The Rise of Securitization: A Recursive Security Design Approach

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Abstract

The global demand of US investment-grade paper increased significantly after the mid-1990's. We lay out a macroeconomic model with endogenous security markets where the consequences of this phenomenon can be studied. Our model economy responds to the increased demand for safe paper by securitizing ever riskier projects. Capital deepening causes output to rise but aggregate productivity to fall. Meanwhile, GDP and investment volatility fall as external capital flows are less sensitive to domestic conditions than domestic investment. Taking on these questions requires a model where the financial structure is endogenous. Specifically, investors take the future supply of various types of securities as given and choose their optimal consumption path accordingly and, in turn, the financial structure investors take as given must fact be profit maximizing for intermediaries given the willingness of investors to pay for various securities, which depends on their consumption decisions. In addition to satisfying standard conditions, equilibria must therefore satisfy a consistency condition a la Allen-Gale (1988). We make such an equilibrium concept specific, establish that an equilibrium must exist in the context of our model and propose an algorithm for computing this class of equilibrium.

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1 Introduction

"*Given the strength of demand for safe U.S. assets, it would have been surprising had there not been a corresponding increase in their supply.*"

Ben Bernanke et al. (2011)

The demand for US investment grade paper started increasing markedly in the late 1990s, fueled in part by the fast emergence of nations with high saving rates and a strong preference for safe paper, a phenomenon then Federal Reserve Board Governor Ben Bernanke termed the *Global Saving Glut*. There should be little doubt that this rise in demand contributed to the growing importance of structured finance during the same period. Structured finance, after all, is inter alia a technology to produce investment grade claims backed by non-investment grade assets.

This paper describes a macroeconomic model in which the quantitative consequences of increased demand for safe assets can be measured. The model is standard on the real side but we allow the security space to respond endogenously to changes in demand for different types of claim. Specifically, financial intermediaries choose what quantity of investment grade claims to produce given investors' willingness to pay for safe assets and for the residual, junior claims that provide credit support for those safe claims.

In a model like ours where the volume of structured finance is endogenous, the supply of safe paper can increase along two margins. First asset types that are already used by securitizers could in principle be used to produce more safe debt. That is, the ratio of the par value of investment grade claims to the the par value of the assets used in securitization could rise. Second and along the extensive margin, the scope of structured finance can be broadened by bringing more assets into the securitization fold. Our model predicts that the supply response comes exclusively from the extensive margin. Because securitization carries a cost, intermediaries always maximize the quantity of safe assets they can extract from the projects they have pooled. Therefore, a supply response to an external shock has to come from the extensive margin.

At the same time and for similar reasons, intermediaries always select the safest projects first for securitization. It follows that the supply response to a increase in demand for the safe asset entails activating ever riskier projects. Our model, therefore, correctly predicts a sharp increase in the default risk associated with bringing more assets into the securitization fold.

Because standard aggregation arguments hold in our model, the effect of the increased demand for safe US assets on macroeconomic variable can be studied readily. Most directly, the saving glut causes capital deepening hence output to rise. At the same time, the activation of marginally productive projects causes TFP to fall (albeit by a small amount.) The risk-free rate, for its part, falls while the premium equity claims earn over the risk-free asset rises as the quantity of subordinated claims rises.

Our quantitative model makes explicit the ways in which costly securitization activities can improve the allocation of resources. First and most obviously, tranching activities result in a menu of securities that tailors to the objectives of heterogenous investors an in so doing, broadens financial markets. But perhaps more importantly, absent securitization, the demand for safe assets could not in and of itself serve to fund risky capital formation. When on the other hand costly securitization is an option, investors who only want to hold safe claims and less risk-averse investors join forces to fund risky projects. The option to securitize obviously results in higher investment and output in stochastic steady state. Our simulations also reveal that, somewhat less intuitively, securitization also causes investment and GDP to become less volatile since external capital flows are less responsive to domestic conditions in general and TFP shocks in particular. At at time when drastic curbs to securitization are being considered given its role in the recent crisis, estimates of the benefits of securitization activities such as those we produce seem particularly helpful.

Our model also makes interesting predictions for subordination patterns. Riskier projects allow for less creation of risky assets so that the ratio of the par value of safe claims to the par value of the asset pool that backs them should fall on average as the supply of safe assets rises. The data show that this is precisely what happened for subprime residential pools, the poster child form of securitization during the housing boom. On the other hand, there is some evidence that subordination levels actually *fell* for deals backed by commercial mortgages between 2000 and 2007. A key difference between these two types of structured finance is that while underwriting standards deteriorated noticeably on subprime residential pools during the boom, commercial standards held fairly steady suggesting that the quality of the asset pools held reasonably constant on the commercial side.

Still, why would subordination levels fall on pools of a given quality during the boom? A potential explanation is that investors began downplaying the potential magnitude of aggregate events. Our model enables us to back out the bout of optimism that could rationalize the observed decline of support levels holding project quality constant and in turn, measure the effect of this rise of this increased complacency on the securitization boom.

Taking on these questions requires a model where the financial structure is endogenous. Specifically, investors choose their optimal consumption path taking the future supply of various types of securities and, in turn, the financial structure investors take as given must be profit maximizing for intermediaries given the willingness of investors to pay for various securities, which depends on their consumption decisions. In addition to satisfying standard conditions, equilibria must therefore satisfy a consistency condition a la Allen-Gale (1988). We make such an equilibrium concept specific, establish that an equilibrium must exist in the context of our model, and propose an algorithm for computing it. In that sense, our paper extends the work of Allen-Gale (1988) to a dynamic environment.

Of course, we are not the first ones to take on the fixed point in state prices that naturally arises in dynamic models with incomplete financial markets (see Telmer, 1993, Alvarez and Jerman, 2001, Guvenen, 2009, Chien, Cole and Lustig, 2011, among others.) Our technical innovation is to take on this problem with costly security creation in each period in the context of a model with a traditional production side.

Our work is also related to Brunnermeier and Sannikov (2012) who introduce a financial sector in a macroeconomic model. Unlike us, they assume that all agents have linear preferences which enables them to derive a number of interesting analytical results on the potential effect of externalities within the financial sector. Our goals are quite different however. We want to quantify the effect of changes in the willingness to pay for safe assets in a traditional macroeconomic model. Gennaioli et. al. (2011) for their part, tell a qualitative story that closely resemble ours in a stylized 2-date model. Our goal is to give a quantitative account of the rise of securitization in a more traditional macroeconomic model.

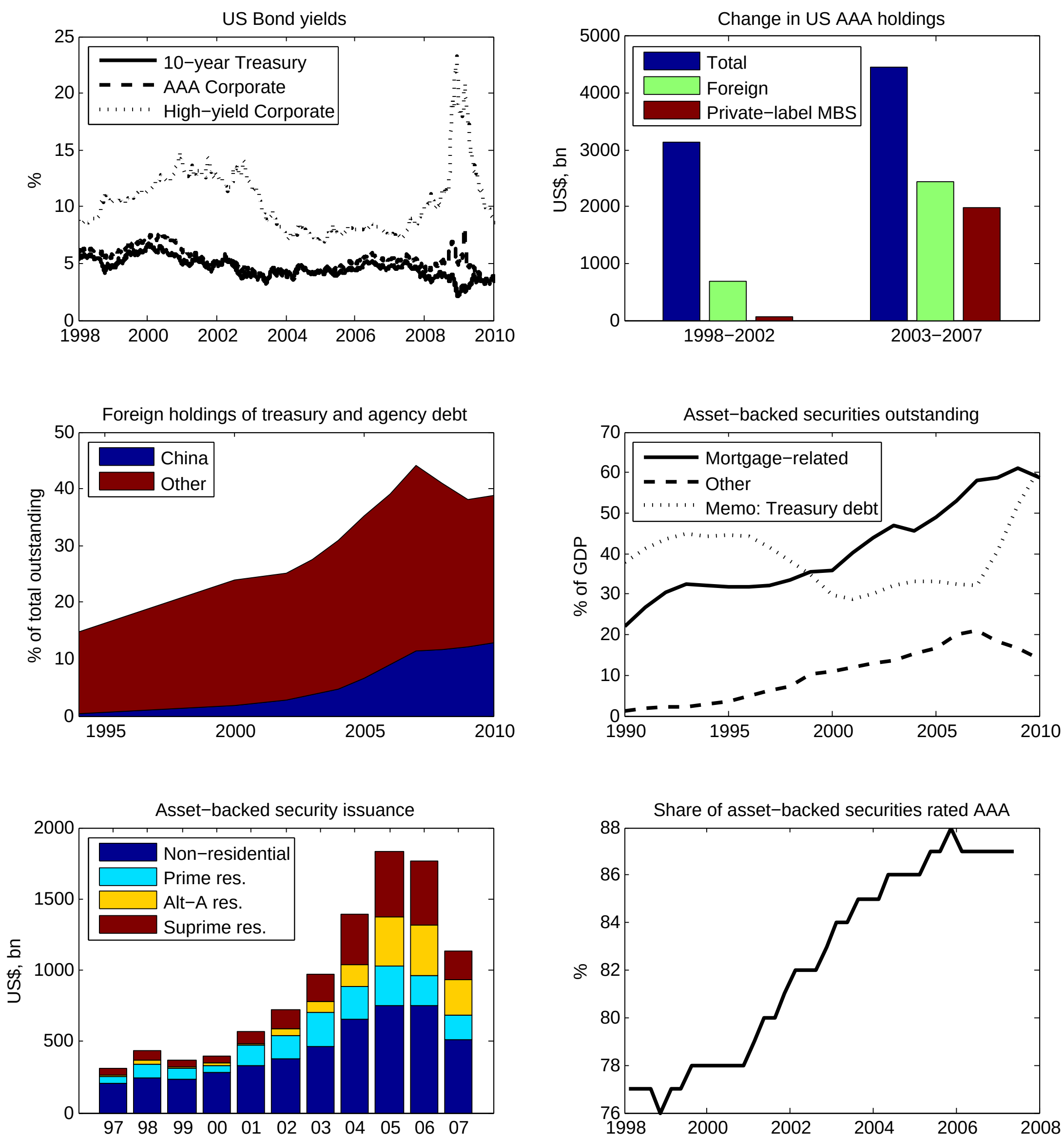
Another related strand of the literature (Gorton and Pennachi, 1990, Winton, 1995, Riddiough, 1997, De Marzo and Duffie, 1999, De Marzo, 2005) provides foundations for the pooling and tranching of asset pools that rely on various forms of asymmetric information such as costly monitoring or adverse selection. These considerations are absent from our paper since we assume that all information is public. Instead, tranching decisions in our model result from a simple, fully informed cost-benefit comparison as in Allen and Gale (1988) and depend on the willingness of agents to pay for safe paper. Securitization, that is, creates value by completing the market and tayloring security supply to the needs of heterogeneous investors. While it conceivable that institutional changes or changes in the macroeconomic environment may have caused the distribution of information or beliefs to change in a way that caused tranching to increase, our goal is to quantify the importance of a much more intuitive demand-based story. Whether informational considerations played a role in the securitization boom is an open question but there is little doubt that the surge of external demand for safe assets was a key factor in this phenomenon.

2 The facts

This section documents the salient features of the rise of structured finance since the mid 1990's. These are quantitative patterns we will use to motivate the design/calibration of our model and to evaluate its quantitative performance.

Fact 1. *Yields on investment-grade bonds have fallen steadily since 2000, even during periods of short-term rate tightening by the Federal Reserve.*

Figure 1: The Rise of Securitization



Sources: Bernanke et. al., 2011, SIFMA.org, Federal Reserve Economic Data (FRED), Inside Mortgage Finance, authors' calculations.

This is the conundrum to which then Federal Reserve Chairman Alan Greenspan famously referred. the first panel of figure 5 illustrates the slow decline of AAA rates both for treasury and corporate instruments. Note also that spreads between high-yield rates and AAA rates fell as well between 2002 and 2008. The risk premia investors require for holding riskier paper, that is, fell during that period. While we will argue that the fall in risk-premia is not easy to explain, an increase in external demand for US safe assets can easily rationalize the fall in AAA rates, at least qualitatively. And the data do suggest that foreign demand for those assets increased a lot during that period:

Fact 2. *The size of the stock of AAA-rated US assets increased sharply between 1995 and 2007. Between 2003 and 2007, foreign holdings account for over half of the increase in that stock.*

Bernanke et. al (2011) document for instance that the share of foreign holdings of US AAA securities jumped from 18% to 25% between 2003 and 2007. While US resident holdings of AAA securities increased by about 25% during that period, foreign holdings nearly doubled. The second panel of figure 5 displays the massive contribution of foreign holdings to the overall increase in US AAA holdings between 2003 and 2007. As shown in the third panel of figure 5, the foreign share of treasury and agency debt rose from 15% to nearly 45% between 1995 and 2000.

Fact 3. *Private-label structured finance accounts for over one-third of the increase in the stock of AAA securities between 2003 and 2007.*

The contribution of private securitization to the increase in the size of the AAA market is displayed in the second panel of figure 5. Between 1998 and 2002 the contribution of private label securitization was minimal. It becomes very significant between 2003 and 2007. Whereas agency-backed debt and securities account for the bulk of the increase in the size of the investment-grade stock between 1998 and 2002, private-label securitization becomes the leading source of AAA debt creation after 2003.

This growing preponderance of private-label activity after 2000is of critical importance because, historically, private-label securitization has relied largely on non-prime assets. The explosion of private activity in after 2000 thus nechanically implies a deterioration of the overall quality of loans used in securitization. But this effect is compounded by the fact that even within the private sphere, the importance of non-prime assets started growing at a fast pace after 2000.

Fact 4. *The share of non-prime mortgage securitization in total residential MBS issuances quadrupled to 40% between 2000 and 2007*

Non-prime issuances include both subprime and Alt-A pools. While the non-prime share sky-rocketed during that period, the GSE share in residential securitization fell from 80% to 50%. The fifth panel of figure 5 show the growing share of subprime residential pools in overall non-agency asset-backed security issuance. Within private-label residential MBS deals, SIFMA data suggests that the share of non-prime loans rises from an already to high 50% in 2000 to almost 80% in 2007 on the eve of the financial collapse.

Fact 5. *The average level of credit-support for investment-grade tranches increased noticeably in deals backed by subprime residential mortgage pools between 2000 and 2007 but* fell *significantly on deals backed by commercial real estate loans.*

These patterns are documented e.g. by Riddiough and Zhu (2011), Stanton and Wallace (2010) and Nadault and Sherlund (2009). Our model predicts that subordination levels should rise as the scope of securitization increases, all else equal. This is the pattern one observes for the loans that account for the bulk of the increase in private securitization after 2000 (non-prime residential pools), but the fact that CMBS deals started providing less support for investment-grade tranches during that same period is much more puzzling, as we will discuss. It suggests that as securitization rates boomed, investors became more complacent about aggregate risk. We will discuss several possible explanations for this increase in complacency. Another manifestation of this complacency is shown in the last panel of figure 5. Even as even riskier assets were used to fuel private-label securitization, the fraction of the resulting securities rated as investment-grade actually *rose* during the boom.

3 The environment

Consider an economy where time is discrete and infinite. The economy is populated by a continuum of two types of consumers, each of mass one, a continuum of producers and a stand-in financial intermediary. The first consumer type (L) has a standard CRRA objective function with curvature parameter $\sigma > 0$. The second type (H) has Epstein-Zin preferences with the same intertemporal willingness to trade consumption as agent L but is much more risk-averse. In fact, we will assume that type H consumer is so risk-averse that he is only willing to hold risk-free assets.[1] Consumer L supplies one unit of labor in each period, but consumer H supplies no labor.

The assumption on the labor supply of H-type agents can be relaxed at no cost but will make it easier to interpret this class of investors as outside investors. Type H agents do receive a fixed endowment $\theta > 0$ of the unique good in each period. In the calibration, we use this parameter to control the size of capital inflows that emanate from these agents.

It will be convenient to assume that consumption takes place at the start of the period. Specifically, agents of type $i \in \{H, L\}$ enter each period t with a certain quantity a_t^i of the consumption good – which results from the investment decisions and shocks realized in period $t-1$ – choose what fraction of that wealth to consume at the start of the period and how to split the balance of their wealth across the securities that happen to be available in that period. Their wealth at the end of period t and the start of period $t+1$ is the payoff of the resulting portfolio. For one advantage, this timing convention means that our economy contains the model of Allen and Gale (1988) as two-period special case. To make the convention more precise, we need to describe investment opportunities more formally.

[1] In appendix 1, we will establish that, in our environment, a consumer with Epstein-Zin preferences chooses to hold nothing but the risk-free asset provided her aversion to risk is high enough.

Consumers of type L are endowed with a continuum of investment projects, each indexed by a probability $p \in (0,1)$ of success. Let $\mu(p)$ be a density function that gives the mass of projects of success likelihood p. We will assume that $\mu(0,1]$ is large enough so that there is always a potential use for available capital. Formally, say, $\mu[p,1] \to +\infty$ as $p \to 0$.

At date t, activating a project of type p requires one unit of the good invested at the start of the period. With probability p the project is successful and yields $Z_t n^\alpha$ in gross operating income if quantity n of labor is employed, where $\alpha \in (0,1)$ and returns fraction $(1-\delta)$ of the capital it employs. The project yields nothing otherwise.[2]

Here, Z_t is common to all projects – we will think of it as an aggregate shock –but is not known when the investment is made. The support $\mathcal{Z} \subset I\!R_+$ for the aggregate shock process is the same in all periods and, to sidestep uninteresting technical considerations, assume throughout that there is only a finite number of possible values for this shock. Of particular interest below will be

$$\underline{Z} = \min(\mathcal{Z}),$$

the worst possible realization of the aggregate shock. This worst-case aggregate shock will dictate the maximum quantity of risk-free claims the economy can support. Denote the time-invariant probability distribution of aggregate shocks by $\nu : \mathcal{Z} \mapsto [0,1]$.

Given the price w_t of labor at date t successful projects generate

$$\Pi(Z_t, w_t) \equiv \max_{n>0} Z_t n^\alpha - n w_t$$

in net operating income. Let

$$n^*(Z_t, w_t) \equiv \arg\max_{n>0} Z_t n^\alpha - n w_t$$

denote profit-maximizing labor use given values of the aggregate shock and the wage. All

[2] Assuming that the project returns the undepreciated capital even when it fails does not complicate the qualitative analysis in any way. We will experiment with both versions in the quantitative section.

successful projects employ the same quantity of labor and standard aggregation algebra will show in the next section that aggregate output must be a standard neoclassical function of aggregate capital and aggregate labor in this environment. The production side of our environment, therefore, is the same as that of a standard RBC model.

Investments in projects are intermediated. Specifically, a stand-in intermediary can buy any given project from the L-type agent for a project-type-specific price $\kappa(p)$ that is determined in equilibrium. The intermediary finances its investments by issuing securities, i.e. claims to the pool's output. Since all project-specific risk is diversified away inside pools, a security is a mapping from the aggregate state to a non-negative dividend.

We require that dividends be non-negative for the same reasons as Allen and Gale (1989.) Allowing negative dividends is formally similar to allowing consumers to short-sell securities. As is well known, doing so can lead to non-existence even in static versions of the environment we describe. More importantly perhaps, securitization cannot generate private profits when short-sales are unlimited in this case since any value created by splitting cash-flows could be arbitraged away in the traditional Modigliani-Miller sense.[3] As a result, no costly securitization would take place in equilibrium.

In principle, the intermediary can create as many securities as it wants, although we will soon make assumptions that greatly simplify the resulting design problem. Given a wage rate $w > 0$, a vector of J securities $r^1, r^2, r^3, \ldots r^n$ is feasible if, for all j, $r^j(Z) \geq 0$ and provided

$$\sum_{i=1}^{n} r^i(Z) = \mu(p)p\big(\Pi(Z, w) + (1 - \delta)\big)$$

for all possible realizations Z of the aggregate shock. The cost of issuing n securities is $(n-1)c$ per project in the pool where $c > 0$. Writing one security type – creating an an all-equity pool, that is – is free, but creating any additional security entails a pool-wide cost $\mu(p)c$.

Note that pooling projects eliminates project-specific risk but aggregate shocks can't be diversified away. Forming a pool of different project type would do nothing to change that

[3]See Allen and Gale, 1991, for the formal version of this argument.

hence there is no loss of generality in assuming that the intermediary forms pools of one given type at a time.[4]

If the intermediary chooses to fund all projects of type $p \in (0,1)$, we will write that the intermediary creates a pool of type p. It finances the cost of this purchase by issuing risk-free debt $b > 0$ and equity promises. The eventual size of the equity payoff is a function of the pool's output and debt usage, hence a function of the realization Z of the aggregate shock and debt choice $b \geq 0$. Specifically, for all $Z > 0$ and b,

$$r^E(Z, b; p, w) = \mu(p)p[\Pi(Z, w) + 1 - \delta] - b.$$

Since the pair (b, p) fully determines the equity payoff it is a sufficient statistic for the pool's capital structure. Write $MV(b; p)$ for the equilibrium-determined market value of a given capital structure choice. While MV must reflect the marginal willingness to pay for securities in equilibrium in a sense we make precise below, the intermediary takes this function as given. Given a pool type p, the intermediary solves:

$$\begin{aligned} \max_{b \geq 0} \quad & MV(b; p) - \mu(p)(1 + \kappa(p) + 1_{\{b>0\}}c) \\ \text{subject to:} \quad & b \leq \mu(p)p[\Pi(\underline{Z}, w) + 1 - \delta], \end{aligned}$$

where $\underline{Z}$ is the lowest possible realization of the aggregate shocks. The constraint states that risk-free debt must be risk-free. In other words, the highest risk-free payout the intermediary can promise out of a given pool is the pool's worst-case cash-flow. Note that the intermediary only incurs per-unit transaction costs c if it chooses to issue strictly positive debt claims against the pool.

[4] There is no role in our model, in particular, for combining claims from different pools to create a new pool and a new set of securities. Our agents can extract the risk-free portion of any combination of assets in one step. In real practice, this process often involves the re-securitization of securities from different pools. Our specification encompasses any and all benefits these activities could yield. Indeed, given the assets that are used for the creation of securities, the intermediary can choose to directly reach the overall bound on the supply of risk-free asset. Our specification thus fully encompasses any value CDO-type practices could generate.

If no feasible capital-structure is such that $MV(b;p) - \mu(p)(1+\kappa(p)+1_{\{b>0\}}c) \geq 0$, there is no profitable way for the intermediary to form a pool of type p and projects of type p remain inactive. As part of an equilibrium, $\kappa(p)$ is such that profits associated with intermediation are non-positive in equilibrium as would have to hold for instance if there is free entry into financial intermediation.

As in Allen and Gale (1988), we will require that in equilibrium $MV(b;p)$ reflects the marginal willingness of agents to pay for securities, an equilibrium requirement they term the *rational conjecture condition.* Specifically and at a given date and history, for $i \in \{H, L\}$, type i's willingness to pay for a marginal unit of the consumption good if state $Z \in \mathcal{Z}$ materializes is

$$p_t^i(Z) = \frac{\beta\nu(Z)U'(c_{t+1}^i(Z))}{U'(c_t^i)}$$

where c_{t-1}^i is the agent's consumption's choice at the start of date t given their realized history while $c_t^i(Z)$ is their consumption choice at the start of date $t+1$ if shock Z materializes.

To understand this expression, notice that investing in a security that delivers a marginal unit of consumption good if state Z occurs in the period raises the agent's utility by $p_t^i(Z)$. Here, $U(c) = \frac{c^{1-\sigma}}{1-\sigma}$ for both agents since they have the same willingness to substitute consumption across time. The key assumption is that the intermediary takes this marginal willingness to pay as given. In that sense, one should think of our intermediary as standing in for a continuum of small intermediaries that, individually, have no marginal impact on the security space.

With this notation in hand, for all project types p, dates t and histories:

$$MV_t(b,p) = b \max_{\{i \in \{H,L\}\}} \int_{\mathcal{Z}} p_t^i(Z)dZ + \int_{\mathcal{Z}} p_t^L(Z) \Big[\mu(p)p\left(\Pi(Z,w) + 1 - \delta\right) - b\Big] dZ.$$

In other words, the intermediary sells each security it creates to the highest bidder, taking the willingness-to-pay vector p_t as given. Recall that equity can only be sold to the agent of type L, by assumption.

To summarize the optimal solution to the intermediary's security design problem at a particular date t and history, write $k_t^S(p) = 1$ if the intermediary choose to fund projects of type $p \in [0,1]$ while $k_t^S(p) = 0$ otherwise and denote by $b_t^S(p) \geq 0$ the quantity of risk-free claims extracted from pools of type p. Similarly denote by

$$q_t^b \equiv \int_{\mathcal{Z}} p_t^i(Z) dZ$$

the price of a risk-free unit promise at date t while for $p \in [0,1]$ such that $k_t(p) = 1$,

$$q_t^e(p) = \int_{\mathcal{Z}} p_t^L(Z) \Big[\mu(p) p \left(\Pi(Z, w) + 1 - \delta \right) - b_t(p) \Big] dZ$$

is the price of the equity tranche in pool $p \in [0,1]$. At a date t and at a particular history,

$$\{k_t^S, b_t^S, q_t^b, \{q_t^e(p) : p \in [0,1] \text{ s. t. } k^S(p) = 1\}\}$$

summarizes the security space available to agents. Let $\mathcal{S}$ be the set of all possible security spaces at a given point in time. $\mathcal{S}$ is time-invariant because the set of potential projects is the same in all periods.

In order to make their consumption plans, agents need to forecast the contingent path of financial structures. To make this precise, denote by $\mathcal{H}_t$ the possible histories of aggregate shocks up to date t. In any equilibrium, agents assume a mapping

$$S_t : \mathcal{H}_t \mapsto \mathcal{S}$$

that gives for every possible date and history the predicted financial structure. In equilibrium of course, we will require this forecast to be correct. Given this forecast and an initial wealth a_0^L, agents of type L choose history-contingent policies $\{c_t, a_t^L, b_{t+1}, (e_t^D(p) : p \in$

$[0,1]$ s. t. $k_t^S(p)=1\}_{t=0}^{+\infty}$ to solve:

$$\max E\sum_{t=0}^{+\infty}\beta^t U^L(c_t)$$

subject to, for every date t and possible history $h_t \in \mathcal{H}_t$:

$$q_t^b b_t + \int_{k_t^S(p)=1} q_t^I(p)e_t^D(p)dp + c_t = a_t^L + \int_{k_t^S(p)=1}\mu(p)\kappa(p)dp,$$

$$a_{t+1}^L = \int_{k_t(p)=1} e_t^D(p)r_t^E(p;h_t,Z)dp + b_t + w_t(h_t,Z),$$

where:

$$\{k_t^S, b_t^S, q_t^b, q_t^e\} = S_t(h_t),$$

and $w_t(h_t,Z))$ is the wage rate that prevails given the history of aggregate shocks. This last object, in equilibrium, is the marginal product of labor which, we will soon argue, only depends on the aggregate stock of capital in use at a particular point in time. Notice that we assume that all objects in the problem satisfy the measurability conditions we need for all integrals to be well defined. As the next section will explain, security spaces have a very simple structures and trivially satisfy the needed conditions.

Type H consumers solve the same problem given initial wealth a_0^H except that they face an additional constraint: $e_t^D(p)=0$ for all dates t and possible histories.

4 Equilibrium

Assume that households of type $i\in\{H,L\}$ enter date 0 with assets $a_0^i>0$. At any given date t, we will write $\{c_t^L, a_{t+1}^L, \{e_t^D(p): p\in S_t\}, b_t^L\}$ for the history-contingent decisions of households of type L while $\{c_t^H, a_{t+1}^H, b_t^H\}$ are the decisions of the H type investors recalling that they hold no equity.

An equilibrium, then, is history-contingent project prices $\{\kappa_t(p)\}_{t=0}^{+\infty}$, wage rates $\{w_t(Z)\}_{t=0}^{+\infty}$

for every Z in the of the distribution of aggregate shocks, market structure values $\{MV_t(b;p) : p \in [0,1]\}_{t=0}^{+\infty}$, a security space mapping S, decision plans (k_t^S, b_t^S) by the intermediary and households decisions for households of both types such that:

1. Decision plans are optimal for the intermediary and both household types;
2. Intermediary profits are non-positive on all project type at all dates and for all possible histories given MV;
3. The market for labor clears at all dates;
4. The market for risk-free debt clears at all dates t and histories:

$$\int_{k_t^S(p)=1} b_t^S(p)dp = b_t^H + b_t^L;$$

5. The market for equity of all types p clears t all dates t and histories:

$$\int_{k_t^S(p)=1} e_t^D(p)dp = 1;$$

6. Market structure values satisfy the Allen-Gale condition for all project types p, dates t and histories:

$$MV_t(b,p) = b \max_{\{i \in \{H,L\}\}} \int_{\mathcal{Z}} p_t^i(Z)dZ + \int_{\mathcal{Z}} p_t^L(Z) \Big[\mu(p)\left(p\Pi(Z,w) + 1 - \delta\right) - b \Big] dZ,$$

   where $p_t^H(Z)$ is the marginal value willingness to pay for a unit of consumption if aggregate state Z materializes at date t given the history of shocks up to date t.

The last condition simply says that the intermediary sells any given security to the agent that is willing to pay the most for it.

5 Aggregate feasibility

As usual the collection of equilibrium conditions above imply that an aggregate feasibility condition must hold each period. This section will show that this constraint has essentially the same structure as what would emanate from a traditional RBC model. Denote by K the aggregate quantity of capital available for production in a given period and let N be the total mass of agents. It should be clear and the next section will demonstrate that only projects above a certain threshold $\underline{p}$ are activated. In equilibrium, we must have, then:

$$K = \int_{\underline{p}}^{1} \mu(p)dp$$

which implicitly defines a threshold $\underline{p}(K)$. Then, by the law of large numbers, fraction

$$E(p) \equiv \frac{\int_{\underline{p}}^{1} p\mu(p)dp}{\int_{\underline{p}}^{1} \mu(p)dp}$$

are successful. It follows that for markets to clear we must have:

$$KE(p)n^{*}(Z,w) = N$$

where Z and w are the current aggregate shock and price of labor, respectively. This defines $w(Z)$ implicitly given K and implies that the employment size of projects in equilibrium must be

$$n^{*}(Z,w) = \frac{N}{KE(p)}.$$

Then, letting

$$F(Z,K,N)$$

be aggregate output given the aggregate shock, aggregate capital and aggregate labor, we have:

$$\begin{aligned} F(Z,K,N) &= \int_{\underline{p}}^{1} \mu(p) p Z n^*(Z,w)^{\alpha} dp \\ &= \int_{\underline{p}}^{1} \mu(p) p Z \left(\frac{N}{KE(p)} \right)^{\alpha} dp \\ &= Z[E(p)]^{1-\alpha} K^{1-\alpha} N^{\alpha} \end{aligned}$$

The second line of derivation simply replaces n^* by what its value must be in equilibrium. The final line uses the fact that:

$$\int_{\underline{p}}^{1} \mu(p) p dp = E(p) \int_{\underline{p}}^{1} p dp = E(p) K.$$

Note that F resembles the standard neoclassical production function used in most macroeconomic exercises with $Z[E(p)]^{1-\alpha}$ playing the role of conventionally-measured TFP.

One important departure from the traditional framework is that capital-deepening – an increase in K holding all else equal, in particular a capital increase not caused by a increase in Z – has a negative effect on TFP by lowering the average quality of activated projects. In particular, scaling up (K,N) increase output less than linearly. This departure from aggregate constant returns to scale is immaterial for our purposes since we hold N fixed throughout this paper. Assuming that μ, the distribution of projects, is linear in N would suffice in any event to guarantee aggregate constant returns to scale hence the existence of balanced growth path in versions of this environment with growing effective labor supply.

Aggregating budget constraints for the two types gives, for all t:

$$C_t + I_t = F(Z_t, K_t, N_t) + (1 - \delta(\underline{p}) K_t$$

where $C_t = c_t^H + c_t^L$ and $I_t \equiv a_{t+1}^H + a_{t+1}^L$ is net investment at date t. Here,

$$\delta(\underline{p}) = \int_{\underline{p}}^{1} [(1-p) + \delta p] dp$$

where $[(1-p) + \delta p]$ is the effective rate of depreciation of pools of type p. For instance, pools that fail with probability 1 also experience 100% depreciation.

Aside from this endogeneity of the depreciation rate (which, again, one could remove if desired by assuming that all projects return the undepreciated part of capital), this expression is exactly what would emanate from a traditional RBC model. However, parts of net investment have to be used to pay for securitization costs and to purchase projects. It follows that :

$$K_{t+1} + \int_{k_t(p)=1} \mu(p)[1_{\{b(p)>0\}} c + \kappa(p)] dp = (1-\delta) K_t + I_t,$$

at all dates t.

This analysis implies among other computational shortcuts that the wage rate and the rental rate of capital can be computed in any given period simply as a function of the aggregate shock and aggregate resources. We will exploit this feature heavily in the quantitative section.

6 Endogenous security markets

This section characterizes optimal securitization choices by the intermediary. First, given the consumers with which we have populated our economy, it is easy to verify that the intermediary never chooses to issue securities other than risk-free debt and residual equity claims, a fact we've already assumed throughout.

Remark 6. *Any pool of type p is such that, in equiibrium, $J \leq 2$, $r^1(Z) = b$ where $b \geq 0$ while $r^2(Z) = \mu(p)p[\Pi(Z, w) + 1 - \delta] - b$ for all possible shocks Z.*

Proof. Type H only wants to hold risk-free debt hence, if that agent type is active at all in securities markets, risk-free debt must be created against some of the pools. Then, in any equilibrium, agent of type L holds all of the pool's output minus the risk-free claims promised to agent H. Since only one type of agent bids for that residual claim, splitting it into several pieces at a cost cannot be profitable. It follows that, as claimed, the intermediary issues at most tow types of claims against the asset pools it creates: risk-free, senior debt, and a residual equity claim. $\square$

The fact that all investment is intermediated greatly simplifies notation and the upcoming analysis. A natural question to ask, however, is whether this entails any loss of generality. It is easy to see that it does not as agents face at least the same constraints on the claims they can issue as the intermediary (as long as their security choice set is not greater than the intermediary's, that is). To see this, consider a consumer of type L who chooses to fund directly a pool of projects of a given type.[5] These consumers could in principle sell securities backed by such a pool but, since we have assumed that intermediation is a zero-profit activity in equilibrium, that option cannot yield any rents beyond what consumers would earn by selling their projects. Of course, instead of issuing securities, consumers could consume the output from the pools they fund. Showing that this option does not increase their choice set requires a more subtle argument. That argument is formalized below:

Remark 7. *In equilibrium, the option for consumers to directly fund pools of projects and retaining the pool's proceeds is weakly dominated by investing in the securities created by the intermediary.*

Proof. For an agent of type L, directly investing a unit of capital in a mass ϵ of projects of type p has utility value $\epsilon\big(\int_{\mathcal{Z}} p\,(\Pi(Z,w)+1-\delta)\,p_t^L(Z)dZ-1\big)$ where $p_t^L(Z)$ is agent L's marginal willingness to pay of a unit of consumption at date t if aggregate state Z materializes. Since the intermediary sells claims to the output of projects to the highest bidder, we have:

$$\int_{\mathcal{Z}} p\,(\Pi(Z,w)+1-\delta)\,p_t^L(Z)dZ-1 \leq MV(b,p)-\mu(p)1_{b>0}c-1=\kappa(p)$$

[5] As is the case for the intermediary, taking projects one type at a time entails no loss of generality.

where $MV(b,p)$ is the market value of the optimal capital structure for a pool of this type in equilibrium. The agent of type L, therefore, is at least as well off selling its projects of type p for $\kappa(p)$ as they are funding it directly. If the project does not get activated in equilibrium then $MV(b,p) - \mu(p)1_{b>0}c - 1 < 0$ and, a fortiori, $\int_{\mathcal{Z}} p\,(\Pi(Z,w) + 1 - \delta)\, p_t^L(Z)dZ - 1 < 0$ so that the agent of type L would not choose to implement the project either. □

In order to proceed, it is useful to record yet another intuitively obvious fact: market structure values increase strictly with project quality.

Remark 8. *In any equilibrium, MV rises strictly with p.*

Proof. Any securitization choice feasible for the intermediary at a given p remains feasible when p rises. Furthermore, holding b the same, equity payoffs rise strictly in all states when p rises. The result then follows directly from the Allen-Gale condition. □

This observation, in turn, implies that the market price of projects rises strictly with project quality among projects that get activated in equilibrium:

Remark 9. *Any equilibrium is such that for all t and possible histories, there exists $\underline{p}_t \in [0,1]$ such that*

1. *$\kappa_t(p) = 0$ if $p \leq \underline{p}_t$;*
2. *κ_t rises strictly on $[\underline{p}_t, 1]$*

Proof. If no profitable way to market a pool of type p exists, this remains true when p falls. On the other hand, keeping profits at zero among active projects as p rises requires that κ_t rises strictly. □

Having recorded these facts, the security space that emerges in any given period can now be characterized quite precisely.

Proposition 10. *The solution to the intermediary is fully described by two thresholds $0 \leq \underline{p}_t \leq \bar{p}_t \leq 1$ such that:*

1. *$k_t^S(p) = 1$ if and only if $p \geq \underline{p}_t$;*

2. *$b_t^S(p) = 0$ if $p < \bar{p}_t$;*

3. *$b_t^S(p) = \mu(p)p(\Pi(\underline{Z}, w) + 1 - \delta)$ if $p > \bar{p}_t$.*

Furthermore, $\underline{p}_t \leq \bar{p}_t$ for all t when $\theta = 0$.

Proof. The first item follows directly from remark 9. Without loss of generality in the context of this proof but to simplify notation, assume that $\mu(p) = 1$ for all p. To see the second item, take a particular history. If $\int_{\mathcal{Z}} p_t^L(Z)dZ \geq \int_{\mathcal{Z}} p_t^H(Z)dZ$ then the intermediary optimally chooses to issue no risk free debt since doing so is costly and, in that case,

$$MV_t(b, p) \leq \int_{\mathcal{Z}} p_t^L(Z)p\left(\Pi(Z, w) + 1 - \delta\right)dZ$$

for all p. The result then holds trivially.

For the rest of this proof then, assume that $\int_{\mathcal{Z}} p_t^L(Z)dZ < \int_{\mathcal{Z}} p_t^H(Z)dZ$. In that case, $V_t(b, p)$ rises strictly with b. If it is profitable to pay the fixed cost for particular project then it is therefore optimal to maximize the production of risk-free debt. Now, maintaining the normalization that $\mu(p) = 1$ for all p, write $MV_t^{b>0}(p)$ for the highest market value conditional on $b = p(\Pi(\underline{Z}, w) + 1 - \delta)$ while $V_t^{b=0}(p)$ is the same under the constraint that $b = 0$. We have:

$$V_t^{b=0}(p) = \int_{\mathcal{Z}} p(\Pi(Z, w) + (1 - \delta))p_t^L(Z)dZ$$

while

$$V_t^{b>0}(p) = p(\Pi(\underline{Z}, w) + 1 - \delta)\int_{\mathcal{Z}} p_t^H(Z)dZ + \int_{\mathcal{Z}} p(\Pi(Z, w) - \Pi(\underline{Z}, w))p_t^L(Z)dZ$$

Under the maintained premise that $\int_{\mathcal{Z}} p_t^L(Z)dZ < \int_{\mathcal{Z}} p_t^H(Z)dZ$, we have:

$$\frac{\partial V_t^{b>0}(p)}{\partial p} > \frac{\partial V_t^{b=0}(p)}{\partial p}$$

and it follows that if $V_t^{b=0}(p) < V_t^{b>0}(p) + c$, this remains true as p rises. This establishes the existence of a second threshold $\bar{p}_t$.

Why must it be the case in equilibrium that $\bar{p}_t > \underline{p}_t$ when $\theta = 0$? If all equity came from tranched pools at a particular history, type L would consume nothing in the lowest state, making their willingness to pay for untranched equity out of any pool unbounded. The intermediary would then opt to tranche no pool, the contradiction we sought. This completes the proof $\square$

While the proof is made simpler by the fact that consumer H has simple preferences, the result holds in full generality. It would hold, in particular, in a setting where consumer H has preferences such that he too chooses to hold both types of securities. The result follows from a fundamental feature of the environment: the intermediary takes state prices as given hence has a linear objective.

In plain English, the intermediary buys all projects above a certain threshold. Second, if it creates any debt at all against a pool, then it maxes out the production of debt. Third, pools above a certain threshold are used to produce investment-grade debt, those below are not. A trivial corollary of these results is that without heterogeneity, no tranching would occur in equilibrium. Tranching cash-flows to sell them to a single agent type would create no value and bearing securitization costs would not make economic sense.

Qualitatively, proposition 6 has one fundamental consequence. An increase in the supply of investment grade debt must come from securitizing more pools – the extensive margin – since the intensive margin is always used up fully because securitization is costly. In addition to its independent interest, by sharply characterizing what the security space must look like in equilibrium,, proposition will greatly simplify the quantitative analysis.

Proposition 6 also implies that, despite the fact that a continuum of different projects are used to produce financial claims, the security space in each period only comprises three types of security. There is, first, a endogenous supply of risk-free promises. Second, notice that equity from a tranched project of type p has payoff

$$\mu(p)p(\Pi(Z,w)-\Pi(\underline{Z},w))dp.$$

It follows that all equity claims from tranched pools yield co-linear payoffs, hence are equivalent to one another. Third and finally there is a supply of equity in untranched projects with a payoff

$$\mu(p)p(\Pi(Z,w)+(1-\delta))$$

but these payoffs are also co-linear in p hence equivalent to one another. In other words, the only distinction of importance from equity claims is whether they come from tranched pools or not. The reason for this is that all equity payoffs from untranched pools are linear in p, and their price is given by a common state price vector, namely the marginal willingness to pay of the agent of type H. The same is true for equity from tranched projects. While this observation is interesting, note that it would no longer hold for untranched equity if the effective depreciation rate were independent of success. Furthermore as we will argue in the quantitative section, it does not change the nature of computations in our model.

7 A recursive approach

7.1 Recursive competitive equilibria

The key difficulty associated with computing the sort of equilibrium we define in this paper is the high-dimensional fixed point that characterizes it. Taking the history-contingent path of financial structures (as given, agents choose an optimal consumption plan. This optimal consumption plan, in turn, implies a history contingent sequence of willingness to pay (marginal rates of substitution) for both types of agent. The financial structure path must then maximize the intermediary's profits given agent's willingness to pay for securities. This is highly reminiscent of and technically similar to the fixed point standard politico-economic equilibria must satisfy and, as in that context, a recursive approach is the most natural way to look for this type of fixed point.[6]

Assuming still that aggregate shocks are i.i.d, the state of the economy at any given point in time is fully summarized by $(A^H, A^L) \in \mathcal{A}^H \times \mathcal{A}^L$, the wealth holdings of agents of each type at the end of any given period. Because $\delta > 0$ and returns to capital are decreasing in the aggregate, it is easy to see that we can confine our attention to a compact set $A^H \times A^L$ of possible aggregate states.

This section proposes a definition of a Markov Recursive Equilibria. An equilibrium consists of the following objects:

- $g^H \times g^L : \mathcal{A}^H \times \mathcal{A}^L \times \mathcal{Z} \mapsto \mathcal{A}^H \times \mathcal{A}^L$ is the law of motion for the aggregate state;
- $p^i : \mathcal{A}^H \times \mathcal{A}^L \times \mathcal{Z} \mapsto I\!R_+$ is agent i's willingness to pay for a marginal unit of consumption if shock Z
- $\underline{p} \times \bar{p} : \mathcal{A}^H \times \mathcal{A}^L \mapsto [0,1]^2$ are the two thresholds that define the financial structure given the current state;
- $r^E : \mathcal{A}^H \times \mathcal{A}^L \times \mathcal{Z} \times S \mapsto I\!R_+$ is the payoff to equity;
- $q^E : \mathcal{A}^H \times \mathcal{A}^L \times [\underline{p}, 1] \mapsto I\!R_+$ is the price of equity;
- $q^B : \mathcal{A}^H \times \mathcal{A}^L \mapsto I\!R_+$ is the price of risk-free debt;
- $MV : \mathcal{A}^H \times \mathcal{A}^L \times I\!R_+ \times [0,1]$ is the market value of all possible structures $(b, p) \in I\!R_+ \times [0,1]$
- $\kappa : \mathcal{A}^H \times \mathcal{A}^L \times [0,1] \mapsto I\!R_+$ gives the price of projects;
- $c^i : \mathcal{A}^H \times \mathcal{A}^L \times \mathcal{A}^i \mapsto I\!R_+$ is agent i's consumption choice;
- $b^i : \mathcal{A}^H \times \mathcal{A}^L \times \mathcal{A}^i \mapsto I\!R_+$ is agent i's demand for risk-free claims;

[6]See e.g. Krusell, Rios-Rull ...

- $e^i : \mathcal{A}^H \times \mathcal{A}^L \times \mathcal{A}^i \times [\underline{p}, 1] \mapsto [0,1]$ is agent i's equity portfolio choice;
- $w : \mathcal{A}^H \times \mathcal{A}^L \times \mathcal{Z} \mapsto I\!R_+$ is the wage rate ;
- $V^i : \mathcal{A}^H \times \mathcal{A}^L \times \mathcal{A}^i \mapsto I\!R$ are value functions for each agent type.

Begin by writing out the functional equation that define V^L given other objects. Given state $(a^H, a^L) \in A^H \times A^L$,

$$V^L(A^H, A^L, a^L) = \max_{\{\{e(p):p\geq\underline{p}\},b>0\}} U^L\left(a^L + \int_{\underline{p}}^1 \kappa(p)dp - \int_{\underline{p}}^1 e(p)q^E(p)dp - q^B b\right) + \beta \int_{\mathcal{Z}} V^L\left(g^H(A^H, A^L, Z), a^{L'}(Z)\right) d\nu(Z)$$

where for all $Z \in \mathcal{Z}$

$$a^{L'}(Z) \equiv b + \int_{\underline{p}}^1 e(p)r^E(A^H, A^L, Z, p)dp + w(A^H, A^L, Z).$$

To understand this expression, notice that agents are atomistic hence take the evolution of the aggregate state as independent of their decisions. The same expression holds for agent H except that, for them, $e(p) = 0$ for all $p \in S$.

Standard arguments imply that the functional equation above defines V^L uniquely in the space of bounded function. Furthermore, under the premise that all underlying objects in the above functional equation are continuous, so is V^L. In fact, under that same premise:

Proposition 11. *Assume that g^H, g^L, κ, $\underline{p} \times \bar{p}$ are continuous. Then V^i is concave and differentiable for $i \in \{H, L\}$ and*

$$V_3^i\left(A^H, A^L, a^L\right) = U^{i'}\left(c^i(A^H, A^L, a^L)\right).$$

Proof. Benveniste-Scheinkman, 1979. □

This envelope property is important because it means that the Allen-Gale approach to pricing all potential security using the marginal rate of substitutability across different periods can be motivated in our infinite horizon context exactly as it can in their two-period environment.

A fact that will greatly aid computations is that, in this environment, agents of type L hold all equity while agents of type H hold all safe debt. Indeed, it only makes economic sense to produce risk-free debt when agent H bids strictly more for it than agent L. It follows that for all $(A^H, A^L, a^L) \in \mathcal{A}^H \times \mathcal{A}^L \times \mathcal{A}^L$, $e^H(A^H, A^L, a^H, p) = 0 = 1 - e^L(a^H, a^L, a^L, p)$ for all p while $b^L(A^H, A^L, a^L) = 0$. Having observed this, we can boil down the search for a RCE to the following set of conditions

$$\begin{aligned}
c^L(A^H, A^L, A^L) &= A^L + \int_{\underline{p}}^1 \kappa(p)dp - \int_{\underline{p}}^1 q^E(A^H, A^L, p)dp && (7.1)\\
c^H(A^H, A^L, A^H) &= A^H - q^B(A^H, A^L)B(A^H, A^L) && (7.2)\\
g^L(A^H, A^L, Z) &= \int_{\underline{p}}^1 r^E(A^H, A^L, Z, p)dp + w(A^H, A^L, Z) && (7.3)\\
g^H(A^H, A^L, Z) &= B(A^H, A^L) && (7.4)\\
p^i(A^H, A^L, Z) &= \frac{\beta\nu(Z)U^{i'}(c^i(g^H, g^L, g^i))}{U^{i'}(c^i(A^H, A^L, A^i))} \text{ for } i \in \{H, L\} && (7.5)
\end{aligned}$$

$$\text{and the financial structure solves the intermediary's problem given } (p^H, p^L) \qquad (7.6)$$

The first two conditions express what consumption must be for each agent given the portfolio they must hold in equilibrium. The next two conditions are consistency conditions: the assumed aggregate law of motion must be correct. Condition (7.5) is the recursive version of the Allen-Gale condition.

The final condition is that the intermediary behave optimally. Recall that the intermediary takes as given w and agents' willingness to pay. Assume, to start the search for a solution to the intermediary's problem, that a mass K of capital is invested in projects. As in the aggregation section, this gives us a threshold $\underline{p}(K)$ above which project are activated and, in turn, $F(Z, K, N)$. The marginal product of labor $F_3(Z, K, 1)$ is the wage rate given state Z. That, in turn, gives us pool payouts $\Pi(Z, w(Z)) + (1-\delta)$ if state Z prevails. Then, we can find

p^E such that $MV(0, p^E) = 1$ and p^B such that $MV(p^B(\pi(\underline{Z}, w(\underline{Z})) + (1-\delta)), p^B) = 1 + c$. In words, p^E is the project-quality threshold above which all equity pools are profitable while p^B is the threshold above which tranched pools are profitable.

In any equilibrium, it must be that $\underline{p}(K) = \min\{p^E, p^B\}$. It easy to see that a unique value of K solves this problem and that it can be found quickly by dichotomy. This first step guarantees that the behavior of the intermediary is optimal and that labor markets clear given the current state. And, given the agents' willingness to pay, we also get r^E and b, the supply of the two types of claims.

The construction so far guarantees that labor markets clear and that the intermediary behaves optimally. We also have all the information we need to get consumption levels by (7.1-7.2). This then gives us the implied willingness to pay by (7.5) . We have an RCE, then,provided the values that come out of that condition are exactly the marginal willingness to pay the intermediary assumed in the first place.

7.2 Algorithm

We have just argued that finding an RCE boils down to solving a fixed point on the willingness to pay vector. In order to locate such a fixed point, we use a version of the algorithm Telmer (1993) employed, only greatly complicated by the fact that the set of securities is endogenous:

1. Start by assuming that agents live for 2-periods ($T = 2$). In the terminal period, agents simply compute their wealth. Finding a solution to conditions $(7.1 - 7.6)$ is a version of the problem studied by Allen-Gale (1988). In particular, that system has a solution. We compute the corresponding fixed point on a grid of the aggregate state space.

2. At the solution, denote by $c^{i,T-1}$ the optimal consumption plan given the aggregate state for agents of type $i \in \{H, L\}$. Apply the same procedure as in step 1 to compute vector of willingness to pay that solves $(7.1 - 7.6)$ as of date $T - 2$, so obtaining $c^{i,T-2}$

3. Let T grow until the optimal consumption policy function become approximately invariant. This gives us the approximate RCE we sought.

8 Quantitative analysis

8.1 Functional forms and calibration

The model is quite parsimonious so there are fairly few objects to parametrize. We will think of a period as a year and set $\delta = 10\%$ while α, the capital share, is set to the standard 0.7. The willingness of agents to trade consumption is $\sigma = 2$. For its part, we set $\beta = 0.95$. We set the endowment θ of H-type agents to 2.

Turning now to the distribution of project quality, recall that we do not require μ to be a probability distribution. Instead, $\mu(p)$ simply denotes the density of projects of type p. Since p is a probability of success however, μ's support is the interval $[0, 1]$. We will work with the following truncated exponential distribution:

$$\mu(p) = \Lambda\lambda e^{-\lambda p} \text{ for all } p \in [0, 1],$$

where $\Lambda > 0$ is a scale parameter while λ is the standard exponential decay parameter. Given this functional from, standard algebra shows that for all $\underline{p} \in [0, 1]$,

$$\mu([\underline{p}, 1]) = \Lambda \left(e^{-\lambda \underline{p}} - e^{-\lambda} \right),$$

while

$$E(p|p \geq \underline{p}) = \Lambda \left[e^{-\underline{p}\lambda} \left(\underline{p} + \frac{1}{\lambda} \right) - e^{-\lambda} \left(1 + \frac{1}{\lambda} \right) \right].$$

For now we choose $\Lambda = 10$ and $\lambda = 5$. Eventually we want to use moments of the size distribution of establishments as a guide or a securitization-related moment. For instance, we could match the average ratio of default losses in securitized pool. Also pending more careful calibration, we assume for now that the aggregate shock can only take two equiprobable values: $\underline{Z}$ and $\bar{Z}$, normalize $\bar{Z} \equiv 1$ and choose $\underline{Z} = 0.8$

There only remains to calibrate c. In the computations described below, we set $c = 2\%$. One natural way to pin down that transaction cost parameter will be to match the fraction of AAA securities to all securitized assets at some start date. But there are many alternatives.

Finally, we bound the wealth of each agent by 2.5 and place 20×20 equally spaced point on $[0, 2.5]^2$, the resulting state space.

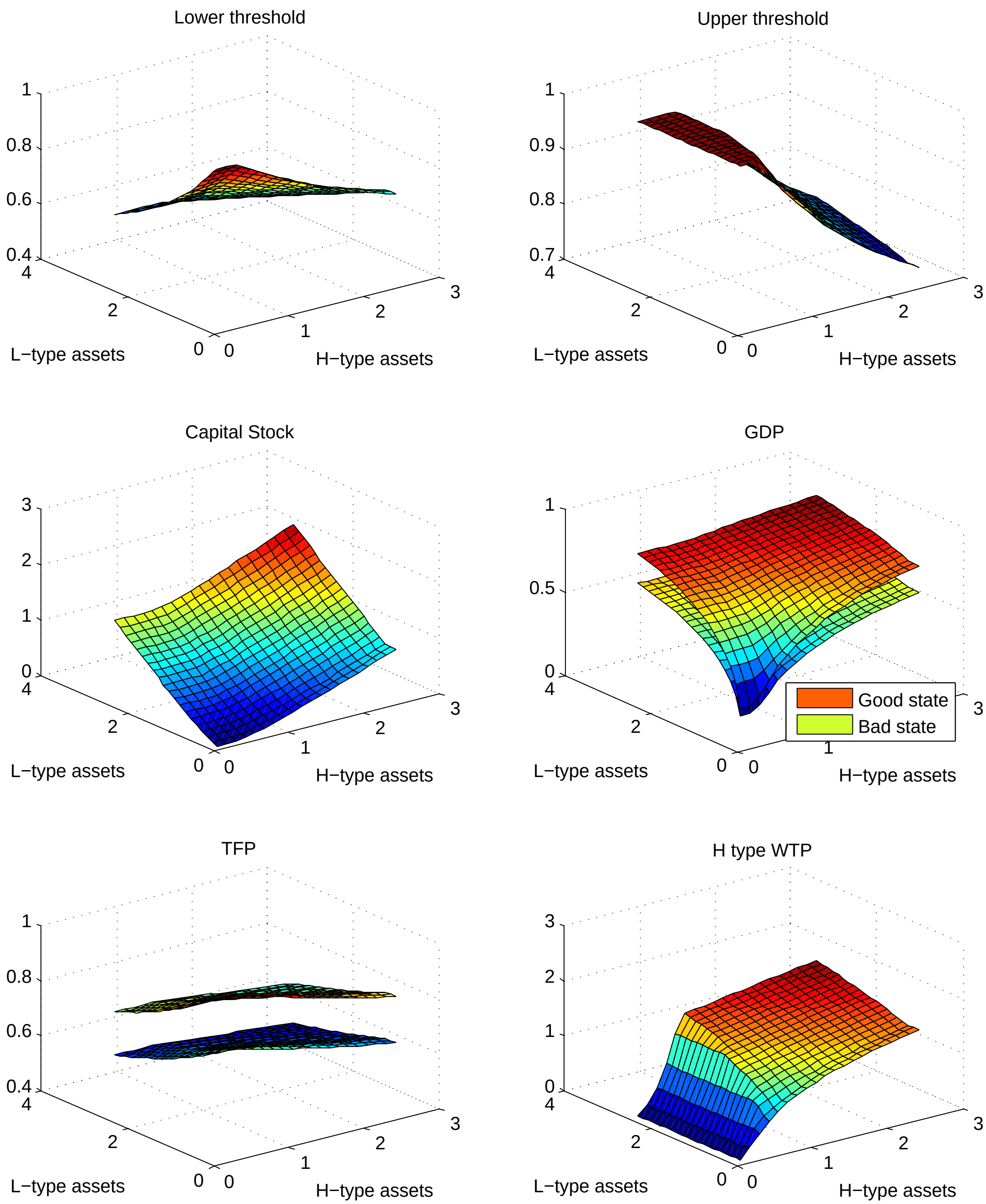
8.2 The Recursive Competitive Equilibrium

Figure 2 shows the main features of the stationary we found equilibrium given the parameters above. On those charts the floor is the state space. As one moves to the right H-type agents become richer. As one moves to the left, L-type agents get richer. The first panel shows the lower threshold – the threshold at which projects get activated. As either agent gets richer, there is more capital invested (as one also sees on the the capital stock panel), hence more projects get activated.

The second chart shows the threshold above which projects are used to produce risk-free debt. That threshold falls as the H-type gets richer and his demand for the risk-free asset increases, but the L-type's wealth matters little, which is quite intuitive. In other words, as the H-type becomes richer, his willingness to pay (WTP) for the risk-free asset (shown in the very last panel) goes up, the supply of the risk-free asset responds which, inevitably, is accomplished by using ever riskier assets as collateral.

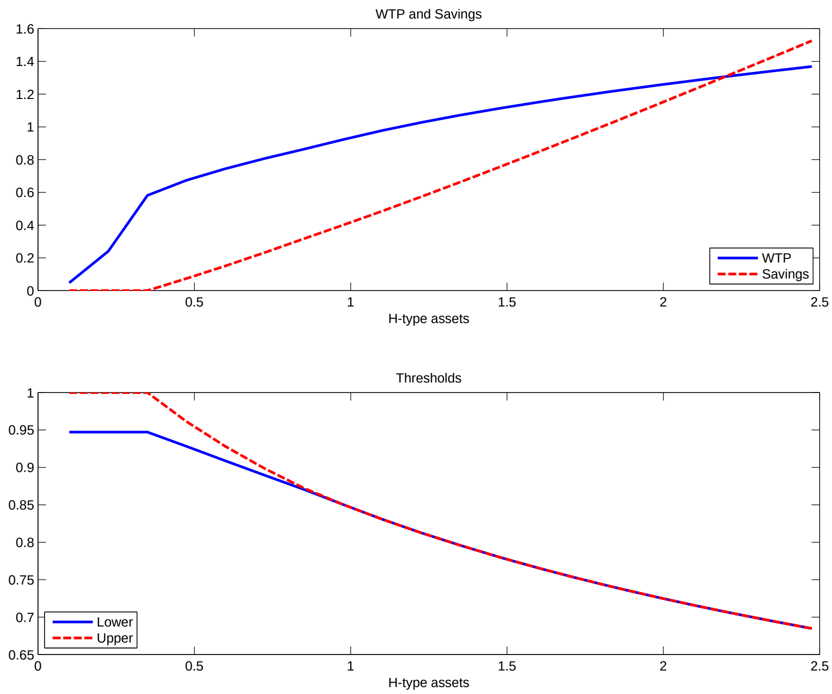
The WTP figure displays an obvious kink. This is natural prediction of our model as figure 3 shows holding type-L agent's assets in the middle of their half of the asset grid. When H-type agents are poor, their willingness to pay relative to that of L-type agent is to low to justify tranching pools. Hence they save nothing until they reach a certain asset threshold. As their wealth rises, so does their consumption at the start of the period hence so does their WTP. At some point their WTP becomes such that the risk-free asset gets produced as the

Figure 2: The stationary RCE



bottom panel of the figure shows, future consumption starts rising, hence further increases in assets begin to affect future consumption as well as current consumption, hence the kink.

Figure 3: The Willingness-To-Pay kink



The fourth panel of Figure 2 shows GDP depending on the realization of the aggregate state. As there is capital deepening, GDP goes up. But the fifth chart shows that TFP goes down because capital deepening leads to the activation of marginal projects.

8.3 The business cycle effects of securitization

This stationary equilibrium enables us measure the effects of endogenous securitization on the level of volatility of GDP. Indeed, it is intuitively clear that our economy has a stochastic steady state which is approached from any point in the state space. To keep the exercize simple

Table 1: Business cycle implications of securitization

	$c = 0.02$	$c = +\infty$	Δ
$E\left(\frac{qb}{K}\right)$	3.02%	0%	–
$E\left(\frac{cost}{GDP}\right)$	1.47%	0%	–
$E(I)$	0.0447	0.0435	+ 2.68%
$E(GDP)$	0.6156	0.6130	+0.42%
$Var(\log(GDP))$	1.34%	1.35%	-0.74%
$E(c^L)$	0.4966	0.4961	+0.10%

for now we assume that the H-type agents are permanently in the middle of their grid (we keep their wealth constant, that is.) This could be formally justified with the appropriately fitted sequence of endowment shocks but is purely assumed for the sake of this experiment. Under that assumption, the L-type agents converges to a stochastic steady state that varies about a long-term asset level depending on aggregate socks.

Table 1 compares the resulting stochastic steady state statistics when $c = 2\%$ and when $c = +\infty$. The second case amounts to banning securitization in our model since no pools ever get tranched. In the first economy, H-type holdings of securities account for 3% of the capital stock on average and securitization costs consume 1.5% of GDP. The participation of H-type agents boosts gross investment by 2.7%, long-term GDP by half a percent, and long-term consumption by 0.1%.Capital inflows naturally have a level effect on production, investment and consumption. Less intuitively perhaps, they also reduce the volatility of GDP because these inflows do not respond dynamically to TFP shocks. TFP shocks have a direct effect on domestic investment through their income effects. External flows are not affected hence have a steadying impact on activity. Naturally then, the flows from type-H agents are countercyclical. The share of H-holding in the capital stock has a correlation of -0.22 with GDP.

8.4 The saving glut

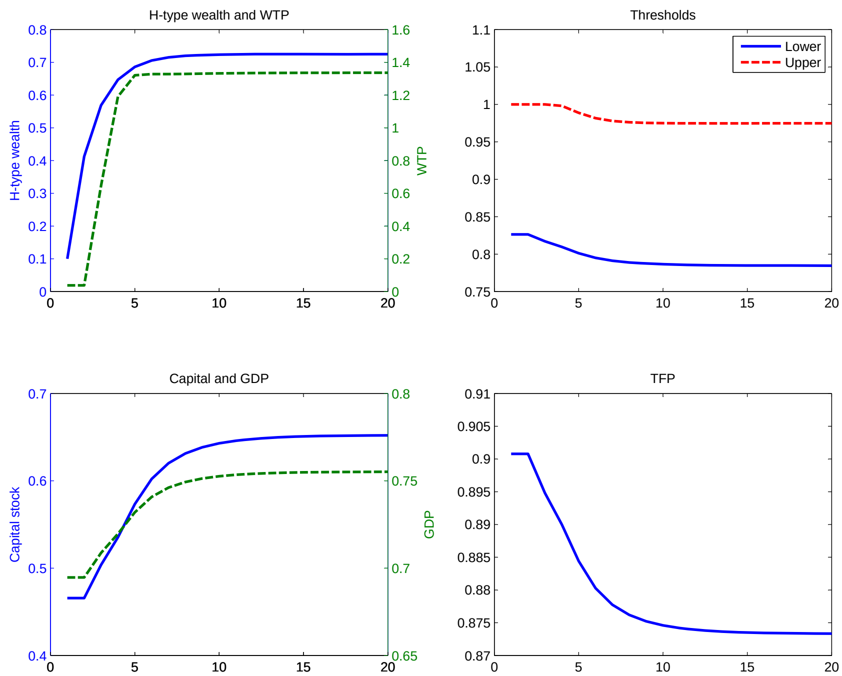
The computed equilibrium confirms all the qualitative effects of increased demand for the safe asset we have discussed in this paper. There only remains to confirm the quantitative importance of these effects. We will now study those by having the H-type agent transit from a low asset position to a higher steady state level. Assume then that (we tweak endowments so that) the H-type agent transits from a low asset position to a high asset position as shown in blue line in the top-right panel of figure 4. Assume further that the L-type agents begins at their long term steady level of assets when H-types are inactive. Assume finally that during the transition only good TFP shocks materialize. The equilibrium policy functions we solved for in the previous section generate the rest of the transition.

Along the transition, the WTP of type-H agents obviously rises so that, after a few periods, it becomes profitable to produce the risk-free asset. Capital deepens and GDP rises, as shown in the bottom-left panel of the figure. TFP for its part falls: an investment boom causes marginal projects to be activated. Unless more traditional booms caused by exogenous TFP shock, there is no offsetting effect and the inflow of capital lowers measured productivity, even as GDP rises.

The boom in this experiment occurs both because of the saving glut and because we are assuming a good sequence of TFP shocks hence the L-type agent accumulates assets. Figure 5 disentangles the two effects. The same phenomenon when $c = +\infty$ does not cause inflows. The red line rises only because of the good sequence of shocks. The chart shows that securitization accounts for roughly half the boom in our experiment.

In conclusion, in a model where securitization is possible, external demand for the risk-free asset affect both business cycle properties and the level of economic activity in the domestic economy. The effects at this points seem quantitatively meaningful but a precisely assessment will require a more careful calibration. [To be continued.]

Figure 4: The Saving Glut



9 Conclusion

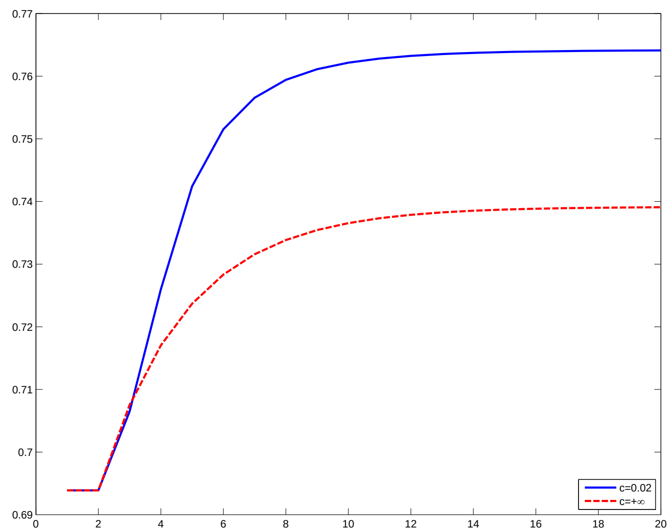
We have described a macroeconomic model with an endogenous security space where equilibria can be computed in manageable time.

There remains to simulate the effects of the recent increase of external demand for US investment-grade paper in a suitably calibrated version of our model. The analysis we have produced already makes it clear that in response to this increase in demand:

1. The yield on investment-grade securities must fall allowing for ever riskier projects to be profitably brought into the securitization fold;

2. Output must rise while aggregate productivity must fall in expected terms;

Figure 5: No securitization, no inflows



3. The level of credit support for newly securitized projects is higher than average, and the ratio of safe claims to junior claims among tranched pools must fall.

The first two predictions are qualitatively consistent with the patterns shown in figure 5 but the third is not. The fact that credit support levels did not rise during the securitization boom suggests that investors became less worried about aggregate shocks. Our model can (eventually) be used to study the effects of this apparent bout of complacency.

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