Make-Whole Clauses as Skin in the Game*

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Abstract

Make-whole provisions first appeared in the United States in the mid-1990s and are now a feature of the vast majority of callable bonds publicly issued by non-financial firms. This rise to prevalence is puzzling because, by design, make-whole clauses give issuers an acceleration option that is almost always out of the money. We provide a rationale for these clauses by showing that requiring issuers to pay above market value to retire existing debt contracts is essential when conditioning those contracts on the arrival and quality of outside options is costly. Those clauses help support the optimal level of effort on the part of issuers and enable issuers with weaker outside options to credibly convey their types to lenders, which translates into better borrowing terms and greater access to lending markets. This view also rationalizes all the salient design features of make-whole clauses.

Keywords: Make-whole Clauses; Optimal Capital Structure; Financial Innovation
JEL codes: G32; D86

Preliminary and incomplete, comments welcome

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1 Introduction

Until the mid 1990s in the United States and 2001 in Western Europe, fixed-rate, callable corporate bonds almost always featured a call price stipulated once and for all at origination. Today, in sharp contrast and as displayed in figure 1, the vast majority of these bonds feature make-whole (or yield-maintenance) clauses.

The popularity of make-whole clauses is not confined to bond markets. For instance, figure 2 shows that among commercial mortgages that are securitized, loans known as CMBS loans, the fraction of contracts that contain yield-maintenance clauses is around 35% on average between 1995 and 2021 according to TreppCRE data. Defeasance clauses – a slight variation of make-whole clauses under which a borrower who prepays a mortgage is required to replace it with a portfolio of treasuries that replicates the initial cash-flows, making lenders at least as well off as they would if the mortgage was not prepaid – are featured in another 25% of securitized real estate loans. All told, make-whole clauses are featured in more than 70% of CMBS loans.[1]

Under the traditional make-whole clause used in bond markets, the call price is calculated as the present value of remaining payments discounted at the yield-to-maturity of a government bond of comparable remaining maturity plus a small premium. Because the discount rate stipulated in that calculation is typically well below the rate the market would require from the underlying instrument, the resulting call price is usually well above market value.[2]

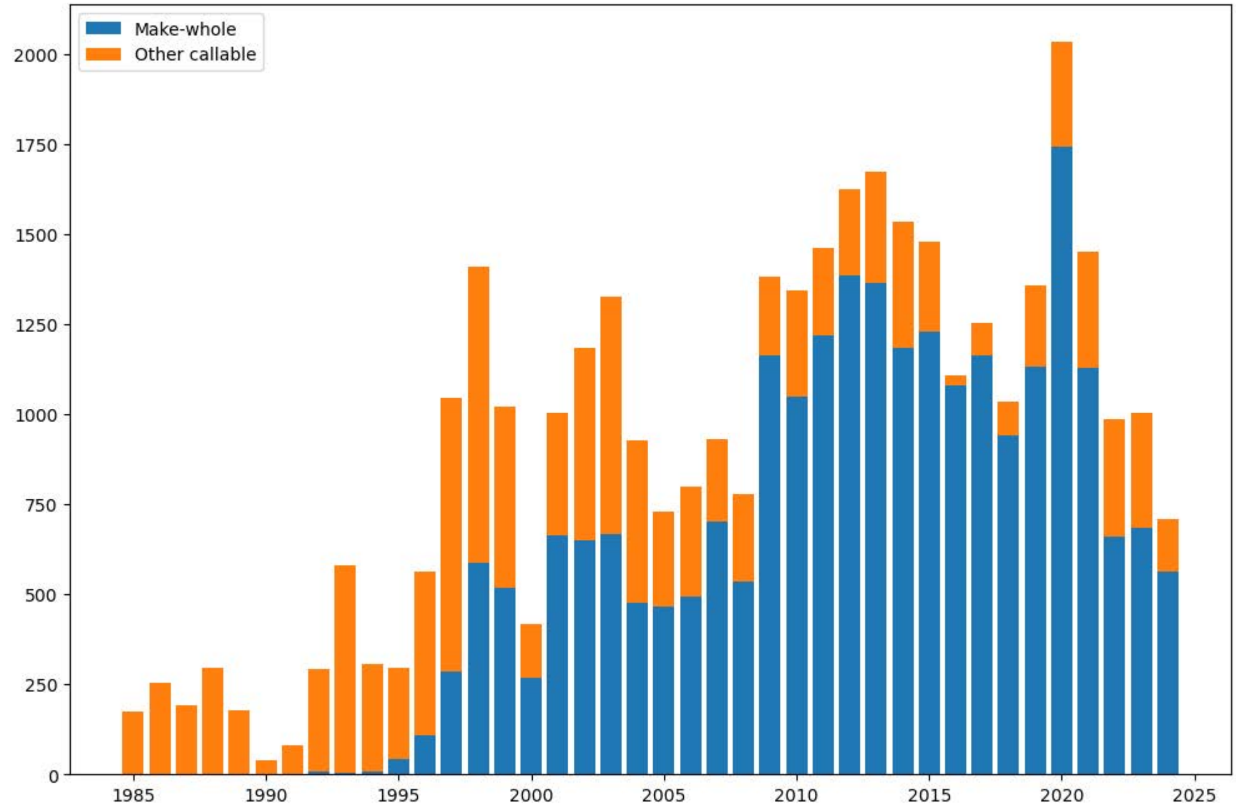
Not surprisingly then and as documented for instance by Mann and Powers [18], issuers who want to retire a bond early may attempt a standard tender offer before invoking a make-whole clause.[3] However, the literature on bond tender-offer premia – see e.g. Mann and Powers [17] – has shown that tender prices are over 5% higher than pre-tender prices on average and that the

[1] Average numbers are similar across real-estate subsectors, Those statistics are available upon request. It is noteworthy that defeasance clauses are not used in bond markets. In addition to making security-holders whole in a securitization context, prepayment by defeasance has the advantage of leaving contractual cash-flows unchanged; in fact, they become more safely guaranteed in nominal terms than they were when collateralized by the initial mortgage since they are generated by treasuries.

[2] Likewise, under a defeasance clause, remaining cash-flows are replicated by the prepaying borrower using a portfolio of treasuries. Since those treasuries usually trade at a lower yield-to-maturity than the incumbent mortgage, the value of the replicating portfolio typically exceeds the market value of the mortgage at call time. So, just like yield-maintenance clauses, defeasance clauses are almost always out-of-the-money.

[3] Calls at make-whole prices are also dominated by open-market purchases but open-market purchases of significant fractions of the underlying volume also have a large price impact and, by SEC regulation 14E, cannot substitute for a formal tender offer if the intent is to retire most of an issue.

Figure 1: **Callable corporate bonds issued by non-financial firms**



Sources and notes: Mergent FISD. These data include all fixed-rate, US-dollar denominated callable bonds issued each year by non-financial firms (two-digit SIC between 10 and 39 and between 50 and 59), and excludes issues with conversion features.

premium gets larger as a higher fraction of the issue is retired. Fees and time costs – including minimum offering period constraints under SEC regulation 14E – render tender premia even more expensive. Furthermore, neither tender offers nor open-market operations are usually successful at retiring all securities issued. So make-whole clauses are in fact invoked in spite of their high inherent price as Ma et al. [15] document, albeit somewhat less often than traditional fixed-price calls tend to be.

How can a deeply out-of-the-money option play an essential role in a debt contract? Existing work on this puzzle (see e.g. Elsaify and Roussanov [13] and Alderson et al. [3]) cast doubt that the traditional rationales emphasized by the literature on callable bonds can be helpful in answering this question. The literature on callable bonds – classical references are Barnea et al. [7] and

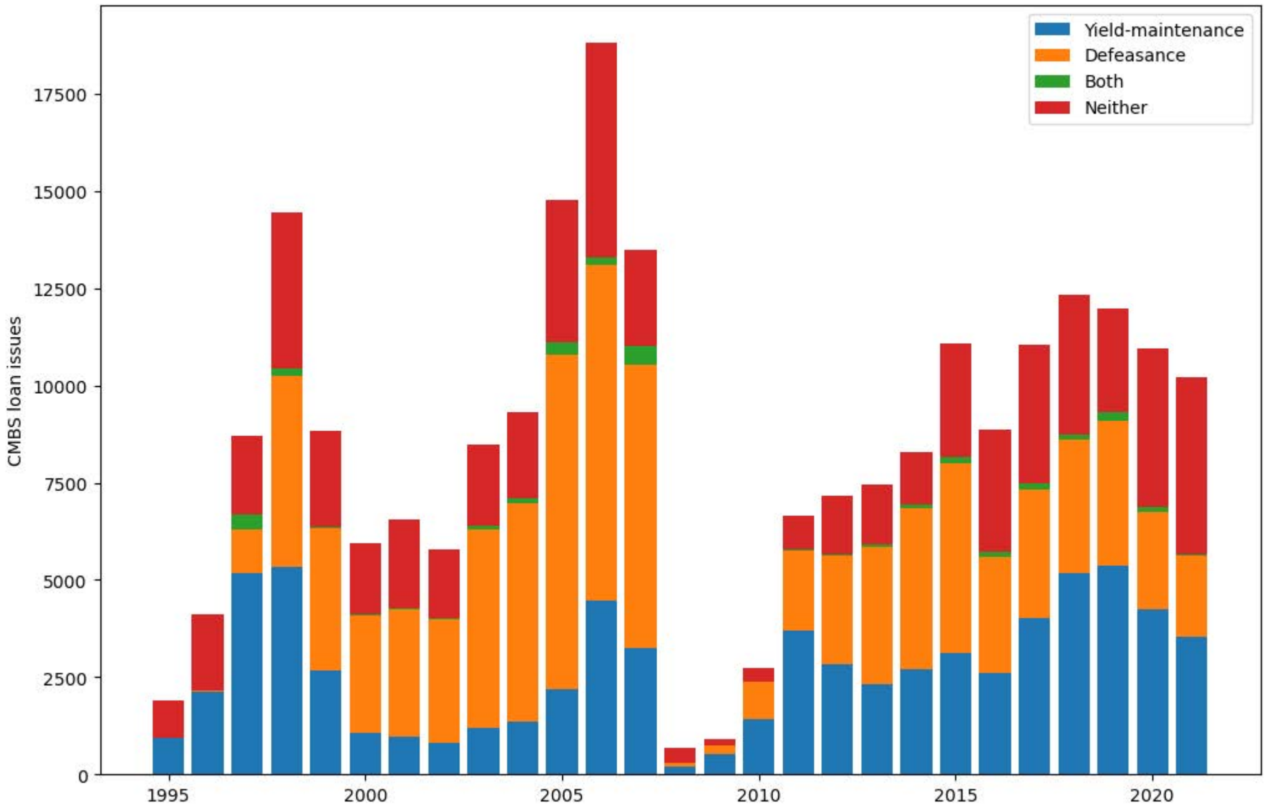
Robbins and Schatzberg [23] – has established that fixed-price calls can mitigate agency costs, for instance by mitigating debt-overhang problems à la Myers [20]. In fact, callable features have been used to test for the presence of agency frictions by an extensive empirical literature.[4] Because they tend to be deeply out-of-the money, make-whole clauses are viewed as unlikely to be as effective at mitigating agency costs as fixed-price calls. As Alderson et al. [3] put it, "*In contrast to fixed price callable bonds however, make whole bonds are largely incapable of mitigating bondholder-shareholder conflicts.*" Elsaify and Roussanov [13] expand on this view: "*Even if the firm's credit profile were to improve, the value of the [make-whole] option would not increase by the same value of the bond, and, in fact, would hardly increase at all. Since these options are virtually never in the money, their value would remain close to zero over the life of the bond and even revelations of positive information are not likely to change that. As such, equity-holders will receive very little compensation for their private information.*"

In this paper, we show that make-whole provisions are in fact a natural way to mitigate agency costs between equity-holders and debt-holders. First, even absent informational frictions, make-whole options have the advantage of eliminating potential refinancing gains for borrowers which come at the expense of lenders. Refinancing gains distort the exercise of outside options hence destroy value because they lead issuers to terminate contracts not because the quality of outside options justifies it but because, absent a make-whole provision, they benefit from low interest rates at the expense of lenders. While this is our most straightforward result, this is also, to our knowledge, the first answer to an age-old question in bond financing that does not depend on ad-hoc assumptions on risk tolerance or the ability to diversify risk: given that refinancing is a zero-sum game, who should bear interest risk? When outside options are present which the lender is unable to contract upon fully, our model says that insuring the lender fully against refinancing risk – which make-whole options accomplish – is optimal.

Once we introduce informational frictions in a model in which outside options arrive randomly, we also get an answer for the bigger puzzle associated with make-whole clauses. When equity-holders have superior information about the quality of the project in which they invest, or when effort is private or un-contractible, it becomes generally optimal for prepayment clauses to impose a call price

[4] See e.g. Thatcher [25], Kish and Livingston [12].

Figure 2: **Yield-maintenance clauses on securitized commercial mortgages**



Sources and notes: TreppCRE.

above market value. This premium constitutes skin-in-the game for project owners which they can use either to credibly convey information about project types or to commit to providing the optimal level of effort.

By resurrecting agency costs as a rationale for make-whole clauses, our theory differs sharply from the few existing explanations that have been proposed for the growing prevalence of make-whole clauses in bond markets. Elsaify and Roussanov [13] and Ma et al. [15] argue that firms value make-whole clauses because they enable them to mitigate and manage rollover risk. Mann and Powers (2003) [16] propose a simpler explanation: "*The make-whole call provision is structured to enable a firm to retire debt should circumstances arise, without relying exclusively on a tender offer.*" Neither view provides an obvious explanation for the fact that make-whole clauses are designed to feature a call price that transfers all refinancing risk to the issuer and is almost always well above market-value. Our view does and is founded on a simple hence robust intuition: setting a deliberately high price for exit gives equity-holders skin-in-the-game, which can improve total surplus when there is a risk that they may be distracted by the arrival of outside options.

Finally, as we review in the next section, make-whole clauses are explicitly designed so that time to maturity and improvement in credit quality both improve the moneyness of the call option. These features, we show, are exactly what a model in which make-whole clauses serve as skin-in-the-game predict. In other words, our contract theoretic view does not just correctly predict the optimal design and moneyness of call features, it also rationalizes the main empirical drivers of the size of the make-whole premium.

2 The facts

As Afik et al. [6] discuss, "[t]he first bond issue to include a make-whole call provision is that of the Canada-based Domtar Inc. on April 15, 1987 when it issued CAN$100 million of 24-year debentures." The first make-whole clause appeared eight years later in the United States in a public issue by Quaker State while the first issue with a make-whole provision in Europe (by AT&T) took place another six years later. Today, almost all investment-grade public bond issued in Canada feature make-whole clauses, and so do the vast majority of US-issued bonds.[5] The theory we present in this paper is consistent with this ubiquity. But it is also consistent with the main facts the existing literature has documented about these clauses, namely:

1. Make-whole options are designed to be almost always out of the money;

2. The moneyness of make-whole clauses improves as time to maturity falls;

3. It also improves as the borrower's credit quality improves;

4. Whereas the bonds issued by investment-grade issuers usually only feature make-whole clauses, those issued by high-yield issuers typically feature both a make-whole clause and a fixed-price call. The make-whole option is available at origination whereas the fixed-price option

[5] See e.g. Mann and Powers [18], Elsaify and Roussanov [13], or Ma et al. (2003).

usually kicks in around halfway to maturity. This combination accelerates the improvement of moneyness over time for initially low-credit quality issuers.[6]

We emphasize these facts both because they are robust across all available studies and because these are features which make-whole clauses are explicitly designed to deliver. A contract-theoretic view like ours, therefore, should be consistent with these patterns. The remainder of this section formalizes and illustrates these empirical regularities.

2.1 Make-whole moneyness

A make-whole option specifies a call price equal to the greater of par and the present value of remaining payments at a make-whole rate equal to the yield on a treasury of similar maturity as the underlying bond plus a make-whole premium.

Figure 3 provides an example of the language used in bond prospectuses with a recent General Motors issue. Figure 8 in the appendix provides the details of how the comparable treasury rate is selected at the redemption date. In a nutshell, the constant maturity treasury rate is used if available at a maturity close to the underlying bond's remaining life. If no closely matching outstanding treasury is available, interpolation is used.

Returning to the general case, denoting by m the sequence of remaining bond payments, consider the case in which the discounted value of remaining payments at the make-whole discount rate is higher than par. The common way to measure the moneyness of the associated call option is:

$$PV(m, ytm) - PV(m, r_{mw})$$

where ytm is the current yield-to-maturity on the underlying bond while r_{mw} is the yield on a comparable treasury plus the make-whole premium. This notion of moneyness can be justified by observing that upon exercising the make-whole option the corporation is retiring a liability of market value $PV(m, ytm)$ at a cost of $PV(m, r_{mw})$.[7]

[6]See for instance by Ma et al. [15] for a systematic documentation of this pattern.

[7]Ma et al. [15] measure moneyness as

$$PV(m, ytm^{NC}) - PV(m, r_{mw})$$

where ytm^{NC} is the yield-to-maturity the bond would earn if it had no call features. They do so to follow "the

Figure 3: **Make-whole clause example**

Optional Redemption

Prior to the applicable Par Call Date, we may redeem the 2030 Notes and the 2035 Notes at our option, in whole or in part, at any time and from time to time, at a redemption price (expressed as a percentage of principal amount and rounded to three decimal places) equal to the greater of:

(i) 100% of the principal amount of the Notes to be redeemed; and

(ii) (a) the sum of the **present values of the remaining scheduled payments** of principal and interest on the Notes being redeemed, **discounted** to the date of redemption (assuming the applicable series of Notes matured on the applicable Par Call Date) on a semi-annual basis **at the Treasury Rate** (as defined below) **plus 15 basis points**, in the case of the 2030 Notes, **or 25 basis points**, in the case of the 2035 Notes, *less* (b) interest accrued to the date of redemption,

plus, in either case, accrued and unpaid interest thereon to the redemption date.

On or after the applicable Par Call Date, we may redeem the 2030 Notes and the 2035 Notes, in whole or in part, at any time and from time to time, at a redemption price equal to 100% of the principal amount of the Notes being redeemed, plus accrued and unpaid interest thereon to, but excluding, the applicable redemption date.

Sources and notes: This redemption language is drawn from SEC filing 333-268704 by General Motors which covers two fixed rates notes and a floating rate note.

Since, by definition, ytm is the sum of the same-maturity treasury yield plus the current G-spread on the bond, it follows that the make-whole option is in the money if and only if the make-whole premium is below the bond's current G-spread. Figure 4 shows, in the first panel, the distribution of the gap between the G-spread and the make-whole premium in a sample of all active fixed-rate bonds issued by US non-financial corporations with make-whole clauses which we downloaded from Bloomberg on 4/1/2025. The second panel shows the same gap on the download date rather than at origination. Both at origination and at the download date, the make-whole premium is much smaller than the G-spread. The average (median) gap is 124bps (100bps) at origination, while it is 65bps (56bps) at the download date.

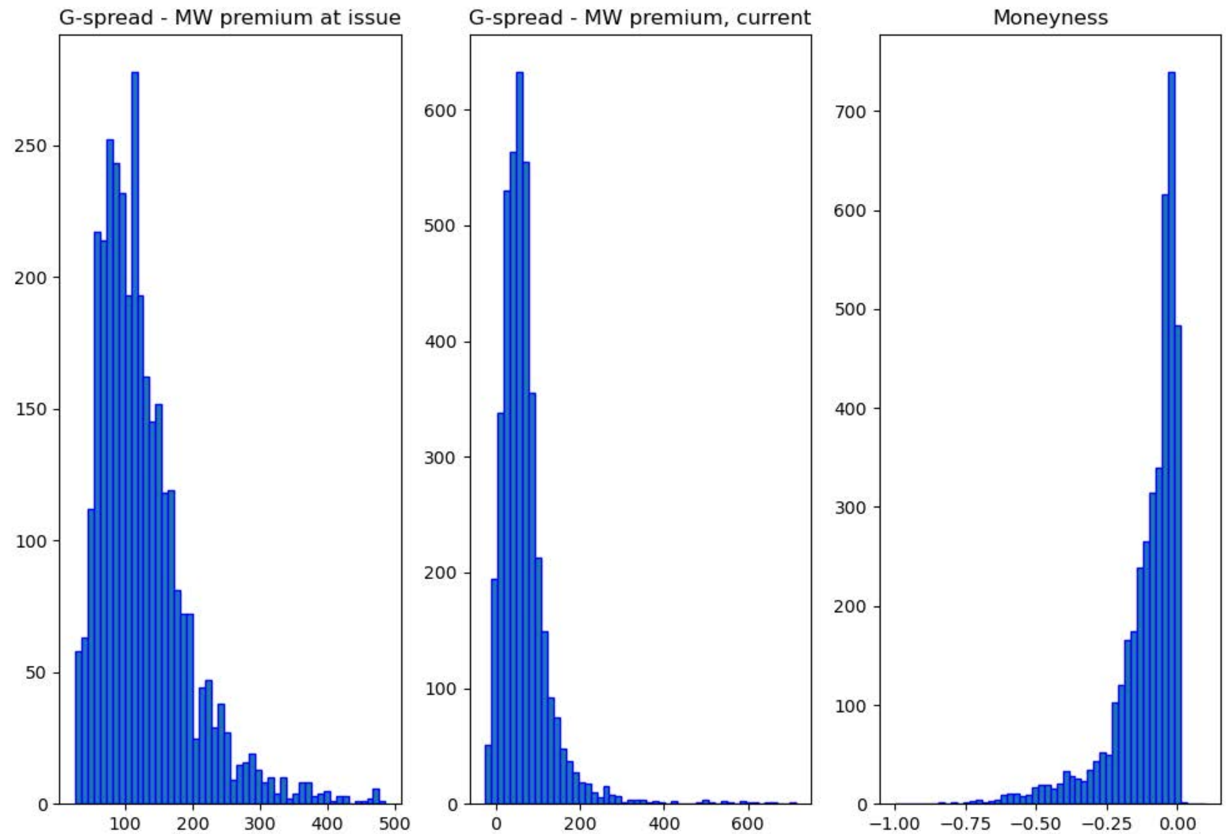
This translates into a significantly negative moneyness of the make-whole option, especially after

equity derivatives' literature" where an option's intrinsic value is measured as if "the option was expiring today." To implement that calculation, since most bonds do feature make-whole clauses, ytm^{NC} needs to be inferred from derivative prices. In complete markets (see Duffie [11] for a discussion) Credit Default Swap (CDS) spreads should equal spreads of uncallable bond of similar maturity. In practice, as a result of transaction and liquidity frictions, a negative basis typically exists between bond spreads and CDS premia. Ma et al. [15] thus use the approximation

$$ytm^{NC} = r^F + CDSspread + basis,$$

where r^F is the pertinent swap rate and basis is an estimate of the relevant basis given the bond's underlying characteristics. For completeness, we replicated this calculation as best our data allow and found largely unchanged results for the subset of bonds for which we have CDS data.

Figure 4: **Moneyness of make-whole call options**



Sources and notes: These data were downloaded from Bloomberg on 4/1/2025. It includes all active, fixed-rates bonds issued by non-financial, public US corporations that stipulate a make-whole call premium and provide g-spread data.

taking into account the fact that, if the make-whole price falls below par, then par is due at exercise. The third panel of the figure shows the moneyness of the make-whole option in our sample as a fraction of current bond value, i.e.:

$$\frac{PV(m, ytm) - PV(m, r_{mw})}{PV(m, ytm)}.$$

The figure confirms that the moneyness is almost always negative on those bonds, and deeply so in many cases. The average (median) relative moneyness is around -15% (-6.3%).

2.2 Determinants of make-whole premia

Most of the current literature on bond call design choice – see e.g. Mann and Powers [18] – has focused on the discrete margin, i.e. the type of call provisions selected by issuers and lenders. Among other findings, relative to firms that issue bonds with fixed-call-price clauses, firms that issue make-whole provisions are typically found to be larger, more highly rated, to feature lower leverage, higher ROA, and slower growth. In short, firms that opt for make-whole provisions appear to be more established and stable.

Interesting though those findings are, the data shown in figures 1 show that there is ever less variation left along the discrete margin with the vast majority of callable public bonds issued by non-financial firms featuring make-whole clauses. Figure 2 shows on the other hand that there is a lot of variation in the depth of the negative moneyness of these call options across issuers. Alderson et al. [3] provide evidence that, at origination, the variation in moneyness displays some systematic correlation with firm and bond characteristics. In their estimation, larger, more profitable, and better rating firms appear to display less negative moneyness at origination.

While Alderson et al. [3] focus on the variation in moneyness across callable bonds at origination, the right two panels of figure 2 display the heterogeneity in moneyness at current, secondary market prices in our Bloomberg data. Table 1 shows similar regressions as in Alderson et al. [3] for this secondary market data, using the covariates we have available. The dependent variable in all specifications is the natural log of 1 minus the moneyness of the option (the variable plotted in the right-hand panel of figure 2 which equals the natural log of the market value of the bond divided by the make-whole price.) Positive coefficients in those estimations, therefore, correspond to variables associated with more negative moneyness of the option.

Like Alderson et al. [3], we find that asset size is associated with higher moneyness. It also appears that issue size is associated with lower moneyness. Credit-worthiness at issue (as proxied by the issue's G-spread at origination) gives a negative coefficient but we do not find it to be significant. Likewise, we do not find profitability measures to have the same effect on our secondary market measure as it does on Alderson et al.'s issue data.

The three covariates which, by far, account for most of the variance in make-whole moneyness are maturity at origination, the current issue G-spread, and remaining maturity. Controlling for maturity at origination in addition to the issue and issuer characteristics results in a specification that accounts for almost 40% of the secondary market variance displayed in the right-hand panel of figure 2. And we account for another 15% in variance by adding the current G-spread and remaining maturity.

That these three last variables are deeply significant should come as no surprise given the mechanics of make-whole clauses we reviewed in the previous section. Call prices vary directly with remaining maturity (which is correlated with maturity at origination)[8] and shocks to the credit worthiness of the issuer. What the results in table 1 suggest is that, in secondary markets, most of the moneyness variation comes from those mechanical factors. Put another way, make-whole call prices are explicitly designed to display less negative moneyness over time and less negative moneyness as the firm's credit quality improves. The theory we provide in this paper provides natural justifications for those key features.

2.3 Timing and pricing facts

Yield-to-maturity at origination on bonds that feature make-whole provisions exceed those of observably similar straight bonds (see [18]). In fact, see [22], this premium appears to exceed what a standard contingent-claims model would predict. As for the timing the timing, frequency, and statistical predictors of call decisions on bonds in the United States using Mergent-FISD data, [18] document a number of interesting facts :

1. Bonds with make-whole provisions are much less likely to be retired early than bonds with fixed-call-price provisions, but twice as likely to be retired as bonds without any call provision so that make-whole provisions are in fact invoked in practice;

2. Calls occur later on average for bonds with make-whole provisions than those with fixed-call-price provisions;

3. The invocation of make-whole provisions is often preceded by a standard tender offer;

4. Firms more likely to be acquisition targets are more likely to retire make-whole bonds early.

[8] As both Alderson et al. [3]) and Brown and Powers [8] emphasize, some bonds feature both make-whole options and a fixed call price that becomes available after a no-call, lockout period. When present, those dual features, by providing a new call price after a seasoning period, accelerate the tendency of the moneyness of the call option to improve as time passes.

Table 1: Determinants of make-whole clause moneyness

	(1)	(2)	(3)
Constant	0.09776***	0.05840***	0.05290***
	(0.01659)	(0.01605)	(0.01434)
Amount issued ($bn)	0.01786***	0.01848***	0.00213
	(0.00298)	(0.00285)	(0.00261)
ln(Revenue)	0.00184	0.00186	0.00454***
	(0.00199)	(0.00191)	(0.00171)
ln(Assets)	-0.00950***	-0.00692***	-0.00619***
	(0.00218)	(0.00210)	(0.00187)
EBITDA/Revenue	1.11194	1.00103	2.86015
	(2.50525)	(2.39778)	(2.14281)
Net Debt to EBITDA	0.00046	0.00012	-0.00011
	(0.00036)	(0.00034)	(0.00031)
G-spread at issue	-0.00010***	-0.00010***	-0.00008***
	(0.00002)	(0.00002)	(0.00001)
Maturity at origination	0.00460***	0.00421***	-0.00069***
	(0.00011)	(0.00011)	(0.00020)
Current G-spread		0.00017***	0.00016***
		(0.00001)	(0.00001)
Remaining maturity			0.00652***
			(0.00023)
Industry controls	Yes	Yes	Yes
Observations	3113	3113	3113
R-squared	0.38	0.43	0.54

*Notes: These data were downloaded from Bloomberg on 4/1/2025. It includes all active, fixed-rates bonds issued by non-financial, public US corporations that stipulate a make-whole call premium and provide g-spread data. Standard errors in parentheses, *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Dependent variable is the log of 1 minus moneyness, so that a positive coefficient implies a more negative moneyness. Excluded industry dummy is Energy.*

3 Static Environment

Consider an economy in which a mass one of risk-neutral borrowers are born at the start of date 0 with no resources other than a risky project of type θ drawn from a known distribution μ. The project requires an upfront investment $I \geq 0$. We will first assume that the project lasts one period since that suffices to derive many of the insights we seek in this paper. We will then generalize our results to the case in which the project is infinitely-lived. If successful and operated to completion, a project of type θ generates output $y > 0$ at the end of the period with probability θ but nothing otherwise.

The economy also contains risk-neutral lenders endowed with a quantity I of capital. They can store their endowment for return R that is drawn from a known distribution with density g and $E(R) = 1$. The realization of the interest rate is discovered after capital I has been committed to projects. Borrowers have access to this storage technology as well but they do not have any resources to invest at the start of the period.

Immediately after discovering the realized interest rate, with probability $\lambda \in [0, 1]$, borrowers receive an outside option with expected value $V \geq 0$. We will think of V as the expected payoff of the best alternative use of the installed capital so that the market value of and payoff to exercising the option is $\frac{V}{R}$ as of the start of the period. We assume that V comes from a distribution with density f and finite expectation, and that R and V are independently drawn. With those two pieces of information in hand, borrowers decide whether to continue the project or liquidate. Unlike borrowers, lenders do not observe the value of the outside option.

One could think of V as the value of a liquidation option for concreteness but our model maps into any environment in which the borrower receives and learns the value of any type of outside option after having committed capital to the original project. The main assumption we are making is that incumbent lenders cannot fully condition transfers from the borrower on the value of the outside option. We view this assumption as a proxy for practical frictions that prevent lenders from funding or monitoring projects that do have the same characteristics as the original project. This includes cases in which the incurrence of financial covenants prevents lenders from funding projects of a different risk type and cases in which lenders do not have the expertise to evaluate, monitor, or enforce payments on projects outside of their area of specialization.

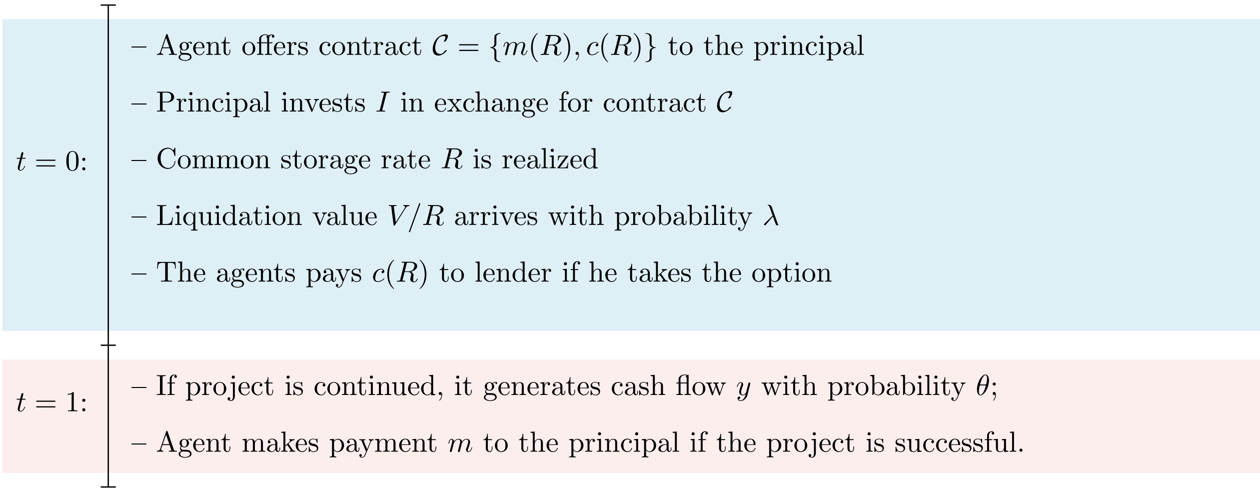


Figure 5: Timeline and summary of the model.

Also for simplicity, we will study the case in which one lender provides the entire funding to a given borrower but assuming that multiple, even uncountably many lenders fund each project would not change the nature of our results. As long as all lenders seek to maximize their expected returns, the contracts we describe in this paper are optimal whether they are bilateral or multilateral.

4 Public Project Types

As a useful benchmark, assume first that project types θ are publicly known. A contract is a payment

$$m(R) \leq y \tag{4.1}$$

made by the borrower to the lender if the project is continued to maturity and successful and if the realized interest rate is $R \geq 0$, and a prepayment $c(R) \geq 0$ if instead the borrower chooses to take advantage of the option. Because incumbent lenders do know not the value of the outside option at separation time, prepayment penalties cannot depend on the value of that option.

Given such a contract, the borrower exercises her outside option given R and V when

$$\left(\frac{V}{R} - c(R)\right) R = V - c(R)R \geq \theta[y - m(R)]. \tag{4.2}$$

Condition (4.2) makes it clear that there are two equivalent ways to interpret the exercise of the liquidation option. Borrowers can either sell existing assets at the start of the period for the market value $\frac{V}{R}$ of the liquidation payoff, use the sale proceeds to pay prepayement cost $c(R)$ and extinguish the original loan, and finally store the net profits at rate R. Or they can borrow $c(R)$ at rate R, retire the original loan with the new loan, and use proceeds V to payoff this new loan at the end of the period. Either interpretation requires a buyer or a new lender who can verify the value of the outside option and enforce payments against that value.

Given this outside option decision rule, participation by lenders requires

$$\int \theta m(R)g(R)dR + \lambda \int \left\{ \int_{\theta[y-m(R)]+c(R)R}^{+\infty} [c(R)R - \theta m(R)]f(V)dV \right\} g(R)dR \geq I. \tag{4.3}$$

To understand this condition, recall that lenders are risk-neutral and expect storage return of 1 over the life of the contract so any feasible contract must at least provide them with that return on their investment I. The first part of the lender payoff under the contract is the payment they expect to receive when the project is held to completion. The second item measures the expected consequences of the arrival of the outside option.

A contract is incentive-feasible if it satisfies feasibility condition (4.1), if it calls for the exercise of the outside option only when (4.2) is met, and if it satisfies lender participation (4.3). An incentive-feasible contract is optimal if it is not possible to find a different incentive-feasible contract that leaves both parties at least as well-off and makes one side strictly better off.

For concreteness, we will assume that lenders behave competitively in the sense that they are willing to accept any contract that generates expected return $E(R) = 1$. This is without any loss of generality. Indeed, to trace the full locus of optimal contracts (as we will do in the dynamic version of the model) it is enough to solve for incentive-feasible contracts that maximize the borrower's value at date zero subject to lenders receiving at least W for all $W \geq I$. But this is equivalent to varying I and so the features we emphasize in this paper are features of all optimal contracts.

It follows that the borrower seeks to maximize:

$$\int \theta(y-m(R))g(R)dR + \lambda \int \left\{ \int_{\theta[y-m(R)]+c(R)R}^{+\infty} [V - c(R)R - \theta(y - m(R))]f(V)dV \right\} g(R)dR \tag{4.4}$$

subject to:

$$\int \theta m(R)g(R)dR + \lambda \int \left\{ \int_{\theta[y-m(R)]+c(R)R}^{+\infty} [c(R)R - \theta m(R)]f(V)dV \right\} g(R)dR \geq I \tag{4.5}$$

and subject to (4.1). Since the constraint must hold as an equality at a solution, and the left-hand side of the constraint appears as is in the objective, we can reduce the borrower's objective to maximizing total surplus:

$$\theta y + \lambda \int \left\{ \int_{\theta[y-m(R)]+c(R)R}^{+\infty} [V - \theta y]f(V)dV \right\} g(R)dR$$

subject to conditions (4.5) and (4.1). Inspection of the objective function shows that, when feasible, maximization is accomplished by setting

$$c(R)R = \theta m(R) \text{ for almost all } R.$$

This can be delivered for instance by making making m constant (i.e. rate independent) and such that

$$\theta m = I,$$

and finally letting

$$c(R) = \frac{\theta m}{R} \text{ for almost all } R.$$

Feasibility requires $m = \frac{I}{\theta} \leq y$ which is the same as

$$\theta y \geq I \tag{4.6}$$

which holds if the project generates an expected return at least at high as that of storage, without taking the outside option into account. Condition (4.6) holds when there is no physical requirement to impose prepayment penalties in order to cover the cost of investment. In that case, fairly priced make-whole call options suffice and are optimal. If, on the other hand, condition (4.6) fails, optimal contracts may not exist and, when they do exist, make-whole clauses must state call prices that exceed the market value of the loan, as we will now establish.

Proposition 1. *When project types are publicly known, an optimal contract exists if and only if*

$$\theta y + \lambda \int_{c}^{+\infty} (c - \theta y) f(V) dV \geq I,$$

for some $c \geq \theta y$. Furthermore, all optimal contracts satisfy

$$c(R) \geq \frac{\theta \cdot m(R)}{R},$$

with equality for all R if and only if $\theta y \geq I$.

Proof. Take first the case in which $\theta y \geq I$. We have already shown in that case that it is possible to deliver the lender his expected payoff I with a payment m that does not depend on R and a prepayment penalty set to $c(R) = \frac{\theta m}{R}$ for all R. With that arrangement, the borrower's payoff is

$$\theta y + \lambda \int_{\theta y}^{+\infty} [V - \theta y] f(V) dV - I$$

Now, for general contracts, the borrower's payoff is

$$\int \theta(y - m(R)) g(R) dR + \lambda \int_{R} \left\{ \int_{\theta[y - m(R)] + c(R)R}^{+\infty} [V - c(R)R - \theta(y - m(R))] f(V) dV \right\} g(R) dR$$

But, for any given R,

$$\int_{\theta[y - m(R)] + c(R)R}^{+\infty} [V - c(R)R - \theta(y - m(R))] f(V) dV < \int_{\theta y}^{+\infty} [V - \theta y] f(V) dV$$

whenever $\theta m(R) \neq c(R)R$. So optimality does require $c(R)R = \theta m(R)$ for almost all R in that first case.

Assume now that $\theta y < I$. A feasible contract exists if the lender's participation constraint

$$\int \theta m(R) g(R) dR + \lambda \int \left\{ \int_{\theta[y - m(R)] + c(R)R}^{+\infty} [c(R)R - \theta m(R)] f(V) dV \right\} g(R) dR \geq I$$

can be met with $m(R) \leq y$ for all R. The left-hand side of the participation constraint is at its highest when the inner (bracketed) expression is maximized for all R. Therefore, if the condition can be met, it can be met with a contract such that $m(R) = m$ for all R and $c(R)R = c$ for all R for some c. In addition, it must be that $c > \theta y$ when $\theta y < I$.

It follows that a feasible contract exists if and only if:

$$\max_{m \leq y, c \geq \theta m} \theta m + \lambda \int_{\theta[y-m]+c}^{+\infty} [c - \theta m] f(V) dV \geq I$$

Setting $m = y$ maximizes the left-hand side of that condition. Indeed, given any $c > \theta y \geq \theta m$, the derivative of the left-hand side with respect to any $m < y$ is

$$\theta \left(1 - \lambda P(V > \theta(y - m) + c)\right) + \theta\lambda[c - \theta m] f(\theta(y - m) + c) > 0.$$

And so, all told, feasibility boils down to:

$$\theta y + \lambda \int_{c}^{+\infty} (c - \theta y) f(V) dV \geq I,$$

as claimed.

To complete the proof, we must also show that any general contract that implements the same payoffs must be such that $c(R)R \geq \theta y$ for almost all R with a strict inequality for a positive mass of R. We already now that we must have $E(c(R)) > \theta y$ for lender participation to hold when $\theta y < I$. So it is enough to show that that no optimal implementation exists with $c(R)R < \theta m(R)$ for a positive mass set of R. Assume it did. Replace that contract with a new contract with $c(R)R = m(R)$ on that set, leaving the rest of the contract unchanged. Then the lender's payoff can only increase. Now decrease both sides of $c(R)R = m(R)$ proportionately until the lender's payoff is I again. Total surplus has increased strictly (it is now maximized for those sets of interest rates), while the lender's payoff is unchanged, and so the original contract could not be optimal. This completes the proof. □

We now record the key consequences of this result for our purposes.

Corollary 2. *When project types are public,*

1. *All optimal contracts can be implemented with a fixed payment contract m and a make-whole clause that stipulates*

$$c(R) = \frac{\theta \cdot m}{R} + \tau \text{ for all } R$$

with $\tau \geq 0$;

2. *The make-whole clause features a strictly positive premium $\tau > 0$ if and only if $\theta y < I$;*

3. *Some optimal contracts can also be implemented with a fixed call price $c > 0$ and floating payments $m(R) = \frac{cR-\tau}{\theta}$.*

Proof. We established the first two items in the course of proving proposition 1. The only issue with the implementation via floating rate debt described in the third item is the possibility that for R large enough, $m(R) > y$. In fact, if g has unbounded support, that issue must eventually arise. But when interest rate uncertainty is essentially bounded, the implementation become possible for y high enough. □

Returning now to the case in which condition (4.6) holds and there is no physical necessity for prepayment penalties to exceed the remaining market value of the incumbent project, the fact that $c(R) = \frac{\theta m}{R}$ for all R is optimal says that allowing the firm to retire its bond via frictionless, fairly-priced open-market operations implements the optimal contract. Another way to do this – which is the relevant case of since open-market operations are not an effective way to retire bonds in practice – is to include a make-whole clause at market value. Under that contract, the outside option is exercised when and only when $V \geq \theta y$, which maximizes total surplus.

To summarize then, in the absence of asymmetric information, the optimal prepayment clause sets the call price to equal the market value of the remaining payments under mild condition (4.6). Anything else would lead to an inefficient, surplus-destroying policy for exercising the outside option. The borrower would not just compare the fundamental value of the two options available to him, but would also and inefficiently consider refinancing gains. Another way to see the optimality of a fairly-prices make-whole clause is to inspect objective (4.4). Such a clause enables the borrower and the lender to optimally share the payoff from the outside option <u>even when</u> the incumbent lender is unable for any reason to request payments contingent on the new project.

Finally, note that this simple version of our model provides (to our knowledge) the first answer to an age-old question in bond finance, an answer that does not depend on ad-hoc assumptions on risk tolerance or the ability to diversify risk: given that refinancing is a zero-sum game, who should bear interest risk? When outside options are present which the lender is unable to contract upon fully, our model says that insuring the lender fully against refinancing risk is optimal. Given the shape of our prepayment clause, prepayment is never associated with refinancing gains for the borrower.

While this simple version of our model delivers a number of useful insights, it does not answer the most puzzling aspect of make-whole clauses: why do bonds specify a clause such that the prepayment cost typically almost always exceeds market value? This puzzle, we will now show, can be addressed once either asymmetric information about product quality θ or about its management effort e is introduced.

5 Unobservable Effort

A positive make-whole clause reduces the value and thus the likelihood of liquidating the project prematurely. Therefore, a make-whole premium increases the fraction of the agent's compensation that stems from the realized project cash flows, i.e., his skin in the game. We show that such incentive effects make make-whole premia necessary even for arbitrarily small financing needs I.

To show this, we extend the baseline model of Section 3 by letting the agent exert private and unobservable effort $e \geq 0$. For concreteness, we assume the agent's private cost of effort is quadratic and equal to $h(e) = \frac{h}{2} \cdot e^2$. (All our results holds for cost functions that are strictly convex and satisfy an Inada condition at $e = 0$.) The agent's private effort choice e implies a project success probability $\theta + e$, i.e., $\tilde{y} = y$ with probability $\theta + e$ and $\tilde{y} = 0$ with the complementary probability $1 - \theta - e$. To ensure that $e \leq 1 - \theta$ assume that $h \cdot (1 - \theta) \leq y$. The agent chooses effort e at the start of the period after the outside investment rate R is realized, but before the liquidation opportunity arises.

5.1 First Best Effort Level

Consider a social planner who chooses an effort level $\bar{e}$ in the absence of agency and financing frictions. Given effort level $\bar{e}$, the social planner would liquidate the project efficiently, resulting in project liquidation if and only if $V \geq \mathrm{E}[\tilde{y} \,|\, \bar{e}] = (\theta + \bar{e}) \cdot y$. Consequently, the socially efficient effort level $\bar{e}$ maximizes total surplus $TS(e)$ assuming the project is liquidated efficiently:

$$TS(e) \stackrel{def}{=} (1 - \lambda) \cdot (\theta + e) \cdot y + \lambda \cdot \mathrm{E}\left[\max\left\{(\theta + e) \cdot y, \quad V\right\}\right] - \frac{h}{2} \cdot e^2 - I. \tag{5.1}$$

Taking the first order condition of (5.1) with respect to effort e, we obtain that the socially optimal effort $\bar{e}$ level solves

$$h \cdot \bar{e} = y \cdot \left[1 - \lambda + \lambda \cdot F\left((\theta + \bar{e}) \cdot y\right)\right]. \tag{5.2}$$

The optimality condition (5.2) shows that the marginal cost of the socially optimal effort must equal its marginal productivity y, scaled down by the probability that it is efficient to continue the project until the end of the period. It is worth pointing out that higher effort $\bar{e}$ increases the marginal value of effort, e.g., the right-hand side of (5.2), by reducing the likelihood that the project will be liquidated. In principle, this may result in multiple solutions to the first-order condition (5.2). Under intuitive regularity conditions such as $F(\cdot)$ being concave, i.e., admitting a decreasing density, the solution to (5.2) will be unique.

A higher probability λ of the liquidation opportunity decreases the socially optimal effort $\bar{e}$, highlighting a trade-off between liquidating and continuing the project. For any given effort e, however, it is efficient to liquidate the project if and only if V exceeds its expected payoff. We show below that the presence of agency frictions changes this intuition.

5.2 Optimality of Prepayment Clauses

A contract $\mathcal{C} = \{m, c, e^*\}$ specifies repayment m, prepayment c if the project is terminated, and the recommended effort e^* chosen by the agent. The repayment amount m and prepayment c both occur after the public discount rate R is realized, and we allow them both to be contingent on R. For notational purposes, we suppress the explicit dependence of m and c on R. If the agent chooses effort e in contract $\mathcal{C}$, his payoff is

$$B(\mathcal{C}, e) = (1-\lambda)(\theta + e)(y - m) + \lambda \mathrm{E}\left[\max\left\{(\theta + e)(y - m),\ V - cR\right\}\right] - \frac{h}{2}e^2. \tag{5.3}$$

We can see from (5.3) that the agent optimally liquidates the project in period $t = 1$ if

$$V \geq (\theta + e) \cdot (y - m) + cR. \tag{5.4}$$

A higher effort e makes continuing the project more valuable, while a higher prepayment c makes liquidation less valuable. Consequently, the liquidation probability decreases in both the agent's effort e and payment c. The recommended effort e^* is optimal for the agent if it equates its marginal cost with the contract's expected pay-for-performance sensitivity, which is equal to the product of the probability that the project survives and the expected payoff to the agent conditional on survival:

$$h \cdot e^* = (y - m) \cdot [1 - \lambda + \lambda \cdot F((\theta + e^*)y + cR - (\theta + e^*)m)]. \tag{5.5}$$

The novel feature of the incentive compatibility constraint (5.5) is that prepayment c increases the effect of pay-for-performance sensitivity by making it more costly for the agent to liquidate the project, thus increasing the effect of pay-for-performance sensitivity $y - m$. If $cR < (\theta + e^*)m$ under the optimal contract, then a marginal increase in c increases the effort e^* sustained by the contract, as well as improving the efficiency of project liquidation. The optimal contract, therefore, should always feature $cR \geq (\theta + e^*)m$. Such a ranking, however, imposes a trade-off in that the increase in the agent's incentives via (5.5) comes at a deadweight loss whenever $cR > (\theta + e^*)m$, which implies that the project is not liquidated as frequently along the recommended effort path, in favor of continuing the project. In what follows, we show that creating such distortions is always optimal in the optimal contract.

Proposition 3 (Make-whole under moral hazard)**.** *Suppose the arrival of the liquidation opportunity λ is sufficiently low.*[9] *There exists a unique optimal contract featuring a fixed rate debt $m < y$, and a make-whole prepayment penalty*

$$c = \frac{(\theta + e) \cdot m}{R \cdot (1 - \tau)} \tag{5.6}$$

that is indexed to the realized interest rate R at $t = 1$, the firm's credit risk $1 - \theta - e$,[10] *and a constant, i.e., independent of R, prepayment penalty τ.*

The make-whole rate τ is strictly positive if and only if $I > 0$.

The magnitude of the make-whole premium will vary with the severity of agency frictions and

[9] These parametric conditions are needed only for technical reasons to ensure that the optimal contract does not involve public randomization.

[10] We can also express this prepayment penalty as $c = \frac{m}{R \cdot (1-\tau) \cdot (1+\alpha)}$, where $\alpha > 0$ is the credit risk of the debt contract.

the amount of capital the agent needs for the project. An increase in investment cost I or expected interest rate $\mathrm{E}[R]$ tightens the principal's participation constraint, requiring a higher make-whole premium τ to satisfy it. An increase in baseline project risk $1-\theta$ reduces the amount of income the agent can pledge to the principal, implying higher repayment and lower pay-for-performance sensitivity. The higher make-whole premium τ is then used to marginally increase the agent's skin in the game.

Corollary 4 (Observable firm risk)**.** *Suppose λ is sufficiently low. The make-whole premium τ is increasing in investment cost I, expected interest rate $\mathrm{E}[R]$, the firm's baseline credit risk $1-\theta$, and the marginal private effort cost h.*

6 A dynamic version

Time is now discrete and infinite. Both the borrower and the lender discount future flows at the same rate $\beta \in (0,1)$ and remain risk-neutral. As long as the project is alive, output in a given period is $y > 0$ with probability $e \in [0,1]$ equal to the effort level selected by the borrower at the start of the period, and zero otherwise. We are simplifying matters somewhat by setting $\theta = 0$ which will guarantee that effort is interior as long as the project is operated. This reduces the number of cases we need to consider below. Once the project fails, output is zero in each subsequent period, so that project death is an absorbing state.

Each period in which the project is active, there arrives an outside option whose value is distributed like it was in the static case, and realizations of this random variable are also independent across time. A history at a date t lists all past loan terms $\{m_s, c_s\}_{s<t}$ and whether the project is still alive. As before, m_t denotes the payment to the lender if the project pays off in a particular period while c_t is the penalty the borrower needs to pay to take advantage of the outside option. We assume as before that c_t is financed with an intra-period loan at deterministic rate $R = \beta^{-1}$.

As usual in this context, see e.g. Spear and Srivastava [24], the resulting contracting game can be given a recursive formulation. Write $B \geq 0$ for the value the borrower expects from the contract at the start of a given period at a specific history and define $Y(B)$ to be the maximum total surplus given that promise. We will begin our characterization of optimal contracts by stating the functional equation Y must solve.

As in the static case, three types of constraints limit the ability of borrowers and lenders to write contracts. First, transfers from the borrower to the lender cannot exceed output when the project is active. Second, borrower effort must be incentive-feasible. And, third, the lender must at least break even in expectations at date 0. To sidestep inconsequential technicalities, as in Clementi and Hopenhayn [10], we will assume that the lender has the ability to commit to any contract at date 0 and has the ability to make transfers of any size to the borrower.

Taking the lender first, since we are working at the level Y of total surplus, their participation boils down to $Y(B_0) - B_0 - I \geq 0$, a test one can easily run once all value functions are defined and computed. As for incentive feasibility, at any given history in which the project is active, the contract specifies a transfer m, a pair continuation promises $(B_y, B_0) \geq (0, 0)$ if the project is continued according to whether the project succeeds or fails, and, finally, a prepayment cost $c \geq 0$. Given those recursive contract terms and the realized value V of the outside option, the borrower exercises her outside option if

$$e(y - m + \beta B_y) + (1 - e)\beta B_0 > V - cR.$$

Therefore, her expected payoff given e is:

$$\begin{aligned} B(e, m, B_y, B_0, c) &= \int_0^{e(y-m+\beta B_y+(1-e)\beta B_0+cR} [e(y - m + \beta B_y) + (1 - e)\beta B_0] f(V) dV \\ &+ \int_{e(y-m+\beta B_y)+(1-e)\beta B_0+cR}^{+\infty} (V - cR) f(V) dV - Ae^2. \end{aligned}$$

Exactly as in the static case then, we get a simple necessary condition for effort

$$\int_0^{e(y-m+\beta B_y)+(1-e)\beta B_0+cR} (y - m + \beta(B_y - B_0) f(V) dV - 2Ae \quad \begin{cases} \leq 0 & \text{if } e = 0 \\ = 0 & \text{if } e \in (0, 1) \\ \geq 0 & \text{if } e = 1 \end{cases} \cdot$$

As is typical in this dynamic moral hazard context (see Spear and Srivastava [24]), we will reduce incentive-compatibility constraints to their above and necessary first-order version. Depending on the shape of f, multiple solutions to the above condition are in principle possible, although restrictions on f would eliminate this technicality. For instance, in the context of our problem, one can simply assume that f follows a uniform distribution. In the simulations we present later, the condition yields a unique candidate for optimal effort at all histories.

As in any dynamic contracting game that features discrete choice, total surplus may exhibit non-convexities hence lotteries over promises may improve surplus. Like Clementi and Hopenhayn [10], we allow lenders to offer those lotteries. Specifically, when a borrower enters a period expecting B in continuation utility, lenders can choose a pair $(B_L, B_H) \in \mathbb{R}_+^2$ and a fraction $\alpha \in [0,1]$ such that $B = \alpha B_L + (1-\alpha) B_H$.

With these preliminaries at hand, we can now state the functional equation which total surplus must solve for all possible levels $B \geq 0$ of borrower expectations. This requires two steps. First, at a particular history at which the project is active, assume that the promise lottery outcome is $\hat{B} \geq 0$ and let $\hat{Y}$ be the maximum total surplus given that outcome. That function solves:

$$\begin{aligned} \hat{Y}(\hat{B}) = \max_{e\in[0,1], m\in[0,y], B_y\geq 0, B_0\geq 0, c\geq 0} & \int_0^{e(y-m+\beta B_y)+(1-e)\beta B_0+cR} e(y+\beta Y(B_y)) f(V) dV \\ & + \int_{e(y-m+\beta B_y)+(1-e)\beta B_0+cR}^{+\infty} V f(V) dV - Ae^2 \end{aligned}$$

subject to:

$$\int_0^{e(y-m+\beta B_y)+(1-e)\beta B_0+cR} [y-m+\beta(B_y-B_0)] f(V) dV - 2Ae \begin{cases} \leq 0 & \text{if } e=0 \\ =0 & \text{if } e\in(0,1) \\ \geq 0 & \text{if } e=1 \end{cases}$$

and:

$$\begin{aligned} B = & \int_0^{e(y-m+\beta B_y)+(1-e)\beta B_0+cR} [e(y-m+\beta B_y)+(1-e)\beta B_0] f(V) dV \\ & + \int_{e(y-m+\beta B_y)+(1-e)\beta B_0+cR}^{+\infty} (V-cR) f(V) dV - Ae^2. \end{aligned}$$

Then, as of the start of the period and recursively, total surplus must solve for all possible pre-lottery promises $B \geq 0$:

$$Y(B) = \max_{\{(B_L, B_H) \in I\!R_+^2, \alpha \in [0,1]\}} \alpha \hat{Y}(B_L) + (1-\alpha)\hat{Y}(B_H),$$

subject to:

$$\alpha B_L + (1-\alpha) B_H = \hat{B}.$$

We can now state and establish:

Lemma 5. *The above, two-step functional equation defines a contraction mapping operator T on the space of bounded real functions equipped with the supnorm, so that Y is well defined. Furthermore, Y is continuous, weakly increasing, and concave in B.*

Proof. The functional operator that defines Y satisfies Blackwell's sufficiency conditions (monotonicity and discounting) hence is a contraction mapping on the Banach space of bounded functions. That the operator preserves continuity follows from Berge's Maximum Theorem. Since the the set of continuous and bounded functions is closed in the supnorm topology, the operator's unique fixed point must be continuous as well.

Likewise, the operator preserves weak monotonicity. Indeed, take any weakly increasing, continuous, bounded function $\tilde{Y}$ and apply the above functional operator to that guess function. A higher level of expectations B can be delivered by equal increases $\epsilon > 0$ in B_y and B_0 and an decrease in c by of $\beta\epsilon$. Recall that we are allowing the lender to make transfers to the borrower so that a decrease in c is always feasible. This joint change leaves optimal effort and exit policies unaffected. Because $\tilde{Y}$, is weakly increasing, this implies that $T\tilde{Y}$ must increase at least weakly as well.

Finally, the two-step functional operator that define Y preserves concavity because any nonconvexity in $\hat{Y}$ is convexified away by the lottery option. □

To proceed, consider the level Y^* of surplus that would prevail if effort were observable and contractible. It solves:

$$Y^* = \max_{e \in [0,1]} \int_0^{e(y+\beta Y^*)} e(y+\beta Y^*) f(V) dV + \int_{e(y+\beta Y^*)}^{+\infty} V f(V) dV - A e^2.$$

Per the same arguments as in the above result, Y^* is well defined and finite. In fact,

Remark 6. *An all-equity firm has value Y^*. Equivalently, $Y^* = Y(Y^*)$.*

Proof. The unique first-best level of effort solves the familiar looking:

$$\int_0^{e(y+\beta Y^*)} (y+\beta Y^*) f(V) dV - 2Ae \quad \begin{cases} \leq 0 & \text{if } e = 0 \\ = 0 & \text{if } e \in (0,1) \\ \geq 0 & \text{if } e = 1 \end{cases} \cdot$$

It follows that Y^* is attained by a contract that sets $m = 0$, $B_y = Y^*$, and $B_0 = 0$ at all histories at which the project is active. □

In finance terms therefore, Y^* is the unlevered value of the firm. While Y^* is achievable when the firm is all-equity financed, incentive-feasibility constraint can also stop binding in principle before equity level $B = B(Y^*)$ can be reached. So define $B^* = \inf \{B \geq 0 : Y(B) = Y^*\}$ to be the level of equity sufficient to achieve the first-best level of surplus. A corollary of our main proposition in this section will be that, in fact, $B^* = B(Y^*)$ but getting to that main result requires several intermediate steps.

Since Y is concave, note that it must increase strictly on $(0, B^*)$, almost surely, otherwise it would achieve a maximum on $(0, B^*)$. Note further that, typically, $Y(0) > 0$. That's because with $A > 0$ it is possible to deliver $B = 0$ even though $e > 0$. But $e > 0$ requires either $m < y$ or $B_y > 0$ which implies that surplus is strictly positive. It follows that while $[B^*, +\infty]$ is an absorbing state at any optimal contract, $B = 0$ is not.

Those properties of the total surplus function are enough to get a specialization of the optimal contract that is common to most dynamic lending problems in which the borrower and the lender discount future flows at the same rate, as discussed in different contexts by Albuquerque and Hopenhyan [2], Monnet and Quintin [19], and Clementi and Hopenhayn [10], among many others.

Lemma 7. *Optimal contracts are such that, at all histories, either $m = y$ or $B_y \geq B^*$.*

Proof. Assume by way of contradiction that at particular history both conditions fail at optimal contract terms. Then it is possible to raise m by $\epsilon > 0$ and, at the same time, increase B_y by $\beta^{-1}\epsilon$.

This leaves both

$$e(y - m + \beta B_y) + (1 - e)\beta B_0 + cR$$

and

$$y - m + \beta(B_y - B_0)$$

unchanged so optimal effort is unchanged as well. Because Y increases strictly $(0, B^*)$, this would raise the problem's value strictly, contradicting optimality. □

Without loss of generality then, we will concentrate on contracts that satisfy the properties of lemma 7. Once B^* is acheived the path of m may become indeterminate but setting $m = y$ until $B = Y^*$ is at least weakly optimal. The resulting, optimal contract has almost surely finite life with constant payments conditional on positive output until, at the most, the very last period where $m \in (0, y]$ may become necessary to deliver exactly the desired level of promise to the lender.

We note in passing that explicit transfers of $m = y$ to the lender at not the only way to implement the above contract. The contract can also stipulate low transfers together with the stipulation that the borrower accumulate other funds in a restricted account to which the lender has a security interest and which can only be used to eventually pay off the nominal value of the loan. For instance, an interest-only bullet together with a sinking fund that requires payments towards principal past a certain date also implement the above contract.

We will now show that our main result – the essential role of make-whole clauses – is fully robust to this dynamic extension. To see this, write $L = Y(B) - B$ for the expectations of the lender given promise $B \geq 0$ to the borrower. In particular, define $L_y \equiv Y(B_y) - B_y$ and $L_0 = -B_0 \leq 0$ for the continuation expectations of the lender at a particular history and depending on the success of the project. We now get the natural extension of our key static result to this dynamic context:

Lemma 8. *At any history and optimal contract with post-lottery lender promise $L > 0$,*

1. $e > 0$
2. $B_y > B_0$ *as long as* $B_y < B^*$
3. $cR \geq e(m + \beta L_y) + (1 - e)\beta L_0$

Proof. If $e = 0$, $L > 0$ requires $c > 0$. But then the only way the borrower would choose $e = 0$ is if $m = y$ and $B_y = B_0$. But now we can lower m slightly which improves the borrower payoff (since the marginal cost of effort is zero at zero) and can also only improve the lender's payoff by the same argument as in the static case. So, indeed, $e = 0$ cannot be optimal.

For the second item, lemma 7 implies that if $B_y < B^*$ then we must have $m = y$. But then $e > 0$ is only possible if $B_y > B_0$.

As for the third item of the lemma, consider any contract such that

$$cR < e(m + \beta L_y) + (1 - e)\beta L_0,$$

so that is possible to increase c strictly while maintaining the inequality. Define

$$\tau = cR - [e(m + \beta L_y) + (1 - e)\beta L_0] < 0$$

as well as

$$\begin{aligned} Y(e,\tau) &= \int_0^{e(y+\beta(Y(B_y)-L_y))-(1-e)\beta L_0)+cR} [e(y + \beta Y(B_y))]f(V)dV \\ &+ \int_{e(y+\beta(Y(B_y)-L_y))-(1-e)L_0)+cR}^{+\infty} Vf(V)dV - Ae^2 \\ &= \int_0^{e(y+\beta(Y(B_y))+\tau} e(y + \beta Y(B_y))f(V)dV \\ &+ \int_{e(y-m+\beta(Y(B_y))+\tau}^{+\infty} Vf(V)dV - Ae^2. \end{aligned}$$

This second function, when evaluated at the level of effort $\bar{e}$ that satisfies incentive compatibility given contract terms, gives the current level $Y(\bar{e}, \tau)$ of total surplus. But, for reasons that will become clear below, we also need to define the function for neighboring levels of effort. Now, for all $e \in [0, 1]$,

$$\begin{aligned} \frac{\partial Y(e,\tau)}{\partial \tau} &= e(y + \beta Y(B_y))f(e(y + \beta Y(B_y)) + \tau) \\ &- [e(y + \beta Y(B_y)) + \tau]f(e(y + \beta Y(B_y)) + \tau) \end{aligned}$$

$$= \quad -\tau f(e(y + \beta Y(B_y)) + \tau) > 0$$

This implies that holding effort at the currently optimal level, a slight increase in c increases surplus. However, that increase also leads the borrower to increase their effort marginally. To sign the impact of this marginal increase in effort, write

$$\bar{V} \equiv \bar{e}(y + \beta Y(B_y)) + \tau,$$

and note that:

$$\begin{aligned}
\frac{\partial Y(\bar{e}, \tau)}{\partial e} &= (y + \beta Y(B_y))[\bar{e}(y + \beta Y(B_y))]f(\bar{V}) \\
&- (y + \beta Y(B_y))[\bar{e}(y + \beta(Y(B_y)) + \tau]f(\bar{V}) \\
&+ \int_0^{\bar{V}} [y + \beta Y(B_y)]f(V)dV - 2A\bar{e} \\
&= -\tau[y + \beta Y(B_y)]f(\bar{V}) \\
&+ \int_0^{\bar{V}} [y + \beta(Y(B_y) - Y(B_0)]f(V)dV - 2A\bar{e} > 0
\end{aligned}$$

The fact that $-\tau(y + \beta Y(B_y))f(\bar{V}) > 0$ follows from the fact that $\tau < 0$. The positivity of the last line follows from several observations. First, $\bar{e}$ is incentive compatible at the original contract, therefore,

$$\int_0^{\bar{V}} [y - m + \beta(B_y - B_0)]f(V)dV - 2A\bar{e} \geq 0.$$

Now

$$\int_0^{\bar{V}} [y - m + \beta(B_y - B_0)]f(V)dV = \int_0^{\bar{V}} [y - m + \beta(Y(B_y) - Y(B_0)) - \beta(L_y - L_0)]f(V)dV.$$

But $cR < e(m + \beta L_y) + (1 - e)\beta L_0$, $L > 0$ and $L_0 \leq 0$, together, imply

$$m + \beta(L_y - L_0) > 0.$$

Therefore, we get

$$\int_0^{\bar{V}} [y + \beta(Y(B_y) - Y(B_0))]f(V)dV - 2A\bar{e} > \int_0^{\bar{V}} [y - m + \beta(B_y - B_0)]f(V)dV - 2A\bar{e} \geq 0,$$

as needed. This means that $cR < e(m + \beta L_y) + (1 - e)\beta L_0$ cannot be optimal, and completes the proof. □

And now, much like we did in the static case, we can establish that an out-of-the-money make-whole clause is part of the optimal contract unless the firm is fully unlevered. Consider a history at which the lender expects $L > 0$. Using the notation $\bar{V} \equiv e(y + \beta Y(B_y)) + \tau$ we introduced for the exit threshold in the preceding proof, we have

$$L = \int_0^{\bar{V}} [e(m + \beta L_y) + (1 - e)L_0]f(V)dV + \int_{\bar{V}}^{+\infty} cRf(V)dV.$$

Assume that the make-whole clause is priced at market-value a particular history so that $\tau = 0$ and $cR = e(m + \beta L_y) + (1 - e)L_0$. Then we have $L = e(m + \beta L_y) + (1 - e)L_0$. Since $L_0 \leq 0$ it follows that $e(m + \beta L_y) \geq L$ hence $m + \beta L_y \geq \frac{L}{e}$. In turn, we get

$$\begin{aligned} y - m + \beta(B_y - B_0) &\leq y - m + \beta B_y \\ &= y + \beta Y(B_y) - (m + \beta L_y) \\ &\leq y + \beta Y(B_y) - \frac{L}{e} \\ &\leq y + \beta Y^* - \frac{L}{e}. \end{aligned}$$

This inequality suffices to obtain a complete extension of the main results we obtained in the static version, namely that an out-of-the-money make-whole clause is optimal unless the firm is fully unlevered.

Proposition 9. *As long as $e^* < 1$ and $L > 0$, a necessary condition for a contract to be incentive-feasible is*

$$cR > e(m + \beta L_y) + (1 - e)\beta L_0.$$

Proof. Given lemma 7, we only need to rule out $cR = e(m + \beta L_y) + (1 - e)\beta L_0$ when $e^* < 1$ and

$L > 0$. Assume that equality did hold at a particular history. Then a first order condition for effort would be:

$$\int_0^{e(y+\beta Y^*)} (y - m + \beta(B_y - B_0))\, f(V)dV - 2Ae \begin{cases} \leq 0 & \text{if } e = 0 \\ = 0 & \text{if } e \in (0,1) \\ \geq 0 & \text{if } e = 1 \end{cases} \cdot$$

But then, since,

$$y - m + \beta(B_y - B_0) < y + \beta Y^*$$

as long as $L > 0$, we have that $e < e^*$ at this history. Now, per the exact same argument as in the static case, a slight increase in c has no first-order effect on the exercise option but causes a first-order increase in effort that does improve surplus. □

So far, we have shown that all our static results survive the introduction of dynamic incentives. But this version of our model also yields new, dynamic predictions, associated with the evolution of equity as the contract gets older. To see this, note that if we focus without any loss of generality on contracts such that $y = m$ until $B \geq B^*$ we get, at any history at which the borrower expects $B \geq 0$,

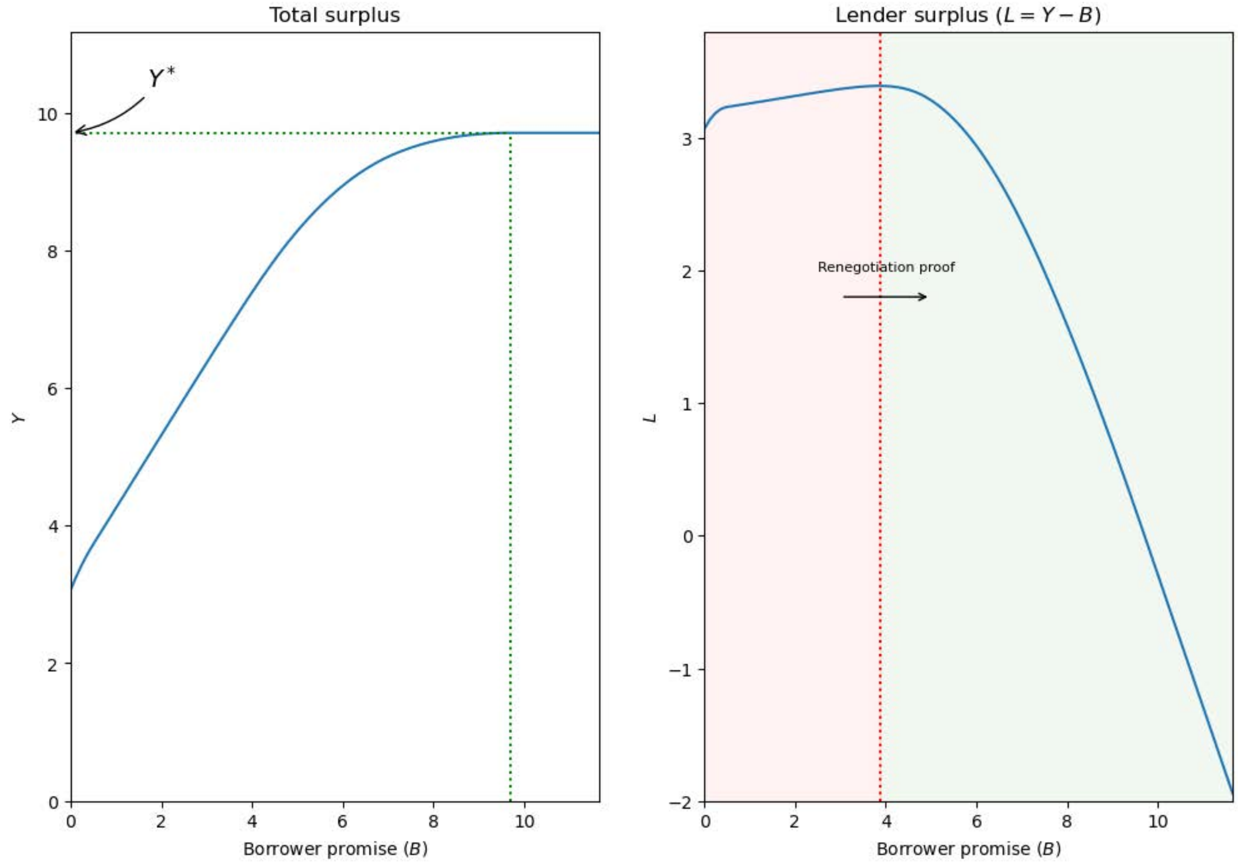
$$B = \int_0^{\bar{V}} \beta(eB_y + (1-e)B_0)f(V)dV + \int_{\bar{V}}^{+\infty} (V - cR)f(V)dV. \tag{6.1}$$

Then, $B_y > B_0$ implies that, as the contract evolves,

$$B_y > \frac{B + P[V > \bar{V}]cR - P[V > \bar{V}]E(V|V > \bar{V})}{\beta P[V < \bar{V}]}.$$

Except for the final part of the numerator which reflects the gross value of the outside option, all other terms would imply that, conditional on the project surviving, $B_y > B$. This is because, inter alia, payments to the lenders are optimally front-loaded as long as the project is active (so that, absent other factors, B_y would need to grow at the rate of time preference) and the borrower has to be compensated for the risk that the project may not survive. Since we need $B_y > B_0$ to sustain effort, insurance against that shock can only be partial. The borrower also needs to be compensated for the risk of incurring a prepayment penalty when it becomes optimal to exercise the outside option.

Figure 6: **Value functions**

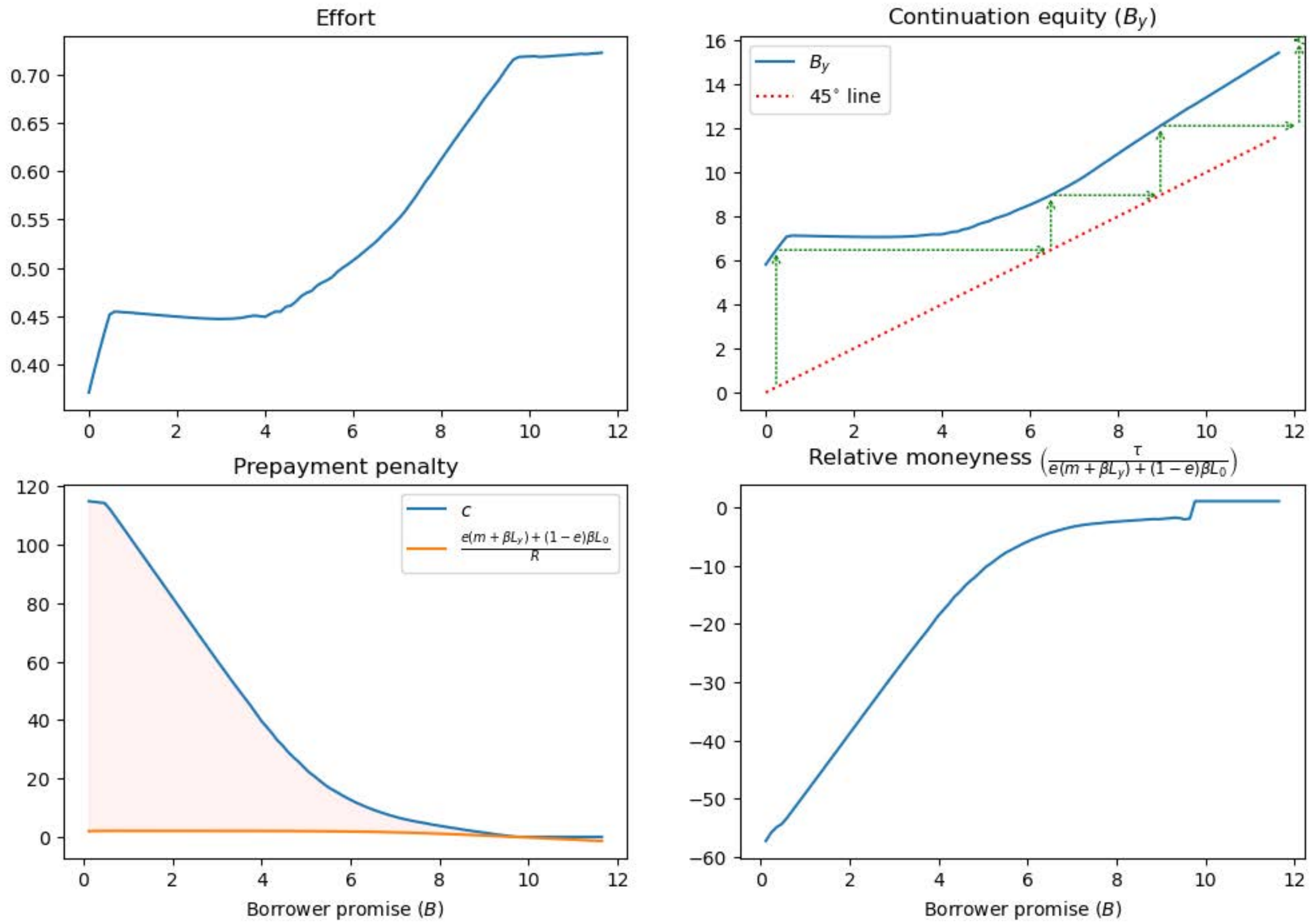


Parametrization: $A = 7$, $y = 3$, $\beta = 0.9$, V is lognormal with location 0 and dispersion parameter 2.

While we expect equity to rise as time passes, the presence of the third term in the numerator of expression (6.1), namely $P[V > \bar{V}]E(V|V > \bar{V})$, makes establishing that B_y follows a submartingale analytically difficult for arbitrary distributions of the outside option. Note in particular that for fat-tailed distributions that term may not decay as the exit threshold rises. So we turn to numerical simulations to explore further the dynamic properties of optimal contracts in this context. For concreteness, consider the parametric case in which $A = 7$, $y = 3$, and $\beta = 0.9$. None of the features we emphasize below appear sensitive to even wide variations in those parameters.[11]

[11]The value function iteration code that generates those figures is available here. We discretize the continuation utility space using 200 equally spaced points on $[0, 1.2Y^*]$ and use linear interpolation throughout. The most time consuming part of those computations lies in the evaluation of the conditional value of the exit options given $\bar{V}$. To make this repeated step faster we evaluate this conditional expectation using linear interpolation. While we allow for the possibility of a convexification step when needed as we iterate, no convexification via lotteries was necessary in any of our simulations. See code for details.

Figure 7: **Policy functions**



Parametrization: $A = 7$, $y = 3$, $\beta = 0.9$, V is lognormal with location 0 and dispersion parameter 2.

The left-hand panel of figure 6 displays total surplus Y as a function of the continuation utility promised to the borrower at a given history. The value function displays all the properties we established in lemma 5, including the fact that it crosses the 45 degree line exactly at the first best level Y^* of total surplus. The right-hand panel shows lender value $L(B) \equiv Y(B) - B$ as a function of the borrower promise. It is hill-shaped since Y is concave and eventually flat. In the uphill portion of the curve contracts are not renegotiation proof since an increase in B also results in an increase in $L(B)$. We would expect contracts to at least start with a promise B_0 in the renegotiation-proof part of the contract curve. In addition, a contract that satisfies the lender's participation constraint exists if and only if the lender's curve exceeds I, at least weakly, at its apex.

Figure 7 shows the policy functions that characterize the optimal contract given the level of borrower promise B at a particular history. While not uniformly monotonic, effort rises for the most part as B rises. Consistently with lemma 8 and as shown in the lower-left panel, the size of the call price always exceeds the market value of promises to the lender, strictly so (in this parametrization) except when the remaining expectations of the lender are exactly zero. In turn, the moneyness of the make-whole option, i.e. $-\tau$ divided by the market value of the loan, rises as borrower promises improve. Of course, and this has been our main point, the moneyness is negative until the firm becomes unconstrained, which holds for instance when it is unlevered.

Dynamic features of the contract depend both on those value functions and on the evolution of borrower equity B as time passes. In this respect the key feature of the contract is in the top-right corner of figure 7. The borrower promise policy is always above the 45 degree line which means that, conditional on project continuation and success, B can only rise. The green arrows illustrate this by tracing the difference equation B follows conditional on success starting from an arbitrary point. Assuming that no outside option arises that would cause an exit and that the project succeeds, equity rises, as does effort overall, while the size of the prepayment penalty falls while the moneyness of the make-whole clause rises as time goes by.

While this version of our model contains a rationale for the mechanical correlation between make-whole moneyness and credit quality we see in practical debt contracts, that connection is deterministic conditional on survival since the evolution of equity is deterministic. Introducing bona fide credit shocks in the model can be done in a number of different ways. We conclude this section by considering a simple variation of the above model with credit shocks where we can treat everything analytically given the results we already have.

Assume that the contract with the firm immune to moral hazard – this is a firm of the highest possible credit quality – but that with probability $\lambda > 0$ the firm may experience a deterioration in its credit quality reflected in hidden effort cost $A > 0$. Letting $Y_{A=0}$ measure total surplus until a credit shock occurs, while $Y_{A>0}$ is the total surplus function following a credit shock, we have, for all equity levels $B \geq 0$:

$$Y_{A=0}(B) = \max_{m, B'_{A=0}, B'_{A>0}, \bar{V} \geq 0} \int_0^{\bar{V}} \left[y + \lambda \beta Y_{A>0}(B_{A>0}) + (1-\lambda)\beta Y_{A=0}(B_{A=0}) \right] f(V) dV$$

$$+ \int_{\bar{V}}^{+\infty} Vf(V)dV$$

subject to:

$$\begin{aligned} m &\leq y, \\ y - m + \beta\left(\lambda B_{A>0} + (1-\lambda)B_{A=0}\right) + cR &= \bar{V}, \\ \int_0^{\bar{V}} \left[y - m + \lambda\beta B_{A>0} + (1-\lambda)\beta B_{A=0}\right] f(V)dV + \int_{\bar{V}}^{+\infty} (V - cR)f(V)dV &= B, \end{aligned}$$

where $B_{A=0}$ and $B_{A>0}$ are borrower promises as of the next period contingent on whether the credit shock occurs or not. The simplification we leverage here is that $A = 0$ implies that $e = 1$ is always at least weakly optimal.

Once a credit shock has occured, we are in exactly the situation we have studied in most of this section and it follows that $Y_{A>0}$ is continuous, increasing, and concave and that an out-of-the-money credit clause is optimal until an equity threshold is reached. In turn, $Y_{A=0}$ is also continuous, increasing, and concave.

As long as we maintain the assumption that the borrower and the lender discount flows at the same rate, it is optimal, strictly so until all risk that the hidden effort constraint will bind in the future to accumulate equity as quickly as possible. This implies that $m = y$ and $B_{A=0}$ are at least weakly optimal so that:

$$B_{A>0} = \frac{B - P(V > \bar{V})(V - cR)}{\beta\lambda}$$

is weakly optimal as well.

As long as the firm has not experienced a credit shock, there is no moral hazard friction hence, per the arguments we have used repeatedly in this paper, setting

$$cR = m + \beta\left[\lambda L_{A>0}(B_{A>0}) + (1-\lambda)L_{A=0}(B_{A=0})\right],$$

is optimal where

$$L_{A>0}(B_{A>0}) = Y_{A>0}(B_{A>0}) - B_{A>0}$$

while

$$L_{A=0}(B_{A=0}) = Y_{A=0}(B_{A=0}) - B_{A=0}.$$

In words, an actuarially fair make-whole clause is optimal before the credit shock but, as we established above, a make-whole wedge becomes necessary in general after the shock. Put another way, credit shocks do cause the moneyness of the prepayment option to jump down.

7 Private Productivity Types

Consider now a case in which the agent has private information about the borrower's project $\theta \in \{\theta_L, \theta_H\}$. The higher quality project generates cash flow y with the higher probability $\theta_H > \theta_L$. As before, a contract is a pair (m, c) stipulating the borrower's repayment and the prepayment penalty c, where both m and c are functions of the (contractable) interest rate R.

It is optimal for the borrower of type θ to pursue his outside option V given interest rate R if and only if its expected value net of the prepayment penalty exceeds the expected value of staying with the project:

$$V - c(R) \cdot R \geq \theta \cdot (y - m(R)).$$

It is worthwhile to note here that better, i.e., higher θ, borrowers, have a higher threshold on their outside option to break the contract. The borrower's expected value from optimally exiting the contract is

$$\max\{V - c(R) \cdot R, \theta \cdot (y - m(R))\}.$$

Lenders compete via the offered contracts – one can think of each lender posting a menu of two contracts. An equilibrium then consists of contracts $m^L = (m^L, c^L)$ and $m^H = (m^H, c^H)$ offered to type L and H borrowers respectively. Lender competition requires that the contract to borrower $i \in \{L, H\}$ maximizes the expected payoff to this borrower

$$\max_{m^i, c^i(\cdot)} \mathrm{E}\left[\max\{\theta_i \cdot (y - m^i(R)),\ V - c^i(R) \cdot R\}\right] \tag{7.1}$$

subject to the lender's break-even constraint

$$\int_R \left\{ \int_0^{\theta_i(y-m^i(R))+c^i(R)R} m^i(R)f(V)dV + \int_{\theta_i(y_2-m^i(R))+c^i(R)R}^{+\infty} c^i(R)Rf(V)dV \right\} g(R)dR = I \quad (7.2)$$

and the incentive compatibility condition for borrower $j \neq i$ to prefer contract j to contract i:

$$\mathrm{E}\left[\max\left\{\theta_j \cdot (y - m^j(R)),\ V - c^j(R)R\right\}\right] \geq +\mathrm{E}\left[\max\left\{\theta_j \cdot (y - m^i(R)),\ V - c^i(R)R\right\}\right]. \quad (7.3)$$

Type L optimal contract. First, note that the incentive constraint (C.4) must be binding for the low type borrower, i.e., $j = L$ and $i = H$. Otherwise, the optimal contract menu would be pinned down by the lender's break-even constraints (C.3) for both $i = L$ and $i = H$ borrowers. In the latter case, the optimal contract would correspond to an efficient full information contract, given by $c^i(R) = \theta_i \cdot I/R$. It can be verified, however, that such a pair of contracts violates the low-type borrower's incentive constraint (C.4): pretending to be a higher type results in lower expected repayments. Consequently, the optimal contract features a binding incentive constraint (C.3) for type L borrower to now wish to accept a type H contract.

It must also be the case that (C.3) must be binding for type L's contract – otherwise, competitive lenders would deviate and reduce required repayments for type L contracts, which would also relax the incentive compatibility condition (C.4).

Maximizing the expected payoff to type L's contract while maintaining the break-even condition (C.3) for the lender results in an efficient contract derived in the previous section, given by $m^L = \frac{I}{\theta_L}$, and $c^L(R) = \frac{I}{R}$. A consequence of this derivation is that the type L contract features a make-whole clause at market rates resulting in efficient outside option exercise.

Type H optimal contract. Having characterized type L's contract, we can now show that the optimal contract for type H borrower features a make-whole clause at below market rates making it out of the money at the time of issuance. Given contract (m^L, c^L) characterized above, the binding incentive compatibility condition for type L borrower to be indifferent between contract L and contract H is

$$\int_R \left\{ \int^{\theta_L(y-m)+c(R)R} \theta_L(y-m) f(V)\, dV + \int_{\theta_L(y-m)+c(R)R} (V - c(R)R)\, f(V)\, dV \right\} g(R)\, dR \tag{7.4}$$
$$= \mathrm{E}\left[\max\{\theta_L y, V\}\right] - I.$$

It is convenient to denote by $\tau \stackrel{def}{=} c(R) \cdot R - \theta_L \cdot m(R)$ as the make-whole wedge for type L borrower if he were to choose type H's contract. It then follows from (C.5) that we can express the necessary repayment m for the high type as a function of the make-whole wedge $\hat{\tau}$:

$$m(\tau) = \frac{1}{\theta_L}\left[\int_R \left\{ \int_0^{\theta_L y+\tau} \theta_L y\, f(V) dV + \int_{\theta_L y+\tau}^{\infty} (V - \tau)\, f(V) dV \right\} g(R)\, dR - \mathrm{E}\left[\max\{\theta_L y, V\}\right] + I\right]. \tag{7.5}$$

At $\tau = 0$, obtain $m(0) = \frac{I}{\theta_L} = m^L > \frac{I}{\theta_H}$ and $c_H(\tau = 0) = \frac{\theta_L \cdot m(\tau=0)}{R} = \frac{I}{R}$. Note that (C.6) only depends on the realization of R to the extent that τ depends on R. Since τ is a contractual term, it is without loss to take it as constant in R but then ex-post verify that it the design problem is concave in τ. For this reason, we will take τ as constant in R, thus suppressing the outer integral in (C.6) with respect to R. The derivative of $m(\cdot)$ with respect to τ is

$$m'(\tau) = -\frac{1}{\theta_L} \cdot \left[1 - F(\theta_L y + \tau)\right] \leq 0.$$

Consequently, type H's repayment amount $m(\cdot)$ is decreasing in τ. The expected payoff to the high type borrower H from contract $m(\tau)$ given by (C.6), and $c(R) = \frac{1}{R}\left[\tau + \theta_L \cdot m(\tau)\right]$, is given by

$$\begin{aligned} G_H(m(\tau), \tau) \stackrel{def}{=} & \int_0^{\theta_H y - (\theta_H - \theta_L) m(\tau) + \tau} \theta_H (y - m(\tau))\, f(V) dV \\ & + \int_{\theta_H y - (\theta_H - \theta_L) m(\tau) + \tau}^{\infty} (V - \tau - \theta_L m(\tau))\, f(V) dV. \end{aligned} \tag{7.6}$$

Note that the uncertain interest rate R factors out in (C.7) as all terms are expressed in terms of present values. Differentiating (C.7) with respect to τ obtain

$$\frac{d}{d\tau} G_H(m(\tau), \tau) = \left[1 - F(\theta_H y - (\theta_H - \theta_L) m + \tau)\right] \cdot \left[-1 + 1 - F(\theta_L y + \tau)\right]$$

$$
\begin{aligned}
&+\frac{\theta_H}{\theta_L}\cdot F(\theta_H y-(\theta_H-\theta_L)m+\tau)\cdot[1-F(\theta_L y+\tau)]\\
&=\frac{\theta_H}{\theta_L}\cdot F(\theta_H y-(\theta_H-\theta_L)m+\tau)\cdot[1-F(\theta_L y+\tau)]\\
&-[1-F(\theta_H y-(\theta_H-\theta_L)m+\tau)]\cdot F(\theta_L y+\tau)\\
&\sim\frac{\theta_H}{\theta_L}\cdot\frac{F(\theta_H y-(\theta_H-\theta_L)m+\tau)}{1-F(\theta_H y-(\theta_H-\theta_L)m+\tau)}-\frac{F(\theta_L y+\tau)}{1-F(\theta_L y+\tau)}\\
&\geq\frac{\theta_H}{\theta_L}\cdot\frac{F(\theta_L y+\tau)}{1-F(\theta_L y+\tau)}-\frac{F(\theta_L y+\tau)}{1-F(\theta_L y+\tau)}>0,
\end{aligned}
$$

where the inequalities above follow from $m \leq y$. Inequality $\frac{dG_H}{d\tau} > 0$ implies that a higher make-whole wedge τ is preferable for type H borrower. This can be because of a lower incentive compatibility distortion, but also due to a reduction in the necessary repayment $m(\tau)$ to the lender. In what follows we show that it is disciplined by the lender's break-even constraint.

Denote by τ^* the unique solution to $\tau = (\theta_H - \theta_L) \cdot m(\tau)$. At $\tau = \tau^*$ it follows that $c(R) \cdot R = \theta_H \cdot m(R)$, implying that the type H borrower will pursue his outside option V only when it is socially efficient. By definition of $m(\tau)$ in (C.6), it follows that

$$
\begin{aligned}
\theta_L\cdot m\left(\tau^*\right)&=\int_0^{\theta_L y+\tau^*}\theta_L y\,dF(V)+\int_{\theta_L y+\tau^*}^{\infty}\left(V-\tau^*\right)\,dF(V)-\mathrm{E}\left[\max\{\theta_L y,V\}\right]+I\\
&=\int_0^{\theta_L y}\theta_L y_2\,dF(V)+\int_{\theta_L y}^{\theta_L y+\tau^*}\theta_L y\,dF(V)-\int_{\theta_L y}^{\theta_L y+\tau^*}\left(V-\tau^*\right)\,dF(V)\\
&+\int_{\theta_L y}^{\infty}\left(V-\tau^*\right)\,dF(V)-\mathrm{E}\left[\max\{\theta_L y,V\}\right]+I\\
&=\int_{\theta_L y}^{\theta_L y+\tau^*}\left[\theta_L y-V+\tau^*\right]\,dF(V)+I\leq\tau^*\cdot\left[F(\theta_L y+\tau^*)-F(\theta_L y)\right]+I\\
&<(\theta_H-\theta_L)\cdot m(\tau^*)+I\qquad\Rightarrow\qquad m(\tau^*)>\frac{I}{\theta_H}.
\end{aligned}
$$

Inequality $m(\tau^*) > \frac{I}{\theta_H}$ implies that at $\tau = (\theta_H - \theta_L) \cdot m(\tau)$, i.e., at $\tau_H = 0$, the incentive compatible contract features a slack creditor participation constraint (C.3) for type H borrower is, however, slack. A profit maximizing borrower then increases the prepayment penalty wedge $\tau > \tau^*$ until the creditor participation constraint (C.3) binds for the high type. This observation leads to the following result.

Proposition 10 (Optimal contract under asymmetric information)**.** *The optimal competitive lending* *contract features a make-whole clause at below-market rates for type H borrower and at precisely market rates for type L borrower.*

Proposition 10 establishes that make-whole provisions issued at below-market rates are carried out by better firms, as ranked by their unobservable private information. Intuitively, higher type borrowers find the outside option relatively less attractive to the current project and find it less costly to adopt more stringent, i.e., out of the money, make-whole provisions.

8 Conclusion

We have established that agency frictions in the form of either moral hazard and/or adverse selection can rationalize the ubiquity of make-whole provisions in bond and other debt contracts, despite the fact that those call options are almost always out of the money. Negative moneyness, we show, plays an essential role whenever conditioning contract terms directly on outside options is costly.

Far from being "incapable" of mitigating agency frictions between equity and debt-holders, make-whole options are in fact the optimal way to address these frictions, and it is not surprising, therefore, that most traded financial debt contracts now feature them.

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Appendix: Proofs and Additional Analysis (in progress)

A.0.1 Make-Whole Clause Example

The redemption prices will be calculated assuming a 360-day year consisting of twelve 30-day months. Our actions and determinations in determining the redemption prices shall be conclusive and binding for all purposes, absent manifest error. For purposes of calculating the redemption prices, the following terms will have the meanings set forth below.

"*Par Call Date*" means, in the case of the 2028 Notes, May 23, 2028 (the date that is one month prior to the stated maturity date for the 2028 Notes), and, in case of the 2033 Notes, October 9, 2032 (the date that is three months prior to the stated maturity date for the 2033 Notes).

"*Treasury Rate*" means, with respect to any redemption date, the yield determined by us in accordance with the following two paragraphs:

(1) the Treasury Rate shall be determined by us after 4:15 p.m., New York City time (or after such time as yields on U.S. government securities are posted daily by the Board of Governors of the Federal Reserve System), on the third Business Day preceding the redemption date based upon the yield or yields for the most recent day that appear after such time on such day in the most recent statistical release published by the Board of Governors of the Federal Reserve System designated as "Selected Interest Rates (Daily) - H.15" (or any successor designation or publication) ("H.15") under the caption "U.S. government securities–Treasury constant maturities–Nominal" (or any successor caption or heading) ("H.15 TCM"). In determining the Treasury Rate, we shall select, as applicable:

(i) the yield for the Treasury constant maturity on H.15 exactly equal to the period from the redemption date to the applicable Par Call Date (such period, the "*Remaining Life*");

(ii) if there is no such Treasury constant maturity on H.15 exactly equal to the Remaining Life, the two yields (one yield corresponding to the Treasury constant maturity on H.15 immediately shorter than, and one yield corresponding to the Treasury constant maturity on H.15 immediately longer than, the Remaining Life) and shall interpolate to the applicable Par Call Date on a straight-line basis (using the actual number of days) using such yields and rounding the result to three decimal places; or

(iii) if there is no such Treasury constant maturity on H.15 shorter than or longer than the Remaining Life, the yield for the single Treasury constant maturity on H.15 closest to the Remaining Life;

(for purposes of this paragraph (1), the applicable Treasury constant maturity or maturities on H.15 shall be deemed to have a maturity date equal to the relevant number of months or years, as applicable, of such Treasury constant maturity from the redemption date); or

(2) if on the third Business Day preceding the redemption date H.15 TCM is no longer published, we shall calculate the Treasury Rate based on the rate *per annum* equal to the semi-annual equivalent yield to maturity at 11:00 a.m., New York City time, on the second Business Day preceding such redemption date of the United States Treasury security maturing on, or with a maturity that is closest to, the applicable Par Call Date, as applicable. If there is no United States Treasury security maturing on the applicable Par Call Date, but there are two or more United States Treasury securities with a maturity date equally distant from the applicable Par Call Date, one with a maturity date preceding the applicable Par Call Date and one with a maturity date following the applicable Par Call Date, we shall select the United States Treasury security with a maturity date preceding the applicable Par Call Date. If there are two or more United States Treasury securities maturing on the applicable Par

S-11

Call Date, or two or more United States Treasury securities meeting the criteria of the preceding sentence, we shall select from among these two or more United States Treasury securities the United States Treasury security that is trading closest to par based upon the average of the bid and asked prices for such United States Treasury securities at 11:00 a.m., New York City time. In determining the Treasury Rate in accordance with the terms of this paragraph, the semi-annual yield to maturity of the applicable United States Treasury security shall be based upon the average of the bid and asked prices (expressed as a percentage of principal amount) at 11:00 a.m., New York City time, of such United States Treasury security, and rounded to three decimal places.

Figure 8: Selection of comparable treasury at redemption time. Sources and notes: This is drawn from SEC filing 333-268704 by General Motors, which covers two fixed-rate notes and a floating-rate note.

A.1 Static Moral Hazard with Continuous Effort

A.1.1 Continuous Effort Model without Shocks

First best. The first-best solution is attained when $w_1 = y$, which implies that $w_0 = \bar{w} = \frac{y^2}{2h}$. Consequently, it is without loss that $w_0 \in [0, \bar{w}] = \left[0, \frac{y^2}{2h}\right]$.

Second best. Consider a quadratic cost $h(a) = \frac{h}{2} \cdot a^2$. Then the creditor's expected value is

$$v(w_0) = a \cdot (y - w_1) \tag{A.1}$$

subject to the incentive compatibility (IC) and promise keeping (PK) constraints

$$\begin{cases} h'(a) = w_1 \qquad \Rightarrow \qquad a = \dfrac{w_1}{h} \\ \quad w_0 = a \cdot w_1 - h(a) \end{cases} \tag{A.2}$$

Binding conditions. Substituting the equilibrium effort from the IC constraint, obtain

$$w_0 = \frac{w_1^2}{h} - \frac{h}{2} \cdot \frac{w_1^2}{h^2} = \frac{w_1^2}{2h} \qquad \Rightarrow \qquad w_1 = \sqrt{2hw_0}, \quad a = \sqrt{\frac{2w_0}{h}}.$$

The expected payoff to the lender is

$$v(w_0) = a \cdot (y - w_1) = \sqrt{\frac{2w_0}{h}} \cdot y - 2w_0.$$

For $w \in [0, \bar{w}]$ we have

$$\frac{\partial}{\partial w_0} = \frac{1}{2} \cdot \frac{y}{\sqrt{2hw_0}} - 2.$$

The second derivative is

$$\frac{\partial^2 v}{\partial w_0^2} = -\frac{3}{4} \cdot \frac{y}{(2hw_0)^{3/2}}.$$

This implies that the Hessian is strictly concave with the second derivative bounded away from 0 for $w_0 \in [0, \bar{w}]$.

A.1.1.1 Lagrange Multipliers

Denote by μ_{IC} the Lagrange multiplier on the incentive constraint and μ_{PK} the Lagrange multiplier on the promise-keeping constraint. The Lagrangian is

$$\mathcal{L}(a, w_1) = a \cdot (y - w_1) + \mu_{IC} \cdot (w_1 - h \cdot a) + \mu_{PK} \cdot \left(a \cdot w_1 - \frac{h}{2} \cdot a^2 - w_0\right). \tag{A.3}$$

The Lagrangian first-order conditions are

$$\begin{cases} 0 = \dfrac{\partial \mathcal{L}}{\partial a} = y - w_1 - \mu_{IC} \cdot h + \mu_{PK} \cdot (w_1 - h \cdot a), \\ 0 = \dfrac{\partial \mathcal{L}}{\partial w_1} = -a + \mu_{IC} + \mu_{PK} \cdot a. \end{cases} \tag{A.4}$$

Substituting the optimal solution from the binding constraints $a = \sqrt{\frac{2w_0}{h}}$ and $w_1 = \sqrt{2hw_0}$ implies

$$\begin{cases} 0 = y - \sqrt{2hw_0} - \mu_{IC} \cdot h \qquad \Rightarrow \qquad \mu_{IC} = \dfrac{y - \sqrt{2hw_0}}{h} = \dfrac{y}{h} - \sqrt{\dfrac{2w_0}{h}}, \\ 0 = -\sqrt{\dfrac{2w_0}{h}} + \mu_{IC} + \mu_{PK} \cdot \sqrt{\dfrac{2w_0}{h}} = -\sqrt{\dfrac{2w_0}{h}} + \dfrac{y}{h} - \sqrt{\dfrac{2w_0}{h}} + \mu_{PK} \cdot \sqrt{\dfrac{2w_0}{h}} \end{cases} \tag{A.5}$$

Solving for μ_{PK} obtain

$$\mu_{IC} = \frac{y}{h} - \sqrt{\frac{2w_0}{h}}, \qquad \mu_{PK} = 2 - \frac{y}{\sqrt{2hw_0}}. \tag{A.6}$$

The incentive constraint multiplier μ_{IC} is decreasing in w_0. The promise-keeping Lagrange multiplier μ_{PK} is increasing in w_0. Note that the promise-keeping multiplier is negative for $w_0 < \frac{y^2}{8h}$ since the principal could increase his payoff by delivering the agent an ex-ante value of greater than w_0.

A.1.2 Probabilistic Liquidation Shock

Suppose the arrival of a liquidation opportunity occurs with probability λ. The principal's objective is

$$v(w_0) = \max_{a, w_1, l} \{a \cdot (y - w_1) \cdot [1 - \lambda + \lambda \cdot F(c + aw_1)] + c \cdot \lambda \cdot [1 - F(c + aw_1)]\}$$

subject to the incentive compatibility (IC) and promise keeping (PK) constraints

$$\begin{cases} h'(a) = w_1 \cdot [1 - \lambda + \lambda \cdot F(c + aw_1)] \\ \quad w_0 = aw_1 \cdot [1 - \lambda + \lambda \cdot F(c + aw_1)] + \lambda \cdot \displaystyle\int_{c + a \cdot w_1} (V - c) \, dF(V) - h(a) \end{cases} \tag{A.7}$$

If $w_1 = y$, then it is efficient to set $c = 0$. The resulting payoff is higher around $\bar{w}$, where we no longer have strict concavity.

Denote $l \stackrel{def}{=} c + a \cdot w_1$ as the equilibrium liquidation threshold. Then $c = l - aw_1$, and we can rewrite the principal's objective as

$$\begin{aligned} v(w_0) &= a \cdot (y - w_1) \cdot [1 - \lambda + \lambda \cdot F(l)] + (l - aw_1) \cdot \lambda \cdot [1 - F(l)] \\ &= ay \cdot [1 - \lambda + \lambda F(l)] - aw_1 + \lambda \cdot l[1 - F(l)]. \end{aligned}$$

subject to

$$\begin{cases} ha = w_1 \cdot [1 - \lambda + \lambda \cdot F(l)] \\ w_0 = aw_1 \cdot [1 - \lambda + \lambda \cdot F(l)] + \lambda \cdot \displaystyle\int_l (V - l + aw_1) \, dF(V) - \frac{h}{2} a^2 \\ \quad = aw_1 - \dfrac{h}{2} a^2 + \lambda \cdot \displaystyle\int_l (V - l) \, dF(V). \end{cases} \tag{A.8}$$

We can express the optimal effort from the IC constraint and substitute it into the PK constraint

$$\begin{cases} a = \dfrac{w_1}{h} \cdot [1 - \lambda + \lambda \cdot F(l)] \\ w_0 = \dfrac{w_1^2}{2h} \cdot [1 - \lambda + \lambda \cdot F(l)]^2 + \lambda \cdot \displaystyle\int_l (V - l)\, dF(V). \end{cases} \tag{A.9}$$

Solve for w_1 from the promise-keeping constraint to obtain

$$\frac{w_1^2}{2h} \cdot [1 - \lambda + \lambda \cdot F(l)]^2 = w_0 - \lambda \cdot \int_l (V - l)\, dF(V)$$

$$w_1 = \frac{\sqrt{2h \cdot \left(w_0 - \lambda \cdot \int_l (V - l)\, dF(V)\right)}}{1 - \lambda + \lambda \cdot F(l)}.$$

The constraint in the above condition is that $w_0 \geq \lambda \cdot \int_l (V - l)\, dF(V)$.

The principal's optimization objective can then be expressed as

$$\begin{aligned} v(w_0) &= \max_{a, w_1, l} \{ay \cdot [1 - \lambda + \lambda F(l)] - a w_1 + \lambda \cdot l[1 - F(l)]\} \\ &= \max_{w_1, l} \left\{ \frac{w_1}{h} \cdot [1 - \lambda + \lambda \cdot F(l)]^2 \cdot y - \frac{w_1^2}{h} \cdot [1 - \lambda + \lambda \cdot F(l)] + \lambda \cdot l[1 - F(l)] \right\} \\ &= \max_{l} \left\{ \sqrt{\frac{2\left[w_0 - \lambda \cdot \int_l (V - l)\, dF(V)\right]}{h}} \cdot [1 - \lambda + \lambda \cdot F(l)] \right. \\ &\qquad \left. - \frac{2 \cdot \left[w_0 - \lambda \cdot \int_l (V - l)\, dF(V)\right]}{1 - \lambda + \lambda \cdot F(l)} + \lambda \cdot l \cdot [1 - F(l)] \right\} \end{aligned}$$

Note that the objective is a concave function of w_0. The necessary promise-keeping constraint is

$$w_0 \geq \lambda \cdot \int_l (V - l)\, dF(V).$$

Lemma 11. *Function $v(w_0)$ is increasing in λ.*

A.1.2.1 Exponential distribution.

Suppose $V \sim Exp(\mu)$. If the PK constraint is not binding, then

$$v(w_0) = \max_{l} \left\{ \sqrt{\frac{2\left(w_0 - \frac{\lambda}{\mu} \cdot e^{-\mu l}\right)}{h}} \cdot \left[1 - \lambda + \lambda \cdot e^{-\mu l}\right] - \frac{2 \cdot \left(w_0 - \frac{\lambda}{\mu} \cdot e^{-\mu l}\right)}{1 - \lambda + \lambda \cdot e^{-\mu l}} + \lambda \cdot l \cdot e^{-\mu l} \right\}$$

Computing the Hessian

$$\frac{\partial v}{\partial w_0} = \frac{1}{\sqrt{2h\left(w_0 - \frac{\lambda}{\mu} \cdot e^{-\mu l}\right)}} \cdot [1 - \lambda + \lambda \cdot e^{-\mu l}] - \frac{2}{1 - \lambda + \lambda \cdot e^{-\mu l}}$$

$$\frac{\partial^2 v}{\partial w_0^2} = -\frac{1}{2} \cdot \frac{1}{\left[2h\left(w_0 - \frac{\lambda}{\mu} \cdot e^{-\mu l}\right)\right]^{3/2}} \cdot [1 - \lambda + \lambda \cdot e^{-\mu l}]$$

$$\frac{\partial^2 v}{\partial w_0 \partial l} = -\frac{1}{2} \cdot \frac{-\lambda \cdot e^{-\mu l}}{\left[2h\left(w_0 - \frac{\lambda}{\mu} \cdot e^{-\mu l}\right)\right]^{3/2}} \cdot [1 - \lambda + \lambda \cdot e^{-\mu l}] - \frac{\lambda \mu e^{-\mu l}}{\left[2h\left(w_0 - \frac{\lambda}{\mu} \cdot e^{-\mu l}\right)\right]^{3/2}} - \frac{2\lambda \mu e^{-\mu l}}{(1 - \lambda + \lambda e^{-\mu l})^2}$$

$$\frac{\partial v}{\partial l} = \frac{2\lambda e^{-\mu l}}{\sqrt{2h\left(w_0 - \frac{\lambda}{\mu} \cdot e^{-\mu l}\right)}} \cdot \left[1 - \lambda + \lambda \cdot e^{-\mu l}\right] - \lambda \mu e^{-\mu l} \cdot \sqrt{\frac{2\left(w_0 - \frac{\lambda}{\mu} \cdot e^{-\mu l}\right)}{h}}$$

$$- \frac{2\lambda e^{-\mu l}}{1 - \lambda + \lambda \cdot e^{-\mu l}} - \frac{2\lambda \mu e^{-\mu l} \cdot \left(w_0 - \frac{\lambda}{\mu} e^{-\mu l}\right)}{(1 - \lambda + \lambda \cdot e^{-\mu l})^2} + \lambda e^{-\mu l} - \lambda \mu \cdot l \cdot e^{-\mu l}.$$

$$\frac{\partial^2 v}{\partial l^2} = \frac{-2\lambda \mu e^{-\mu l}}{\sqrt{2h\left(w_0 - \frac{\lambda}{\mu} \cdot e^{-\mu l}\right)}} \cdot \left[1 - \lambda + \lambda \cdot e^{-\mu l}\right] - \frac{2\lambda^2 \cdot \mu e^{-2\mu l}}{\sqrt{2h\left(w_0 - \frac{\lambda}{\mu} \cdot e^{-\mu l}\right)}}$$

$$- \frac{2\lambda e^{-\mu l} \cdot 2h\lambda e^{-\mu l}}{\left[2h\left(w_0 - \frac{\lambda}{\mu} \cdot e^{-\mu l}\right)\right]^{3/2}} + \lambda \mu^2 e^{-\mu l} \cdot \sqrt{\frac{2\left(w_0 - \frac{\lambda}{\mu} \cdot e^{-\mu l}\right)}{h}} - \frac{\lambda^2}{\mu^2} \cdot 2\mu e^{-\mu l} \cdot \frac{1}{\sqrt{2h\left(w_0 - \frac{\lambda}{\mu} \cdot e^{-\mu l}\right)}}$$

$$+ \frac{2\lambda \mu \cdot e^{-\mu l}}{1 - \lambda + \lambda e^{-\mu l}} - \frac{2\lambda^2 \cdot e^{-2\mu l}}{(1 - \lambda + \lambda \cdot e^{-\mu l})^2} + \frac{-2\lambda \mu^2 e^{-\mu l} \cdot w_0 + 2\lambda^2 \mu \cdot e^{-2\mu l}}{(1 - \lambda + \lambda \cdot e^{-\mu l})^2}$$

$$- \frac{4\lambda^2 \mu^2 e^{-2\mu l}\left(w_0 - \frac{\lambda}{\mu} e^{-\mu l}\right)}{(1 - \lambda + \lambda \cdot e^{-\mu l})^3} - 2\lambda \mu \cdot e^{-\mu l} + \lambda \mu^2 \cdot l e^{-\mu l}.$$

Note that

$$\frac{\partial^2 v}{\partial l^2} = \lambda \cdot \frac{\partial^2 v}{\partial l^2}(w_0, 0) + O(\lambda^2).$$

We can compute

$$\frac{1}{\lambda} \frac{\partial^2 v}{\partial l^2}(w_0, 0) = -\frac{-2\mu e^{-\mu l}}{\sqrt{2hw_0}} + \mu^2 e^{-\mu l} \cdot \sqrt{\frac{2w_0}{h}} + 2\mu \cdot e^{-\mu l} - 2\mu e^{-\mu l} \cdot w_0 - 2\mu \cdot e^{-\mu l} + \mu^2 \cdot l e^{-\mu l}$$

$$= \frac{2\mu e^{-\mu l}}{\sqrt{2hw_0}} + \mu^2 e^{-\mu l} \cdot \sqrt{\frac{2w_0}{h}} - 2\mu^2 e^{-\mu l} w_0 + \mu^2 \cdot l \cdot e^{-\mu l}.$$

In our situation, we have $\frac{\partial^2 v}{\partial w_0} < 0$ while all other second-order derivatives are of the order $O(\lambda)$.

A.1.2.2 Lagrangian at $\lambda = 0$

The Lagrangian can then be written as

$$\mathcal{L}(a, w_1, l) = a(y - w_1) \cdot [1 - \lambda + \lambda \cdot F(l)] + (l - aw_1) \cdot \lambda \cdot [1 - F(l)]$$

$$+\mu_{IC}\cdot[w_1\cdot(1-\lambda+\lambda\cdot F(l))-h\cdot a]$$
$$+\mu_{PK}\cdot\left[aw_1\cdot[1-\lambda+\lambda F(l)]+\lambda\cdot\int_l V\,dF(V)-\frac{h}{2}a^2-w_0\right]$$

The first order conditions are

$$0=\frac{\partial\mathcal{L}}{\partial a}=(y-w_1)\cdot[1-\lambda+\lambda\cdot F(l)]-w_1\cdot\lambda\cdot[1-F(l)]-h\cdot\mu_{IC}$$
$$0=\frac{\partial\mathcal{L}}{\partial w_1}=-a\cdot[1-\lambda+\lambda F(l)]-a\cdot\lambda[1-F(l)]+\mu_{IC}\cdot[1-\lambda+\lambda F(l)]+\mu_{PK}\cdot a\cdot[1-\lambda+\lambda F(l)]$$
$$0=\frac{\partial\mathcal{L}}{\partial l}=a(y-w_1)\cdot\lambda f(l)+\lambda[1-F(l)]-(l-aw_1)\cdot\lambda f(l)+\mu_{IC}\cdot w_1\cdot\lambda f(l)+\mu_{PK}\cdot\lambda\cdot(aw_1-l)\cdot f(l).$$

The first-order optimality condition in l is homogeneous in λ, implying

$$\begin{aligned}0&=a(y-w_1)\cdot f(l)+1-F(l)-(l-aw_1)\cdot f(l)+\mu_{IC}\cdot w_1\cdot f(l)+\mu_{PK}\cdot(aw_1-l)f(l)\\&\sim ay+\frac{1-F(l)}{f(l)}-l+\mu_{IC}\cdot w_1+\mu_{PK}\cdot(aw_1-l)\\&=ay+\mu_{IC}\cdot w_1+\mu_{PK}\cdot aw_1+\frac{1-F(l)}{f(l)}+(\mu_{PK}+1)\cdot l.\end{aligned}\tag{A.10}$$

Substitute the local solutions for μ_{IC} and μ_{PK} at $\lambda=0$ to obtain

$$0=\sqrt{\frac{2w_0}{h}}\cdot y+\left(\frac{y}{h}-\sqrt{\frac{2w_0}{h}}\right)\cdot\sqrt{2hw_0}+\frac{1-F(l)}{f(l)}-\left(3-\frac{y}{\sqrt{2hw_0}}\right)\cdot l$$
$$0=\underbrace{2\sqrt{\frac{2w_0}{h}}\cdot y-2w_0+\frac{1-F(l)}{f(l)}}_{\geq 0\ \forall\ w_0\in[0,\bar{w}]}+\left(\frac{y}{\sqrt{2hw_0}}-3\right)\cdot l \tag{A.11}$$

If w_0 is low, then there is no positive solution to the problem above. If w_0 is large, then there exists an interior solution.

If $\mu_{PK}+1\leq 0$, captured by $\sqrt{2hw_0}\leq\frac{y}{3}$, then (A.11) is positive for all l, implying that it is optimal to set $l=+\infty$. The intuition is that the promise-keeping constraint is too tight, and it is optimal to increase l to allow for more effort.

Suppose $\sqrt{2hw_0}>\frac{y}{3}$, then there exists an l such that (A.11) is satisfied with equality.

Uniform distribution. Suppose $V\sim U[0,\bar{v}]$. For $l\notin[0,\bar{v}]$ the Lagrangian is constant in l. Suppose $\sqrt{2hw_0}>\frac{y}{3}$. Then

$$0=\sqrt{\frac{2w_0}{h}}\cdot(y-1)+\frac{y}{h}+\frac{1-l/\bar{v}}{1/\bar{v}}+\left(\frac{y}{\sqrt{2hw_0}}-3\right)\cdot l$$
$$0=\sqrt{\frac{2w_0}{h}}\cdot(y-1)+\frac{y}{h}+\bar{v}-l+\left(\frac{y}{\sqrt{2hw_0}}-3\right)\cdot l$$
$$0=\sqrt{\frac{2w_0}{h}}\cdot(y-1)+\frac{y}{h}+\bar{v}+\left(\frac{y}{\sqrt{2hw_0}}-4\right)\cdot l$$

$$l = \frac{\sqrt{\frac{2w_0}{h}} \cdot (y-1) + \frac{y}{h} + \bar{v}}{4 - \frac{y}{\sqrt{2hw_0}}}.$$

Since there is an additional constraint that $l \leq \bar{v}$, it implies that

$$l = \min\left\{ \frac{\sqrt{\frac{2w_0}{h}} \cdot (y-1) + \frac{y}{h} + \bar{v}}{4 - \frac{y}{\sqrt{2hw_0}}}, \quad \bar{v} \right\}.$$

Exponential distribution. Suppose $V \sim Exp(\mu)$. Then the optimal threshold is always finite whenever $\sqrt{2hw_0} > \frac{y}{3}$:

$$0 = \sqrt{\frac{2w_0}{h}} \cdot (y-1) + \frac{y}{h} + \frac{1}{\mu} + \left(\frac{y}{\sqrt{2hw_0}} - 3 \right) \cdot l$$

$$l = \frac{\sqrt{\frac{2w_0}{h}} \cdot (y-1) + \frac{y}{h} + \frac{1}{\mu}}{3 - \frac{y}{\sqrt{2hw_0}}}.$$

This solution is always interior due to unbounded upper support. Note that it could be infinite. Given that $l = c + a \cdot w_1$ we have

$$c(w_0) = l(w_0) - a(w_0) \cdot w_1(w_0) = \frac{\sqrt{\frac{2w_0}{h}} \cdot (y-1) + \frac{y}{h} + \frac{1}{\mu}}{3 - \frac{y}{\sqrt{2hw_0}}} - 2w_0.$$

A.1.2.3 Concavity Preservation

I want to show that $v(w_0, \lambda)$ is concave for sufficiently small λ. To show this, it is sufficient to show that $\frac{\partial v}{\partial w_0}(w_0, \lambda)$ is decreasing in λ. To establish this, it is sufficient to show that

$$\frac{\partial^3 v}{\partial^2 w_0 \partial \lambda}(w_0, \lambda)$$

is uniformly bounded in w_0 and λ for sufficiently low λ.

Hypothesis 12. *The controls a, l, w_1 are $C^2(w_0, \lambda)$.*

We know that

$$\frac{\partial \mathcal{L}}{\partial w_0} = -\mu_{PK}.$$

To establish concavity, we need to show that μ_{PK} is decreasing in w_0 for λ sufficiently low. Taking $\lambda = 0$ is not sufficient unless we establish some sort of continuity in λ.

The principal's problem is continuous in λ. Using the Envelope theorem, the derivative with respect to λ of the Lagrangian equals[12]

$$\frac{\partial \mathcal{L}}{\partial \lambda} = a(y - w_1) \cdot [F(l) - 1] + (l - aw_1) \cdot [1 - F(l)] + \mu_{IC} \cdot w_1 \cdot [F(l) - 1] + \mu_{PK} \cdot \int_l V \, dF(V)$$

12

$$= (l - ay - \mu_{IC} \cdot w_1) \cdot [1 - F(l)] + \mu_{PK} \cdot \int_l V \, dF(V)$$
$$= \frac{[1 - F(l)]^2}{f(l)} - \mu_{PK} \cdot l \cdot [1 - F(l)] + \mu_{PK} \cdot \int_l V \, dF(V)$$
$$= \frac{[1 - F(l)]^2}{f(l)} + \mu_{PK} \cdot \int_l (V - l) \, dF(V)$$

With the exponential distribution, the above derivative can be simplified to

$$\frac{\partial \mathcal{L}}{\partial \lambda} = e^{-\mu l} \cdot \frac{1}{\mu} + \mu_{PK} \cdot \frac{1}{\mu} \cdot e^{-\mu l} = \frac{1}{\mu} \cdot e^{-\mu l} \cdot [1 + \mu_{PK}].$$

The derivative with respect to w_0 is equal to

This implies that The derivative with respect to w_0 is equal to

$$-\frac{\partial \mu_{PK}}{\partial \lambda} = \frac{\partial^2 \mathcal{L}}{\partial \lambda \partial w_0} = \frac{\partial}{\partial w_0} \left[e^{-\mu l} \cdot \left[l - ay - \mu_{IC} \cdot w_1 + \mu_{PK} \cdot \left(l + \frac{1}{\mu} \right) \right] \right]$$

I would only need to establish that $\frac{\partial^2 \mu_{PK}}{\partial \lambda \partial w_0}$ is bounded above. The challenge is, again, that I would need to do so at the interior solutions.

A.2 Continuous Time Model [ongoing analysis]

Suppose effort cost $h(a) = h \cdot a^2/2$. The project generates flow output y per unit of time. The project is terminated with intensity $1 - a_t$ in period t following the arrival of the counting process $N = (N_t)_{t \geq 0}$. The outside value V arrives with exogenous intensity λ and is distributed according to $F(\cdot)$.

A.2.1 Agent First Best

Suppose the agent receives all cash flows from the project. His continuation value $\bar{w}$ implies an effort level $\bar{a} = \frac{\bar{w}}{h}$. The expected value to the project is then given by

$$\bar{w} = \frac{y - \frac{\bar{w}^2}{2h} + \lambda \cdot \int_{\bar{w}} v \, dF(v)}{r + \lambda \cdot [1 - F(\bar{w}] + 1 - \frac{\bar{w}}{h}}. \tag{B.1}$$

At $\lambda = 0$ have

$$\bar{w} = \frac{y - \frac{\bar{w}^2}{2h}}{r + 1 - \frac{\bar{w}}{h}}.$$

At $\bar{w} = 0$, the right-hand side of the above expression is negative. For $\bar{w} < h$ we must have

$$0 < -y + (1 + r) \cdot h - \frac{h^2}{2h} - \lambda \cdot \left(\int_h v \, dF(v) - h \right)$$
$$= -y + \left(\frac{1}{2} + r + \lambda \right) \cdot h - \lambda \int_h v \, dF(v)$$
$$\Rightarrow \qquad y < \left(\frac{1}{2} + r + \lambda \right) \cdot h - \lambda \int_h v \, dF(v). \tag{B.2}$$

A.2.2 Agent's Problem

Consider a contract $\mathcal{C} = \{(a_t, w_t, c_t)_{t\geq 0}\}$. The agent's continuation value follows

$$dw_t = rw_t\,dt + h(a_t)\,dt - w_t \cdot [dN_t - (1-a_t)\,dt] + \left(\int_{c_t+w_t} (v - c_t - w_t)\,dF(v)\right) \cdot [d\Lambda_t - \lambda\,dt], \quad \text{(B.3)}$$

where we accounted for the fact that the agent receives 0 when process N arrives. The local incentive compatibility condition then reads as

$$h'(a_t) = w_t \quad \Rightarrow \quad a_t = \frac{w_t}{h}. \quad \text{(B.4)}$$

Note that the arrival of outside liquidation opportunities does not affect (B.4). Substituting (B.4) into (B.3) we have

$$dw_t = rw_t\,dt + \frac{w_t^2}{2h}\,dt - w_t \cdot \left[dN_t - \left(1 - \frac{w_t}{h}\right)\,dt\right] + \left(\int_{c_t+w_t} (v - c_t - w_t)\,dF(v)\right) \cdot [d\Lambda_t - \lambda\,dt].$$

Along the path of no arrivals $dN_t = d\Lambda_t = 0$, the continuation value process follows

$$\dot{w}_t = (1+r) \cdot w_t - \frac{w_t^2}{2h} - \lambda \cdot \left(\int_{c_t+w_t} (v - c_t - w_t)\,dF(v)\right). \quad \text{(B.5)}$$

If $\lambda = 0$ then the continuation value is increasing since $w_t \leq h$ for any $w_t \leq \bar{w}$ where $\bar{w}$ was defined by (B.1).

Suppose $c_t \geq 0$ and $\dot{w}_t = 0$. Then

$$\begin{aligned} 0 &= rw_t + \frac{w_t^2}{2h} + w_t \cdot \left(1 - \frac{w_t}{h}\right) - \lambda \cdot \left(\int_{c_t+w_t} (v - c_t - w_t)\,dF(v)\right) \\ &\leq y - y + rw_t + \frac{w_t^2}{2h} + w_t \cdot \left(1 - \frac{w_t}{h}\right) - \lambda \cdot \left(\int_{w_t} (v - w_t)\,dF(v)\right) \end{aligned}$$

A.2.3 Principal's Problem

Given contract $\mathcal{C} = \{a, w, c\}$, the principal's value function satisfies

$$\begin{aligned} rv(w) &= a \cdot y - v(w) \cdot (1-a) + [c - v(w)] \cdot \lambda \cdot \mathrm{P}(V > c + w) \\ &\quad + v'(w) \cdot \left[rw + h(a) + w \cdot (1-a) - \lambda \cdot \int_{c+w} (v - c - w)\,dF(v)\right] \end{aligned}$$

Substituting $a = \frac{w}{h}$ from (B.4) the above simplifies to

$$\begin{aligned} rv(w) &= \frac{w}{h} \cdot y - v(w) \cdot \left(1 - \frac{w}{h}\right) + [c - v(w)] \cdot \lambda \cdot [1 - F(c+w)] \\ &\quad + v'(w) \cdot \left[rw + \frac{w^2}{2h} + w \cdot \left(1 - \frac{w}{h}\right) - \lambda \cdot \int_{c+w} (v - c - w)\,dF(v)\right] \end{aligned} \quad \text{(B.6)}$$

Prepayment c is optimal if it maximizes the right-hand side of (B.6). The first order condition with respect to c is given by

$$\begin{aligned} 0 &= \lambda[1 - F(c+w)] - [c - v(w)] \cdot \lambda f(c+w) + v'(w) \cdot \lambda[1 - F(c+w)] \\ &\sim -[c - v(w)] \cdot f(c+w) + [v'(w)+1] \cdot [1 - F(c+w)] \\ \Rightarrow \qquad c(w) &= v(w) + \frac{1 - F(c(w)+w)}{f(c(w)+w)} \cdot [v'(w)+1]. \end{aligned} \tag{B.7}$$

The optimal initial condition requires that $v'(w_0^*) = 0$. Substituting (B.7) obtain

$$rv(w_0^*) = \frac{w_0^*}{h} \cdot y - v(w_0^*) \cdot \left(1 - \frac{w_0^*}{h}\right) + \lambda \cdot \frac{[1 - F(c(w_0^*) + w_0^*)]^2}{f(c(w_0^*) + w_0^*)}.$$

Lemma 13. *The make-whole premium is always positive:* $c(w_t) \geq v(w_t)$.

Note that $v(w)$ is decreasing in w for $w \geq w_0^*$ and $v'(w) + 1$ is positive and decreasing due to concavity of the value function. Consequently, if $\frac{1-F(x)}{f(x)}$ is decreasing in x, then $c(w)$ is decreasing in w.

From here, we see that c_t exceeds the expected value of the contract to the lender, i.e., $c_t \geq v(w_t)$.

Denote by $g(x)$ to be the hazard rate of $F(\cdot)$:

$$g(x) = \frac{f(x)}{1 - F(x)}$$

Then the fixed point problem is given by

$$c(w) = v(w) + \frac{1}{g(c(w)+w)} \cdot [v'(w)+1].$$

Suppose $c'(w) = 0$. Then

$$\begin{aligned} 0 &= v'(w) - \frac{g'(c(w)+w)}{g(c(w)+w)^2} \cdot [v'(w)+1] + \frac{1}{g(c(w)+w)} \cdot v''(w) \\ &< v'(w) - \frac{g'(c(w)+w)}{g(c(w)+w)^2} \cdot [v'(w)+1]. \end{aligned}$$

A.2.4 Prepayment Penalty Dynamics

Consider the identity

$$c(w) = v(w) + g(c(w)+w) \cdot [v'(w)+1].$$

Suppose $c'(w) = 0$. Then

$$0 = v'(w) + g'(c(w)+w) \cdot [v'(w)+1] + g(c(w)+w_t) \cdot v''(w) \geq v'(w) + g'(c(w)+w) \cdot [v'(w)+1].$$

The necessary condition for this is $g'(c(w_t)+w_t) > 0$.

Suppose $V \sim Exp(\mu)$. Then

$$c(w_t) = v(w_t) + \frac{1}{\mu} \cdot [v'(w_t)+1]$$

The derivative of $c(w_t)$ with respect to t is then given by

$$c'(w_t) = v'(w_t) + \frac{1}{\mu} \cdot v''(w_t) \leq 0,$$

since $v(w_t)$ is decreasing and concave for $w_t \geq w_0^*$.

A.3 Two Type Adverse Selection Model

Suppose $\theta \in \{\theta_L, \theta_H\}$ – the probability that the project succeeds in period $t = 2$ and generates y. The distribution of types is $\mathrm{P}(\theta = \theta_H) = \pi$. The project generates no cash flow at $t = 1$. The agent knows his type at $t = 0$. A contract is (m, c) a repayment amount m in the event of the project's success in period $t = 2$ and a severance penalty c in the event of early project termination at $t = 1$. The project requires an up-front investment $I \geq 0$ and the agent has an outside option $K \geq 0$.

The agent learns his outside option V at $t = 1$, distributed according to a cdf $F(\cdot)$. Interest rate $R > 0$ is assumed to be constant, and the agent can borrow/lend freely at this rate. The expected value to agent $i \in \{L, H\}$ from contract $\mathcal{C} = (m, c)$ is

$$G_i(m) = \int \max\{\theta_i(y - m),\ V - cR\}\, dF(V).$$

The agent chooses his outside option whenever

$$V - cR \geq \theta_i \cdot (y - m) \qquad \Leftrightarrow \qquad V \geq \theta_i \cdot (y - m) + cR.$$

The expected value of a contract (m, c) to the agent of type $i \in \{L, H\}$ is

$$G_i(m, c) \stackrel{def}{=} \int_0^{\theta_i(y-m)+cR} \theta_i(y - m)\, dF(V) + \int_{\theta_i(y-m)+cR}^{\infty} (V - cR)\, dF(V).$$

Full Information Efficient Contracts. An efficient contract maximizes joint surplus, resulting in

$$\max_{m,c} \left\{ \int_0^{\theta_i(y-m)+cR} \theta y\, dF(V) + \int_{\theta_i(y-m)+cR}^{\infty} V\, dF(V) \right\}. \tag{C.1}$$

Differentiating this expression with respect to c obtain

$$R \cdot [\theta_i \cdot y] \cdot f(\theta_i(y - m) + cR) - R \cdot [\theta_i \cdot (y - m) + cR] \cdot f(\theta_i(y - m) + cR) = 0$$
$$\theta_i \cdot y - [\theta_i \cdot (y - m) + cR] = 0 \qquad \Leftrightarrow \qquad c_i^* = \frac{\theta_i \cdot m}{R}.$$

Given a contract, an efficient prepayment penalty has to satisfy $c_i^* = \frac{\theta_i \cdot m_i}{R}$.

The lowest cost financing for the debtor results in $\theta_i \cdot m_i = I$, where I is the up-front cost of the project. Consequently, a full information efficient contract features $c_i^* = \frac{I}{R}$ and $m_i^* = \frac{I}{\theta_i}$. The incentive compatibility problem is that a lower θ type strictly prefers a higher θ contract.

A.3.1 Competitive Creditors: Lowest Cost Separating Contract

Consider now a case in which the agent has private information about the borrower's project $\theta \in \{\theta_L, \theta_H\}$. Either project generates cash flow y_1 at $t = 1$, however the higher quality project generates cash flow y_2 with the higher probability $\theta_H > \theta_L$. A contract is still a pair (m_1, m_2, c) stipulating the borrower's repayment in both periods and the prepayment penalty c, where both $m_2(\cdot)$ and $c(\cdot)$ are functions of the (contractable) interest rate R in period $t = 1$.

It is optimal for the borrower of type θ to pursue his outside option V given interest rate R if and only if the expected value to the outside option net of the prepayment penalty exceeds the expected value of staying with the project:

$$V - c(R) \cdot R \geq \theta \cdot (y_2 - m_2(R)).$$

It is worthwhile to note here that better, i.e., higher θ, borrowers have a higher threshold on their outside option to break the contract. The borrower's expected value from optimally exiting the contract is given By

$$\max\{V - c(R) \cdot R, \theta \cdot (y_2 - m_2(R))\}.$$

Lenders compete via the offered contracts – one can think of each lender posting a menu of two contracts. An equilibrium then consists of contracts $m^L = (m_1^L, m_2^L, c^L)$ and $m^H = (m_1^H, m_2^H, c^H)$ offered to type L and H borrowers respectively. Lender competition requires that the contract to borrower $i \in \{L, H\}$ maximizes the expected payoff to this borrower

$$\max_{m_1^i, m_2^i, c^i(\cdot)} \left\{ y_1 - m_1^i + \mathrm{E}\left[\max\{\theta_i \cdot (y_2 - m_2^i(R)),\ V - c^i(R) \cdot R\}\right] \right\} \tag{C.2}$$

subject to the lender's break-even constraint

$$m_1^i + \int_R \left\{ \int_0^{\theta_i(y_2 - m_2^i(R)) + c^i(R)R} m_2^i(R) f(V) dV + \int_{\theta_i(y_2 - m_2^i(R)) + c^i(R)R}^{+\infty} c^i(R) R f(V) dV \right\} g(R) dR = I \tag{C.3}$$

and the incentive compatibility condition for borrower $j \neq i$ to prefer contract j to contract i:

$$\begin{aligned} & y_1 - m_1^j + \mathrm{E}\left[\max\left\{\theta_j \cdot (y_2 - m_2^j(R)),\ V - c^j(R)R\right\}\right] \\ \geq & y_1 - m_1^i + \mathrm{E}\left[\max\left\{\theta_j \cdot (y_2 - m_2^i(R)),\ V - c^i(R)R\right\}\right]. \end{aligned} \tag{C.4}$$

Type L optimal contract. First, note that the incentive constraint (C.4) must be binding for the low type borrower, i.e., $j = L$ and $i = H$. Otherwise, the optimal contract menu would be pinned down by the lender's break-even constraints (C.3) for both $i = L$ and $i = H$ borrowers. In the latter case, the optimal contract would correspond to an efficient full information contract, given by $m_1 = y_1$ and $c^i(R) = \theta_i \cdot (I - m_1)/R$. It can be verified, however, that such a pair of contracts violates the low-type borrower's incentive constraint (C.4): pretending to be a higher type results in lower expected repayments. Consequently, the optimal contract features a binding incentive constraint (C.3) for type L borrower to now wish to accept a type H contract.

It must also be the case that (C.3) must be binding for type L's contract – otherwise, competitive lenders would deviate and reduce required repayments for type L contracts, which would also relax the incentive compatibility condition (C.4).

Maximizing the expected payoff to type L's contract while maintaining the break-even condition (C.3) for the lender results in an efficient contract derived in the previous section, given by $m_1^L = y_1$, $m_2^L = \frac{I-y_1}{\theta_L}$, and $c^L(R) = \frac{I-y_1}{R}$. A consequence of this derivation is that the type L contract features a make-whole clause at market rates resulting in efficient outside option exercise.

Type H optimal contract. Having characterized type L's contract, we can now show that the optimal contract for type H borrower features a make-whole clause at below market rates making it, effectively, out of the money at the time of issuance. Given contract (m_1^L, m_2^L, c^L) characterized above, the binding incentive compatibility condition for type L borrower to be indifferent between contract L and contract H is

$$m_1 + \int_R \left\{ \int^{\theta_L(y-m_2)+c(R)R} \theta_L(y-m_2) f(V)\, dV + \int_{\theta_L(y-m_2)+c(R)R} (V - c(R)R)\, f(V)\, dV \right\} g(R)\, dR$$
$$= y_1 + \mathrm{E}\left[\max\{\theta_L y, V\}\right] - I. \tag{C.5}$$

It is convenient to denote by $\tau \stackrel{def}{=} c(R)\cdot R - \theta_L \cdot m_2(R)$ as the make-whole wedge for type L borrower if he were to choose type H's contract. It is without loss to set $m_1 = y_1$ in (C.5). It then follows from (C.5) that we can express the necessary repayment m_2 for the high type as a function of the make-whole wedge $\hat{\tau}$:

$$m_2(\tau) = \frac{1}{\theta_L}\left[\int_R \left\{ \int_0^{\theta_L y_2+\hat{\tau}} \theta_L y_2\, f(V) dV + \int_{\theta_L y_2+\hat{\tau}}^{\infty} (V-\tau)\; f(V) dV \right\} g(R)\, dR - \mathrm{E}\left[\max\{\theta_L y_2, V\}\right] + I\right]. \tag{C.6}$$

At $\tau = 0$, obtain $m_2(0) = \frac{I-y_1}{\theta_L} = m_2^L > \frac{I-y_1}{\theta_H}$ and $c_H(\tau=0) = \frac{\theta_L \cdot m_2(\tau=0)}{R} = \frac{I-y_1}{R}$. The derivative of $m_2(\cdot)$ with respect to τ is

$$m_2'(\tau) = -\frac{1}{\theta_L}\cdot\left[1 - F(\theta_L y_2 + \tau)\right] \leq 0.$$

Consequently, type H's repayment amount $m_2(\cdot)$ is decreasing in τ. The expected payoff to the high type borrower H from contract $m_1 = y_1$, $m_2(\tau)$ given by (C.6), and $c(R) = \frac{1}{R}\left[\tau + \theta_L \cdot m_2(\tau)\right]$, is given by

$$\begin{aligned} G_H(m_2(\tau), \tau) = &\int_0^{\theta_H y_2 - (\theta_H-\theta_L)m_2(\tau)+\tau} \theta_H(y - m_2(\tau))\, f(V) dV \\ &+ \int_{\theta_H y_2 - (\theta_H-\theta_L)m_2(\tau)+\tau}^{\infty} (V - \tau - \theta_L m_2(\tau))\, f(V) dV. \end{aligned} \tag{C.7}$$

Note that the uncertain interest rate R factors out in (C.7) as all terms are expressed in terms of present values. Differentiating (C.7) with respect to τ obtain

$$\begin{aligned} \frac{d}{d\tau} G_H(m_2(\tau), \tau) = &\left[1 - F(\theta_H y_2 - (\theta_H-\theta_L)m_2 + \tau)\right]\cdot\left[-1 + 1 - F(\theta_L y_2 + \tau)\right] \\ &+ \frac{\theta_H}{\theta_L}\cdot F(\theta_H y_2 - (\theta_H-\theta_L)m_2 + \tau)\cdot\left[1 - F(\theta_L y_2 + \tau)\right] \end{aligned}$$

$$
\begin{aligned}
&= \frac{\theta_H}{\theta_L} \cdot F(\theta_H y_2 - (\theta_H - \theta_L)m_2 + \tau) \cdot [1 - F(\theta_L y_2 + \tau)] \\
&- [1 - F(\theta_H y_2 - (\theta_H - \theta_L)m_2 + \tau)] \cdot F(\theta_L y_2 + \tau) \\
&\sim \frac{\theta_H}{\theta_L} \cdot \frac{F(\theta_H y_2 - (\theta_H - \theta_L)m_2 + \tau)}{1 - F(\theta_H y_2 - (\theta_H - \theta_L)m_2 + \tau)} - \frac{F(\theta_L y_2 + \tau)}{1 - F(\theta_L y_2 + \tau)} \\
&\geq \frac{\theta_H}{\theta_L} \cdot \frac{F(\theta_L y_2 + \tau)}{1 - F(\theta_L y_2 + \tau)} - \frac{F(\theta_L y_2 + \tau)}{1 - F(\theta_L y_2 + \tau)} > 0,
\end{aligned}
$$

where the inequalities above follow from $m_2 \leq y$. Inequality $\frac{d\hat{G}_H}{d\tau} > 0$ implies that a higher make-whole wedge τ makes type H borrower better off. This can be because of a lower incentive compatibility distortion, but also due to a reduction in the necessary repayment $m_2(\tau)$ to the lender.

Denote by τ^* the unique solution to $\tau = (\theta_H - \theta_L) \cdot m_2(\tau)$. At $\tau = \tau^*$ it follows that $c(R) \cdot R = \theta_H \cdot m_2(R)$, implying that the type H borrower will exit the contract efficiently. Social surplus $W(\tau)$ (the sum of the borrower and lender's expected payoffs) is increasing in $\tau < \tau^*$ and decreasing otherwise. By definition of $m_2(\tau)$ in (C.6), it follows that

$$
\begin{aligned}
\theta_L \cdot m_2(\tau^*) &= \int_0^{\theta_L y_2 + \tau^*} \theta_L y_2 \, dF(V) + \int_{\theta_L y_2 + \tau^*}^{\infty} (V - \tau^*) \, dF(V) - \mathrm{E}[\max\{\theta_L y_2, V\}] + I \\
&= \int_0^{\theta_L y_2} \theta_L y_2 \, dF(V) + \int_{\theta_L y_2}^{\theta_L y_2 + \tau^*} \theta_L y_2 \, dF(V) - \int_{\theta_L y}^{\theta_L y_2 + \tau^*} (V - \tau^*) \, dF(V) \\
&+ \int_{\theta_L y}^{\infty} (V - \tau^*) \, dF(V) - \mathrm{E}[\max\{\theta_L y, V\}] + I \\
&= \int_{\theta_L y}^{\theta_L y_2 + \tau^*} [\theta_L y_2 - V + \tau^*] \, dF(V) + I \leq \tau^* \cdot [F(\theta_L y_2 + \tau^*) - F(\theta_L y)] + I \\
&< (\theta_H - \theta_L) \cdot m_2(\tau^*) + I \qquad \Rightarrow \qquad m_2(\tau^*) > \frac{I}{\theta_H}.
\end{aligned}
$$

Inequality $m_2(\tau^*) > \frac{I}{\theta_H}$ implies that at $\tau = (\theta_H - \theta_L) \cdot m_2(\tau)$, i.e., at $\tau_H = 0$, the incentive compatible contract features a slack creditor participation constraint (C.3) for type H borrower is, however, slack. A profit maximizing borrower then increases the prepayment penalty wedge $\tau > \tau^*$ until the creditor participation constraint (**??**) binds for the high type. Consequently, under the optimal type H contract it must be the case that $c(R) \cdot R \geq \theta_H \cdot m_2(R)$, which implies that the make-whole provision is calculated at below market rates.

Lemma 14 (Optimal contract under asymmetric information)**.** *The optimal competitive lending contract features a prepayment penalty for type H contract and a competitive prepayment penalty for type L contract.*