{c}$. She would instead receive $\bar{c}_L$ if she waits until period 2 to withdraw. We can, therefore, identify fragility with a situation in which this investor has a strict incentive to participate in the run, that is, in which $c_E > \bar{c}_L$ holds. I will say that the financial system of an economy is \emph{fragile under the $q$-efficient allocation} if $c_E > \bar{c}_L^*$ holds for some $q > 0$.

Let $\Phi^*$ denote the set of parameter values $(R, \pi, u, v)$ such that the financial system is fragile under the $q$-efficient allocation of resources. From the first-order conditions above, we have

\begin{align*}
    u'(c_E^*) &= (1 - q) \mu^* + q \mu^* \\
    u'(\bar{c}_L^*) &= \frac{1}{R} \mu^*
\end{align*}

The condition $c_E^* > \bar{c}_L^*$ can, therefore, be written as

\[ (1 - q) \mu^* + q \mu^* < \frac{1}{R} \mu^* \]

or

\[ \mu^* - \frac{R^{-1} - q}{1 - q} \mu^* < 0. \tag{7} \]
The financial system is fragile in the $q$-efficient allocation if this inequality holds for some $q > 0$.

4 Equilibrium under Discretion

In this section, I study the allocation of resources that emerges in an equilibrium of the decentralized economy and compare this outcome to the $q$-efficient allocation derived above. The equilibrium is constructed by working backward, beginning with the division of resources among the remaining investors in the event of a run.

4.1 The post-run allocation

Suppose the realized state is $s_2$ and a run occurs. A fraction $\pi$ of investors will withdraw before the intermediaries and the policy maker are able to infer the state; investors withdrawing from intermediary $j$ each receive an amount $c_{E,j}$. Let $\phi_j$ denote intermediary $j$’s remaining resources, per remaining investor, after the first $\pi$ withdrawals have taken place, that is,

$$\phi_j = \frac{1 - \tau - \pi c_{E,j}}{1 - \pi}.$$

Suppose that the bailout policy has already been decided, with intermediary $j$ receiving an amount $b_j \geq 0$ per remaining investor, and let

$$\psi_j = \phi_j + b_j$$

be the amount of resources, per remaining investor, available to intermediary $j$. Once it discovers that a run is underway, the intermediary will intervene in a way that leads to the efficient distribution of these remaining resources among its remaining investors. Letting $\hat{c}_{E,j}$ denote the consumption of the remaining impatient investors and $\hat{c}_{L,j}$ the consumption of the remaining patient investors, the efficient continuation allocation solves

$$\max \pi u(\hat{c}_{E,j}) + (1 - \pi) u(\hat{c}_{L,j})$$

subject to

$$\pi \hat{c}_{E,j} + (1 - \pi) \frac{\hat{c}_{L,j}}{R} \leq \psi_j \quad \text{and}$$

$$\hat{c}_{L,j} \geq \hat{c}_{E,j}.$$
The solution to this problem is characterized by the first-order conditions

\[ u'(\hat{c}_{E,j}) = Ru'(\hat{c}_{L,j}) = \hat{\mu}_j, \]  

(10)

where \( \hat{\mu}_j \) is the multiplier on the resource constraint (9). Let \((\hat{c}_{E,j}^D, \hat{c}_{L,j}^D)\) denote the solution to this problem, where the “D” superscript refers to the discretionary policy regime. Define the value function

\[ \hat{V}_c(\psi_j) = \pi u(\hat{c}_{E,j}^D) + (1 - \pi) u(\hat{c}_{L,j}^D). \]  

(11)

This function measures the average utility from private consumption of the remaining investors given the amount of resources (per remaining investor) available to the intermediary.

### 4.2 The bailout policy

In the event of a run, the resources available to the policy maker can be divided between provision of the public good and bailout payments to the financial system. These payments are allocated among the intermediaries in an ex post efficient manner. Recall that \( \sigma_j \) represents the fraction of investors in the economy that have deposited with intermediary \( j \). Then the policy maker’s budget constraint is

\[ \hat{g} + \sum_j \sigma_j (1 - \pi) b_j = \tau. \]  

(12)

We can write the problem of choosing the optimal bailout policy in terms of dividing the remaining resources between private and public consumption, that is,

\[ \max_{\{b_j, \hat{g}\}} \sum_j \sigma_j (1 - \pi) \hat{V}_c(\phi_j + b_j) + v(\hat{g}) \]

subject to the budget constraint (12).

The solution to this problem is characterized by first-order conditions

\[ \hat{V}_c'(\phi_j + b_j) = u'(\hat{g}) \quad \text{for all } j. \]

These conditions immediately imply

\[ \phi_j + b_j = \phi_{j'} + b_{j'} \quad \text{for all } j \text{ and } j'. \]  

(13)

In other words, the resources available for private consumption should be the same in all interme-
diaries. The incentive problems that will be caused by this bailout policy are clear: an intermediary with fewer remaining resources \( \phi_j \) (because it chose a higher value of \( c_{E,j} \)) will receive a larger bailout \( b_j \). In equilibrium, of course, all intermediaries will choose the same value of \( c_{E,j} \) and receive the same bailout payment.\(^6\)

Let \( \{b_j^D, \hat{g}^D\} \) denote the solution to the problem above. The total size of the bailout payments, per remaining investor, is then given by

\[
b^D \equiv \sum_j \sigma_j b_j^D = \frac{\tau - \hat{g}^D}{1 - \pi}.
\]

Let \( \psi \) denote the common level of \( \psi_j = \phi_j + b_j \) from equation (13), and let \( \phi \) denote the average level of resources per investor remaining in an intermediary, \( \sum_j \sigma_j \phi_j \). Define the value function

\[
\hat{V} \left( (1 - \pi) \phi + \tau \right) \equiv (1 - \pi) \hat{V}_c (\psi) + v (\hat{g}).
\]

(14)

This function represents the contribution to total welfare of the private consumption of the remaining \( (1 - \pi) \) investors and the public consumption of all investors, given that the total remaining resources \( (1 - \pi) \phi + \tau \) will be divided optimally among the competing uses.

4.3 The equilibrium allocation

The remaining elements to be determined are the payments given by intermediaries to the first \( \pi \) investors who withdraw and the tax rate. Since all intermediaries face the same decision problem, I will omit the \( j \) subscript and use \( c_E \) to denote the payment offered by a representative intermediary. In choosing this payment, the intermediary takes as given the level of public good provision in both states and the allocation of private consumption among the remaining \( 1 - \pi \) investors in the event of a run. The property of the bailout policy highlighted in equation (13) implies that the consumption of an intermediary’s remaining \( (1 - \pi) \) investors will depend only on the total amount of resources left in the economy, not on the amount held by their own intermediary. Of course, the decisions of the intermediaries collectively determine this total amount. This external effect of an individual intermediary’s choice on the consumption of other intermediaries’ investors in the event of a run is

\(^6\) Note that, in principle, a similar incentive problem could arise in state \( s_1 \) if the policy maker made bailout payments to intermediaries that chose an unusually high level of \( c_E \) in that state as well. I assume that bailout payments are only made in the event of a financial crisis. This assumption could be justified by reputation concerns, which will be significant for decisions made in normal times but much less important for a policy maker facing a rare event like a financial crisis.
the source of the distortion in the decentralized economy.

The equilibrium value of $c_E$ solves the following maximization problem

$$\max_{c_E, c_L} \ (1-q) \left( \pi u(c_E) + (1-\pi)u(c_L) + v(\tau) \right) + q \left( \pi u(c_E) + \hat{V} \right)$$

(15)

subject to

$$\pi c_E + (1-\pi)\frac{c_L}{R} = 1 - \tau,$$

and

$$c_L \geq c_E.$$  

(16)

The first-order condition that characterizes the solution to this problem when the incentive-compatibility constraint does not bind is

$$u'(c_E) = (1-q)Ru'(c_L) = (1-q)\mu,$$  

(17)

where $(1-q)\mu$ is the multiplier on the resource constraint. The distortion of incentives is clear from the first equality: the equilibrium payment $c_E$ will balance the marginal value of resources in the early period against the marginal value of resources in the late period in the no-run state, ignoring the value of resources in the event of a run. The larger the probability of a run $q$ is, the more the distorted the allocation of resources becomes.

We can also see from this expression that the incentive compatibility constraint will be satisfied at the interior solution as long as

$$q \leq \frac{R-1}{R},$$

(18)

but will otherwise be violated. When the constraint does bind, the solution is determined by the condition $c_L = c_E$ together with the resource constraint. Let $(c_E^D, c_L^D)$ denote the solution to this problem and define the value function

$$V^D(\tau) = \pi u(c_E) + (1-q) \left( (1-\pi)u(c_L) + v(\tau) \right) + q\hat{V} \left( 1 - \pi c_E \right).$$

(19)

At the beginning of period 0, the policy maker will choose the tax rate $\tau$ to maximize the function $V^D$. The first-order condition for this problem can be written as

$$v'(\tau) = \frac{1}{1-q} \mu + \frac{q}{1-q} \bar{\mu} \pi \frac{dc_E}{d\tau}.$$

If the probability of a crisis ($q$) were zero, the tax rate would be set to equate the marginal utility
of the public good with the marginal value of goods used for private consumption \( \mu \). When \( q \) is positive, however, the policy maker must also take into account the fact that changes in \( \tau \) will lead to changes in the equilibrium level of \( c_E \), which in turn affects the total quantity of resources available in the event of a run. In this event, resources have a higher marginal value, \( \hat{\mu} \). Letting \( \tau^D \) denote the solution to this problem, welfare in the competitive equilibrium is given by

\[
\hat{W}^D = V^D (\tau^D).
\]

Let \( c^D \) denote the complete equilibrium allocation. It is straightforward to show that this solution varies continuously with the probability of a crisis \( q \) and converges to the efficient allocation as \( q \) goes to zero.

### 4.4 Illiquidity and fragility

In the CRRA case, the degree of illiquidity in the financial system can be derived in closed form (see Appendix B) and delivers the following result.

**Proposition 3** Under (2), \( \rho^D \) is strictly increasing in \( q \) for \( q < (R - 1)/R \) and constant for larger values of \( q \).

In other words, as a financial crisis becomes more likely, the financial system adopts a less liquid position. The anticipation that they will be bailed out in the event of a crisis leads investors to prefer a higher short-run return, which implies a more illiquid financial system. Recall that this is the opposite of what happens in the efficient allocation of resources, where an increase in the probability of a crisis leads to a more liquid financial system (see Proposition 1). Combining these results with the fact that the two allocations are equal when \( q \approx 0 \) gives the following corollary.

**Corollary 2** Under (2), \( q > 0 \) implies \( \rho^D > \rho^* \).

In other words, whenever \( q \) is positive, the degree of illiquidity in the equilibrium allocation is strictly greater than that in the efficient allocation.

This higher degree of illiquidity increases the scope for financial fragility in the model. Let \( \Phi^D \) denote the set of parameter values \( (R, \pi, u, v) \) such that \( c^D_E > c^D_L \) holds for some \( q > 0 \). Then the following strict inclusion relationship obtains, a proof of which is given in Appendix A.
Proposition 4  Under (2), $\Phi^D \supset \Phi^*$ holds.

This result gives a precise sense in which the incentive problem caused by bailouts can make the economy more susceptible to self-fulfilling financial crises. For some parameter values, the $q$-efficient allocation of resources is such that a patient investor has no incentive to withdraw early, even if he believes everyone else will try to do so. As a result, the financial system is stable. In the competitive equilibrium, however, intermediaries become more illiquid than in the efficient allocation and, as a result, investors would find themselves in a worse position in the event of a run. This fact increases the incentive for a patient investor to withdraw early in be believes others will run. In some cases, this increase is large enough to make joining the run an optimal response. In these cases, the ex ante distortions created by the bailout policy introduce the possibility of a self-fulfilling financial crisis.

5 Committing to No Bailouts

In this section and the next, I analyze two policy measures designed to mitigate the incentive problem and potentially improve welfare compared to the discretionary regime described above. The first policy is one that has received considerable attention in the popular press and elsewhere: a commitment to not providing any bailout payments, that is, to setting $b = 0$ in all states of nature. Whether or not such commitment is feasible in reality is debatable. The question here is whether such a policy – if feasible – would be desirable.

5.1 Equilibrium

In the event of a run, each intermediary responds to implement the efficient allocation of its remaining resources among its investors, as in problem (8). The value of these resources is, therefore, given by the function $\hat{V}_c$ defined in (11). Intermediaries choose the payments $c_E$ and $c_L$ to solve

$$\max_{\{c_E, c_L\}} \pi u(c_E) + (1 - \pi) \left( (1 - q) u(c_L) + q \hat{V}_c \left( \frac{1 - \tau - \pi c_E}{1 - \pi} \right) \right) + v(\tau)$$

subject to

$$\pi c_E + (1 - \pi) c_L \leq 1 - \tau,$$

and

$$c_L \geq c_E.$$
As indicated in the objective function above, the level of the public good will be equal to tax revenue $\tau$ in both states. Intermediaries and investors take the level of $\tau$ as given when making their individual decisions. Note that the value function $\tilde{V}_c$ is evaluated at the level of resources (per investor) that the intermediary will have after $\pi$ withdrawals, a quantity that depends on the intermediary’s choice of $c_E$. Because of the no-bailout policy, intermediaries and investors now recognize that, in the event of a run, the only resources that will be available for the private consumption of the remaining investors will be those funds still held by the intermediary.

Letting $(1 - q) \mu$ be the multiplier on the intermediary’s budget constraint, the solution to this problem is characterized by the first-order conditions

$$u'(c_E) = (1 - q) \mu + q \tilde{V}'_c \left( \frac{1 - \tau - \pi c_E}{1 - \pi} \right)$$

and

$$Ru'(c_L) = \mu.$$  

Using the envelope condition $\tilde{V}'_c = \hat{\mu}$, where $\hat{\mu}$ is the multiplier on the intermediary’s post-run budget constraint (9), we can write

$$u'(c_E) = (1 - q) \mu + q\hat{\mu}.$$  

Comparing this condition with (17) shows the effect of the no-bailout policy and how it mitigates the incentive problem. Under this policy, an intermediary must balance the value of the promised payment $c_E$ not only against the value of late consumption in the no-run state $\mu$, but also against the value of resources in the run state $\hat{\mu}$. Let $(c^{NB}_E, c^{NB}_L)$ denote the solution to this problem.

Define the value function

$$V^{NB}(\tau) = \pi u(c^{NB}_E) + (1 - \pi) \left( (1 - q) u(c^{NB}_L) + q V_P \left( \frac{1 - \tau - \pi c^{NB}_E}{1 - \pi} \right) \right) + v(\tau).$$

The policy maker will choose the tax rate to maximize this value; let $\tau^{NB}$ denote the policy maker’s optimal choice.

### 5.2 Illiquidity and Fragility

For the CRRA case, it is straightforward to show that $\rho^{NB}$ is strictly decreasing in $q$. Recall that this result is the opposite of that obtained in the previous section. When intermediaries and investors
anticipate a bailout in the event of a run, an increase in the probability of a run leads them to adopt a more illiquid position. Here, in contrast, an increase in the probability of a run leads intermediaries to adopt a more liquid position. In this sense, the no-bailout policy is successful in eliminating the distortion of ex ante incentives.

Comparing $\rho^{NB}$ to the degree of illiquidity in the $q$-efficient allocation, however, shows that the no-bailout policy actually leads intermediaries to be too liquid.

**Proposition 5** Under (2), $q > 0$ implies $\rho^{NB} < \rho^*$.  

This result shows that the no-bailout policy introduces a new distortion in ex ante incentives. (A proof is given in the appendix.) Instead of performing too much maturity transformation, and taking on too much illiquidity, intermediaries perform too little under this policy. The reason is that intermediaries must now completely self-insure against the possibility of a run. In the $q$-efficient allocation, in contrast, the bailout policy provides intermediaries with some insurance against this event, as established in Corollary 1.

Despite making financial intermediaries more liquid, the no-bailout policy actually increases the scope for financial fragility relative to the $q$-efficient allocation. Letting $\Phi^{NB}$ denote the set of economies for which $c_E^{NB} > c_L^{NB}$ holds under the no-bailout policy for some $q > 0$, we have the following result.

**Proposition 6** Under (2), $\Phi^{NB} \supset \Phi^*$ holds.

A proof is given in Appendix A. The intuition for the result can be seen by considering the limiting case as $q$ goes to zero. The components of the allocation that apply to the no-run state ($c_E, c_L$, and $\tau$) converge to the corresponding components of the $q$-efficient allocation. However, the post-run components of the allocation ($\hat{c}_E, \hat{c}_L$, and $\hat{g}$) do not. Because no bailout payments are made, the level of the public good is higher than in the $q$-efficient allocation and the private consumption levels $\hat{c}_E$ and $\hat{c}_L$ are lower. It follows that the condition $c_E > \hat{c}_L$ must hold for a strictly larger set of parameter values.

This same logic can be used to show that some economies that are not fragile under the discretionary policy regime become fragile when a no-bailout policy is implemented.


**Proposition 7**  Some economies in the set $\Phi^{NB}$ are not in the set $\Phi^g$.

This result is somewhat surprising in light of the arguments made by many commentators during the recent financial crisis and the subsequent debate over financial regulatory reform. While committing to a no-bailout policy increases the liquidity of the financial system, it can simultaneously leave the system more vulnerable to a run. The intuition behind this result is clear: by increasing $\tilde{c}_L$, bailouts reduce the cost to an investor of leaving her funds deposited in the event of a run. In other words, the anticipation of a bailout also has a positive effect on ex ante incentives in that it encourages investors to keep their funds deposited in the financial system. The no-bailout policy removes this positive effect and, as a result, can create financial fragility. This phenomenon tends to occur when investors place a high value on the public good, which implies that the resources available to the policy maker are relatively large.

5.3 An example

Figure 1 illustrates the results for a particular example. The CRRA utility function (2) is used and the parameter values are given by $(R, \pi, \gamma, \delta) = (1.1, 0.5, 6, 0.01)$. For these values, the financial system is fragile under the $q$-efficient allocation of resources and, hence, is fragile under both the discretionary and the no-bailout policy regimes. Panel (a) in the figure shows the degree of illiquidity in each case as a function of the probability of a crisis $q$. When $q = 0$, the first-best allocation obtains in all three scenarios. As a crisis becomes more likely, the degree of illiquidity in the efficient allocation declines, in accordance with Proposition 1. Under the no-bailout policy, illiquidity declines even faster, as intermediaries adopt more conservative positions, in line with Proposition 5. Under the discretionary policy, in contrast, illiquidity rises rapidly. The kink in this curve corresponds to point where the incentive compatibility constraint begins to bind in problem (15). Beyond this point, intermediaries’ choice of $c_E$ is insensitive to further increases in $q$ and the degree of illiquidity stays constant, as stated in Proposition 3.

Panel (b) of the figure compares equilibrium welfare under the discretionary and no-bailout policy regime. The curve plotted in the figure represents the benefit of the discretionary regime over the no-bailouts regime, $W^D - W^{NB}$. Two competing forces are at work in determining the shape of this curve. The ex ante distortion – as depicted in panel (a) – is larger in the discretionary case; this fact tends to make the no-bailout policy attractive. However, the no-bailout regime also
leads to an ex post inefficient allocation of resources in the event of a run. For small enough values of \( q \), these ex post concerns dominate and the discretionary policy yields higher welfare. As \( q \) increases further and the ex ante distortions become larger, however, the former effect eventually dominates. For values of \( q \) above approximately 0.08, the curve becomes negative and welfare is higher under the no bailout policy.

Once \( q \) passes the threshold level \( (R - 1) / R \), however, the incentive compatibility constraint binds in the discretionary equilibrium. As a result, the ex ante distortion in the discretionary case remains constant as \( q \) increases further. For the no-bailout policy, however, the welfare loss from having an inefficient allocation of resources in the event of a run continues to grow as the probability of this event increases. For values of \( q \) above 0.12, curve becomes positive and the discretionary policy again yields higher welfare. This example demonstrates that committing to a no bailout policy may either raise or lower welfare relative to a purely discretionary regime, depending on parameter values and the likelihood of a crisis.

6 Taxing short-term liabilities

Another policy option is to place no restrictions on the bailout policy, but to offset the distortion it creates through regulation or some other ex ante intervention. To illustrate the effects of such an intervention, I now allow the policy maker to impose a tax on intermediaries’ short-term liabilities; this policy can also be thought of as a tax on the activity of maturity transformation. This particular tax is one of several possible policies that would have equivalent effects in the simple model studied.
here, including directly imposing an appropriately-chosen cap on short-term liabilities. The goal
is to investigate the effectiveness of a policy regime that aims to influence intermediaries’ choices
through ex ante intervention rather than through restrictions on the ex post bailout payments. A
Pigouvian tax on short-term liabilities is one way to illustrate the results of such an approach.

Suppose that each intermediary must pay a fee that is proportional to the total value of its short-
term liabilities,

\[
fee_j = \eta \pi \sigma_j c_E,
\]

where, as above, \( \sigma_j \) denotes the fraction of investors who deposit with intermediary \( j \). The tax
rate is this policy is \( \eta \pi \), where \( \eta \) is chosen by the policy maker. For simplicity, I make the policy
revenue neutral by giving each intermediary a lump-sum transfer \( N \sigma_j (1 - \tau) \), where \( N \) is equal
to the average fee collected per unit of deposits. This assumption is only to facilitate comparison
with the earlier cases.

### 6.1 Equilibrium

Under this policy, the equilibrium payment \( c_E \) will solve

\[
\max_{\{c_E, c_L\}} \pi u (c_E) + (1 - q) ((1 - \pi) u (c_L) + v (\tau)) + qV
\]

subject to

\[
\pi c_E + (1 - \pi) \frac{c_L}{R} \leq 1 - \tau - \eta \pi c_E + N (1 - \tau),
\]

where the \( j \) subscripts have been omitted since all intermediaries face the same decision problem.

Notice that I have already used the fact that deposits will equal \( (1 - \tau) \) per investor; this is only to
avoid introducing additional notation. As in the previous section, investors and intermediaries take
as given the level of provision of the public good in all states, as well as the allocation of private
consumption to the remaining investors in the event of a crisis.

The first-order conditions of this problem are

\[
u' (c_E) = (1 + \eta) (1 - q) Ru' (c_L) = (1 + \eta) (1 - q) \mu,
\]

where \( (1 - q) \mu \) is again the multiplier on the resource constraint. We know that the post-run
allocation of resources will be efficient, and hence will satisfy the usual first-order conditions

\[
u' (\bar{c}_E) = Ru' (\bar{c}_L) = v' (\bar{g}) = \bar{\mu}.
\]
Revenue neutrality implies

\[ N(1 - \tau) = \eta \pi c_E. \]

Substituting this condition into the budget set of the representative intermediary yields the standard resource constraint for the no-run state.

### 6.2 The optimal tax rate

Can the tax rate \( \eta \) be set so that the equilibrium allocation with ex ante intervention matches the \( q \)-efficient allocation? In the \( q \)-efficient allocation, we have

\[ u'(c_E^*) = (1 - q) Ru'(c_L^*) + q Ru'(\hat{c}_L^*) \]

In order for the equilibrium allocation to be efficient, therefore we need

\[ \eta (1 - q) Ru'(c_L^*) = q Ru'(\hat{c}_L^*) \]

or

\[ \eta = \frac{q \hat{\mu}^*}{(1 - q) \mu^*} \equiv \eta^* \quad (20) \]

where \( (1 - q) \mu^* \) and \( q \hat{\mu}^* \) are the multipliers on the resource constraints (4) and (5), respectively, evaluated at the \( q \)-efficient allocation. In other words, the tax rate \( \eta^* \) induces each intermediary to place an additional value on period-2 resources that is based on the marginal social value of resources in the event of a run, rather than in the no-run state. When \( \eta \) is set equal to \( \eta^* \), the competitive equilibrium allocation will satisfy all of the conditions characterizing the \( q \)-efficient allocation. Since these conditions uniquely determine the efficient allocation, we have the following result.

**Proposition 8**  
When the tax rate \( \eta \) is set according to (20), the equilibrium allocation with a tax on short-term liabilities is equal to the \( q \)-efficient allocation.

This result shows how ex ante intervention can be a powerful policy tool in the environment studied here. An appropriately chosen tax rate allows the policy maker to follow the efficient bailout policy while correcting the distortion this policy creates. The bailout policy is thus able to provide investors with the optimal level of insurance against the losses associated with a financial crisis without leading intermediaries to choose excessively high levels of illiquidity. Importantly,
the set of economies for which the financial system is fragile is the same as that in the $q$-efficient allocation, $\Phi^*$. In other words, the optimal tax policy decreases financial fragility relative to either the discretionary or the no-bailouts regime.

Of course, other types of ex ante intervention could be equally effective in the simple environment studied here. The policy maker could, for example, simply impose a ceiling of $c^*_E$ on the level of short-term liabilities per investor. The model is not designed to distinguish between different types of ex ante policy interventions; a richer environment in which intermediaries face a higher-dimensional decision problem would be needed for that purpose. Rather, the model here highlights the benefits of using some ex ante intervention together with the ex post optimal bailout policy. Compared to a no-bailouts regime, this combination not only leads to a more efficient allocation of resources, it also increases financial stability.

7 Concluding Remarks

There is widespread agreement that the anticipation of a receiving a public-sector bailout in the event of a crisis distorts the incentives of financial institutions and other investors. By partially insulating these agents from the effects of a negative outcome, bailouts diminish their incentive to provision for such outcomes and encourage excessively risky behavior. Such concerns have featured prominently in the recent debate on financial regulatory reform and have lead some commentators to argue that governments and central banks should aim to make credible commitments to not providing any future bailouts.

The model presented here shows that there is another side to this issue, however, and that the anticipation of a bailout can have positive ex ante effects as well. These positive effects appear in two distinct forms. First, bailouts are part of an efficient insurance arrangement. The logic behind this result is quite general and seems likely to be present in a wide range of environments. A financial crisis leads to a misallocation of resources that lowers consumption for at least some agents and thus raises the marginal social value of private consumption. The optimal response for a policy maker is to decrease public consumption, using these resources to augment the private consumption of agents facing losses. This “bailout” policy raises ex ante welfare by providing risk-averse agents with insurance against the losses associated with a crisis.

In addition, the insurance provided by a bailout policy can have a stabilizing effect on the
financial system. Financial crises are commonly thought to have an important self-fulfilling component, with individual investors each withdrawing funds in part because they fear the withdrawals of others will deepen the crisis and create further losses. The anticipation of a bailout lessens the potential loss an investor faces if she does not withdraw her funds. As such, it decreases the incentive for investors to withdraw, which, in turn, makes the financial system less susceptible to a crisis. Committing to a no-bailouts policy removes this insurance and, in some cases, can actually create fragility in the financial system.

The analysis also highlights the importance of efforts to clearly delineate the bounds of the so-called financial safety net. To the extent that investors are uncertain about how much, if any, of a bailout payment they will receive, the insurance benefits of the bailout policy will be lost. While efforts to insert “constructive ambiguity” into policy may lessen the moral hazard problems associated with anticipated bailouts, they will also undermine the positive effects of the bailout policy. In the environment studied here, introducing randomness into the size or scope of the bailout policy can only reduce welfare.

It should be emphasized that the bailout policies studied here are efficient; they do not lead to rent-seeking behavior, nor are they motivated by outside political considerations. In reality, these types of distortions are important concerns. The message of the paper is not that any type of bailout policy is acceptable as long as the ex ante effects are offset through taxation. Limits on the ability of policy makers to undertake inefficient redistribution during a crisis may well be desirable. Rather, the message here is that restrictions on bailouts are not sufficient to ensure that investors face the correct ex ante incentives. The efficient allocation of resources requires that investors receive some insurance in the form of a bailout. Providing this insurance distorts incentives, however, and this distortion must be corrected. Policies that aim to prevent inefficient bailouts must, therefore, be used in conjunction with some form of regulation or other ex ante policy intervention.

Extending the analysis to richer environments may generate insight into the relative merits of different types of ex ante intervention. In the model presented here, taxing short-term liabilities and imposing a cap on such liabilities are equally effective policies. In a setting where intermediaries make additional decisions and, perhaps, take unobserved actions (such as portfolio allocations, effort in monitoring investments, etc.), this equivalence may no longer hold. Studying such environments using the approach developed here seems a promising avenue for future research.
Appendix A. Proofs of Propositions

**Proposition 2** \( \hat{\mu}^* > \mu^* \) holds if and only if \( \rho^* > 1 \).

**Proof:** By definition, \( \rho^* > 1 \) is equivalent to \( c_E^* > 1 - g^* \). Straightforward algebra shows that this latter inequality holds if and only if

\[
1 - g^* > \frac{1 - \pi c_E^* - \hat{g}^*}{1 - \pi}.
\]

The resource constraints (4) and (5) will both hold with equality at the \( q \)-efficient allocation and can be written as

\[
\begin{align*}
\pi c_E^* + (1 - \pi) \frac{c_L^*}{R} &= 1 - g^* \quad \text{and} \\
\pi \hat{c}_E^* + (1 - \pi) \frac{\hat{c}_L^*}{R} &= \frac{1 - \pi c_E^* - \hat{g}^*}{1 - \pi}.
\end{align*}
\]

Comparing these expressions with the inequality above shows that \( c_E^* > 1 - g^* \) holds if and only if setting \((\hat{c}_E, \hat{c}_L, \hat{g}) = (c_E^*, c_L^*, g^*)\) would violate (5). The first-order conditions (6) imply that

\[
(c_E^*, c_L^*, g^*) \begin{cases} > \end{cases} (\hat{c}_E^*, \hat{c}_L^*, \hat{g}^*) \quad \text{as} \quad \mu^* \begin{cases} < \end{cases} \hat{\mu}^*,
\]

where the first comparison is a vector inequality. The values \((c_E^*, c_L^*, g^*)\) will, therefore, violate the constraint (5) if and only if \( \hat{\mu}^* > \mu^* \), which establishes the result. \( \blacksquare \)

**Proposition 4** Under (2), \( \Phi^D \supset \Phi^* \) holds.

**Proof:** I begin by stating a technical lemma that shows that as the probability of a run becomes very small, the discretionary equilibrium allocation converges to the \( q \)-efficient allocation. The proof of this lemma is relatively straightforward and is omitted.

**Lemma 1** \( \lim_{q \to 0} c^D(q) = \lim_{q \to 0} c^*(q) \).

Next, consider any economy in the set \( \Phi^* \). Straightforward calculations show that, for the CRRA utility function (2), the ratio \( c_E^*/\hat{c}_L^* \) is strictly decreasing in \( q \). Therefore, for any economy in \( \Phi^* \), we have

\[
\lim_{q \to 0} c_E^*(q) > \lim_{q \to 0} \hat{c}_L^*(q).
\]
Lemma 1 then implies
\[ \lim_{q \to 0} c_E^D (q) > \lim_{q \to 0} \tilde{c}_L^D (q), \]
which implies that the financial system is also fragile in the competitive equilibrium, that is, the economy also belongs to the set \( \Phi^g \).

To show that the inclusion relationship is strict, it suffices to produce an example of any economy in \( \Phi^D \) that is not an element of \( \Phi^* \). Such examples are easy to generate; \( (R, \pi, \gamma, \delta) = (1.1, 0.3, 6, 0.01) \) is one. For this economy, \( c_E^D (q) > \tilde{c}_L^D (q) \) holds if and only if \( q > 0.08 \). \( \blacksquare \)

**Proposition 5** Under (2) and a no-bailout policy, \( q > 0 \) implies \( \rho^D < \rho^* \).

**Proof:** Using expressions (21) and (23) in Appendix B, we have that \( q > 0 \) implies \( \rho^D < \rho^* \) if and only if
\[ \frac{\alpha_1}{R^{1-\gamma}} > \frac{\alpha_2}{\alpha_3} \]
or
\[ \frac{(\alpha_1)^{\frac{1}{\gamma}}}{R^{1-\gamma}} > \frac{\delta^{\frac{1}{\gamma}} + (1 - \pi) (\alpha_1)^{\frac{1}{\gamma}}}{\delta^{\frac{1}{\gamma}} + (1 - \pi) R^{1-\gamma}}. \]
The assumption \( \gamma > 1 \) implies that \( (\alpha_1)^{\frac{1}{\gamma}} > R^{1-\gamma} \) holds, which in turn implies that the above inequality necessarily holds. \( \blacksquare \)

**Proposition 6** Under (2), the relationship \( \Phi^{NB} \supset \Phi^* \) holds.

**Proof:** I begin by stating a technical lemma that shows that as \( q \to 0 \), the ex ante components of the equilibrium allocation converge to the corresponding components of the \( q \)-efficient allocation. The proof of this lemma is relatively straightforward and is omitted.

**Lemma 2** \( \lim_{q \to 0} \left( c_E^{NB} (q), c_L^{NB} (q), g^{NB} (q) \right) = \lim_{q \to 0} \left( c_E^* (q), c_L^* (q), g^* (q) \right) \).

These conditions imply that when \( q \) is small, the total resources available in the event of a run \( (1 - \pi c_E) \) will be the same in the two cases. From Corollary 1, we know that \( \tilde{g}^{NB} > \tilde{g}^* \) always holds. It then follows that
\[ \lim_{q \to 0} \tilde{c}_L^{NB} (q) < \lim_{q \to 0} \tilde{c}_L^* (q). \]
Consider any economy in the set $\Phi^*$. As in the proof of Proposition 4, we know
\[
\lim_{q \to 0} c^*_E(q) > \lim_{q \to 0} \hat{c}^*_L(q).
\]
These conditions immediately imply that the economy is also in $\Phi^{NB}$.

To show that the inclusion relationship is strict, it suffices to produce an example of any economy in $\Phi^{NB}$ that is not an element of $\Phi^*$. Such examples are easy to generate; $(R, \pi, \gamma, \delta) = (1.1, 0.5, 6, 1)$ is one.

\textbf{Proposition 7} Some economies in the set $\Phi_{NB}$ are not in the set $\Phi_g$.

\textbf{Proof:} The parameter values $(R, \pi, \gamma, \delta) = (1.1, 0.5, 6, 100)$ are an example of an economy that is in $\Phi_{NB}$ but not in $\Phi^D$. \hfill \blacksquare
Appendix B. The Solution with CRRA Preferences

B.1 The q-efficient allocation

For the CRRA utility function in (2), the q-efficient allocation of resources is given by

\[ c^*_E = \left( \frac{1}{(1-q)\alpha_4 + q\alpha_5} \right)^{\frac{1}{\gamma}}, \quad c^*_L = \left( \frac{R}{\alpha_4} \right)^{\frac{1}{\gamma}}, \quad g^* = \left( \frac{\delta}{\alpha_5} \right)^{\frac{1}{\gamma}} \]

and

\[ \widehat{c}^*_E = \left( \frac{1}{\alpha_5} \right)^{\frac{1}{\gamma}}, \quad \widehat{c}^*_L = \left( \frac{R}{\alpha_5} \right)^{\frac{1}{\gamma}}, \quad \widehat{g}^* = \left( \frac{\delta}{\alpha_5} \right)^{\frac{1}{\gamma}}, \]

where the constants \( \alpha_4 \) and \( \alpha_5 \) are equal to the multipliers on the resource constraints (4) and (5) evaluated at the solution. The values of these constants are given by

\[ \alpha_4 \equiv \mu^* = \frac{\alpha_3}{(1-q)\alpha_3 + q\alpha_2} \left( \pi + \left[ (1-q)\alpha_3 + q\alpha_2 \right]^{\frac{1}{\gamma}} \right)^{\gamma} \] and

\[ \alpha_5 \equiv \widehat{\mu}^* = \frac{\alpha_2}{(1-q)\alpha_3 + q\alpha_2} \left( \pi + \left[ (1-q)\alpha_3 + q\alpha_2 \right]^{\frac{1}{\gamma}} \right)^{\gamma}, \]

where

\[ \alpha_1 \equiv \left( \pi + (1-\pi)R^{\frac{1-\gamma}{\gamma}} \right)^{\gamma}, \quad \alpha_2 = \left( \frac{\delta^{\frac{1}{\gamma}} + (1-\pi)\alpha_1^{\frac{1}{\gamma}}}{\delta^{\frac{1}{\gamma}} + (1-\pi)\alpha_2^{\frac{1}{\gamma}}} \right)^{\gamma}, \quad \text{and} \]

\[ \alpha_3 \equiv \left( \delta^{\frac{1}{\gamma}} + (1-\pi)R^{\frac{1-\gamma}{\gamma}} \right)^{\gamma}. \]

The degree of illiquidity in the q-efficient allocation is then given by

\[ \rho^* = \frac{c^*_E}{1-g^*} = \frac{1}{\left( \pi + (1-\pi)R^{\frac{1-\gamma}{\gamma}} \left( (1-q) + q\alpha_2^{\frac{1}{\gamma}} \right) \right)^{\gamma}}. \] (21)

It is straightforward to show that \( \gamma > 1 \) implies \( \alpha_2 > \alpha_3 \), which in turn implies that \( \rho^* \) is strictly decreasing in \( q \).

It is also straightforward to show that the ratio \( \frac{c^*_E}{\widehat{c}^*_L} \) is strictly decreasing in \( q \); this result is a reflection of the fact that the q-efficient allocation becomes less illiquid as the probability of a run increases. If \( c^*_E > \widehat{c}^*_L \) holds for some value \( \bar{q} \), therefore, it must also hold for all \( q < \bar{q} \). This result allows us to characterize the set \( \Phi^* \) by looking at condition (7) in the limit as \( q \) goes to zero, which yields

\[ R^{-1} > \frac{\mu^*}{\widehat{\mu}^*} = \frac{\alpha_3}{\alpha_2} = \left( \frac{\delta^{\frac{1}{\gamma}} + (1-\pi)R^{\frac{1-\gamma}{\gamma}}}{\delta^{\frac{1}{\gamma}} + (1-\pi)\alpha_1^{\frac{1}{\gamma}}} \right)^{\gamma}. \]
Using this expression, one can show that the set $\Phi^*$ is nonempty.\footnote{When $\delta = 0$, meaning that agents do not value the public good, this inequality reduces to the condition for the existence of bank-run equilibria given in Ennis and Keister [10, p. 411].}

### B.2 Equilibrium under discretion

The competitive equilibrium allocation under discretion can be derived by working backward. First, in the event of a run intermediary $j$ with remaining resources per investor $\psi_j$ will distribute them according to

\[
\tilde{c}_{E,j}^D = \left( \frac{1}{\tilde{\mu}_j^D} \right)^{\frac{1}{\gamma}} \quad \text{and} \quad \tilde{c}_{L,j}^D = \left( \frac{R}{\tilde{\mu}_j^D} \right)^{\frac{1}{\gamma}},
\]

with

\[
\tilde{\mu}_j^D = \left( \pi + (1 - \pi) R^{1-\gamma} \right)^\gamma \psi_j^{-\gamma}. \tag{22}
\]

Note that we can use the constant $\alpha_1$ defined above to write $\tilde{\mu}_j^D = \alpha_1 \psi_j^{-\gamma}$. Straightforward algebra then shows

\[
\tilde{V}_c = \alpha_1 \frac{\psi_j^{1-\gamma}}{1-\gamma}.
\]

Next, the equilibrium bailout policy will set

\[
b^D = \left( \frac{\alpha_1}{\alpha_2} \right)^{\frac{1}{\gamma}} ((1 - \pi) \phi + \tau) - \phi,
\]

which leads to the value function

\[
\tilde{V} ((1 - \pi) \phi + \tau) = \alpha_2 \frac{(1 - \pi) \phi + \tau)^{1-\gamma}}{1-\gamma}.
\]

For a given tax rate $\tau$, the solution to the representative intermediary’s ex ante choice problem for the case of $q < (R - 1) / R$ can be written as

\[
c_{E}^D = \left( \frac{1}{\alpha_6} \right)^{\frac{1}{\gamma}} (1 - \tau), \quad c_{L}^D = \left( \frac{(1 - q) R}{\alpha_6} \right)^{\frac{1}{\gamma}} (1 - \tau), \quad \mu^D = \frac{\alpha_6}{1 - q} (1 - \tau)^{-\gamma}
\]

where

\[
\alpha_6 = \left( \pi + (1 - \pi) (1 - q)^{\frac{1}{\gamma}} R^{1-\gamma} \right)^\gamma. \tag{22}
\]

For the case of $q \geq (R - 1) / R$, the solution is

\[
c_{E}^D = c_{L}^D = \left( \frac{1}{\alpha_7} \right)^{\frac{1}{\gamma}} (1 - \tau),
\]
where
\[ \alpha_7 = \left( \pi + (1 - \pi) R^{-1} \right)^\gamma. \]

The equilibrium tax rate cannot be derived in closed form under this regime, but it can easily be solved for numerically. The degree of illiquidity \( \hat{\rho} \) can, however, be derived in closed form because it is independent of the tax rate in this case. Using the government budget constraint in the no-run state, \( g = \tau \), we have
\[ \rho^D = \frac{c^D_E}{1 - q^D} = \left( \frac{1}{\alpha_6} \right)^{\frac{1}{\gamma}} \]
when \( q < (R - 1)/R \) holds. Note that the constant \( \alpha_6 \) is strictly decreasing in the probability of a crisis \( q \), which implies that \( \rho^D \) is strictly increasing in \( q \). For larger values of \( q \), the incentive compatibility constraint binds and \( \alpha_6 \) is replaced with \( \alpha_7 \), which is independent of \( q \). Once the incentive compatibility constraint binds, changes in \( q \) no longer effect the degree of illiquidity.

### B.3 Equilibrium under a no-bailout policy

The competitive equilibrium allocation in the no-bailout regime is given by
\[ c_{NB}^E = \left( \frac{1}{\alpha_8} \right)^{\frac{1}{\gamma}} (1 - \tau) \quad \text{and} \quad c_{NB}^L = R \left( \frac{(1 - q) R^{1-\gamma} + q \alpha_1}{\alpha_8} \right)^{\frac{1}{\gamma}} (1 - \tau), \]
where
\[ \alpha_8 = \left( \pi + (1 - \pi) \left( (1 - q) R^{1-\gamma} + q \alpha_1 \right)^\frac{1}{\gamma} \right)^\gamma. \]

The post-run consumption allocation is
\[ \tilde{c}_{NB}^E = \left( \frac{1}{\alpha_1} \right)^{\frac{1}{\gamma}} \left( \frac{1 - \tau - \pi \tilde{c}_E}{1 - \pi} \right) \quad \text{and} \quad \tilde{c}_{NB}^L = \left( \frac{R}{\alpha_1} \right)^{\frac{1}{\gamma}} \left( \frac{1 - \tau - \pi \tilde{c}_E}{1 - \pi} \right). \]

In this case, the equilibrium tax rate can be derived in closed form,
\[ \tau_{NB} = \frac{\delta^{\frac{1}{\gamma}}}{\delta^{\frac{1}{\gamma}} + (\alpha_8)^{\frac{1}{\gamma}}}, \]
and the degree of illiquidity is given by
\[ \rho_{NB} = \left( \frac{1}{\alpha_8} \right)^{\frac{1}{\gamma}} = \frac{1}{\pi + (1 - \pi) \left( (1 - q) R^{1-\gamma} + q \alpha_1 \right)^\frac{1}{\gamma}}. \]

It is straightforward to show that \( \alpha_1 > R^{1-\gamma} \) and, hence, that \( \rho_{NB} \) is strictly decreasing in \( q \).
References


