# Rational Opacity\*

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#### Abstract

We present an environment where long term investors sometimes choose to remain uninformed about the value of their long-term investment. They do so to preserve the liquidity of their investment in secondary markets. When and only when there is a risk that secondary markets may be shallow, more information can reduce the expected payoff of agents who need to cash out. Even given direct and costless control over information design, stakeholders choose to incentivize managers to withhold interim information. In such an environment, imposing transparency can lower investment and welfare.

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# 1 Introduction

We present an environment where long term investors sometimes choose to remain uninformed about the value of their long-term investment. This is in contrast to the traditional view that attributes the lack of communication between investors and managers as a result of agency problems. The presumption is that managers have information that would be valuable to stakeholders but that it is too costly to set up incentives for managers to share this information.<sup>1</sup> In our model, investors have full control over the design of the information policy and yet they choose to be kept in the dark to preserve the liquidity of their investment in secondary markets. Investors choose to introduce agency frictions between themselves and managers to restrict their access to information.

To make this point, we present an environment where more information leads to better divesting decisions but increases the risk that stakeholders may lose value if they must liquidate their positions early. If the key benefit of opacity is to mitigate potential liquidation losses, incentives to restrict information flows should depend on the expected depth of secondary markets. In our model, stakeholders choose to limit the revelation of new information on asset quality when and only when the depth of secondary markets may affect the liquidation value – a situation Allen and Gale (2005) describe as cash-in-the-market pricing.

We develop our argument in a simple model of liquidity needs in the spirit of Diamond and Dybvig (1983) and Jacklin (1987). Agents can invest in a long-term project but face the risk that they may need to consume at an interim stage, before the project matures. When they need to liquidate their investment early, they can either scrap the project or, instead, sell it to more patient agents as in Jacklin (1987). Our model differs in several key aspects from the canonical Jacklin framework. First, our agents are risk neutral. Second, the long-term project is risky and its probability of success – its *quality* – is drawn at the interim stage. Third, when they make the original investment, agents can design how much information they would like to receive on the project quality at the interim stage. Information is free so that agents can choose at no cost full public information, no information at all, or anything

<sup>&</sup>lt;sup>1</sup>See for instance Milgrom and Roberts (1988) for a review of the traditional literature on agency costs, information, and compensation contracts. They present a canonical model where "[...] it is always optimal for the firm to adjust its promotion criteria and information collection rules from what would otherwise be optimal." Along related lines, the cheap-talk literature started by Crawford and Sobel (1982) shows that when there is any misalignment of preferences between an informed expert and a principal, all Bayesian-perfect equilibria feature some information loss. Even if the principal can write incentive contracts, full revelation is generally suboptimal. Implementing direct revelation, even when feasible, requires the provision of incentives whose cost can outweigh the benefits. See Krishna and Morgan (2008) for a review of these ideas.

in between these two extremes.

The optimal information design becomes more opaque, in a sense we make precise, as the risk of early liquidation rises. While more information allows investors to scrap early when ex-post efficient, it can also reduce the expected payoff when agents are constrained to liquidate their investments. Indeed, investors may be forced to liquidate at a price that does not reflect the fundamental value of the project when secondary markets are shallow. Therefore, cash-in-the-market pricing in the sense of Allen and Gale (2005) imposes an upper bound on the investment's liquidation value in some states, thus making our risk neutral investors effectively risk averse. Coarser information provides some insurance to those early investors who have to liquidate their project.

Given this logic, it would seem that the optimal situation for investors would be to observe project quality privately at the interim stage in order to make efficient scrapping decisions without incurring the risk of liquidation losses. That intuition turns out to be correct from an individual point of view, but wrong in equilibrium. As in Milgrom and Stokey (1982) all private information is revealed when projects trade in secondary markets. As a result, private information can hurt investors if they cannot commit to restrict it. It is optimal, therefore, for investors to restrict their access to information in some fashion.

One natural way to implement the desired solution is for investors to delegate the project continuation decision to a manager. The manager's compensation scheme should induce him to reveal the desired level of information. We show that the compensation scheme that implements the constrained optimal scrapping policy features a participation in revenues when the project matures and a severance payment if the manager chooses to scrap it early. In other words, the natural implementation of the optimal contract in our environment involves imposing a veil between investors and investment managers.

Doing so, we draw a connection between the literature on the optimal level of information pioneered by Hirshleifer (1971, 1972) and the cash-in-the-market pricing literature introduced by Allen and Gale (1994, 2005). Allen and Gale concentrate their attention on how the depth of secondary markets may affect asset price volatility. We focus instead on the consequences of cash-in-the-market pricing on the optimal control of the fundamental information investors receive. Hirshleifer (1971) shows that the prospect of interim information can make agents with long-term investment and consumption plans worse off by introducing "redistributive risk" once the new information emerges. Along different lines, Hirshleifer (1972) shows that interim or, in his terminology, "emergent" information can lower the market value of longterm projects unless the project "can be converted into money at a price representing only a time-discount of the value at maturity." In our model, original investors recognize that the possibility of cash-in-the-market pricing makes their investment subject to this second type of "Hirshleifer effect" and, rationally, choose the level of information that maximizes their ex-ante welfare.

A link between information and cash-in-the-market pricing is also present in Bolton, Santos and Scheinkman (2011). They consider a model of asymmetric information between short and long-term investors with cash-in-the-market pricing. Short-term investors may sell an asset either because of a liquidity shock or because they know the quality of the asset is low. The more short-term investors wait to sell their asset, the more likely they are to receive information about its quality, which leads to an ever increasing discount on the asset price. These investors take the asymmetry of information as given whereas our short-term investors recognize the trade-off they face and devise their information structure in an optimal way. In a related vein, Zetlin-Jones (2013) describes a model where, at the optimal contract, more opaque firms tend to emphasize short-term sources of finance. One could think of our result as the converse: corporations whose stakeholders value liquidity highly are more likely to be opaque. Von-Thadden (1995) shows that the possibility of asymmetric interim information between investors and firms can cause the optimal contract to feature "short-termism" in the sense that short-term investments are preferred to more productive investments. Our model can generate the same outcome, but for different reasons.

Our paper is also related to the banking literature where banks are usually seen as especially opaque.<sup>2</sup> Using a Diamond and Dybvig (1983) argument Kaplan (2006) holds the view that banks retain information because revealing bad news makes it more costly to prevent a bank run. In a similar vein, Goldstein and Sapra (2014) argued that revealing too much information about the results of a stress test may induce a run on banks.<sup>3</sup> Given the roll-over risk of banks, Bouvard, Chaigneau, and De Motta (2014) study the optimal disclosure policy of banks. They show that the inability of regulators to commit to reveal information in crisis times gives rise to excessive opacity. Like us, Goldstein and Leitner (2013) relate optimal information disclosure to a possible Hirshleifer effect. They propose a model of interbank loans where information disclosure may prevent a market freeze but eliminates risk sharing opportunities in the interbank market.

<sup>&</sup>lt;sup>2</sup>See Morgan (2002) and Flannery et. al. (2004, 2013).

 $<sup>^{3}</sup>$ Goldstein and Sapra (2014) survey the literature on the cost and benefits of disclosing stress test results and they conclude that full disclosure is rarely desirable.

Still in the banking literature, Dang et. al. (2013) study an environment where markets do not implement the first best allocation because the possibility of bad news cause investors to face liquidation losses. They show that a bank that commits not to share its project continuation decisions with late investors and finances early withdrawals by selling information-insensitive securities to these late investors can implement the first best allocation.<sup>4</sup> We discuss their results further and compare them to ours in Section 7.3.

In Diamond (1984), opacity is a side effect of any banking activity: the bank is opaque because it is too costly for each depositor to monitor the borrowers. A similar effect is at play in Diamond (1985) in the context of the release of information by a firm. There information disclosure is optimal because stockholders then save the cost of acquiring information. More recently, Andolfatto, Berentsen and Waller (2014) show that the threat of undue diligence – the possibility that agents may decide to acquire private information – can influence the socially optimal disclosure policy.<sup>5</sup>

On the technical side, the information design problem we solve is similar to the Bayesian persuasion game studied by Kamenica and Gentzkow (KG, 2011). Our original investors design messages knowing how receivers will act given the information they get. The optimal information design takes the form of what KG call a "straightforward signal" which amounts to a recommendation to the receiver to take a specific action. In our setting this is the recommendation to either scrap the project or hold on to it. One difference between our environment and KG's is that our senders – the early investors at date 0 – know that they will receive the message at date 1 but that secondary market investors will receive it as well. The fact that our setting contains multiple receivers turns out to be inconsequential however since all receivers interpret the message in the same way.

Our implementation of the desired solution via delegation is reminiscent of Aghion, Bolton, and Tirole (2004). They consider a set-up where an entrepreneur can either work or shirk, but will work if he is monitored by an investor who may have a need for liquidity at an interim stage. A trade-off exists between monitoring and liquidity since the investor has private information on the quality of the firm and may use it to his advantage. Instead, we design a mechanism that induces the manager to keep most information to themselves.

<sup>&</sup>lt;sup>4</sup>See also Gorton and Pennachi (1990), Breton (2007), Dang, Holmstrom and Gorton (2012) and Siegert (2012).

<sup>&</sup>lt;sup>5</sup>For the literature on disclosure regulation see Leuz and Wysocki (2008) and the references therein. In particular, Kurlat and Veldkamp (2013) argue that information disclosure can reduce investors' payoffs as it decreases asset return.

The remainder of the paper is organized as follows. Section 2 introduces the environment and defines an equilibrium. Section 3 characterizes pricing given the information made available in secondary markets. In Section 4, we show that some opacity is typically rational and we discuss the key, testable implications of this result. In Section 5 we consider the case with private information and we describe a natural implementation of the optimal information design that involves delegating project and information control to a manager. Several extensions of our basic framework are presented in Section 6 while Section 7 presents various applications of our theory. Section 8 concludes.

### 2 The environment

**Investment opportunities and preferences** Consider an economy with three dates t = 0, 1, 2, and unit measures of two types of agents. The first type of agents are early investors who are endowed with one unit of a consumption good at t = 0. The second type are late investors who appear at date t = 1 with an endowment A > 0.

As will soon become clear, the size of the endowment of late investors pins down the size of secondary markets in our model. Therefore, we will think of A as capturing the expected depth of secondary markets when early investor select their information disclosure policy. When A is low, secondary markets are shallow, and, as we will argue below, assets are more likely ex ante to sell at a price that is below their expected payoff, as in Allen and Gale (2005). Our main result will be that this leads early investors to opt for a more opaque information policy. One simplifying assumption we are making for now is that A is deterministic. This shortens several of the upcoming arguments but dealing with the stochastic case does not present major technical difficulties or change the nature of our results, as we explain in Section 6.2.

A fraction  $\pi \in [0, 1]$  of early investors and a fraction  $1 - \pi$  of late investors have the desire to consume at date 1 while other agents want to consume at date 2. In other words, half of all agents consume at date 1, while the other half want to consume at date 2.<sup>6</sup> We will refer to  $\pi$  as the liquidity risk for early investors. As of date 0, early investors do not yet know

<sup>&</sup>lt;sup>6</sup>This symmetric assumption on the risk of early consumption for early and late investors simplifies notation in the upcoming analysis by implying that a mass  $\pi$  of agents want to liquidate their projects a date 1 (namely early investors who turn out to be early consumers) and the same mass of agents are willing to buy projects at date 1 (namely late investors who turn out to be late consumers.) Even though this pins down the number of potential buyers in secondary markets, we can still vary the depth of secondary markets at will by varying A. Doing so, in fact, gives us one of the main comparative statics we establish in this paper, see Corollary 4.2.

whether they will want to consume early or late hence they seek to maximize:

$$u(c_1, c_2; \pi) \equiv \pi c_1 + (1 - \pi)c_2$$

where  $c_1$  is their expected consumption in period t = 1 conditional on being an early consumer while  $c_2$  is expected consumption at t = 2 conditional on being a late consumer.<sup>7</sup>

Agents have the option to store the consumption good across dates. The economy also comprises a risky project that, if continued at full scale until date 2, yields either R >1 or nothing. Activating the project requires an aggregate investment of one unit of the consumption good at date 0. In particular, all early investors must commit their endowment to the risky project in order to activate it. When they do so, early investors receive a claim to the project's output which is proportional to their contribution to the initial capital.

As of date 0, early investors know that the success probability  $q \in [0, 1]$  will be drawn at date 1 from a distribution F with a continuous and strictly positive density in [0, 1]. At date 1, any investor can scrap their portion of the project for a payoff  $S \in (0, 1)$  which is independent of q. When fraction  $\kappa \in [0, 1]$  of the project is scrapped at date 1, the total project payoff at date 2 is  $(1 - \kappa)R$  when the project is successful, zero otherwise. The scrapping decision captures the option to interrupt, downsize or re-purpose long-term investment projects in which cases S is the value of the next best use of the invested capital, net of re-purposing costs.<sup>8</sup>

Parameters could in principle be such that early investors are always better off storing their endowment but we focus on the more interesting case where early investors choose to invest in the risky project. Specifically, we assume throughout that

$$1 < \pi \min\left(A, \int qRdF\right) + (1-\pi)\int qRdF.$$
(2.1)

As will become clear below, (2.1) implies that early investors choose to invest in the risky project even when they have no information about project quality. Of course and as we

<sup>&</sup>lt;sup>7</sup>We assume here that a law of large number holds:  $\pi$  is both the fraction of early investors who turn out to be early consumers and the likelihood that a particular early investor will become an early consumer.

<sup>&</sup>lt;sup>8</sup>As we will explain below, it turns out in this environment that if it is optimal for one primary investor to scrap their share of the project, it is optimal for all investors to do so, so that either the entire project is scrapped or it is continued at full scale. Be that as it may, the specification of scrapping options we use embeds a constant return-to-scale assumption. One could imagine that scrapping by some investors diminishes the returns of remaining investors. This would only increase incentives by remaining investors to scrap as well, which would reinforce the prediction that either the entire project is scrapped or it is continued at full scale.

will discuss at length in this paper, early investors can typically do better by releasing some information about project quality at date 1.

Also to shorten the exposition we will assume throughout that A > S. This will imply that there are always enough resources in secondary markets to pay at least the scrapping value of the project. In addition, only one price clears the Walrasian market for project shares that we now describe.<sup>9</sup>

**Market for project shares** At date 1, agents can buy or sell claims to the risky project's output in a Walrasian market. Agents take the equilibrium share price as given. They buy or sell shares to maximize their expected utility given the information they have.

In appendix 9.1 we show that our model with Walrasian trade makes the exact same predictions as a model where early investors who wish to consume early are matched with exactly one late investor who wish to consume late and the former gets to make the latter a take-it-or-leave it offer. The transaction we model in the secondary market is also isomorphic to a secured debt contract between early and late investors that gives early investors the share price p(m(q)) at date 1 in exchange for a payment of R contingent on the project being successful.<sup>10</sup>

While both the option to scrap and the option to sell project shares in secondary markets enable early investors to get an early payoff, they are very different in nature. Scrapping a share of the project eliminates the possibility of a project payment at date 2. One should think of it as a redeployment of the capital invested in the risky project to a different use and more information allows investors to exercise that option more efficiently. In contrast, secondary markets enable investors to sell claims to date 2 payoffs. More information does not raise the ex-ante value of that option but it can lower it as we will show. The value of the scrapping option could depend on new information about q but we assume for simplicity and without loss of generality for our purposes that it is independent of q.

**Information** This paper is principally about what early investors choose to know about q once it is drawn at date 1. To learn about q, early investors can choose to activate an information technology at date 0. This technology sends a message m once q is realized at

<sup>&</sup>lt;sup>9</sup>When A < S, the scrapping option dominates what secondary markets can offer regardless of what information is available at date 1. Secondary markets are irrelevant, therefore, and full information is always best for original investors.

<sup>&</sup>lt;sup>10</sup> See appendix 9.8.

date 1. Early investors are free to choose any message function in the following set:

$$\{m: [0,1] \mapsto \mathcal{B}([0,1]): q \in m(q) \text{ for almost all } q \in [0,1]\}$$

where  $\mathcal{B}([0,1])$  is the space of Borel subsets of [0,1]. Restricting the choice of message functions to satisfy  $q \in m(q)$  is without loss of generality<sup>11</sup> and has the advantage that the technology can be thought of as announcing a subset of [0,1] to which q belongs. In Section 5, we will discuss the option for early investors to keep information to themselves and argue that this does not affect any of our results. Finally, restricting our attention to deterministic message functions is also without loss of generality as we will show when we fully characterize the optimal information design choice of early investors.<sup>12</sup>

Agents are free to become fully informed about the project quality by setting  $m(q) = \{q\}$ for all  $q \in [0, 1]$ . One of our main results, however, is that early investors usually opt for much coarser information technology designs, unless they know they will consume late, that is unless  $\pi = 0$ . Choosing no information -m(q) = [0, 1] for all  $q \in [0, 1]$  – is always an option as well, but is not optimal either unless  $\pi = 1$ .

**Equilibrium** At date 0, early investors establish a message function and decide whether or not to commit their endowments to the risky project. At the start of date 1, late investors appear, all consumption types are revealed, and a message  $m \in \mathcal{B}([0, 1])$  becomes available. Agents immediately and correctly translate this message into an expected likelihood of success for the long-term project,

$$E(q|m) = \frac{\int_m q dF}{\int_m dF}.$$

Given those expectations, we show in the next section that a unique price clears the Walrasian market for shares at date 1. Given this price, early agents decide, first, whether to scrap their share of the project.<sup>13</sup> Agents who do not scrap their project shares decide whether to buy and sell their claim to output at date 2. At date 2, all agents consume the proceeds from

<sup>&</sup>lt;sup>11</sup>To see why this is without loss of generality take any Borel-measurable mapping h from [0, 1] to an arbitrary message space. Then the set-valued mapping  $m : [0, 1] \mapsto \mathcal{B}([0, 1])$  defined for all  $q \in [0, 1]$  by  $m(q) = h^{-1} \circ h(q)$  has the desired properties and conveys exactly the same information as h. In other words, as long as all agents understand the selected design of the information technology, they can invert any message into a subset of [0, 1].

 $<sup>^{12}</sup>$ See the proof of Proposition 4.1.

<sup>&</sup>lt;sup>13</sup>The next section shows that that there is no disagreement on this decision between early and late consumers. In fact, in all equilibria, either the entire project is scrapped or it is continued at its original scale.

their claims to the risky project or their storage investments.

In this context, an equilibrium is a decision by early investors whether or not to activate the risky project, a message function, and, for each possible message at date 1, a share price, scrapping decisions, share trading decisions by early and late investors, and consumption plans, such that:

- 1. Given the message function, all agent decisions at date 1 are optimal and the Walrasian market for shares clears for every possible message;
- 2. No other message function and associated Walrasian price schedule gives early investors a higher expected payoff as of date 0.

In Appendix 9.8 we show that the allocation that obtains in this equilibrium is the one that a social planner who seeks to maximize the welfare of early investors would select, as long as the planner must abide by minimal participation constraints and cannot preclude early and late investors from entering into side-trades at date t = 1. In particular, the equilibrium we characterize is constrained-efficient.

### 3 Market for project shares

Given  $\pi \in [0,1]$  and A > 0, let p(m(q)) be the price of a project share when the message m(q) is issued at the start of period 1. To keep notation simple we do not make explicit the dependence of market prices on the model's parameters. If  $E(q|m(q))R \leq S$  then secondary market buyers are willing to pay no more than S per project share and no early investors is willing to accept less since they could always scrap their share of the project. In that case, it must be that p(m(q)) = S for markets to clear.

Assume, on the other hand, that E(q|m(q))R > S. If p(m(q)) > E(q|m(q))R then all early investors sell but no late investors are willing to buy since they are better off storing their endowment. So we must have  $p(m(q)) \le E(q|m(q))R$ . If the inequality is strict only early consumers may sell ( $\pi$  shares are supplied at the most). Late investors for their part, consume their endowment if they turn out to be early consumers at date 1. Those who are late consumers spend all their endowment on projects if p(m(q)) < E(q|m(q))R making demand  $\frac{\pi A}{p(m(q))}$  which is consistent with market clearing only if p(m(q)) = A which, in turn, is consistent with the premise that p(m(q)) < E(q|m(q))R only if A < E(q|m(q))R. When  $A \ge E(q|m(q))R$  the same argument leads to p(m(q)) = E(q|m(q))R as the only market clearing price. In summary,

**Proposition 3.1.** Given a message function m, at the Walrasian stage and for almost all  $q \in [0, 1]$ ,

$$p(m(q)) = \max \left[ S, \min\{E(q|m(q))R, A\} \right].$$

The argument is illustrated in Figure 1. Given the message, the share price must fall between S and the expected payoff E(q|m(q))R. As long as the price is the expected payoff, late investors who want to consume late are willing to spend their entire endowment on project shares making demand (drawn in dashed blue) the entire interval between 0 and  $\frac{\pi A}{p} = \frac{\pi A}{E(q|m(q))R}$ . To move beyond that demand level, the price must fall and demand becomes  $\frac{\pi A}{p}$  for  $p \in (0, E(q|m(q)R)$ . Supply, shown in solid red, is  $[0, \pi]$  when p = S, is exactly  $\pi$  when  $p \in (S, E(q|m(q))R)$  and becomes  $[\pi, 1]$  when p = E(q|m(q))R since in that case even late consumers are willing to sell their share.

The figure shows the case where there are not enough resources to purchase all the shares at fair value, that is when

$$\frac{\pi A}{E(q|m(q))R} < \pi \Longleftrightarrow E(q|m(q))R > A.$$

Then the only equilibrium price is p(m(q)) = A. In other words, the price is dictated by the resources available in the market rather than the project's expected payoff. This is a situation Allen and Gale (2005) describe as cash-in-the-market pricing. On the other hand, if  $\frac{\pi A}{E(q|m(q))R} \ge \pi$ , then the equilibrium price is the expected payoff. We will show that cashin-the-market pricing implies a trade-off between information and liquidity.

One simple consequence of this result is that Walrasian prices always exceed the proceeds early investors receive when the project is scrapped. This implies that there is no conflict of interest between patient and impatient investors when it comes to the continuing decision. If patient investors wish to continue the project at full scale, impatient investors are at least as well off agreeing with this decision as they would be if the project is scrapped. In any equilibrium then, either the project is scrapped in full with all investors agreeing with this decision or it is continued at full scale.

We assume that A is fixed but q varies while Allen and Gale (2005) assume the reverse. We need q to vary to create an interesting information problem. The key aspect of both environments, however, is that projects may sell at a discount when A is small relative to q.



Figure 1: Walrasian market for project shares under cash-in-the-market pricing

We keep A fixed for simplicity but making both q and A stochastic is easy and does not alter any of our results, as we show in section 6.2.<sup>14</sup>

Observe also that the payoff early investors obtain when projects trade is the same as when early investors who experience liquidity shocks are matched with one late investor who wishes to consume late and make a take-it or leave-it offer to these agents. It follows that all the results we establish below regarding the rational choice of information technology hold in a simple search environment exactly as they do in a Walrasian environment. In appendix 9.1, we also show that proportional bargaining does not change the qualitative nature of our results.

# 4 Rational Opacity

We are now in a position to characterize the information design decisions of early investors. It will be instructive to first consider a parametric example where the trade-off between information and liquidity is transparent. We will then characterize the general solution to our information problem.

<sup>&</sup>lt;sup>14</sup>One could also assume that the measure of late investors who desire early consumption is stochastic.

### 4.1 An illustrative example

Assume that technological parameters are such that

$$A < \int qRdF. \tag{4.1}$$

In this particular part of the parameter space, there is cash-in-the-market pricing in secondary markets when no information is provided as the share price cannot be above A, as we explained above. In this case, the fact that A > S implies that information reduces the sellers' expected payoff from secondary markets since late investors are already willing to pay A when no information is provided. In other words, the Walrasian price is as shown in Figure 2 for the two polar information cases: The black solid line shows the price with full information and the dashed red line shows the price with no information. In this specific case, information cannot have any positive effect on liquidation value but when the news is bad, if q is low, it can have a negative effect on secondary market prices.



Figure 2: Price under full information (solid) and no information (dashed).

To make clear the resulting trade-off between information and liquidity, observe that if

early investors opt for no information, their ex-ante payoff is

$$\pi A + (1 - \pi) \int_0^1 q R dF.$$
 (4.2)

Indeed, they can sell their share of the project for A in secondary markets when they must consume early and, if they turn out to be late consumers then they keep their shares to maturity, as no new information becomes available at date 1. If on the other hand early investors opt for full information, their expected payoff is

$$\pi \left( \int_0^{\frac{S}{R}} SdF + \int_{\frac{S}{R}}^{\frac{A}{R}} qRdF + \int_{\frac{A}{R}}^1 AdF \right) + (1 - \pi) \left( \int_0^{\frac{S}{R}} SdF + \int_{\frac{S}{R}}^1 qRdF \right).$$
(4.3)

Information is valuable ex-post for late consumers because it enables them to make efficient scrapping decisions, thus obtaining a higher expected payoff,

$$\int_0^{\frac{S}{R}} SdF + \int_{\frac{S}{R}}^1 qRdF > \int qRdF,$$

but it is costly for early consumers because it reduces the expected liquidation value of project shares at date 1:

$$\int_0^{\frac{S}{R}} SdF + \int_{\frac{S}{R}}^{\frac{A}{R}} qRdF + \int_{\frac{A}{R}}^{1} AdF < A.$$

A trivial consequence of these observations is that given only a choice between full information and no information, early investors would only opt for full information if their liquidity risk is low enough.

We can say much more. Assume that agents can design the message function in any way they wish. Revealing information cannot improve early investors' payoff if they must consume early, as condition (4.1) implies that their payoff is already at its maximum if they do not receive any information. The only point of revealing some information, then, is to make better scrapping decisions. It follows that there is no need for the message function to partition [0, 1] in more than two subsets: scrap or hold. While late consumers would like to be informed when qR < S, there is no value in having more information than just  $q \geq \frac{S}{R}$ . To summarize, while the value of marginal information in  $[\frac{S}{R}, 1]$  is non-negative, revealing that information could reduce early investors' payoff when they have to consume early.<sup>15</sup>

<sup>&</sup>lt;sup>15</sup>A formal proof of this claim as well as other claims we make in this intuitive discussion are provided in

These simple observations give us the first source of opacity. It is not rational for early investors to reveal any information beyond what is strictly necessary to induce efficient scrapping decisions. Late investors, for their part, would value finer information, but they have no means to induce the original investors to provide it.

We will show in full generality below that the two subsets, scrap and hold, are nonoverlapping intervals.<sup>16</sup> We can thus restrict our search for optimal message functions to the following class of functions, indexed by  $\bar{q} \in [0, 1]$ : for  $q \in [0, 1]$ ,

$$m(q) = \begin{cases} & [0,\bar{q}] & \text{if } q < \bar{q} \\ & (\bar{q},1] & \text{otherwise.} \end{cases}$$

At date zero then, early investors need only choose  $\bar{q}$ . We refer to  $\bar{q}$  as the scrapping threshold. An obvious possibility is to set  $\bar{q} = \frac{S}{R}$  which would enable late consumers to always make the ex-post efficient scrapping choice. In this case, the message is designed to convey the most information subject to the constraints we have outlined above. This design, however, turns out to be optimal only when  $\pi = 0$  and early investors know they will consume late.

To characterize the optimal design, notice that the early investors' payoff is

$$V(\bar{q}) \equiv \pi \left( \int_{0}^{\bar{q}} SdF + \int_{\bar{q}}^{1} AdF \right) + (1 - \pi) \left( \int_{0}^{\bar{q}} SdF + \int_{\bar{q}}^{1} qRdF \right).$$
(4.4)

Since V is continuous on a compact set, an optimal  $\bar{q}$  exists for all  $\pi \in [0, 1]$ . While the payoff function is not necessarily concave in the scrapping threshold for arbitrary density functions, it is hill-shaped with a single peak so that the optimal threshold is in fact unique. Furthermore, V is strictly submodular: the early investor's marginal payoff is decreasing in  $\pi$ . Therefore, the higher the liquidity risk, the greater the cost of increasing the scrapping threshold. Intuitively, there is a tradeoff between the desire to scrap when it is efficient to do so and the fact that better information can lower the project's resale value. This implies that investors who face a relatively low liquidity risk will choose a higher scrapping threshold. And, inversely, investors facing a high liquidity risk will prefer a lower scrapping threshold and possibly no information at all.

the next section where we take on the optimal design problem in full generality.

<sup>&</sup>lt;sup>16</sup>Our arguments in this respect are similar to those of Kamenica and Gentzkow (2011). See the proof of Proposition 4.1 for details.

In this simple parametric case, one can show<sup>17</sup> that the optimal scrapping cut-off is

$$\bar{q} = \max\left\{\frac{S - \pi A}{(1 - \pi)R}, 0\right\}.$$

Indeed, we argue below that if the optimal scrapping solution is interior it must satisfy the following first order condition:

$$\pi A + (1 - \pi)\bar{q}R = S.$$

Any  $q > \bar{q}$  such that the left hand side of the equality exceeds the right-hand side should be included in the holding message, as holding then dominates scrapping. Of course, consistency requires that the holding strategy be optimal for late consumers given the message. But (4.1) guarantees that they are willing to hold on to their shares if no new information is revealed, so they remain willing to do so upon learning the good news that  $q \ge \bar{q}$ . For the same reason, since late investors are willing to pay A before hearing that  $q \ge \bar{q}$ , this remains true after learning the good news.

This result implies in particular that the optimal  $\bar{q}$  is zero on  $(\frac{S}{A}, 1]$  and decreases strictly on  $[0, \frac{S}{A}]$ . More liquidity-minded (high  $\pi$ ) early investors thus opt to reveal less information. It also suggests that deeper secondary markets – a higher A – causes early investors to opt for more opacity. But, as will now see, this only holds in the particular part of the parameter space on which this section focuses. The relationship between the depth of secondary markets and opacity turns out to be more complicated than this simple example would suggest. To see this, we now turn to the general solution of the information design problem.

#### 4.2 The general solution

This section provides the general solution to our problem. Intuitively and as discussed in the example above, a trade-off only exists between liquidity and information when project shares sell at a price below their expected value. Otherwise, it is never optimal to withhold information. Capturing this idea is the main direction in which we need to generalize the

$$V(\bar{q}) \equiv \bar{q}S + \pi(1-\bar{q})A + \frac{(1-\pi)R}{2} \left(1-\bar{q}^2\right).$$

This function is strictly concave in  $\bar{q}$  and its derivative vanishes at  $\frac{S-\pi A}{(1-\pi)R}$ .

<sup>&</sup>lt;sup>17</sup>For a concrete example, assume that  $\overline{F}$  is uniform. Then

example. To shorten the exposition, we will proceed assuming that  $\pi \in (0, 1)$ .<sup>18</sup>

The previous example assumed that  $A < \int qRdF$  which holds when A is low. When  $A < \int qRdF$  lowering the scrapping threshold would eventually mean that projects sell at their expected value in the holding region, so that cash-in-the-market pricing no longer holds, hence there is no remaining reason to withhold information. Therefore, in the general case, a key quality cutoff is the threshold  $\tilde{q}(A)$  past which, if the message  $q \geq \tilde{q}(A)$  is emitted at date 1, projects sell at price A, below their expected value. This threshold is defined by

$$\tilde{q}(A) = \max\left\{\tilde{q} \in [0, \frac{S}{R}] : E(qR|q \ge \tilde{q}) \le A\right\}$$
(4.5)

with the understanding that  $\tilde{q}(A) = 0$  if E(qR) > A. Original investors have no incentive to shrink the scrapping message beyond that threshold. Formally,

**Proposition 4.1.** The optimal information design consists of a scrapping message and a holding message. The scrapping message is F-essentially an interval  $[0, \bar{q}(\pi, A)]$  where

$$\bar{q}(\pi, A) = \max\left\{\frac{S - \pi A}{(1 - \pi)R}, \tilde{q}(A)\right\}.$$
(4.6)

The proof provided in the appendix consists of several steps. First we show that we can restrict the search for the optimal message function to binary functions – scrap or hold – and that these functions are two non-overlapping intervals with no gaps. This implies the existence of a scrapping threshold  $\bar{q}$  such that agents receive the scrapping message whenever  $\bar{q} < q$  and the holding message otherwise. Second, we show that  $\bar{q} \geq \tilde{q}(A)$ . Otherwise, and given (4.5), raising  $\bar{q}$  would strictly raise the early investors' expected payoff. Therefore their problem is to maximize their expected payoff (4.4) subject to  $\bar{q} \geq \tilde{q}(A)$ , which yields (4.6). Finally, we show that random messages would not help early investors in achieving a higher expected payoff.

Cash-in-the-market pricing – the possibility that market price may depend on available resources on the demand side for projects – plays a critical role in our results. It introduces a cap on prices hence on the early consumer's payoff, thus making their payoff function nonlinear in m(q). As a consequence, even though agents are risk neutral, liquidity concerns can make them behave as if they were risk-averse.

<sup>&</sup>lt;sup>18</sup> When  $\pi = 0$  the secondary market can play no role and early investors opt for full information. If  $\pi = 1$ , information has no value and choosing no information is always optimal.

#### 4.3 Key implications

This general result has several immediate consequences. First, it yields the main comparative statics results we seek to establish in this paper.

**Corollary 4.2.** At the optimal information design:

- 1.  $\bar{q}(\pi, A)$  decreases weakly with  $\pi$ , strictly so if and only if  $\bar{q}(\pi, A) \in (\tilde{q}(A), \frac{S}{R})$ .
- 2.  $\bar{q}(\pi, A)$  is U-shaped in A. Given  $\pi \in [0, 1]$ , there exists  $\bar{A}(\pi) \leq \int_{\frac{S}{R}}^{1} qRdF$  such that  $\bar{q}(\pi) = \frac{S}{R}$  if  $A \geq \bar{A}(\pi)$ , and the optimal scrapping thresholds first decreases and then increases on  $[S, \bar{A}(\pi)]$ .

The first item states that more liquidity-concerned investors choose a lower scrapping threshold. A testable version of this prediction is that organizations whose stakeholders value liquidity highly should be especially opaque. This is the converse of the main point made by Zetlin-Jones (2013). The negative relationship between the liquidity risk and information revelation comes from the basic trade-off between liquidity and information we discussed earlier. The second item says that the trade-off is only operative when the market price of projects is affected by the endowment of late investors. It should be clear that scrapping low quality projects is always optimal when  $A \leq S$ . Hence, in this case  $\bar{q}(\pi, A) = \frac{S}{R}$ . At the opposite end, when A is so large that shares always sell at their expected payoff, information cannot affect liquidation value and there is no need to take the risk of holding the project when it would be efficient to scrap, so that again  $\bar{q}(\pi, A) = \frac{S}{R}$ . In between these two thresholds, there is cash-in-the-market pricing in secondary markets and the scrapping threshold does depend on A.

Notice that a lower threshold  $\bar{q}(\pi, A)$  is an increase in opacity: as the threshold decreases, the set of project quality for which all investors receive the same information is larger. Put another way, original investors become more prone to curtail the release of bad news. One testable version of this prediction is that investments for which secondary market opportunities are ample should feature few if any curbs to the release of interim information about fundamentals.

One direct way to test this prediction is to study the relationship between the size of secondary markets for a particular project and proxies for transparency.<sup>19</sup> More indirectly, opacity should be more prevalent in industries where barriers to entry into secondary markets

<sup>&</sup>lt;sup>19</sup>Morgan (2002) or Flannery et al. (2013) propose various ways to proxy for the opacity of corporations.

- legal restriction or the cost of learning about complex investment projects, for instance – are high. As should be clear and as Section 6.3 will formalize, industries with high entry costs are more likely to feature cash-in-the-market pricing. Another indirect way to test this basic prediction of our model is that opacity should be especially prevalent when secondary markets are in their infancy as they tend to be for new industries.

The risk of shallow secondary markets, in our world, is a necessary condition for opacity to serve a purpose. But Corollary 4.2 also says that the relationship between the expected depth of secondary markets is not globally monotonic. To make this stark, if secondary markets do not exist, full information is obviously optimal. As secondary markets grow from insignificant and start becoming relevant opacity initially worsens but eventually falls. In other words, our model produces a Kuznets-curve-like relationship between secondary market development and transparency.

More fundamentally, Proposition 4.1 also implies that equilibria can be inefficient. In cases where  $\bar{q}(\pi, A) < \frac{S}{R}$ , early investors choose a scrapping threshold that induces them to keep the project in some states of the world when they should not. Therefore, total expected output is strictly below what would prevail under full information as is, therefore, aggregate expected consumption. The inefficiency arises from the fact that ignorance is bliss for those agents who must sell their project. In summary:

**Corollary 4.3.** The equilibrium allocation under rational information design can be Pareto inefficient.

Finally, an important result is that while information distortions may lower expected output below its potential, this does not imply that imposing transparency necessarily causes output to rise. In fact, yet another consequence of proposition 4.1 is that doing so may lead to a decrease in expected output.

**Corollary 4.4.** Imposing full information can lead early investors to opt for storage rather than the risky project. In particular, it can cause expected output and expected consumption to fall.

As in Andolfatto et. al. (2014) therefore, more transparency can imply less investment and hence destroys total surplus. Here, this occurs because imposing full information lead liquidity-minded investors to opt for less productive projects with safer short-term returns. Our result is also reminiscent of Goldstein and Sapra (2014) who claim that disclosing too much information on stress test results could trigger runs and destroy value. Von-Thadden (1995) also presents a model where the possibility of asymmetric interim information between investors and firms can cause the optimal contract to feature "short-termism" in the sense that short-term investments are preferred to more productive investments. The mechanism behind this aspect of our model is quite different however: stakeholders are concerned about their ability to liquidate their investment at a good price and transparency, therefore, can reduce the value of entering into long-term investment projects.

## 5 Private information

So far we have assumed that if information is made available to some agents, then it is public information. Our results seem to suggest that early investors would prefer to observe project quality privately at date 1 to make efficient scrapping decisions without incurring the risk of liquidation losses. This section shows that this intuition is wrong. While it is true that each investor has an incentive to be better informed than other agents, this is true for all agents, and general equilibrium arguments imply that acquiring private information can only hurt investors. Since they are unable to commit not to act on their private information, their willingness to trade in secondary markets will make public any private information. Therefore private information can only hurt investors if they cannot commit to restrict it. One solution to this paradox is to delegate the reception of information to a representative investor with the right incentives.

### 5.1 Trade reveals all private information

Assume that early investors always observe the interim signal perfectly but privately. In that case, as long as late investors observe the supply of project shares, the Walrasian market reveals all private information which means that the equilibrium allocation is the same as in the full information case.

**Remark 5.1.** If early investors observe project quality privately then the only equilibrium allocation is the full information equilibrium allocation.

This observation should not come as a surprise: agents' willingness to trade at the interim stage reveals all private information in this environment as in Milgrom and Stokey (1982).

Since unbridled access to private information can lead to an inferior allocation from the ex-ante point of view of early investors, they have an incentive to observably commit to remaining ignorant. To explore this possibility, assume now that agents can make the *design* of private information they select observable to late investors, or, equivalently, that they can somehow commit to it. In this case, late investors observe what information design early investors selected. At the trading stage, late investors can infer all information early investors received from their willingness to trade shares and, therefore, the equilibrium is the same as when the signal is public.

**Remark 5.2.** If the design of the information technology is observable, the rational information design choice is the same regardless of whether the message is private or public.

Put another way, all the results we established in the previous section go through unaffected when information is private rather than public. In addition, this section says that investors who must confront liquidity risk have incentives to observably commit to reveal any information they have (say, via delegated monitoring) or to not trade on the basis of that information (say via regulations that ban trading on the basis of undisclosed information.)

#### 5.2 Implementation via delegation

The analysis above suggests that agents have an incentive to find ways to commit to ignore – or at least not to act upon – their private information. In this section we show that a natural way to implement the desired solution is to delegate the project continuation decision to a risk-neutral representative agent (e.g. a manager, operating entity or General Partner) with the right incentives. Assume then that the coalition of early investors hire an agent with no holdings in the project and give her the authority to scrap the project at date 1. Assume further that only this agent is given full access to the signal at date t = 1.

Consider the class of compensation scheme whereby the manager receives a fixed payment M > 0 if the project is scrapped – think of it as a severance payment – and, if the project is continued, receives a payment  $\alpha R$  if the projects succeeds– think of this part of her compensation as a participation in revenues. For simplicity, we assume that the manager has no mass so that, in particular, the payment she receives does not affect the expected surplus generated by the project. We now have:

**Proposition 5.3.** Let  $\bar{q}(\pi)$  be the optimal scrapping threshold given  $\pi \in [0,1]$ . Let the manager's compensation scheme  $(M, \alpha)$  be such that

$$M = \alpha \bar{q}(\pi)R.$$

Then the manager implements the optimal scrapping policy and, correspondingly, early investors expect the constrained-efficient payoff.

Investors can implement the ex-ante optimal allocation and information design by creating ex-post conflict of interests between a manager and at least some of the stakeholders. Late consumers would prefer upon discovering their type that all information be revealed. By committing to delegation with a carefully designed set of incentives, stakeholders are committing to the ex-ante optimal information environment. Far from being a friction that ought to be addressed as it is in traditional models, agency costs serve to implement the constrained optimal solution.

Note that the proposition does not pin down the level of the compensation scheme so that in principle, the entire one-dimensional space of schemes that satisfies the desired property implement the optimal policy. Since the manager has no mass, investors are indifferent across such schemes as long as they involve finite payments. In Appendix 9.5 we introduce moral hazard and we show that – among other insights – doing so provides a natural way to pin down the level of the optimal compensation scheme.

### 6 Extensions

This section considers several important variations on the model we have used to establish our main results.

### 6.1 Continuous project control variable

The coarse nature of the optimal information design that obtains in the model we have used so far is not simply a consequence of the assumption that the only meaningful decision that takes place at date 1 is whether to scrap or hold. To see this, generalize our model by assuming that at date 1 and after project quality q has been drawn the expected project payoff is qR(y,q) where y is a control variable and R is a function that rises continuously with q for all y. We assume that the choice of control at date 1 cannot be hidden from late investors. So knowing q is beneficial as it allows to choose the optimal  $y^{20}$  We write  $y(m) = \arg \max_y E[qR(y,q) \mid q \in m]$  for the action that maximizes the expected payoff of a holder of the project if investors receive some message m in period 1. Notice that  $qR(y(\{q\}),q) \ge qR(y(m),q)$  for any q whenever  $q \in m$ , so that investors always prefer to have full information.

We assume there is cash in the market pricing. That is we define  $\tilde{q}$  as the solution to

$$E\left[qR\left(y([\tilde{q},1]);q\right) \mid q \geq \tilde{q}\right] = A,$$

so that whenever investors receive message [q, 1] where  $q > \tilde{q}$  then the project will sell for A and we further assume that  $\tilde{q} < 1$ .

This modeling change should affect the optimal design of information since knowing q now matters not only for continuation decisions but also for optimal operation choices. It turns out the optimal information design now reveals full information at the bottom and at the top of the quality interval, while giving no information in the middle, in a way the following proposition makes precise.

**Proposition 6.1.** Suppose  $\pi < 1$ . Then there is  $q_0 > \tilde{q}$  and  $q_1 \in (q_0, 1]$  such that the optimal message structure is m(q) = q for all  $q \in [0, q_0) \bigcup (q_1, 1]$  and  $m(q) = [q_0, q_1)$  for all  $q \in [q_0, q_1)$ . Furthermore,

$$E\left[qR\left(y([q_0, q_1]); q\right) \mid q \in [q_0, q_1]\right] = A.$$

When  $\pi = 1$ ,  $q_0 = \tilde{q}$  and  $q_1 = 1$ .

As before, the region where there is no information is a consequence of cash-in-the-market pricing, and the region becomes larger as the probability to consume early increases. The intuition is simple. Information is useful as it allows to choose the optimal control y. When investors know they want to consume early ( $\pi = 1$ ), they do not care about information details for high quality projects as they know they will not operate the project. So they choose to bundle high quality projects with as many low quality projects as possible, and our previous information structure obtains. When  $\pi < 1$  however early investors trade-off liquidity and optimal control. For q large, full information is optimal because they know the

$$R(y,q) = q^{1-\alpha}y^{\alpha} - yw$$

<sup>&</sup>lt;sup>20</sup>For instance, one could specify

where w > 0 is a unit cost and  $\alpha \in (0, 1)$ . One could then think of y as labor input or as capacity utilization in the sense of Greenwood et. al (1988).

project would sell for A if they want to consume early, while they can choose the optimal control variable when they turn late consumers. For q low, investors know the project sells for S as it should be scrapped and full information is also (weakly) optimal there. For middle range quality, investors trade-off liquidity and optimal control just like in the benchmark version of our model, and the range depends on the probability to consume early. The higher the probability the larger the range.

In turn, the same delegation approach with the appropriate compensation scheme implements the optimal allocation. Optimal delegation of control thus implements the optimal solution even when continuous decisions are present. The bottom line is that introducing continuous control variables attenuates but does not eliminate incentives by investors to control information flows.

#### 6.2 Stochastic secondary market depth

We have assumed for simplicity that the size A of secondary markets is known and independent of the project's quality q when early investors make their information design choice at date 0. A specific consequence of this simplifying assumption is that cash-in-the-market binds when the project is of high quality, while the project trades at its fundamental value when it is of fairly low expected value. This need not be the case. This section provides a simple example where (a) A is correlated with the expected value of the project, and (b) sales below the project's fundamental value occur when the project is expected to be of low value. One natural interpretation of this last event is that bad news about the project tend to produce fire-sale discounts.

Consider once again the environment we studied in Section 4.1 but suppose that the quality of the project is randomly drawn from distribution  $F_B$  with probability  $\eta \in (0, 1]$  and  $F_G$  with probability  $1 - \eta$ , where  $F_G$  first order stochastically dominates  $F_B$ . In other words, agents expect a better project outcome if the G ("good") distribution is active. In contrast to the foregoing analysis, assume that the endowment of late investors is correlated with the distribution of the project's quality: it is A > 0 with probability  $\eta \in (0, 1]$  – where A continues to satisfy assumption 4.1 – while with probability  $1 - \eta$  – when the distribution of project's value is  $F_G$  – the endowment is so large that project shares always sell at the expected payoff when they are not scrapped. In this environment, the state of secondary markets is correlated with project outcomes.

Given this modeling change, we need to ask whether the message can depend on the

realized state of the world (Good or Bad) hence on the depth of secondary markets. Both cases are manageable but for concreteness we will deal with the case where the message cannot be made contingent on the date 1 state. It is easy to see that the optimal message function is once again binary – scrap or hold – and that the scrapping message is an interval that includes q = 0. Letting  $\bar{q}$  denote as before the upper bound of the scrapping interval, the optimal information design choice must now maximize:

$$\begin{split} V(\bar{q};\eta) &\equiv \eta \left[ \pi \left( \int_{0}^{\bar{q}} SdF_{B} + \int_{\bar{q}}^{1} AdF_{B} \right) + (1-\pi) \left( \int_{0}^{\bar{q}} SdF_{B} + \int_{\bar{q}}^{1} qRdF_{B} \right) \right] \\ &+ (1-\eta) \left[ \int_{0}^{\bar{q}} SdF_{G} + \int_{\bar{q}}^{1} qRdF_{G} \right] \\ &= \eta \pi \left( \int_{0}^{\bar{q}} SdF_{B} + \int_{\bar{q}}^{1} AdF_{B} \right) \\ &+ \left( \int_{0}^{\bar{q}} S \left[ (\eta(1-\pi)dF_{B} + (1-\eta)dF_{G} \right] + \int_{\bar{q}}^{1} qR \left[ (\eta(1-\pi)dF_{B} + (1-\eta)dF_{G} \right] \right) \end{split}$$

The maximization problem is very similar to the case where the size of secondary markets is deterministic except that investors now worry about the joint event that they may be early consumers and that, as the same time, secondary markets may be shallow, which occurs with probability  $\eta\pi$ . As long as  $\eta > 0$  – that is, as long as there is a risk that secondary markets may be shallow – V remains submodular and, in particular, some opacity is optimal. In other words, the assumption we have implicitly maintained that A is independent of project quality is made only for convenience and plays no role in our comparative statics results. The key assumption is that when early investors select the optimal information design, they attach positive probability to the event that secondary market prices will be determined by market depth rather than by project quality. All that matters for our purposes, therefore, is that market depth and project quality not be perfectly correlated. Credit freezes and more generally financial crises are natural examples of events that affect secondary markets beyond what changes in fundamental project quality alone would justify.

#### 6.3 Endogenous market depth

So far we have treated market depth as independent of information design. However, in the model, opacity impacts the rents secondary market buyers generate which creates a feedback effect from opacity to market depth. To make this clear, this subsection embeds our one-project model into a broader framework where the size of secondary markets is fully endogenous.

Consider an economy that contains a unit interval of locations each containing one investment project and a unit mass of early investors both exactly as described in section 2. The economy also contains an unbounded mass of late investors who appear at date 1 each endowed with exactly one unit of the consumption good. Those late investors can choose to store their endowment. They can also choose to join one location which enables them to participate in Walrasian markets for project shares in that location when they open at date 1. For concreteness we assume that the decision to join a particular market takes place before any information is revealed at date 1.<sup>21</sup> This simplifies the analysis by making the size of secondary markets independent of the message issued although, of course, that size depends on the information design selected by primary investors.

Joining a location is potentially costly however and we denote by  $c_i \ge 0$  the cost associated with joining location  $i \in [0, 1]$ . For simplicity but without loss of generality, we take this cost to be a utility cost so that late investors who enter a location all have their unit of endowment available for purchasing project shares. We interpret this cost as capturing the time and resources necessary to locate a particular market and learn its characteristics.

A general equilibrium in this context is a location decision for each late investor – allowing for the possibility that a given investor participates in no market – and, at each location, an information design choice by early investors and Walrasian prices for projects at date 1 such that:

- 1. The information design maximizes the ex-ante payoff of early investors given Walrasian prices at each location;
- 2. Secondary markets clear at all locations, i.e Walrasian prices are as defined in 3.1 where A now stands for the mass of late investors in a given location;
- 3. Net of entry costs, late investors earn the the same payoff (namely 1) as they would if they stored their endowment.

To understand why the third condition must hold, observe that if late investors earned a return net of learning costs that exceed storage returns in some locations, they would keep

<sup>&</sup>lt;sup>21</sup>We make this assumption for simplicity only. It would still be the case that cash-in-the-market must prevail when entry decisions are made after the date 1 message is issued. The complication is that the size of markets may now depend on the message.

entering locations with the highest return since they are available in unbounded numbers. Entry must eventually drive all returns net of entry costs down to the storage return. The following results characterizes equilibria in this extension of our basic model.

**Proposition 6.2.** Equilibria are such that all markets where entry costs are strictly positive feature cash-in-the-market pricing and incomplete information.

The proof we provide in the appendix formalizes the two-way connection between market depth and opacity. Given a potential size  $A_i$  of secondary markets in location i, only one information policy – i..e only one scrapping level  $\bar{q}_i(A_i) \leq \frac{S}{R}$  – is optimal. The associated rents for secondary market investors are

$$F(\bar{q}_i(A_i)) + [1 - F(\bar{q}_i(A_i)] \frac{\pi E(q|q \ge \bar{q}_i(A_i))R}{A_i} - 1.$$

Indeed, if  $q < \bar{q}_i(A_i)$  then the project is scrapped and either sold at price S to late investors or the proceeds of scrapping are consumer by all early investors. In that case investors earn no more than they would from storing their endowment. If, on the other hand, the message  $q < \bar{q}_i(A_i)$  is issued then the project is continued and late investors expect return  $\frac{\pi E(q|q \geq \bar{q}_i(A_i))R}{A_i}$ .

Holding  $A_i$  constant, a decrease in  $\bar{q}_i$  means lower rents for late investors. To see this, remember that continuing the project when  $q < \frac{S}{R}$  yields negative returns for project holders. In equilibrium then, more opacity in the sense of a marginal decrease in  $\bar{q}_i$  must be associated with smaller secondary markets to preserve secondary market investors' rents. In that sense, this environment with endogenous secondary markets exhibits a feedback effect from opacity to market depth.

The key consequence of introducing endogenous and costly entry, however, is that some measure of cash-in-market pricing hence some opacity must characterize all markets whose entry cost is strictly positive so that gross rents that exactly offset learning costs are generated. Secondary markets that carry learning costs must feature some cash-in-the-market pricing in this environment.

### 6.4 Optimal storage by early investors

It is interesting to compare the approach to endogenizing secondary market depth we just described to the approach adopted by Allen and Gale (1994). That paper relies on essentially

the same set of investment opportunities as we do and also contains similar needs for liquidity. However, unlike us, they allow early investors to simultaneously invest in risky projects and storage and the depth of markets at date 1, in fact, is pinned down entirely by initial storage decisions. In particular, the possibility of cash-in-the-market pricing and the associated rents endogenously raise the returns to storage. The crux of our argument is that there is a conflict of interest between early and late investors when it comes to the design of information. Imposing a clear separation between primary investors and secondary market participants enables us to focus on that conflict.

To see that some separation between primary and secondary investors is necessary, suppose that our primary investors have aggregate endowments in excess of what is needed to fund the project and that this excess endowment is the only source of secondary market funds. Then the solution where all primary agents hold the same portfolio and collectively agree to opt for full information generates the highest possible payoff since it maximizes expected output. Put trivially, agents do not have any ex-ante incentives to withhold information from their future selves. But the moment secondary markets feature agents not involved in the original design and there is a possibility that cash-in-the-market pricing may prevail, then the incentives to control information we have emphasized in this paper are present.

To see this more formally and returning for simplicity to the one project case, assume that our original investors are no longer constrained to invest all of their funds in the project but can choose a scale  $1 - a \in [0, 1]$  at which to activate the project. They store the remaining a units. Imposing that  $a \ge 0$  amounts to assuming that agents cannot short the project. As before, early investors maximize the payoff of a representative coalition member by choosing the level of investment at date 0 and the level of opacity. As in our basic framework, there is no reason why the coalition would choose more than two messages, scrap and hold. With storage, the cash in the market becomes  $\pi A + (1 - \pi)a$ , since early and patient investors can spend a on buying projects. Assuming that parameters are such that cash-in-the-market prevails, the price for each project sold in this case is  $p = \frac{\pi A + (1 - \pi)a}{\pi(1-a)}$ . Then, given the level of opacity  $\bar{q}$ , the expected return per unit of capital invested in the project is

$$V(\bar{q};a) \equiv \pi \left( \int_{0}^{\bar{q}} SdF + \int_{\bar{q}}^{1} pdF \right) + (1-\pi) \left( \int_{0}^{\bar{q}} SdF + \int_{\bar{q}}^{1} qRdF \right).$$
  
=  $\pi \left[ \int_{0}^{\bar{q}} SdF + \int_{\bar{q}}^{1} \left( A + \frac{1-\pi}{\pi} a \right) dF \right] + (1-\pi) \left( \int_{0}^{\bar{q}} SdF + \int_{\bar{q}}^{1} qRdF \right).$ 

In addition, at date 1, early and patient investors receive the gross return from purchasing the project in secondary markets, namely:

$$r(\bar{q};a) = \frac{E\left[qR|q \ge \bar{q}\right]}{p}$$

so that the expected payoff of a member of the coalition of early investors when investing 1 - a and choosing  $\bar{q}$  is

$$W(\bar{q}, a) = (1 - a)V(\bar{q}, p) + \pi a + (1 - \pi)ar(\bar{q}; p)$$

When A = 0, i.e. when early investors are also the only secondary market participants, note that

$$W(\bar{q},a) = (1-a)\left(\int_0^{\bar{q}} SdF + \int_{\bar{q}}^1 qRdF\right)$$

where we have used resource feasibility since, under the premise that there is cash-in-themarket pricing, all resources available to late consumers are invested in the project at date 1 rather than stored. But then the optimal information design is the one that maximizes expected output regardless of a and, therefore, full information, i.e.  $\bar{q} = S/R$ , is optimal. In words, early investors have no reason to withhold information from themselves.

Now suppose A > 0. If they use storage, early investors who are late consumers can invest in the secondary market when the project is not scrapped as they expect a higher return than storage. But at this stage, they would rather be informed about the true quality of the project and this effect pushes  $\bar{q}$  toward the efficient level S/R. However as the secondary market price is higher with storage, early investors may prefer more opacity to capture this rent in more states when they need to consume early. So at the time of choosing the optimal information structure, investors contrast the benefit of transparency – increasing return of interim investment – with the usual gain from opacity. The effect of a on  $\bar{q}$  is in general indeterminate, but we can show<sup>22</sup> that locally around a = 0, opacity increases if and only if the return from the secondary market is not too sensitive to opacity. That is  $\frac{\partial \bar{q}}{\partial a}|_{a=0} < 0$  if

<sup>&</sup>lt;sup>22</sup>The details of that argument are available upon request.

and only if  $\frac{\partial r}{\partial \bar{q}} < 1$ . In particular, if q is uniformly distributed,

$$1 - \frac{S - \pi A}{(1 - \pi)} < A \left[ 1 - \frac{S - \pi A}{(1 - \pi)R} \right]^2$$

In words, opacity is increasing with savings if and only if rents are not too sensitive to opacity. Notice that we already solved for  $\bar{q}$  at a = 0 so that the condition only depends on the deep parameter of the model and is not an equilibrium condition.

# 7 Applications

This section briefly describes several possible interpretations of our framework and discusses our model's predictions in each of these contexts.

### 7.1 Private equity markets

Private equity markets have at least two salient, distinguishing features vis-a-vis their public counterparts. They are illiquid – opportunities to liquidate partnership interests are restricted – and they are opaque – under the typical arrangement, most original investors only receive from fund operators the bare-minimum information needed to compute the distributions to which they are contractually entitled. Our model establishes a clear connection between these two features.<sup>23</sup>

To be sure, private funds tend to be opaque for a number of different reasons. The vast majority of private equity funds are structured as Limited Liability Companies or Limited Partnerships and feature a collection of passive investors (Limited Partners, or LPs) and a designated fund manager (General Partner, GP) who exercises sole control over the fund's operations. In fact, under the Limited Partnership Act, it is only by relinquishing all control to the GP that LPs are guaranteed limited liability protection. Any action that merely suggests a management role by LPs exposes them to lawsuits in the event of under-performance

<sup>&</sup>lt;sup>23</sup>Metrick and Yasuda (2010) provide a description of the typical purposes and performance of private equity funds. Kaplan and Stromberg (2009) discuss how private equity funds built for the purpose of pursuing leveraged-buyout (LBO) opportunities have evolved since the LBO wave of the 1980s. Gompers and Lerner (1999) described the typical organization of Venture Capital (VC) Funds emphasizing in particular the distance and veil that exist under the standard partnership contract between operators or general partners on the one hand and passive investors on the other hand.

or mismanagement. In addition – and further creating distance between LPs and operational matters – asset management is typically delegated by GPs to an operating entity (the "investment adviser") of their choosing and whose compensation they design.<sup>24</sup>

In our model, investors give up control over operational matters as a natural way to limit their access to information. They do so not to limit their exposure but, instead, to preserve the liquidity of their positions in secondary markets. Until two decades ago secondary transactions of private interests were infrequent and confined to OTC markets. In the past two decades however, intermediaries that specialize in secondary market purchases have emerged. In a 2012 survey of 212 Private Equity LPs (see https://www.seic.com/docs/IMS/SEI-PE-Liquidity-Challenge\_US.pdf) carried out by SEU, 58% of respondents reported that they have bought or sold assets in secondary markets while 33% of the same respondents confirmed that the market is "more liquid than it used to be." Furthermore and as Galfetti et. al (2014) explain, secondary market participants tend to be specialists suggesting that barriers to entry into these markets remain costly.

Recent surveys of private equity investors<sup>25</sup> suggest that even though opacity remains a dominant feature transparency is slowly on the rise in private equity markets. In part, this is a consequence of the massive losses institutional investors suffered during the recent crisis prompting many of them to ask for at least some experimentation with a new equity fund model. But our model points to another possible explanation for enhanced communication between operators, GPs and LPs. Secondary market options are becoming broader and deeper. As more and larger investors enter secondary markets, our model suggests that transparency should improve under the optimal contracting arrangement. While we have argued that, importantly, the relationship is not monotone, deeper secondary market options should eventually result in more transparency.

The optimal implementation we propose in section 5.2 requires highly contingent agreements between passive investors, managers, and operating entities that provide financial rewards when the investment project performs well and severance payments when the investment project must be shut down early. Partnership statutes provide precisely the freedom GPs and LPs need to enter into detailed contracts. As is well known, the majority of US private equity funds are incorporated in Delaware. One key advantage of registering a partnership in that State is that under Delaware law governing partnership agreements, fiduciary

<sup>&</sup>lt;sup>24</sup>See Naidech (2011) for a thorough description of the typical GP-LP setup in the United States.

 $<sup>\</sup>label{eq:second} {}^{25} See \ http://www.seic.com/docs/IMS/IMS-PE_Whitepaper_US_FINAL.pdf?cmpid=im-pe3-pr-11 \ for one of many examples.}$ 

duties are narrowly defined to cover default responsibilities. In that sense, Delaware provides for the maximum "freedom of contract" to partners in unincorporated entities and, as a result, the GP's obligations can be defined as narrowly as needed. With respect to transparency and communication between GPs, operating entities and LPs, this means in particular that many partnership agreements merely require the release of cash flow information necessary to the computation of distributions to LPs and, second, that any bookkeeping documents maintained by the GP or fund sponsor be available on demand. Stated or mandatory fiduciary responsibilities of partnership operators usually do not require the release to investors of soft information that operating entities receive over time about the fund's prospects.

The contract flexibility afforded by partnership statutes manifests itself in detailed incentive clauses for GPs and operating entities. So called "Carried Interest" clauses describe the performance-based part of the GP's compensation. The most typical incentive scheme takes the form of a "promote" structure whereby the GPs or operator's share of profits rises when certain internal rate return thresholds are met by LPs, together with claw-back periods when late losses reduce earlier returns. In the event of early termination of the fund and/or the fund manager, financial obligations of all partners are described in a separation agreement that can and often does feature severance payments.

Many partnership agreements require that investors get the approval of other partners before selling their interests, which seems potentially inconsistent with our Walrasian market set-up. However, the transaction we model in the secondary market is isomorphic to a contingent debt contract between early and late investors that gives early investors the share price p(m(q)) at date 1 in exchange for a payment of R contingent on the project being successful. For more on those side-trades, see appendix 9.8. Under that contract, early investors formally keep their partnership interest, receive p at date 1, and a net zero payoff at date 2 whether or not the project succeeds.

### 7.2 IPO markets

Alternatively, one could think of the secondary markets in our framework as Initial Public Offering (IPO) markets. Under this interpretation, our investors play the role of founders and initial investors who get their first opportunity to cash out their investments in public markets at date 1. The fraction of  $\pi$  of shares can be interpreted as the fraction of initial investors who experience a liquidity shocks. Alternatively, one could assume that all initial investors prefer to cash out their investment at date 1 but that they must abide by lockup

 $constraints.^{26}$ 

The fact that disclosure is limited in primary asset markets is discussed by Pagano and Volpin (2012) among many other papers. Pagano and Volpin (2012) and Monnet and Quintin (2016) show that limited disclosure can mitigate the adverse selection issues that result from the coexistence of expert and non-expert markets in those markets. Our model point to a different, complementary motivation for carefully managing the release of fundamental information about assets, namely the fact that the depth of IPO markets is uncertain. As we have argued, when cash-in-the-market pricing is a possibility, it becomes optimal to scramble information and bundle bad news with good news.

In this context, the implementation we propose in section 5.2 is best interpreted as the delegation of marketing decisions to an underwriter who receives a fixed proportion of IPO proceeds  $q(\bar{\pi})R$ . As discussed for instance by Ljungqvist (2004), the level of disclosure by underwriters varies a lot across IPOs. The literature has also found an empirical correlation between the level of disclosure and the level of underpricing. This finding is broadly consistent with our model in the sense investors who are concerned that their shares may sell at a discount vis-a-vis the value that would prevail when markets are deep opt for more opaque disclosure designs.

#### 7.3 Banks

A traditional interpretation of a framework such as ours in the spirit of Diamond and Dybvig (1983) is to think of our set of early investors as forming a bank for the purpose of creating liquid claims backed by illiquid but productive assets. This is the interpretation adopted for instance by Dang et al. (2013) in an environment that shares several key features with ours.<sup>27</sup> The traditional Diamond and Dybvig bank contract involves storing part of the resources invested at date 0. We rule that solution out by assumption since the project requires all available funds to be activated at date 0. Instead, liquidity is provided by secondary market investors which, in this banking interpretation, could be thought of as agents who invest equity into the bank at date 1.

However, as is well known in this context (see Jacklin, C., 1987) and is especially clear in a model like ours where project shares trade according to a Walrasian protocol, the inter-

 $<sup>^{26}</sup>$ See Brav and Gompers (2003) for a discussion for a detailed discussion of lockup provisions in IPO markets.

 $<sup>^{27}\</sup>mathrm{See}$  also Breton (2007).

pretation of the implicit two-period contract as a banking contract is arbitrary. When trade is possible at date 2, markets suffice to deliver the constrained optimal allocation. Breton (2007) and Dang et al. (2013) make the case that what makes banks essential is their ability to conceal information. They are "optimally opaque institutions."

Their banks are, in fact, fully opaque. Even though their framework is very similar to ours, they find that full opacity is optimal as opposed to the partial disclosure solution that emanates in our model. The reason for this difference is the fact that our model contains a scrapping option of potentially positive value and that whether this shut-down information is employed at date t = 1 is public information. If we allowed project managers to hide scrapping decisions, to store scrapping proceeds when they are positive, and compensate the manager with a carefully chosen fraction of proceeds at maturity, then it is easy to show that the manager would scrap when and only when  $q \leq \frac{S}{R}$ , as needed to maximize surplus. In that environment, secondary markets always pay the no-information price – scrapping decisions, since they are unobserved, have no consequences on liquidation values – exactly as in Dang et al. (2013). While low-quality projects may have been scrapped, secondary markets buyers only discover that they bought bad projects when proceeds are distributed at maturity.

The assumption we make that scrapping choices are observable can be justified in most contexts on at least two distinct grounds. First, at the optimal contract, when scrapping turns out to be optimal then the entire project is shut down, an action which seems difficult to conceal in practice. Second, the implementation of the first-best outcome described above would involve managers selling to date-1 investors projects which they know have failed. In most contexts, this amounts to defrauding investors. While recent events have shown that fund and trust sponsors do defraud investors on occasions, they also show that doing so comes with the risk of significant punishment.

However, in the banking context, the primary focus of Dang et al. (2013), the assumption that projects can be shut down unbeknownst to stakeholders seems more reasonable, for two reasons. First, banks' key stakeholders are its diffuse depositor base, each of whom has comparatively little exposure to the performance of each of the bank's long-term investments. Second, banks hold large and complex portfolios of positions making the monitoring of projectspecific actions more costly for depositors.

# 8 Conclusion

In this paper we argue that curtailing the flow of interim information about expected payoffs can be a rational choice for long-term investors who are concerned about secondary market depth because of a natural trade-off between information and liquidity. A natural way to restrict their own access to information is to delegate project management to agents whose compensation provides them with incentives that differ from the ex-post incentives of original investors. In our model therefore and far from being a friction that ought to be addressed, agency costs serve to implement the constrained optimal solution. Imposing transparency may lower welfare.

## 9 Appendix

#### 9.1 Bargaining

Here we analyze the case where early consumers and newborn are bilaterally matched in period 1 and bargain over project shares following the realization of the public signal m(q). To shorten the analysis, we focus on the parametric example considered in Section 4.1.

Assume that agents split the surplus from trade using proportional bargaining. We denote by  $\theta$  the share of the surplus of early consumers. Since m(q) is public, all agents expect the long-term project to return  $\max[E(q|m(q))R; S]$ . Let  $p(m(q)) \leq A$  be the agreed price between the early consumers and the newborn. With proportional bargaining, p(m(q)) has to satisfy

$$(1 - \theta) [p(m(q)) - S] = \theta \{ \max[E(q|m(q)) R; S] - p(m(q)) \}$$

and arranging, together with the resource constraint  $p(m) \leq A$ , we have

$$p(m(q)) = \min \{\theta \max[E(q|m(q)) R; S] + (1-\theta)S; A\}$$

As usual in the context of proportional bargaining, newborns extract more of the surplus as their bargaining power  $1 - \theta$  increases, in which case the price decreases to S. Agents then expect payoff

$$\pi \int p(m(q))dF + (1-\pi) \int \max\left[S, E(q|m(q))R\right]dF.$$

If agents choose to reveal nothing, then their payoff is:

$$\pi \min\left\{\theta \int qRdF + (1-\theta)S, A\right\} + (1-\pi)\int qRdF.$$
(9.1)

If they choose to provide full information, then their payoff is:

$$\pi \left\{ \int_{0}^{\frac{S}{R}} SdF + \int_{\frac{S}{R}}^{\frac{1-(1-\theta)S}{\theta R}} \left[\theta qR + (1-\theta)S\right] dF + \int_{\frac{1-(1-\theta)S}{\theta R}}^{1} AdF \right\} + (1-\pi) \left\{ \int_{0}^{\frac{S}{R}} SdF + \int_{\frac{S}{R}}^{1} qRdF \right\}$$
(9.2)

With take-it or leave-it offers from early consumers (i.e.  $\theta = 1$ ) we obtain the same expressions as in main text. As before then, given a choice only between full information and no information, early investors choose the former if and only if their liquidity risk is below a certain threshold.

More generally, consider now the same general class of messages as in the main text. At date zero then, agents of type  $\pi \in [0, 1]$  choose  $\bar{q}$ .

**Proposition 9.1.** The optimal information level  $\bar{q}(\pi, \theta)$  for agents of type  $\pi \in [0, 1]$  decreases strictly with  $\pi$  and  $\theta$ . Furthermore,  $\bar{q}(0, \theta) = \bar{q}(\pi, 0) = \frac{S}{R}$  and  $\bar{q}(1, \theta) = 0$  for all  $\theta > 0$ .

*Proof.* Fix  $\pi \in [0, 1]$ . Given  $\bar{q} \leq \frac{S}{R}$  the agent's payoff is:

$$\pi \left\{ \int_0^{\bar{q}} SdF + \int_{\bar{q}}^1 \min \left\{ \theta E[qR|q \ge \bar{q}] + (1-\theta)S; A \right\} dF \right\} + (1-\pi) \left\{ \int_0^{\bar{q}} SdF + \int_{\bar{q}}^1 qRdF \right\}.$$

Differentiating this expression with respect to  $\bar{q}$  yields

$$Sf(\bar{q}) - \pi \min \left\{ \theta E[qR|q \ge \bar{q}] + (1-\theta)S; A \right\} f(\bar{q})$$

$$-\mathbb{I}_{\left\{ \theta E[qR|q \ge \bar{q}] + (1-\theta)S < 1 \right\}} \theta \pi \int_{\bar{q}}^{1} \bar{q}Rf(\bar{q})dF - (1-\pi)\bar{q}Rf(\bar{q}).$$
(9.3)

where I is an indicator function. Since min  $\{\theta E[qR|q \ge \bar{q}] + (1-\theta)S; A\} > S$ , this expression is strictly negative when  $\bar{q} = \frac{S}{R}$  unless  $\pi = 0$ . Therefore only when  $\pi = 0$  do agents choose to reveal the efficient level of information. If  $\pi = 1$  then (9.3) is strictly negative even if  $\bar{q} = 0$  so that no information is revealed. Also, since  $\bar{q} < \frac{S}{R}$  for all interior  $\pi$  the derivative is uniformly decreasing as  $\pi$  rises through (0,1) which implies that  $\bar{q}$  decreases strictly, as claimed. Turning to the effect of  $\theta$ , when  $\theta = 0$  the derivative is strictly positive if  $\bar{q} < \frac{S}{R}$  and strictly negative if  $\bar{q} > \frac{S}{R}$ . Therefore, the maximum is attained at  $\bar{q} = \frac{S}{R}$ , and all agents prefer more information when they have no bargaining power. The case with  $\theta = 1$  is as in the text. Finally, the derivative is uniformly decreasing as  $\theta$  rises through (0, 1) which implies that  $\bar{q}$  decreases strictly.

We leave aside the case with bargaining under private information as it is substantially more difficult.

#### 9.2 **Proof of Proposition 3.1**

Proof. Take any q and associated message m(q). If p(m(q)) > E(q|m(q))R then all early investors would sell their shares at date 1 while there are no buyers. If, on the other hand, S < p(m(q)) < E(q|m(q))R then only early investors who need to consume early sell their projects. In that case, all late investors who need to consume at date 2 buy as many projects as they can afford, namely  $\frac{A}{p(m(q))}$  so that demand equals supply if and only if

$$\pi \frac{A}{p(m(q); \pi, A)} = \pi \iff p(m(q); \pi, A) = A.$$

Finally, if  $E(q|m(q))R \leq S$  then the optimal strategy for any project shareholder is to scrap it. Therefore we must have p(m(q)) = S so that all agents are indifferent between buying or selling project shares.

#### 9.3 **Proof of Proposition 4.1**

*Proof.* To ease the exposition in the context of this proof, we will dispense with all "F-essentially" qualifiers. Statements we make below about various subsets of project quality levels are understood to apply except possibly on sets of F-measure zero.

We will first characterize the solution under the assumption that the messages must be deterministic but will then argue that this assumption can be dropped without changing the solution. When the message function is deterministic, information takes the form of a partition of the interval (namely the range of the inverse of the message function) and we can assume that for all possible q the message is a subset m(q) of [0, 1] that contains q itself.

Consider any solution to the early investors' information design problem (i.e. consider any optimal message function.) Let  $\mathcal{H}$  be the union of messages  $m \subset [0, 1]$  that induce late consumers to hold their shares with probability one, while S is the complement set, i.e. the union of messages that induce scrapping with strictly positive probability. The expected payoff for early investors given this messaging strategy can be written as:

$$\pi\left\{F(\mathcal{S})S + \int_{\{q:m(q)\subset\mathcal{H}\}} \min\left(E(qR|m(q)), A\right) dF\right\} + (1-\pi)\left\{F(\mathcal{S})S + F(\mathcal{H})E(qR|\mathcal{H})\right\}$$

To understand this expression, note that project shares are scrapped with positive probability by late consumers only if  $E(qR|m(q)) \leq S$ . Given the same message then, secondary markets are willing to pay no more than S for shares, so that the expected payoff is the same whether shares are sold or scrapped by early investors. If the project is continued on the other hand, late consumers get as payoff the expected date 2 revenue. Early consumers get min (E(qR|m(q)), A) from secondary markets. But note that

$$\int_{\{q:m(q)\subset\mathcal{H}\}} \min\left(E(qR|m(q)),A\right)dF \leq \min\left(\int_{\{q:m(q)\subset\mathcal{H}\}} E(qR|m(q))dF,F(\mathcal{H})A\right)$$
$$= F(\mathcal{H})\min\left(E(qR|\mathcal{H}),A\right)$$

so that merging all messages that lead late consumers to hold into one hold message can only raise the expected payoff of early investors. Henceforth then we can restrict our search for the optimal message functions to binary functions: hold or scrap.

Next we show that S is an interval that contains the origin. If this is not the case then there are two sets  $M_1$  and  $M_2$  of equal and strictly positive F-mass such that the first set is in  $\mathcal{H}$ , the second set is in S, and  $M_1 < M_2$ . Moving  $M_2$  to  $\mathcal{H}$  and  $M_1$  to S leaves the scrapping part of the expected payoff unchanged but, since the q's are higher in  $M_2$  than in  $M_1$ , this strictly raises the payoff conditional on holding. If follows, then, that we must have  $S = [0, \bar{q}]$ and  $\mathcal{H} = (\bar{q}, 1]$  for some  $\bar{q} \in [0, 1]$ .

Next assume (yet again by way of contradiction) that  $\bar{q} < \tilde{q}(A)$  which implies, in particular, that  $\bar{q} < \frac{S}{R}$ . Then secondary markets pay  $E(qR|\mathcal{H})$  when the hold message is issued. Indeed, the definition of  $\tilde{q}(A)$  implies that when  $\bar{q} < \tilde{q}(A)$ ,  $E(qR|q \ge \bar{q}) < A$  so that shares trade at their expected value in secondary markets when the hold message is issued. It follows that the ex-ante expected payoff for date-0 agents is

$$\int_0^{\bar{q}} SdF + \int_{\bar{q}}^1 E(qR|q \ge \bar{q})dF.$$

But since the scrapping threshold is such that  $\bar{q} < \frac{S}{R}$  so that  $\bar{q}R < S$ , raising  $\bar{q}$  marginally would strictly increase the payoff, contradicting the premise that the messaging strategy was optimal.

These results, taken together, imply that the optimal scrapping threshold maximizes:

$$V(\bar{q};A) \equiv \pi \left\{ \int_{0}^{\bar{q}} SdF + \int_{\bar{q}}^{1} AdF \right\} + (1-\pi) \left\{ \int_{0}^{\bar{q}} SdF + \int_{\bar{q}}^{1} qRdF \right\}$$

subject to:

 $\bar{q} \ge \tilde{q}(A).$ 

The unconstrained maximizer of V is easily seen to be  $\max\left\{\frac{S-\pi A}{(1-\pi)R},0\right\}$ . If the constraint does bind, the solution is  $\tilde{q}(A)$  instead.

To complete the proof, we now need to argue that the suggested information design remains optimal even if random messages are allowed. Consider then general message functions hdefined from [0, 1] to the set of probability distributions on a given message space  $\mathcal{M}$  that includes at least the set of all Borel measurable subsets of [0, 1] so that, in particular, the optimal deterministic solution remains feasible. We will require that h be such that for any subset  $\mathcal{P}$  of  $\mathcal{M}$  that has a positive mass in the distribution induced by  $F \circ h$ ,  $E(qR|\mathcal{P})$  is well defined. The same Jensen inequality argument as in the deterministic case implies that we may restrict our attention to a binary message space, scrap or hold, and we denote each message as before by  $\mathcal{S}$  and  $\mathcal{H}$ , respectively. The complication is that date-0 agents may now randomize over those two possibilities for a set of  $q \in [0, 1]$ .

Assume, first, that at the optimal messaging policy  $E(qR|\mathcal{H}) > A$  but that there is a set of positive mass in  $[0, \max\left\{\frac{S-\pi A}{(1-\pi)R}, 0\right\})$  such that the probability that  $\mathcal{H}$  is emitted given almost any q in that set is strictly positive. Take a subset of those q's sufficiently small that  $E(qR|\mathcal{H}) > A$  continues to hold even if we change the message to scrap for those q's. Since  $\pi A + (1-\pi)qR < S$  by construction for those quality levels, the ex-ante payoff for date-0 agents rises strictly when we do make that policy change. This implies that q's in  $[0, \max\left\{\frac{S-\pi A}{(1-\pi)R}, 0\right\})$  must trigger the scrap message with probability one as before. The same argument implies that if  $E(qR|\mathcal{H}) > A$ , q's in  $\left(\max\left\{\frac{S-\pi A}{(1-\pi)R}, 0\right\}, 1\right]$  trigger the hold message with probability one. If  $E(qR|\mathcal{H}) > A$  then, the messaging policy is deterministic.

If  $E(qR|\mathcal{H}) < A$  then early investors expect the same payoff regardless of whether they turn out to be early or late consumers. In that case, if the scrapping policy is not the fullinformation one, the payoff can be strictly raised by changing the message policy as above, without perturbing the fact that  $E(qR|\mathcal{H}) < A$ , a contradiction. In particular, the message policy is once again deterministic.

Finally, conditional on  $E(qR|\mathcal{H}) = A$ , it is easy to see that the payoff is at its highest possible level when messages are deterministic and  $\bar{q} = \tilde{q}(A)$ .

These three scenarios for  $E(qR|\mathcal{H})$  cover all possibilities and, in all cases, the message function is deterministic. This completes the proof.

### 9.4 Proof of corollary 4.2

*Proof.* The first item is obvious. As for the second item, note first that if A is sufficiently high,  $\tilde{q}(A) = \frac{S}{R}$ , and the optimal threshold is  $\frac{S}{R}$ . As A falls  $\tilde{q}(A)$  falls below  $\frac{S}{R}$ , the threshold initially traces  $\tilde{q}(A)$  an increasing function of A. As A falls further, it starts tracing  $\frac{S-\pi A}{(1-\pi)R}$  instead, a decreasing function, until that function becomes exactly  $\frac{S}{R}$  which occurs at A = S.

#### 9.5 Proof of corollary 4.4

*Proof.* Assume that parameters satisfy:

$$S < \int \max(S, \min(qR, A)) dF < 1 < A < \int qR dF.$$

In other words, the expected payoff from fully informed secondary markets is dominated by the storage payoff, but it continues to be the case that selling to uninformed secondary markets dominates storage. Now consider early investors with a high liquidity risk. If  $\pi$  is high enough, constraining early investors to provide full information will cause them to opt for storage, thus causing a decline in investing activity since those same agents would choose to invest if they could opt for no (or, more generally, less) information.

### 9.6 Proof of proposition 6.1

To make upcoming arguments easier, we will first show that the optimal message structure is a partition of [0, 1] formed of non-overlapping intervals. To that end, we show that the strategic communication results of Crawford and Sobel (1982, CS) apply to our set-up. CS write the sender's and receiver's utility as $U^{S}(y, m, b)$  and  $U^{R}(y, m)$ , respectively, where y is the action taken by the receiver, m is a random variable (our q) and b is a scalar to measure how different the sender and the receiver are from each other. The sender observes his type, m and then has to communicate it to the receiver. In our model, the sender is the early investor in period 0, who devises the message structure. The receiver is the same investor in period 1, when he has to take the action on the scale of the project. That is, we can define

$$U^{S}(y,q,\pi) = \pi \min \{ \max[qR(y,q),S]; A\} + (1-\pi)U^{R}(y,q) \}$$

and

$$U^{R}(y,q) = \max\left[qR(y,q),S\right]$$

Notice that  $\pi$  plays the role of b in CS. One difference is that our sender does not observe q. But this does not matter since he's not the one taking the action y. Notice we have

$$U^{S}(y,q,\pi) = \pi \min \left\{ U^{R}(y,q); A \right\} + (1-\pi)U^{R}(y,q)$$

Given  $\pi$  and q, we assume that there is a unique maximum y for S and R, and we assume a sorting condition  $\frac{\partial^2 qR(y,q)}{\partial y \partial q} > 0$  so that our payoff functions satisfy the assumptions imposed by CS. They show that the optimal message structure is an interval-partition of [0, 1].

Knowing that the optimal message structure is non-overlapping intervals  $[0, q_1)$ ;  $[q_1, q_2)$ ; ..;  $[q_n, 1]$ allows us to use the lower bound of each interval as a sufficient statistics for that interval. We write  $\bar{q}_i = [q_i, q_{i+1})$ . Therefore, when agents receive message  $q \in [q_i, q_{i+1}]$ , we can write the optimal investment decision as

$$y(\bar{q}_i) = \arg\max_{y} E\left[qR(y;q) \mid q \in \bar{q}_i\right],$$

Then we write  $R(\bar{q}_i, q) = R(y(\bar{q}_i), q)$  so that the expected payoff at the optimal level of input given message  $\bar{q}_i$  is

$$E\left[qR(\bar{q}_i;q) \mid q \in \bar{q}_i\right].$$

We begin with a simple observation:

**Remark 9.2.** Let  $\pi = 1$ . Then optimally, the message is m(q) = q for all  $q < \tilde{q}$  and  $m(q) = [\tilde{q}, 1]$  for all  $q \ge \tilde{q}$ .

The argument is intuitive: When investors are sure to sell their project, they only care about its scale to the extent that it increases its value. But whenever the message is such that  $E[qR(\bar{q}_i;q) \mid q \in \bar{q}_i] > A$ , investors only sell it for A. So there is no gain in designing such a message. Instead, investors prefer to bundle all project types above  $\tilde{q}$  such that  $E[qR([\tilde{q},1];q) \mid q \in [\tilde{q},1]] = A$ . For all  $q < \tilde{q}$ , full disclosure increases the value of each project as they all sell for qR(q;q) < A. We now turn to the case where  $\pi < 1$ .

**Lemma 9.3.** Suppose  $\pi < 1$ . Then optimally, there is  $q_0 > \tilde{q}$  and  $q_1 \in (q_0, 1]$  such that the message is m(q) = q for all  $q < q_0$  or  $q \ge q_1$  and  $m(q) = [q_0, q_1)$  for all  $q \in [q_0, q_1)$ . Furthermore,

$$E\left[qR\left(\bar{q}_{0};q\right)\mid q\in\bar{q}_{0}\right]=A.$$

*Proof.* That full information is optimal for  $q < \tilde{q}$  follows from the fact that the payoff of the sender and receiver coincide in that region. Now we concentrate on  $q_0$  and we show  $q_0 \ge \tilde{q}$ . First, notice that  $q_0$  is such that  $E[qR(\bar{q}_0;q) | q \in \bar{q}_0] \ge A$ . By way of contradiction, suppose  $q_0$  satisfies

$$E\left[qR\left(\bar{q}_{0};q\right) \mid q \in \bar{q}_{0}\right] dF(q) < A$$

It should be clear that  $q_1R(y(q_1); q_1) > A$  as otherwise, the receiver and sender's payoff would coincide on the interval  $[q_0, q_1)$ . Hence,  $q_0R(y(q_0); q_0) < A$  and the receiver and sender's payoff coincide in a neighborhood of  $q_0$ . Therefore, the message is dominated by  $\tilde{m}$  such that  $\tilde{m}(q) = q$  for all  $q < q_0 + \varepsilon$  and  $\tilde{m}(q) = [q_0 + \varepsilon, q_1)$  for all  $q \in [q_0 + \varepsilon, q_1)$  and  $\tilde{m}(q) = m(q)$  for all  $q > q_1$ , where  $\varepsilon$  is chosen such that

$$E\left[qR\left(\overline{q_0+\varepsilon};q\right) \mid q\in\overline{q_0+\varepsilon}\right] \le A.$$

This contradicts that our original message was optimal. Hence,

$$E\left[qR\left(\bar{q}_{0};q\right)\mid q\in\bar{q}_{0}\right]\geq A.$$

Above, we showed that  $q_1 = 1$  when  $\pi = 1$ . When  $\pi = 0$  however,  $q_1 < 1$ . Indeed in this case,  $m(q) = \{q\}$  is optimal (i.e.  $q_1 = q_0$ ). Hence, by continuity, we necessarily have  $q_1 \leq 1$  whenever  $\pi < 1$  (and with strict equality for  $\pi$  sufficiently below 1). This implies  $q_0 \geq \tilde{q}$ . Hence  $q_0 \geq \tilde{q}$  with strict equality for  $\pi < 1$ .

It remains to show that for  $q > q_1$  it is optimal to reveal the information. We showed that

$$E\left[qR\left(\bar{q}_{0};q\right)\mid q\in\bar{q}_{0}\right]\geq A,$$

so that  $q_1R(y(q_1), q_1) > A$ . Therefore for any  $q \in m_i = [q_i, q_{i+1})$  and  $q_i \ge q_1$  we have, qR(y(q); q) > A, as well as

$$E\left[qR\left(\bar{q}_i;q\right) \mid q \in m_i\right] \ge A.$$

Hence, the impatient agent does not lose anything if the information is revealed on that interval as he gets A in any case, while the patient agent prefers to obtain the information as he can choose his action optimally. So any positive interval m above  $[q_0, q_1)$  is dominated by  $m(q) = \{q\}$ . Finally, we show that  $E[qR(\bar{q}_0;q) \mid q \in \bar{q}_0] = A$ . Suppose  $E[qR(\bar{q}_0;q) \mid q \in \bar{q}_0] > A$ . Then there is a  $\varepsilon$  such that the message  $\hat{m}(q) = [q_0, q_1 - \varepsilon)$  is such that  $E[qR(y(\hat{m});q) \mid q \in \hat{m}] \ge A$  and qR(y(q),q) > A for all  $q \in [q_1 - \varepsilon, q_1]$ . Then the original message is dominated by message  $\hat{m}$  where  $\hat{m}(q) = [q_0, q_1 - \varepsilon)$  for all q in that interval and  $\hat{m}(q) = q$  for all  $q \in [q_1 - \varepsilon, q_1]$ .

#### 9.7 Proof of proposition 6.2

Because the potential supply of secondary market investors is infinite by assumption, it is enough to show that for almost each market i, a size  $A_i$  of secondary markets exists such that given the associated optimal information design, the expected rents for entrants are exactly  $c_i$ . This, we will argue, leads a fixed point problem on  $A_i$  which satisfies standard conditions hence has at least one solution. This will establish existence.

To begin the loop then, start from a guess for  $A_i$ . The analysis of the one-market case we have carried out in this paper implies that only one optimal design policy exists given this size of secondary markets, and that this policy is fully characterized by a threshold  $\bar{q}_i$ below which the project is scrapped. Furthermore, the mapping from  $A_i$  to  $\bar{q}_i$  is continuous (and, incidentally, fully characterized in proposition 4.1.) This, in turn, implies a continuous mapping from  $A_i$  to expected rents. Either that mapping contains  $c_i$  in its image, in which case an equilibrium with active secondary markets exists, or the mapping does not achieve  $c_i$ in which case the only equilibrium is one where  $A_i = 0$  and full information prevails.

At any equilibrium where secondary markets are in fact active, there must be a probability that cash-in-the market pricing must prevail. Indeed, otherwise, rents are zero and entrants cannot recovery their entry costs. This establishes that cash-in-the-market pricing in any market where entry costs are strictly positive and completes the proof.

#### 9.8 Constrained efficiency

Could a social planner propose a different information arrangement at date 0 that improves the lot of agents alive at that date and implement that arrangement via carefully designed sets of transfers across agents? The answer turns out to be no as long as the planner cannot exclude agents from entering into side-trades and must abide by the resulting participation constraints.

To see this, consider a social planner who seeks to maximize the ex-ante welfare of early investors. We assume throughout that parameters are such that the planner is better off investing all date 0 resources in the risky project. Once a message function m is set and given a specific message  $\bar{m} = m(q)$  for some  $q \in [0, 1]$ , the planner chooses the consumption of early investors if they turn out to be early consumers,  $c_1^E(\bar{m})$ , the consumption of late investors who are early consumers,  $c_1^L(\bar{m})$ , the expected consumption of early investors who turn out to be late consumers,  $c_2^E(\bar{m})$ , and the expected consumption of late investors who are late consumers,  $c_2^L(\bar{m})$ . The planner also chooses whether or not to scrap the project given  $\bar{m}$ . We will write  $x(\bar{m}) = 1$  if the project is scrapped and  $x(\bar{m}) = 0$  otherwise. Finally, the planner must choose a quantity  $k(\bar{m}) \geq 0$  of resources to store at date 1. These choices must first be resource feasible at date 1, for all possible messages  $\bar{m}$ :

$$\pi c_1^E(\bar{m}) + (1 - \pi) c_1^L(\bar{m}) + k(\bar{m}) \le A + x(\bar{m})S.$$
(9.4)

Indeed, the only resources available for consumption at date 1 are the endowment of late investors and the proceeds from scrapping the project. Likewise, the expected payoffs for late consumers given  $\bar{m}$  must be feasible:

$$(1-\pi)c_2^E(\bar{m}) + \pi c_2^L(\bar{m}) = (1-x(\bar{m}))E(qR|\bar{m}) + k(\bar{m})$$
(9.5)

Since late investors can always consume their endowment immediately or store it, the plan must also satisfy the following participation constraints:

$$c_1^L(\bar{m}), c_2^L(\bar{m}) \ge A \tag{9.6}$$

Finally we require that early investors who are early consumers be willing to participate in the arrangement upon discovering their consumption type. We assume that types are either unverifiable or unobservable so that any agent can claim either  $c_1^E(\bar{m})$  or  $c_2^E(\bar{m})$ . Hence, early consumers who pretend to be late consumers can always sell claims to  $c_2^E(\bar{m})$  to late investors. An early investor who chooses to sell her individual claims to late investors must offer at least the same return as the one offered by the planner, namely

$$r(\bar{m}) \equiv \frac{c_2^L(\bar{m})}{A} - 1$$

Therefore the planner faces the additional constraint that an early investor should not be better off by selling his claim to late consumption rather than taking the proposed consumption for early consumers:

$$c_1^E(\bar{m}) \ge \frac{c_2^E(\bar{m})}{1+r(\bar{m})}.$$
(9.7)

Here, a key observation is that while the planner must internalize aggregate resource constraint (9.4) in establishing a consumption vector for all agents, individual deviators are not constrained in that fashion. In particular, since each early investor is small, she would face unbounded demand for claims to late consumption remunerated at a transformation rate infinitesimally higher than the social planner.

Then, given a message function  $m: [0,1] \mapsto \mathcal{B}[0,1]$ , the planner solves:

$$SP(m) = \max \pi \int c_1^E(m(q))dF + (1-\pi) \int c_2^E(m(q))dF$$

subject to (9.4), (9.5), (9.6) and (9.7) holding for every possible message m(q) emitted at date 1. We will now show that the message strategy which early investors select in the decentralized environment maximizes SP(m), hence is constrained-efficient.

**Proposition 9.4.** The messaging strategy early investors select in the decentralized environment is constrained-efficient.

Proof. Given a message function, we will show that the allocation described in section 3 is feasible for the planner and achieves SP(m). Since the planner seeks to maximize the welfare of early investors and  $c_1^L(\bar{m})$  only matters through its effect on available resources, any solution must feature  $c_1^L(\bar{m}) = A$  for every possible message  $\bar{m}$  emitted at date 1, so that (9.4) becomes  $\pi c_1^E(\bar{m}) + k(\bar{m}) \leq \pi A + x(\bar{m})S$ . Next, if  $S \geq E(qR|\bar{m})$ , setting  $x(\bar{m}) = 1$ ,  $c_1^L(\bar{m}) = c_2^L(\bar{m}) = A$ , and  $c_1^E(\bar{m}) = c_2^E(\bar{m}) = S$ , which is the decentralized solution, obviously solves the social planner's problem. Likewise, if A < S, the presence of late investors is irrelevant and the proposition holds trivially.

So assume henceforth that  $S < E(qR|\bar{m})$  and that S < A. In that case  $x(\bar{m}) = 0$  is easily seen to be optimal which means that (9.4) becomes  $c_1^E(\bar{m}) \leq A$ . But this, together with (9.7), implies

$$A(1+r(\bar{m}) \ge c_2^E(\bar{m}) \iff c_2^L(\bar{m}) \ge c_2^E(\bar{m})$$
 for all possible messages  $\bar{m}$ .

This inequality is the linchpin of the proof. It says that the fact that early investors can enter into side trades ends up implying that – even though he does not value their welfare directly – the planner has to deliver a payoff to late investors who are late consumers that is as high as that of original stakeholders. If the planner tries to reduce the rate of transformation late investors receive in the arrangement, individual deviators can offer them a better deal.

To conclude the proof, observe that (9.4) and (9.5), together with the fact that  $x(\bar{m}) = 0$ , and the fact that  $c_1^L(\bar{m}) = A$  imply that

$$(1-\pi)c_2^E(\bar{m}) + \pi c_1^E(\bar{m}) \le \pi A - \pi c_2^L(\bar{m}) + E(qR|\bar{m}).$$
(9.8)

Now we only need to consider two simple subcases. If  $A > E(qR|\bar{m}) > S$ , the decentralized solution calls for a hold for early investors who are late consumers and for a sale of project shares at price  $E(qR|\bar{m})$  for early investors who are early consumers. Late investors who are late consumers expect payoff  $E(qR|\bar{m})$  at date 2. That allocation satisfies all of the planner's constraints and makes the payoff  $E(qR|\bar{m})$  which, given (9.8) and the fact that  $c_2^L(\bar{m}) \ge A$ , is the highest payoff the planner can achieve in this case.

If  $S < A < E(qR|\bar{m})$ , the decentralized solution calls once again for a hold and for a sale of their share at price A for early investors who are early consumers. Late investors who are late consumers expect payoff  $E(qR|\bar{m})$  at date 2. That allocation is feasible for the planner and makes  $(1 - \pi)c_2^E(\bar{m}) + \pi c_1^E(\bar{m}) = (1 - \pi)E(qR|\bar{m}) + \pi A$ . We will show that this payoff cannot be beat by the planner. Since we must have  $c_1^E(\bar{m}) \leq A$ , the only way to beat it is to have  $c_2^E(\bar{m}) > E(qR|\bar{m})$ . Since  $c_2^L(\bar{m}) \geq c_2^E(\bar{m})$ , that would imply  $c_1^E(\bar{m}) < A$ . But in that case it feasible to reduce both  $c_2^L(\bar{m})$  and  $c_2^E(\bar{m})$  by a marginal  $\epsilon > 0$ , increase  $c_1^E(\bar{m})$  by  $\frac{\epsilon}{\pi}$ , which changes the objective by  $-(1 - \pi)\epsilon + \pi \frac{\epsilon}{\pi} > 0$ . Hence the decentralized allocation is optimal in that subcase as well, which completes the proof.

Given any message strategy m, the decentralized allocation we characterize in section 3 gives

early investors a payoff of exactly SP(m). If follows that date-0 investors always select a messaging strategy m that maximizes SP(m) and, as a result, is constrained-efficient. Because early investors can enter into side-trades, the planner must deliver the same consumption to all late consumers, whether they were early or late investors. Hence, although the planner does not value the welfare of late investors, they still receive consumption when they consume late, and that consumption must be higher than what they would obtain from storing their endowment. It follows that the marginal return to raising  $c_2^E(m(q)) \ge A$  is below the implied resource cost. On the other hand, while the marginal return to raising early consumption  $c_1^E(m(q))$  equals its marginal resource cost, early consumption cannot be higher than the available resources A. At any optimum therefore, if A is relatively low – i.e. when S < A <E(qR|m) – then the planner would like but cannot increase the early consumption of early investors beyond A, and so  $c_1^E(m(q)) = A$ . If A is high – i.e. when A > E(qR|m) > S – then the planner can shift consumption towards early consumers until the constraint on late consumption binds, so that  $c_2^E(m(q)) = A$ . But this is exactly what the decentralized solution delivers, and the result follows.

#### 9.9 Proof of remark 5.1 and remark 5

*Proof.* Consider a candidate price schedule p(q) for project shares at date 1 where, in the context of this proof, the premise is that q is only observed by early investors. We know that  $p(q) \geq S$  in any equilibrium and for almost all q. If S < p(q) < qR then only early consumers supply their project shares and, upon observing that demand, potential buyers infer that q is distributed with strictly positive continuous density over  $\left[\frac{p(q)}{R}, 1\right]$ . It follows that demand for project shares is  $\pi \frac{A}{p(q)}$ . The only case in which this is an equilibrium, therefore, has p(q) = A and qR > A. If p(q) > qR then all potential sellers sell, from which buyers infer that q is distributed with strictly positive continuous density over  $[0, \frac{p(q)}{R}]$  so that demand is zero, which can not be an equilibrium. The only equilibrium, then, has  $p(q) = \min(S, qR)$  if qR < A and p(q) = A if  $qR \geq A$  exactly as in the full information case.

The argument is the same for remark 5 with E(q|m(q))R playing the role of qR

#### 9.10 Proof of proposition 5.3

*Proof.* Assume that the (fully but privately informed) manager observes that  $q < \bar{q}(\pi)$ . Then, since  $\alpha q R < M$ , she chooses to scrap, as desired. The converse holds by the exact same logic

and the compensation scheme, therefore, leads to exactly the desired policy.

#### 9.11 Delegation with moral hazard

The delegation scheme we considered in the text assumes that the manager has no impact on the project's outcome. Assume instead and more realistically that success requires a certain level of attention, or effort, on the part of the manager. In this section, we show that, counterintuitively, opacity about asset quality may *lower* the cost of inducing managers to expend the optimal level of effort on the project they oversee.

Precisely, we assume the manager can affect the quality of the project by exerting an unobservable effort  $e \in [0,1]$  before nature draws the quality of the project. If she exerts effort e and nature draws quality q then the probability the project succeeds is  $e.q \in [0,q]$ . Therefore, we can think of the manager's action as affecting the distribution from which nature draws a level of quality. To make the problem interesting, we assume that effort is costly and the per unit cost of effort for the manager is B. Notice that the structure of the economy is the same as the one we have already studied when e = 1.<sup>28</sup>

Since the effort level is unobservable and unverifiable, agents who observe e.q are unable to distinguish e from q. Therefore, given effort e, the analysis proceeds as before: agents select a liquidation threshold  $\bar{q}_h$  and, when if  $eq < \bar{q}_h$ , then the delegate scraps the project, while if  $eq \ge \bar{q}_h$  then the project is held to maturity. Agents need to design a compensation scheme such that (1) the delegate chooses to liquidate the project if and only if  $eq < \bar{q}_h$ , and (2) the manager chooses to the optimal level of effort. Below we show that the unique compensation scheme that satisfies both requirements belongs in the class of schemes we have used before: the manager receives a fixed severance payment or, when the project is held to maturity, a participation in revenues. In particular then, the manager gets paid a positive amount whenever he has to scrap the project and this gives him an incentive to shirk, since in that case pay is unaffected by effort. Hence, transparency – in the sense of a higher  $\bar{q}_h$  – will increase the cost of maintaining the manager's incentives to work. Formally:

**Proposition 9.5.** With moral hazard, more transparency is costly, as it requires a higher severance payment and a higher share of the project's revenue to incentivize the manager to work.

 $<sup>^{28} \</sup>text{Assuming that the other agents incur no cost of exerting effort, or that the minimum <math display="inline">\pi$  is high enough that agents always exert efforts.

Proof. Consider first the incentives of the manager to exert the right amount of effort, given  $\bar{q}$ . Take a payment scheme  $P(eq, \rho, a)$  where e is the manager's effort, q is the quality of the project known only to the manager,  $\rho \in \{s, f\}$  is the outcome of the project (success or failure) and  $a \in \{S, H\}$  is the interim announcement of the manager (scrap, S or hold, H). Since the project should be scrapped whenever  $eq \leq \bar{q}$ , the payment cannot be conditioned on the final outcome of the project, success or failure, so that  $P(eq, \rho, S) = P(eq, S)$  for all  $eq \leq \bar{q}$ . Truth-full revelation of eq in the range  $[0, \bar{q}]$  implies that P(eq, S) = P(S), as otherwise the manager would always choose to reveal the quality eq that gives him the highest payoff. In particular, notice that P(S) does not depend on the effort level chosen by the manager.

We now turn to the case where  $eq > \bar{q}$ . In this case the project should be kept to maturity, so that the payoff can depend on the outcome s or f. Now, for any  $eq > \bar{q}$ , the manager has to prefer to say just H than revealing another  $q' \in [\bar{q}, 1]$  as otherwise the payment scheme would not satisfy the requirement that it is optimal that the manager only communicates scrap or hold, S or H. Hence, the payment scheme has to satisfy for any  $eq, q' \in [\bar{q}, 1]$ ,

$$eqP(eq, s, H) + (1 - eq)P(eq, f, H) = eqP(q', s, H) + (1 - eq)P(q', f, H),$$

i.e. whatever q nature draws, the manager is indifferent between revealing any q' as long as it implies H. Therefore, combining the incentive compatibility constraint to reveal eq instead of any q' as well as the constraint to reveal q' instead of eq we obtain

$$(eq - q')(P(eq, s, H) - P(q', s, H)) = (eq - q')(P(eq, f, H) - P(q', f, H))$$

Hence, P(eq, s, H) - P(q', s, H) = P(eq, f, H) - P(q', f, H). As it would be more expensive to compensate the manager more in one case than in another<sup>29</sup> we conclude that P(eq, s, H) = P(q', s, H) = P(q', s, H) = P(q', f, H) = P(f, H).

Finally, we need to insure that the manager announces S (scrap) for all  $eq < \bar{q}$  and H (hold) otherwise. That is the payment scheme should satisfy,

$$eqP(s,H) + (1-eq)P(f,H) \ge P(S), \text{ if } eq > \bar{q},$$

and

$$P(S) \ge eqP(s,H) + (1-eq)P(f,H), \text{ if } eq < \bar{q}.$$

<sup>&</sup>lt;sup>29</sup>If P(eq, s, H) = P(q', sH) yields the desired result, then there is no reason to incur the additional cost of setting P(eq, s, H) > P(q', sH).

Since the payoff function when H is increasing in eq whenever P(s, H) > P(f, H) and decreasing otherwise, we need to set P(s, H) > P(f, H). Finally, payment is minimized whenever

$$P(S) = \bar{q}P(s,H) + (1-\bar{q})P(f,H).$$
(9.9)

We can now derive the incentive constraint on the effort level. Given  $\bar{q}$ , the manager's payoff of exerting effort e is simply

$$\int_{0}^{\bar{q}/e} P(S)dF(q) + \int_{\bar{q}/e}^{1} \left[ eqP(s,H) + (1-eq)P(f,H) \right] dF(q) - Be$$

which is convex in e as P(s, H) > P(f, H), that is the marginal payoff is

$$\int_{\bar{q}/e}^{1} q \left[ P(s,H) - P(f,H) \right] dF(q) - B$$

which is negative at e = 0 and increasing in e. Hence, the manager will choose either e = 0 or e = 1. In other words, the manager exerts effort if and only if

$$\int_{0}^{\bar{q}} P(S)dF(q) + \int_{\bar{q}}^{1} \left[ qP(s,H) + (1-q)P(f,H) \right] dF(q) - B \ge P(S)$$

on the left hand side of this incentive constraint is the payoff when e = 1 while on the righthand side is the payoff when e = 0. In this case notice that the manager always gets P(S), as the project is always scrapped. Arranging terms and using (9.9) we can rewrite this incentive constraint as

$$\int_{\bar{q}}^{1} \{ (q - \bar{q}) P(s, H) + (\bar{q} - q) P(f, H) \} \, dF(q) \ge B$$

As  $\bar{q} < q$  in the range of integration, it is optimal to set P(f, H) = 0 and P(s, H) such that

$$P(s,H) = \frac{B}{\int_{\bar{q}} (q-\bar{q})dF(q)}$$

$$(9.10)$$

Notice that the payment is increasing in  $\bar{q}$ . Therefore, more transparency (in the sense of a higher  $\bar{q}$ ) is costly, as it requires a higher compensation scheme to incentivize the manager.

When, as in the text, we set  $P(s, H) = \alpha R$  for a given  $\alpha$  then the manager exerts effort

if and only if

$$\int_{\bar{q}}^{1} (q - \bar{q}) dF(q) \ge \frac{B}{\alpha R}$$
(9.11)

On the left-hand side is the gains from working, while on the right-hand side is the relative gains from shirking. Notice that the left-hand side is decreasing in  $\bar{q}$  and there is a pair  $\hat{q}(\alpha)$ and  $\alpha$  such that (9.11) is satisfied for all  $\bar{q} \leq \hat{q}(\alpha)$ . Hence, agents will choose  $\bar{q}$  to maximize

$$V(\bar{q};\pi) \equiv \int_{0}^{\bar{q}} SdF + \dots \int_{\bar{q}}^{1} qRdF - \pi \int_{\bar{q}}^{1} (qR - A) dF, \qquad (9.12)$$

subject to (9.11) and  $\bar{q} \leq \hat{q}(\alpha)$ . Therefore moral hazard will increase opacity (weakly), as by decreasing  $\bar{q}$  the agent decreases the region where the manager gets paid  $\alpha \bar{q}R$  while exerting no effort.

Notice that the contracting problem bears a strong resemblance to the standard costly state verification problem of Townsend (1979), with scrapping playing the same role in our problem as audit does in Townsend's. However, the payment structure is essentially reversed: it is debt when the manager scraps the project and equity otherwise. This occurs because outsiders cannot tell whether nature selected a low q or whether the manager shirked.

#### References

Aghion, P., Bolton, P. and J. Tirole (2004), "Exit Options in Corporate Finance: Liquidity versus Incentives," *Review of Finance* 8, 327-353.

Allen, F. and D. Gale (1994), "Limited Market Participation and Volatility of Asset Prices," *American Economic Review*, 84(4), 933-955.

Allen, F. and D. Gale (2005), "From Cash-in-the-Market Pricing to Financial Fragility," *Journal of the European Economic Association* 3 (2-3), 535-546.

Andolfatto, D., Berentsen, A., and C. Waller (2014), "Optimal Disclosure Policy and Undue Diligence," *Journal of Economic Theory* 149, 128-152.

Bolton, P., T. Santos, and J. A. Scheinkman (2011), "Inside And Outside Liquidity," *Quarterly Journal of Economics* 126, 259-321.

Bouvard, M., P. Chaigneau, and A. de Motta (2015), "Transparency in the Financial System: Rollover Risk and Crises," *Journal of Finance*, forthcoming.

Breton, R. (2007), "Monitoring and the Acceptability of Bank Money," *mimeo*, Banque de France.

Dang, T. V., Gorton, G. Holmstrom B. and G. Ordonez (2013), "Banks As Secret Keepers," *mimeo*, Yale University.

Dang, T.V, Gorton, G. and Holmstrom B. (2012), "The Information Sensitivity of a Security," *mimeo*, Yale University.

Diamond, D. (1984), "Financial Intermediation and Delegated Monitoring," *Review of Economic Studies* 51, 393-414.

Diamond, D. (1985), "Optimal Release of Information By Firms," *Journal of Finance* 40(4), 1071-1094.

Diamond, D. and P. Dybvig (1983), "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy* 91, 401–419.

Flannery, M. J., Kwan, S. H. and M. Nimalendran (2004), "Market Evidence on the Opaqueness of Banking Firms' Assets," *Journal of Financial Economics* 71(3), 419-460.

Flannery, M. J., Kwan, S. H. and M. Nimalendran (2013), "The 2007–2009 Financial Crisis and Bank Opaqueness," *Journal of Financial Intermediation* 22(1), 55-84.

Gorton, G. and G. Pennacchi (1990), "Financial Intermediaries and Liquidity Creation," *Journal of Finance* 45(1), 49-71.

Hirshleifer, J. (1971), "The Private and Social Value of Information and the Reward to

Inventive Activity," American Economic Review 61(4), 561–574.

Hirshleifer, J. (1972), "Liquidity, Uncertainty, and the Accumulation of Information," in C.F. Carter and J.L. Ford (eds.), Uncertainty and Expectations in Economics, Essay in Honour of G.L.S. Shackle, Basil Blackwell, Oxford, 136-147.

Jacklin, C. (1987), "Demand Deposits, Trading Restrictions, and Risk-Sharing," in Ed Prescott and Neil Wallace (eds.), *Contractual Arrangements for Intertemporal Trade*, University of Minnesota Press, Minneapolis, 26-47.

Galfetti, S., Perembetov, K. and J. B. Marks (2014), "Private Equity Secondaries: Roadmap for Small to Mid-sized Investors to Successfully Access Secondaries," Capital Dynamics White Paper (http://www.capdyn.com/media/1752/white-paper-private-equity-secondaries-second-editionkap-and-jbm-1.pdf)

Goldstein, I. and Y. Leitner (2013), "Stress Tests and Information Disclosure," Federal Reserve Bank of Philadelphia Working Paper No. 13-26.

Goldstein, I. and H. Sapra (2014), "Should Banks' Stress Test Results be Disclosed? An Analysis of the Costs and Benefits," *Foundations and Trends in Finance* 8(1), 1-54.

Gompers, P. and J. Lerner (1999), "An Analysis of Compensation in the U.S. Venture Capital Partnership," *Journal of Financial Economics* 51(1), 3-44.

Greenwood, J., Hercowitz, Z. and G. W. Huffman (1988), "Investment, Capacity Utilization, And The Real Business Cycle," *American Economic Review* 78(3), 402-17.

Kaplan, T. (2006), "Why Banks Should Keep Secrets," *Economic Theory* 27(2), 341-357. Kaplan, S. N. and Per Stromberg (2009), "Leveraged Buyouts and Private Equity," *Journal of Economic Perspectives* 23(1), 121-46.

Kurlat, P. and L. Veldkamp (2013), "Should We Regulate Financial Information?" *mimeo*, New York University.

Kamenica, E. and M. Gentzkow (2011), "Bayesian Persuasion," American Economic Review 101, 2590-2615.

Leuz, C. and P. Wysocki (2008), "Economic Consequences of Financial Reporting and Disclosure Regulation: A Review and Suggestions for Future Research," *mimeo*, Chicago Booth, University of Chicago.

Ljungqvist, A. (2007), "IPO underpricing: a survey," In: Eckbo, B.E. (Ed.), Handbook of Corporate Finance, North-Holland, Amsterdam.

Metrick, A. and A. Yasuda (2010), "The Economics of Private Equity Funds," *Review of Financial Studies* 23 (6), 2303-41.

Milgrom, P. and N. Stokey (1982), "Information, Trade and Common Knowledge" *Journal* of Economic Theory 26(1), 17-27.

Morgan, D. (2002), "Rating Banks: Risks and Uncertainty in an Opaque Industry," *American Economic Review* 92(4), 874-888.

Naidech, S. W. (2011), "Private Equity Fund Formation," Practical Law Company Practice Note 3-509-1324.

Siegert, C. (2012), "Optimal Opacity and Market Support," *mimeo*, University of Munich. Townsend, R. (1979), "Optimal Contracts and Competitive Markets with Costly State

Verification," Journal of Economic Theory 21, 265-293.

Von Thadden, E. (1995), "Long-Term Contracts, Short-Term Investment and Monitoring," *Review of Economic Studies* 62(4), 557-75.

Zetlin-Jones, A. (2013), "Efficient Financial Crises," mimeo, Carnegie-Mellon.