The Rise of Securitization: A Recursive Security Design Approach

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"*Given the strength of demand for safe U.S. assets, it would have been surprising had there not been a corresponding increase in their supply.*"

Bernanke et al. (2011)

Motivation

- The demand for safe US debt rose a lot starting in the mid to late 1990s...
- ...fueled by foreign demand ▸ Data
- That demand was met in significant part by a rise of asset-backed securitization
- What are the consequences of this phenomenon on:
  1. Output (level and volatility),
  2. Productivity,
  3. Asset prices (the risk-free rate and risk premia),
  4. The scope of securitization activities?
- To answer those questions, we lay out a macroeconomic model that is standard on the real side but where the security space responds endogenously to changes in fundamentals

Methodology: the fixed point problem

- Taking the contingent path of financial structures as given, agents choose an optimal consumption policy
- This consumption path, in turn, determines agents' willingness to pay for different securities
- Taking this willingness to pay as given, financial intermediaries issue menus of securities that maximize their profits
- The resulting financial structure must coincide with the guess agents made in the first place

We extend Allen and Gale's static "Optimal Security Design" framework to a dynamic setting.

Pooling + Tranching

- *Pooling*: Financial intermediaries acquire a pool of risky assets, i.e. projects that yield stochastic cash-flows
- *Tranching*: They use the resulting pool to create securities
-

$$\begin{aligned} \text{Profits} &= \text{Market Value of securities} \\ &- \text{Cost of Funding the Pool} \\ &- \text{Cost of Creating Securities} \end{aligned}$$

- The intermediary sells each type of security to the bidder with the highest willingness to pay (WTP)
- A pool is created if and only if there is a profitable way to fund it

Implications of increase in demand for the safe asset

An increase in WTP for the safe asset causes:

1. The MV of risk-free claims to rise (the risk-free yield to fall)
2. Ever riskier projects to be brought into the securitization fold
3. The average quantity of risk-free debt created from pools to fall
4. Output to go up (via capital deepening) but TFP to fall
5. Output becomes less volatile (!)

Counterfactual: On banning costly security creation

- What if securitization were banned or prohibitively costly?
- Greater demand for the safe asset can no longer help fund risky projects
- The capital deepening effect would be muted. How much?
- What are the welfare consequences?
- What are the consequences on the cyclicality of output?

Literature

1. Tranching to mitigate asymmetric information: Riddiough (1997), DeMarzo and Duffie (1999), De Marzo (2005), Winton (1995) . . .
2. Tranching to meet greater demand for safe assets: Bernanke et al. (2011), Gennaioli et al. (2011)
3. Empirical aspects of the recent rise of securitization: Riddiough and Zhu (2011), Stanton and Wallace (2010), Nadault and Sherlund (2009)
4. Methodology: Krusell, Quadrini, Rios-Rull (1997)
5. Fixed point problem: Telmer (1993), Guvenen (2010)

The model

- Time is discrete and infinite
- A mass one of two types of consumers: L, H
- Time separable preferences with period utility U^L, U^H and discount rate β
- Both types enter each period with some wealth, which they split between consumption and investments in available securities
- Type L consumers supply one unit of labor each period
- Type H receive an endowment $\theta \geq 0$ each period
- Next period's wealth is the sum of labor and investment income

Projects (= potential uses for invesment funds)

- Type L are endowed with a distribution μ of projects indexed by a probability $p \in [0, 1]$ of success ▸ graph
- Activating a project requires one unit of capital
- If successful, a project yields

$$Z_t n^\alpha + (1 - \delta)$$

  if labor n is employed
- Failed projects return nothing
- $Z_t \in \mathcal{Z}$ is common to all projects, success is idiosyncratic
- Define $\Pi(Z_t, w_t) \equiv \max_{n>0} Z_t n^\alpha - n w_t$

Aggregation

- Assume that in a particular period the aggregate shock is Z, aggregate capital use is K, and aggregate labor use is N
- In equilibrium, the marginal product of labor is equated across successful projects
- This implies that aggregate output must be:

$$
\begin{aligned}
F(Z, K, N) &= \int_{\underline{p}}^{1} \mu(p) p Z \left(\frac{N}{K E(p)} \right)^{\alpha} dp \\
&= Z [E(p)]^{1-\alpha} K^{1-\alpha} N^{\alpha}
\end{aligned}
$$

where $\underline{p}$ is given by:

$$\int_{\underline{p}}^{1} \mu(p) dp = K$$

and

$$E(p) = \frac{\int_{\underline{p}}^{1} p\mu(p) dp}{\int_{\underline{p}}^{1} \mu(p) dp}$$

The rise of securitization and TFP

- Greater demand for the safe asset causes K to rise hence $E(p)$ to fall as marginal projects become funded
- TFP (i.e. $Z[E(p)]^{1-\alpha}$) falls all else equal

Securities

- An intermediary can buy any given project from L-type agent for price $\kappa(p)$
- It funds those projects (invest one unit of capital in each) and sells claims to project pools' output
- A vector of J securities $r^1, r^2, r^3, \ldots r^J$ is feasible if $r^j(Z) \geq 0$ for all j and

$$\sum_{i=1}^{J} r^i(Z) = \mu(p)p\big(\Pi(Z, w) + (1 - \delta)\big)$$

  for all possible realizations Z of the aggregate shock.
- Issuing J securities against a pool carries cost $\mu(p)(J-1)c$

Demand for safe debt

- Assume type H only holds risk-free assets
- Then the intermediary creates risk-free securities,

$$r^B \equiv b \geq 0$$

and equity

$$r^E(Z, b; p, w) = \mu(p)p[\Pi(Z, w) + 1 - \delta] - b$$

- The intermediary solves:

$$\begin{aligned} \max_{b \geq 0} \quad & MV(b; p) - \mu(p)(1 + \kappa(p) + 1_{\{b>0\}}c) \\ \text{subject to:} \quad & b \leq \mu(p)p[\Pi(\underline{Z}, w) + 1 - \delta], \end{aligned}$$

where $\underline{Z}$ is the worst possible realization of the aggregate shock

Allen-Gale condition

$$MV_t(b,p) = b \max_{\{i\in\{H,L\}\}} \int_{\mathcal{Z}} p_t^i(Z)d\nu(Z) + \int_{\mathcal{Z}} p_t^L(Z)\Big[\mu(p)p\left(\Pi(Z,w)+1-\delta\right)-b\Big]d\nu(Z),$$

where

$$p_t^i(Z) = \frac{\beta\nu(Z)U'(c_{t+1}^i(Z))}{U'(c_t^i)}$$

▸ Mixing types can't create value

Financial Intermediary's policy

- Write $k(p) = 1$ if projects of type p get funded ...
- ... and $b^S(p)$ for the quantity of risk-free debt created from pools of type p

Solution to the intermediary's problem

Proposition

The solution to the intermediary is fully described by two thresholds $0 \leq \underline{p}_t \leq \bar{p}_t \leq 1$ *such that:*

1. $k_t(p) = 1$ *if and only if* $p \geq \underline{p}_t$*;*
2. $b_t^S(p) = 0$ *if* $p < \bar{p}_t$*;*
3. $b_t^S(p) = \mu(p)p(\Pi(\underline{Z}, w) + 1 - \delta)$ *if* $p > \bar{p}_t$*;*

Financial structure

The financial structure at any point is fully summarized by:

$$S_t = \{\underline{p}_t, \bar{p}_t, q_t^b, \{q_t^e(p) : p \in [\underline{p}_t, 1]\}\}$$

where

- $q_t^b \equiv \max_{\{i \in \{H,L\}\}} \int_{\mathcal{Z}} p_t^i(Z) d\nu(Z)$ is the price of a risk-free claim to one unit of the good at date t
- $q_t^e(p) \equiv \int_{\mathcal{Z}} p_t^L(Z) \left[\mu(p) p \left(\Pi(Z, w) + 1 - \delta \right) - b \right] d\nu(Z)$ is the price of equity in pools of type p
- Write $\mathcal{S}$ for the space of possible structures at any given point in time

Contingent path of financial structures

- $\mathcal{H}_t$: possible histories of aggregate shocks up to date t
- Agents assume a mapping $\mathcal{H}_t :\mapsto \mathcal{S}$

Consumer L's problem

Type L consumers choose history-contingent policies $\{c_t, a_t^L, b_{t+1}, (e^D(p) : p \geq \bar{p}_t\}_{t=0}^{+\infty}$ to solve:

$$\max E \sum_{t=0}^{+\infty} \beta^t U^L(c_t)$$

subject to, for every date t and possible history $h_t \in \mathcal{H}_t$:

$$q_t^b b_t + \int_{p \geq \bar{p}_t} q_t^l(p) e_t^D(p) dp + c_t = a_t^L + \int_{p \geq \underline{p}_t} \mu(p) \kappa(p) dp,$$

$$a_{t+1}^L = \int_{p \geq \bar{p}_t} e_t^D(p) r_t^E(p; h_t, Z) dp + b_t + w_t(h_t, Z),$$

where: $(\underline{p}_t, \bar{p}_t, q_t^b, q_{t+1}^e) = S_t(h_t)$

Equilibrium (in words, first)

1. Consumers take as given the contingent path of financial structures (the set of securities that will be traded at very possible history of aggregate shocks)
2. They choose a consumption plan accordingly
3. This optimal consumption plan implies their willingness to pay for various securities at all nodes
4. The intermediary chooses the financial structure that maximizes its profits at every node given this willingness to pay
5. The solution to the intermediary's problem must coincide with the path of financial structures consumers assumed

A recursive formulation

Aggregate state: $(A^H, A^L) \in \mathcal{A}^H \times \mathcal{A}^L$ is the wealth distribution at the start of a period . A RCE is:

- $g^H \times g^L : \mathcal{A}^H \times \mathcal{A}^L \times \mathcal{Z} \mapsto \mathcal{A}^H \times \mathcal{A}^L$ is the law of motion for the aggregate state ;
- $p^i : \mathcal{A}^H \times \mathcal{A}^L \times \mathcal{Z} \mapsto R_+$ is agent i's willingness to pay for a marginal unit of consumption if shock Z;
- $\underline{p} \times \bar{p} : \mathcal{A}^H \times \mathcal{A}^L \mapsto [0, 1]^2$ are the two thresholds that define the financial structure given the current state;
- $r^E : \mathcal{A}^H \times \mathcal{A}^L \times \mathcal{Z} \times S \mapsto R_+$ is the payoff to equity;
- $q^E : \mathcal{A}^H \times \mathcal{A}^L \times [\underline{p}, 1] \mapsto R_+$ is the price of equity;
- $q^B : \mathcal{A}^H \times \mathcal{A}^L \mapsto R_+$ is the price of risk-free debt;
- $MV : \mathcal{A}^H \times \mathcal{A}^L \times R_+ \times [0, 1]$ is the market value of all possible structures $(b, p) \in R_+ \times [0, 1]$

- $\kappa : \mathcal{A}^H \times \mathcal{A}^L \times [0,1] \mapsto R_+$ gives the price of projects;
- $c^i : \mathcal{A}^H \times \mathcal{A}^L \times \mathcal{A}^i \mapsto R_+$ is agent i's consumption choice;
- $b^i : \mathcal{A}^H \times \mathcal{A}^L \times \mathcal{A}^i \mapsto R_+$ is agent i's demand for risk-free claims;
- $e^i : \mathcal{A}^H \times \mathcal{A}^L \times \mathcal{A}^i \times [\underline{p}, 1] \mapsto [0,1]$ is agent i's equity portfolio choice;
- $w : \mathcal{A}^H \times \mathcal{A}^L \times \mathcal{Z} \mapsto R_+$ is the wage rate
- $V^i : \mathcal{A}^H \times \mathcal{A}^L \times \mathcal{A}^i \mapsto R$ are value functions

Value functions

$$V^L(A^H, A^L, a^L) =$$

$$\max_{\{\{e(p):p\geq\underline{p}\},b>0\}} U^L\left(a^L + \int_{\underline{p}}^{1} \kappa(p)dp - \int_{\underline{p}}^{1} e(p)q^E(p)dp - q^B b\right)$$
$$+\beta \int_{\mathcal{Z}} V^L\left(g^H(A^H, A^L, Z), a^{L'}(Z)\right) d\nu(Z)$$

where for all $Z \in \mathcal{Z}$

$$a^{L'}(Z) \equiv b + \int_{\underline{p}}^{1} e(p)r^E(A^H, A^L, Z, p)dp + w(A^H, A^L, Z).$$

The Allen-Gale condition

Proposition

Assume that g^H and g^L are continuous. Then V^i is concave and differentiable for $i \in \{H, L\}$ and

$$V_3^i\left(A^H, A^L, a^L\right) = U^{i'}\left(c^i(A^H, A^L, a^L)\right).$$

At equilibrium:

$$
\begin{aligned}
c^L(A^H, A^L, A^L) &= A^L + \int_{\underline{p}}^{1} \kappa(p)dp - \int_{\underline{p}}^{1} q^E(A^H, A^L, p)dp \\
c^H(A^H, A^L, A^H) &= A^H - q^B(A^H, A^L)B(A^H, A^L) \\
g^L(A^H, A^L, Z) &= \int_{\underline{p}}^{1} r^E(A^H, A^L, Z, p)dp + w(A^H, A^L, Z) \\
g^H(A^H, A^L, Z) &= B(A^H, A^L) \\
p^i(A^H, A^L, Z) &= \frac{\beta\nu(Z)U^{i\prime}(c^i(g^H, g^L, g^i))}{U^{i\prime}(c^i(A^H, A^L, A^i))}
\end{aligned}
$$

and, the financial structure solves the intermediary's problem given (p^H, p^L).

An algorithm

Discretize $\mathcal{A}^H \times \mathcal{A}^L$.

1. Guess (p^H, p^L)
2. This implies (c^H, c^L) hence a new (p^H, p^L)
3. Update
4. Iterate until convergence

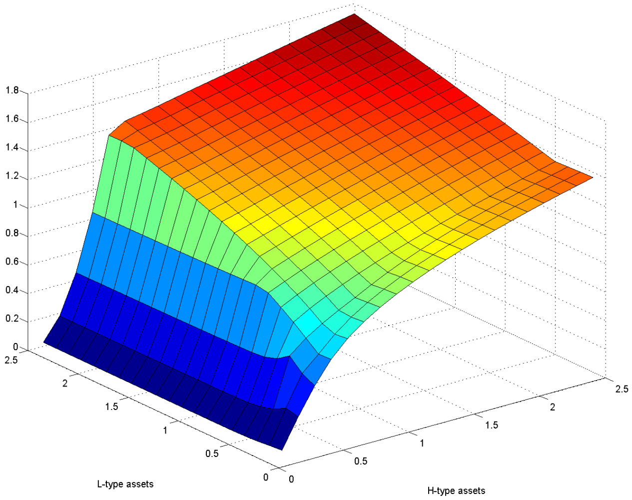
In practice, use Telmer's trick:

1. Solve the problem for finite horizon case, starting with $T = 2$
2. Let T grow large
3. Check that (p^H, p^L) is (essentially) invariant

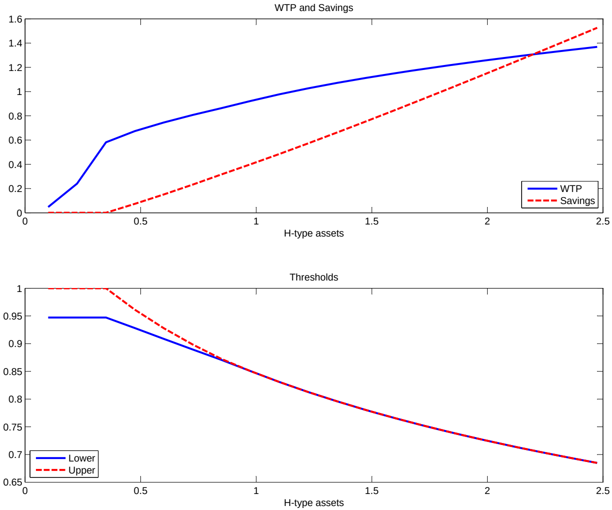
Parameters

- $\mu(p) = 50e^{-5p}$ for all $p \in [0, 1]$
- $\alpha = 0.7$
- $\delta = 0.1$
- $\sigma = 2$
- $\beta = 0.95$
- $\mathcal{Z} = \{0.8, 1\}$
- $c = 0.02$

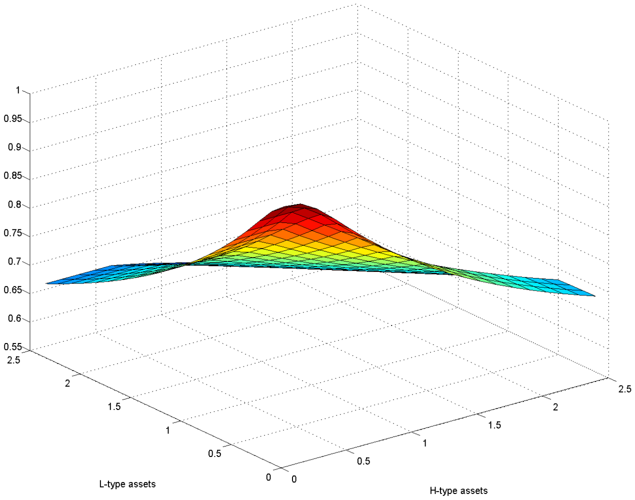
WTP for safe assets rises as H-agent becomes richer



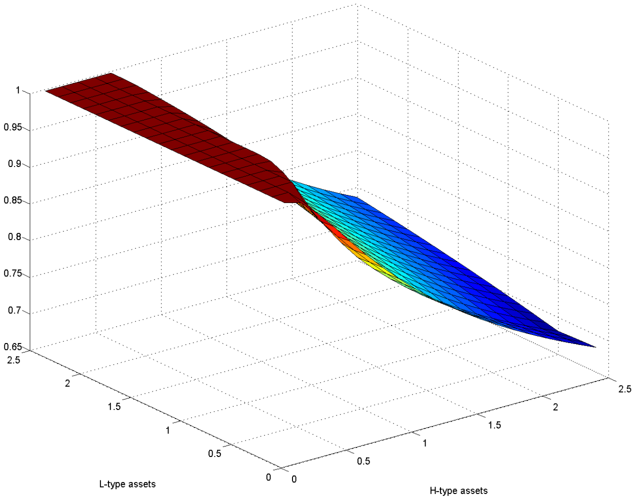
The kink occurs when H-type agents become active



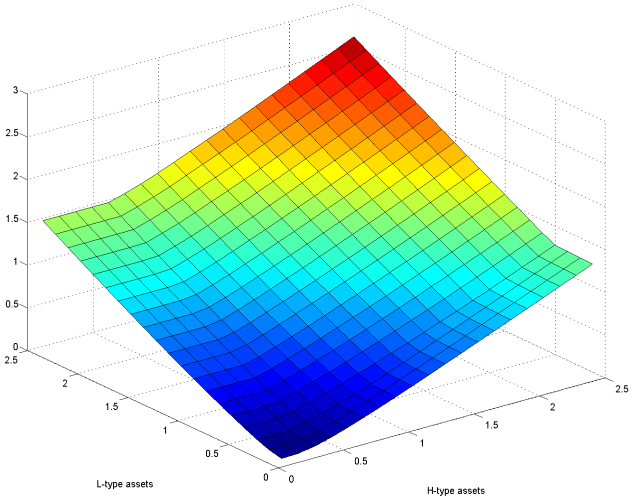
Lower threshold falls as agents get richer



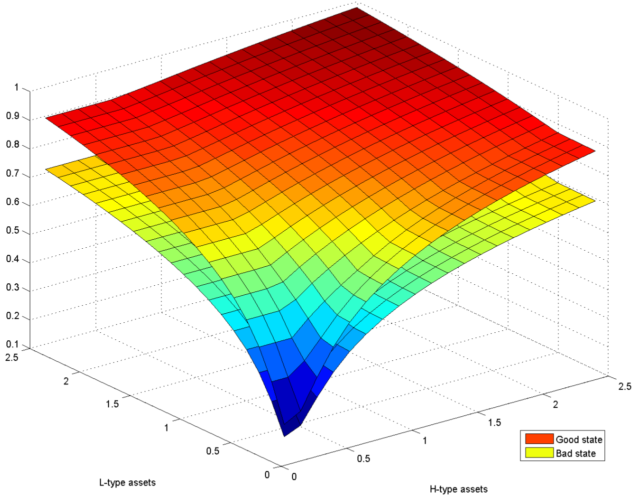
Upper threshold falls with demand for the safe asset



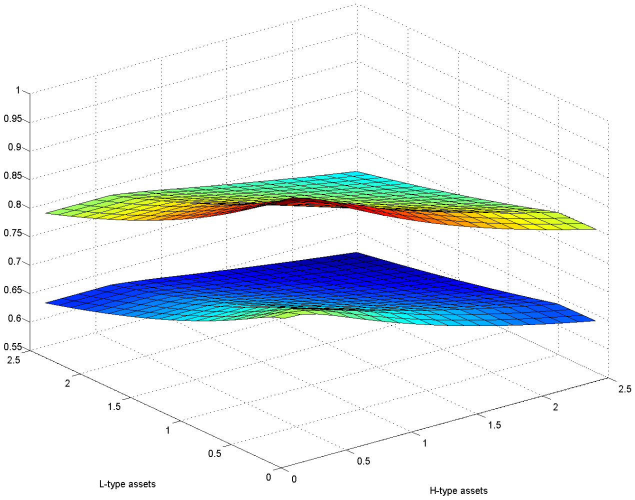
Capital stock rises as either agent gets richer



GDP goes up . . .


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
2.5
2
1.5
1
0.5
0
L-type assets
0
0.5
1
1.5
2
2.5
H-type assets
Good state
Bad state

. . . but TFP falls as marginal projects become activated



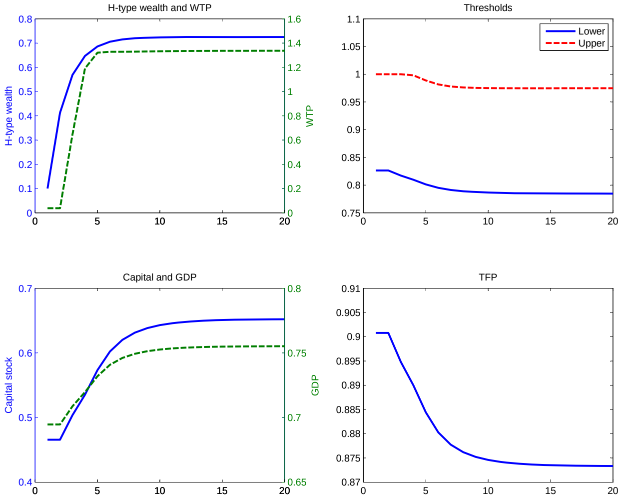
Stochastic steady state

- External demand for the safe asset raises investment, GDP and lowers volatility:

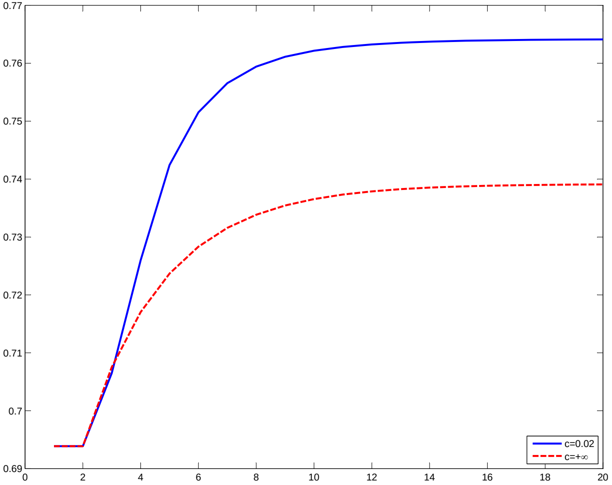
	$c = 0.02$	$c = +\infty$	Δ
$E\left(\frac{qb}{K}\right)$	3.02%	0%	–
$E\left(\frac{cost}{GDP}\right)$	1.47%	0%	–
$E(I)$	0.0447	0.0435	+ 2.68%
$E(GDP)$	0.6156	0.6130	+0.42%
$Var(\log(GDP))$	1.34%	1.35%	-0.74%
$E(c^L)$	0.4966	0.4961	+0.10%

- Leverage is countercyclical: $\rho\left(\frac{qb}{K}, GDP\right) = -0.2195$
- As is the securitization cost: $\rho\left(\frac{cost}{GDP}, GDP\right) = -0.5791$

The saving-glut



On banning securitization: GDP

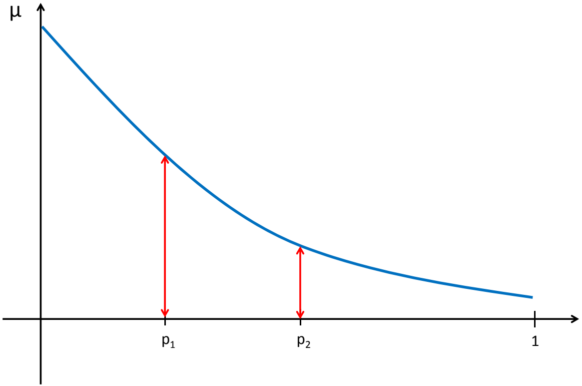


Summary

We (are in the process of) build(ing):

1. A down-the-fairway macro model with an endogenous financial structure
2. A method to compute equilibria
3. A model where Bernanke's saving glut story can be told ...
4. ... and tested.

Potential investment projects



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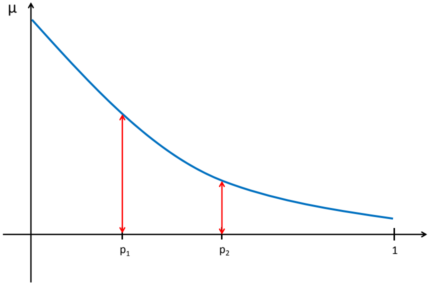
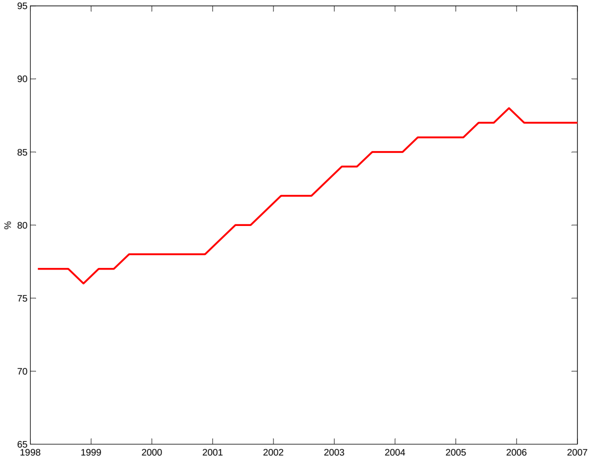
Mixing project types can't add to profits



- Mixing projects has no diversification advantage
- $\text{Profits}(p_1 + p_2) \leq \text{Profits}(p_1)\text{+}\text{Profits}(p_2)$
- Mixing cannot create value. It can destroy value.

Go back

Share of asset-backed securities rated AAA



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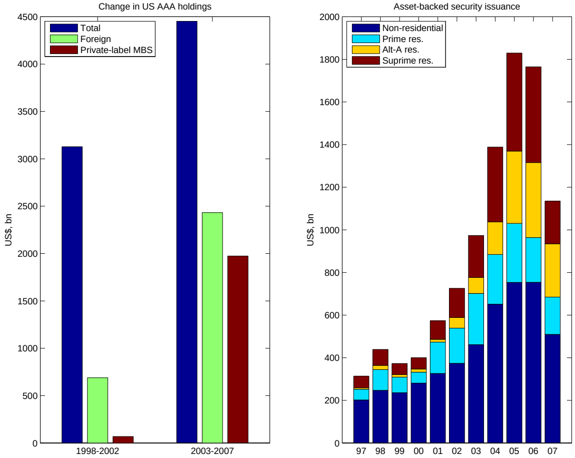
The rise of securitization



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