# On Existence in Equilibrium Models with Endogenous Default

Erwan Quintin*
Wisconsin School of Business

July 31, 2012

**Abstract**

Establishing that market-clearing equilibria exist is notoriously difficult in models of financial economies where agents can choose to default on their financial obligations. In a seminal paper, Dubey, Geanakoplos and Shubik (DGS, 2005) propose an equilibrium concept for this class of models under which, remarkably, existence obtains universally. The key aspects of this concept are that all agents take both asset prices and default rates as given. I show that the resulting equilibrium concept generates outcomes where lenders could increase their profits simply by lowering the rates they charge, making price-taking an untenable assumption. What's more, the DGS equilibrium set may contain nothing but these outcomes, hence the universal existence argument hinges critically on the assumption that lenders ignore glaring profit opportunities. Put another way, the argument that underlies the conventional wisdom that prevailed before DGS (1990, 2000, 2005) fully applies to the DGS environment as well.

Keywords: Default; General Equilibrium; Existence; Credit Rationing
JEL codes: D52; E44; G12

*Real Estate Department, University of Wisconsin - Madison, equintin@bus.wisc.edu. I wish to thank Marek Weretka, Marzena Rostek, Peter Ritz, Abdullah Yavas, François Ortalo-Magné, Morris Davis, Dean Corbae, Stefan Krasa, Anne Villamil, Guillaume Rocheteau as well as seminar participants at the 2010 Midwest Economic Theory Meetings, Sciences Po Paris, the University of California at Irvine, Duke University, the University of Windsor, Ecole Polytechnique and the University of Illinois for useful comments and suggestions.

# 1 Introduction

Most loan contracts are written between counterparties who understand that default is a possibility. Lenders take this possibility into account when deciding whether to issue loans and what terms to offer borrowers. In the residential mortgage market for instance, observable characteristics such as income and credit scores affect both approval decisions and interest rates. In general equilibrium therefore, the possibility of default should matter for the aggregate quantity of financial intermediation and the terms at which different agents borrow and lend.

Studying the theoretical implications of endogenous default requires a general equilibrium model where borrowers have the option to renege on their financial promises and where the consequences of exercising this option are fully specified. Among the most natural choices for this purpose is the seminal model of Dubey, Geanakoplos and Shubik (DGS, 1990, 2000, 2005.) The DGS framework builds on the standard general equilibrium model with incomplete market by allowing for default. Agents take commodity and asset prices as given as in the standard model, but they also take as given delivery rates hence effective returns on assets.

DGS establish that equilibria always exist in this environment, a remarkable result since endogenizing asset payoffs and allowing heterogenous agents to self-select into financial contracts is known to make existence problematic in competitive models. The universality of this existence result stems from the nature of the equilibrium concept DGS employ: agents take both asset prices and delivery rates as given, and delivery choices by agents must be consistent with those expectations. Existence then boils down to a fairly standard fixed point problem and, under standard conditions on preferences and budget sets, the usual arguments apply (see Bisin et al., 2010, for additional illustrations of the benefits of this equilibrium concept.)

This paper shows that while the DGS equilibrium concept guarantees existence, it also tends to generate equilibria that make the assumption that agents take asset prices as given impossible to defend. Specifically, since by assumption lenders ignore the impact of loan terms on delivery rates, an equilibrium may be such that by lowering stated returns on assets – by paying a higher price for a given asset, that is – lenders could raise effective, net-of-default returns. The intuition for this is simple: lowering stated returns, which borrowers obviously welcome, can cause delivery rates to rise more than proportionately. The question, then, is what could prevent lenders from proposing borrowers terms that would make both parties better off?

After showing that this type of equilibrium can arise in the DGS environment, I define and study a procedure that discards these equilibria. In general however, excluding these outcomes compromises existence as a simple example demonstrates. This means that DGS' existence argument depends critically on calling equilibria a set of contract terms such that lenders choose to forego obvious profit opportunities. In fact, in my main counter-example, equilibria may involve some credit-rationing in the sense of Stiglitz and Weiss (1981).

In this sense, my findings favor the conventional wisdom that prevailed before DGS' existence result. DGS (2005, p15) describe this conventional wisdom as "the historical tendency to associate default with disequilibrium ..." They go on to write that "... the endogeneity of the asset payoff structure is known to complicate the existence of equilibrium with incomplete markets. But we show that no new existence problems arise from the endogeneity of the asset payoffs due to default." One way to interpret my results is that the conventional wisdom is in fact exactly right.

In a paper that applies a similar equilibrium concept as DGS (2005) to the canonical insurance problem studied by Rothschild and Stiglitz (1976), Dubey and Geanakoplos (2002, pp 1549-50) argue that exploiting profit opportunities by proposing contracts outside the equilibrium set seems "implausible [...] in a large economy." The same objection could be made to rule out the class of deviations on which my exclusion procedure relies. In a literal sense, atomistic agents have no choice but to take prices and contracts as given. But in the return-dominated equilibria the DGS concept fosters, all lenders have incentives to deviate from the equilibrium pricing scheme by lowering their rates and an incentive to form coalitions (financial intermediaries, in effect) to exploit that opportunity if necessary. My results say that modeling lenders as agents who for purely technical reasons cannot lower the rates they charge – in addition to flying in the face of the reality of credit markets we are trying to approximate – is an indefensible modeling assumption. Modeling lenders as in Stiglitz and Weiss (1981) as agents who understand that their pricing decisions have consequences on default rates seems not only natural but, given my findings, the only justifiable way to model loan markets. The DGS concept circumvents the existence issues that naturally arise in models with endogenous default by arbitrarily ruling out the option to respond to the very incentives that cause these issues in the first place.

To put this in more general terms, price taking can be justified in the standard framework with no default on trivial, compelling grounds: in equilibrium, no borrower would accept a rate higher than the equilibrium rate from any given lender and lenders have no reason to offer a rate below the equilibrium rate. Price taking is a natural assumption in such a context regardless of whether lenders are allowed to form coalitions of positive mass. They simply have no incentives to form such coalitions. In the DGS model, while the first part of the standard argument remains correct, the second part fails to hold: buyers may now be able to raise delivery rates by lowering the rate they charge borrowers, which borrowers obviously welcome. Nothing prevents agents from taking advantage of this option when it is present. Competition among lenders is bound to eliminate this glaring profit opportunity. In practical terms, given the resources large financial intermediaries devote to estimating default models,[1] it seems difficult to argue that they fail to understand a relationship as trivial and fundamental as that between the size of loan payments and the likelihood of default.

Dubey and Geanakoplos (2002) also question the plausibility of profitable deviations from equilibrium contracts on the grounds that "households lack the knowledge and computing power to infer the composition [of deliveries] when expectations are far from equilibrium." This only says, however, that agents (intermediaries) would naturally emerge in this environment that carry out these calculations on behalf of lenders, very much as they do in actual credit markets. Furthermore, while I make the examples in this paper as simple as possible

[1] See e.g. Gerardi et al., 2008, for a discussion

by focusing on discrete deviations, these examples are trivially extended to generate DGS equilibria where marginal deviations are profitable (see Quintin, 2012, for a specific example.) In those cases agents need only understand the local shape of the price-default relationship, they do not need to consider distant contracts.

Dubey and Geanakoplos (2002) and DGS (2005) argue more practically that assuming that lenders take delivery rates as given is justified in contexts where assets are pooled by intermediaries and sold to large anonymous pools of lenders, as is particularly common in mortgage markets. This portrayal of the securitization process and of the limits it imposes on agents seems peremptory, at best. Anonymous as these markets may be, if it is possible to change loan terms across the board to increase cash-flows in all states while making all borrowers better off, someone – the securitizing intermediary, for one, the banks that hold loans on their books, for another – should recognize and take advantage of this costless profit opportunity. Even in a hypothetical world where all assets of particular type are securitized, return-dominated equilibria are bound to be arbitraged away.

A version of the DGS concept has also been used to study environments where asset purchases must be collateralized.[2] As is well known, many loans used to finance the purchase of a durable good are collateralized by that good. When liability is limited to that collateral and borrowers incur no additional default penalty, they default when and only when the value of their debt exceeds the value of that collateral. This provides a theoretical framework in which the popular notion of "strategic default" can be formalized. In this simpler environment, it should be fairly obvious that lowering stated payoffs cannot increase expected delivery rates and in that sense, the issues raised in this paper become less problematic.

Nevertheless, environments where at least as a first approximation borrowers incur no default cost beyond a specific collateral loss are likely to be few and far between. Take once again the canonical example of residential mortgages. Even in the few US states (California, e.g.) where liability is effectively limited to the home, borrowers incur myriad other costs

[2]The pioneering paper is Geanakoplos and Zame (2005). See also Araujo et al. (1998) and Araujo et al. (2005).

when they default, including the impact on their credit history and ability to borrow and become home-owners again. In fact, while popular accounts would have one believe that ruthless, strategic default is common, the facts do not bear that out. In the United States, even in the midst of the most severe housing downturn on record, most households with negative equity choose to continue meeting their financial obligations.[3] This suggests that residential borrowers do incur costs beyond their collateral, in which cases the issues I have formalized in this paper become relevant, as shown in Quintin (2012).

Finally, the arguments this paper employs to show that endogenous default can make existence problematic are obviously related to the seminal work of Stiglitz and Weiss (1981), the paper that initially motivated the conventional view that default and disequilibrium go hand-in-hand. As in Stiglitz and Weiss (1981), the natural equilibrium that arises in this paper is one where some agents are excluded from borrowing even though they would be willing to accept higher borrowing rates. However, the intuition for the rationing outcome in this paper differs substantially from the intuition behind the findings of Stiglitz and Weiss (1981). The issue here need not be that the composition of the pool of borrowers changes adversely when rates increase. Instead, my examples rely on the intuitively obvious fact that identical borrowers become more prone to default when their financial obligations increase. Recent work (see Arnold and Riley, 2009) has argued that the possibility of rationing in the Stiglitz-Weiss framework may be fragile. This paper's findings suggest that rationing may prevail in models of default for more robust reasons.

## 2 The basic idea

Before delving into the notation-heavy details of the DGS framework, it may be useful to sketch the basic issues the DGS concept raises in the context of a simple, informal example. Consider a static economy where a large number of identical lenders have funds they can lend to a large number of identical borrowers. Assume that the opportunity cost of those funds is

[3] See, e.g., Gerardi et al. (2007), Gerardi et al. (2009a) and Gerardi et al. (2009b).

fixed, for instance because their only alternative use is storage at a deterministic and strictly positive return. Assume further that borrowers have the option to default on their loan and that their incentives to default rise with the size of the payment they owe at the end of the period.

Lenders must decide whether to lend funds to borrowers. As the next section will make precise, the DGS model assumes that lenders take as given both the rate such loans earn and the fraction of the stipulated payment that will in fact be paid by borrowers.

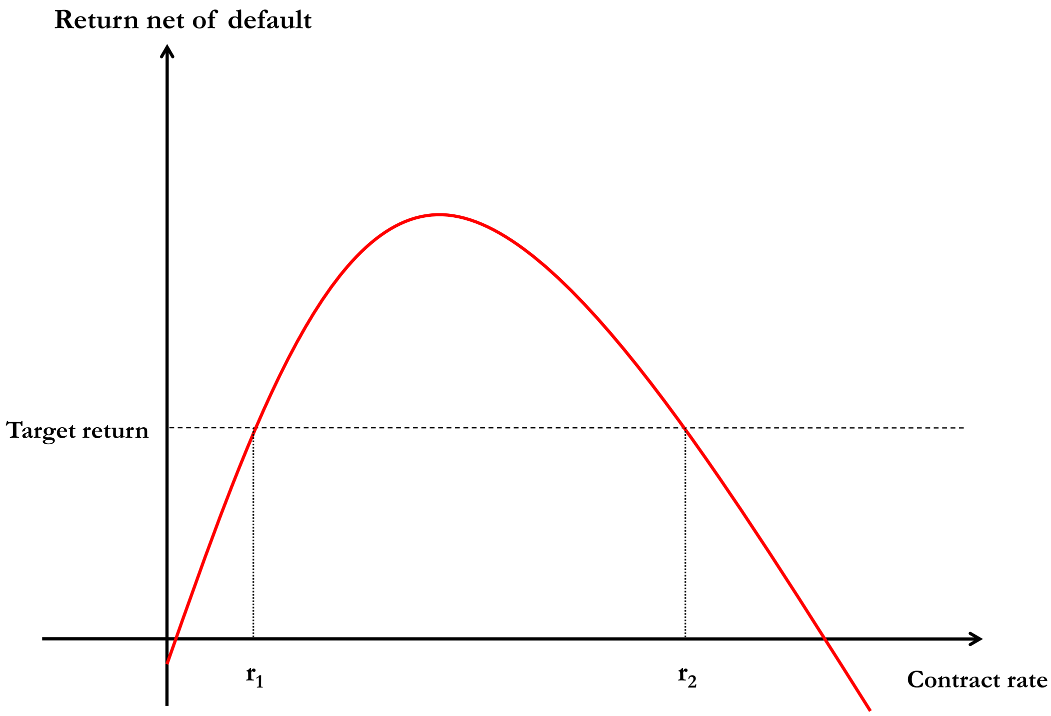
At a sufficiently low interest rate, the return on the loan cannot match the opportunity cost of funds and lenders would choose to store funds rather than loan them out. As rates rise, so does the size of the payment and so may, therefore, the return net of default. On the other hand, the probability of default or, equivalently, expected default losses also rise. It may in fact be the case that past a certain level, raising the interest rate further causes the effective return to fall. The relationship between stated contract rates and effective return may therefore look as depicted in figure 1.[4]

In that case, two rates cause effective loan returns to exactly equal the target return. If lenders have funds in excess of what borrowers need, or if there is free entry into lending activities, standard competitive arguments show that $r_1$ and $r_2$ are the only two possible equilibrium rates. Under the DGS concept where lenders take default rates are given, both rates are in fact equilibria.

Yet, it should be obvious that $r_2$ could not possibly persist in a competitive environment. Indeed, lenders would recognize or learn that lowering the returns they require from their loans (which all borrowers would obviously welcome) would raise profits. The argument for assuming price taking one would invoke in a version of this economy without default – that lenders would like to raise rates but would find no takers while they have no incentive to lower rates since they can fund as many loans as they want at the equilibrium rate – breaks down since they can now raise profits by lowering the rates they charge.

[4]For the complete description of an economy that yields exactly this type of relationship, see Quintin (2012).

Figure 1: Effective returns vs. stated returns




This paper's message can be quickly summarized in the context of this informal example. The next section shows that the DGS model does generate equilibria like $r_2$, outcomes I term return-dominated equilibria. An obvious way to solve this issue is to exclude this type of equilibrium from the DGS set and only retain $r_1$-like equilibria. What I demonstrate however is that the DGS model may generate nothing but return-dominated equilibria. In other words, universal existence only holds in DGS (2005) because the concept calls equilibria outcomes that hardly seem sustainable.

## 3 The DGS equilibrium concept

DGS (2005) describes a financial economy with exogenously incomplete markets where agents can choose to renege on their financial obligations. Specifically, they consider an environment with two dates – 0 and 1 – and $S$ possible states at date 1. For simplicity, assume that only one commodity is traded at each date since this entails no loss of generality for my purposes.

The economy contains an equal mass of a finite set $H$ of agent types with utility function $u^h : \mathbb{R}_+^{S+1} \mapsto \mathbb{R}$ which is continuous, concave and strictly increasing for all $h \in H$. Letting $i$ index agents on the unit interval, order agents so that agent $i \in (\frac{h-1}{H}, \frac{h}{H}]$ is of type $h \in \{1, 2, \ldots H\}$. For expositional simplicity, DGS (2005) focus on the case where the number of agents is finite, but doing so without compromising existence requires imposing stringent restrictions on punishment technologies such that the each agent's problem remains convex. As DGS (2005) explain,[5] an existence argument that holds only for a linear specification of default punishment would be of little interest or generality. In this paper, I need to work with a broader class of punishment functions.

Agents of type $h \in H$ are endowed with $e^h \in \mathbb{R}_+^{S+1}$. Agents trade the commodity at each state $s \in \{0\} \cup S$ at a spot price I normalize to be $p_s \equiv 1$ in all states.[6] Agents also trade a set $J$ of assets and $R_j \in \mathbb{R}^S$ denotes the quantity of the commodity to be delivered by asset $j$ at each possible state at date 1, while $\pi_j$ denotes the price of asset $J$ at date 0. Agents of type $h \in H$ face a short-sale constraint $Q_j^h$ on asset $j \in J$.

As they would in standard financial economies, agents choose a consumption vector $x \in \mathbb{R}_+^{S+1}$, asset purchases $\theta \in \mathbb{R}_+^J$, and asset sales $\psi \in \mathbb{R}_+^J$ taking all prices as given. However, they must also choose what quantities $D \in \mathbb{R}_+^{S \times J}$ to deliver on each asset at each possible

[5] See page 9: "One could easily imagine a legal system that imposes penalties that are nonconcave and even discontinuous in the size of the default, for example, trigger penalties that jump to a minimum level at the first infinitesimal default. One could also imagine confiscation of commodities in case of default. Our model does not explicitly allow for these possibilities. But as we show in our working paper (Dubey, Geanakoplos, and Shubik (2000)), with a continuum of households, such modifications to the default penalties do not destroy the existence of equilibrium."

[6] This is the notational simplification the one-commodity assumption makes possible.

state at date 1. If an agent chooses to deliver

$$D_{sj} < \psi_j R_{sj},$$

then the agent is defaulting on her promise on asset $j$ in state $h$.

Agents of type $h \in H$ who default experience a punishment measured in utility terms and equal to $g^h(\psi, D; R)$ where $g$ rises with $\psi$ and falls with $D$. As discussed above, while DGS (2005) focus on the case where $g^h$ is linear, their existence result can easily be extended to the general case with discontinuous or non-concave technologies provided the economy comprises a continuum of households or households can resort to a randomization device.

Asset buyers realize that they may not be paid in full at date 1. The heart of the DGS model, in fact, is a set of assumptions about expected delivery rates. First, buyers take delivery rates as given. For instance, they cannot attempt to lower asset returns in order to boost delivery rates. Second, they cannot opt to trade only with types that deliver at higher rates. In fact, returns are pooled and all buyers receive a pro-rata share of total deliveries on a given asset.

Given expected delivery rates $K \in [0,1]^{S\times J}$ and given asset prices $\pi \in I\!R^J_+$, the budget set $B^h(\pi, K)$ of an agent of type $h \in H$ is:

$$\begin{array}{rcl} B^h(\pi,K) = \Big\{ (x,\theta,\psi,D) & \in & I\!R^{S+1}_+ \times I\!R^J_+ \times I\!R^J_+ \times I\!R^{J\times S}_+ : \\ x_0 - e^h_0 + \pi\cdot(\theta-\psi) & \leq & 0, \\ \psi_j & \leq & Q^h_j \text{ for } j \in J, \\ x_s - e^h_s + \sum_{j\in J} D_{sj} & \leq & \sum_{j \in J} \theta_j K_{sj} \cdot R_{sj} \text{ for } s \in S \Big\}. \end{array}$$

A DGS equilibrium in this context is a list $\left(\pi, K, \{x^i, \theta^i, \psi^i, D^i\}_{i\in[0,1]}\right)$ such that:

1. For all $h \in H$ and almost all $i \in (\frac{h-1}{H}, \frac{h}{H}]$,

$$(x^i, \theta^i, \psi^i, D^i) \in \arg\max_{B^h(\pi,K)} u^h(x^i) - g^h(\psi, D; R),$$

2. $\int_i (x^i - e^i)di = 0$,

3. $\int_i (\theta^i - \psi^i)di = 0$,

4. $K_{sj} = \frac{\int_i D^i_{sj} di}{\int_i R_{sj}\psi^i_j di}$ if $\int_i R_{sj}\psi^i_j di > 0$ for all $(s,j) \in S \times J$.

The only non-standard aspect of this definition is the final condition which states that the beliefs agents form about delivery rates prior to choosing their consumption and asset holding plans must be borne out in equilibrium. It should be evident that degenerate equilibria supported by beliefs so pessimistic that no asset is actively traded always exist. DGS (2000, 2005) provide a simple trembling-hand refinement that rules out degenerate equilibria of that sort and show that an equilibrium, in this refined set, must exist.

While DGS (2000, 2005) rule out equilibria where asset markets are shut down by excessive pessimism, beliefs continue to matter critically in equilibrium. Intuitively, buyers who anticipate low delivery rates are likely to require low asset prices to be willing to participate in a given asset market and, in turn, low asset prices could cause low delivery rates. This creates an environment propitious to multiple equilibria. In fact, the next section shows by way of example that equilibria exist where, holding all other prices and quantities traded the same, raising the price of an asset – lowering the stated return, that is – can increase delivery rates and effective rates of returns. These equilibria, I will argue, make DGS's price-taking assumption impossible to defend.

# 4 Return-dominated equilibria

This section shows that the DGS model can generate equilibria such that lowering the stated return on a given asset raises the effective return. The intuition for this possibility is trivial:

lowering stated returns can lower default rates more than proportionately. In the example below, this happens because some agents begin defaulting only once required payments become large enough. The simplicity of the construction makes it clear that return-dominated equilibria are a robust outcome in the DGS model. The next section will show, in fact, that it is only because the concept allows for these equilibria that existence obtains universally in DGS (1990, 2005).

Consider a version of the economy described above with equal masses of two agents types ($H = 2$) and only one state at date 1 ($S = 1$). There is one asset ($J = 1$) with payoff $R = 1$ at date 1. Agents of type 1 are endowed with $e^1 = (1, 0)$ (one unit of the commodity at date 0, and zero at date 1), while agents of type 2 are endowed with $e^2 = (0, B)$ where $B > 0$.

Agents of type 1 only care about consumption at date 1. As a result, these agents always save their endowment at date 0, and there is no need to define $g^1$ precisely. Furthermore, letting $\theta^1$ denote their holdings of the asset, any equilibrium in this environment must feature $\theta^1 = \frac{1}{\pi_1}$ where $\pi_1 > 0$ is the price of the one asset.

Given $\pi_1$ and an anticipated delivery rate $K \in [0, 1]$ on the one asset, agents of type 2 choose $(x_0, x_1, \theta, \psi, D)$ to maximize:

$$A \min \{x_0, 1\} + x_1 - \frac{\lambda}{2} (\psi R - \eta D)^2 - \tau 1_{\{D < \psi R\}}$$

subject to:

$$\begin{aligned} x_0 + \theta\pi_1 - \psi\pi_1 &= 0 \\ x_1 + D &= B + \theta K R \end{aligned}$$

where $A > 0$ $\lambda > 0$, $\tau > 0$, and $\eta > 1$. Notice that the punishment technology features a fixed default cost ($\tau > 0$) which will play an important role in this example. Agents of type 2 only default provided the associated savings are high enough which only occurs provided $\pi_1$ is low enough so that the effective return on the asset is high.

Strictly speaking, the two types' utility functions violate DGS' strict monotonicity assumption but this is done for convenience only. Rather than assuming that agents of type 1 do not value consumption at date 0, it is enough to assume that they discount consumption at date zero at a sufficiently high rate. Likewise, assuming that agents of type 2 derive utility out of date 0 consumption according to $A\min\{x_0, 1\} + \epsilon x_0$ where $\epsilon$ is positive and small would not affect my conclusions below. The only consequence would be that equilibrium asset returns must exceed $\epsilon$, where $\epsilon$ can be made as small as needed. Those variations would make both utility functions strictly monotonic without altering my arguments in any way.[7]

Notice also that as specified above $g^2$ only declines with $D$ up to $\frac{\psi R}{\eta}$. At the cost of complicating notation, global monotonicity could be guaranteed by truncating the quadratic part of $g^2$ without any consequences on optimal delivery choices. In fact, globally strict monotonicity can be guaranteed by adding a linear piece to $g^2$ but this complication would not change the qualitative nature of the example this section develops.

As long as $A$ is high enough, agents of type 2 have no reason to purchase the asset, so that $\theta^2 = 0$. Then, any equilibrium in this example must feature $\psi^2 = \frac{1}{\pi_1}$. Again, as long as $A$ is high enough, it is in fact optimal for agents of type 2 to set $x_0 = 1$ and finance this consumption by selling quantity $\frac{1}{\pi_1}$ of the asset hence market clearing trivially obtains at any price.[8] Agents of type 2 need only decide whether or not to default on their obligations at date 2 and, when they choose to default, how much to deliver.

Standard manipulations of first-order conditions[9] show that if they default, it is optimal

[7]DGS (2005) make the same simplifying assumption in several of the examples they study. As they explain in footnote 20, their existence theorem is unaffected by this shortcut.

[8]This is where capping utility at 1 for agents of type 2 simplifies the example.

[9]Conditional on defaulting, agents maximize an objective function that is strictly concave in $D$. Provided the optimal delivery level is interior, the first-order condition for the optimal delivery choice is:

$$\begin{aligned} -1 + \lambda\eta\left(\frac{1}{\pi_1} - \eta D\right) &= 0 \\ \iff D = \frac{1}{\eta}\left(\frac{1}{\pi_1} - \frac{1}{\eta\lambda}\right). \end{aligned}$$

for agents of type 2 to select:

$$D = \frac{1}{\eta}\left(\psi R - \frac{1}{\lambda\eta}\right) = \frac{1}{\eta}\left(\frac{1}{\pi_1} - \frac{1}{\lambda\eta}\right)$$

Note that $D < \frac{1}{\pi_1}$, naturally, and that $D \geq 0$ provided $\frac{1}{\pi_1} > \frac{1}{\lambda\eta}$. When $D > 0$, the punishment penalty equals

$$\tau + \frac{\lambda}{2}\left(\frac{1}{\lambda\eta}\right)^2,$$

while overall utility is:

$$A + \left(B - \frac{1}{\eta}\left(\frac{1}{\pi_1} - \frac{1}{\lambda\eta}\right)\right) - \tau - \frac{\lambda}{2}\left(\frac{1}{\lambda\eta}\right)^2. \tag{1}$$

If, on the other hand, agents choose to deliver on all their promises then they incur no punishment and utility is

$$A + B - \frac{1}{\pi_1} \tag{2}$$

Agents default when (1)> (2) i.e. when:

$$\frac{1}{\pi_1}\left(1 - \frac{1}{\eta}\right) > \frac{\lambda}{2}\left(\frac{1}{\lambda\eta}\right)^2 - \frac{1}{\lambda\eta^2} + \tau.$$

Since $\eta > 1$, the left-hand side of the inequality above rises with $\frac{1}{\pi_1}$. Let $\pi_1^*$ be such that the default condition holds as an equality and assume for simplicity that parameters are such that $\frac{1}{\pi_1^*} > \frac{1}{\lambda\eta}$, which can be guaranteed for instance by making $\tau$ large enough.

Therefore, this economy generate a continuum of DGS equilibria indexed by $\pi_1 > 0$. When $\frac{1}{\pi_1} < \frac{1}{\pi_1^*}$, agents of type 2 deliver in full in period 1. When $\frac{1}{\pi_1} = \frac{1}{\pi_1^*}$, agents of type 2 are indifferent between delivering $\bar{D} = \frac{1}{\pi_1^*}$ and delivering $\underline{D} = \frac{1}{\eta}\left(\frac{1}{\pi_1^*} - \frac{1}{\lambda\eta}\right)$. Assigning various masses of type 2 agents to those two choices can produce any equilibrium at that asset price with $K \in [\underline{D}\pi_1^*, 1]$. Finally, when $\frac{1}{\pi_1} \in (\frac{1}{\pi_1^*}, B]$, agents deliver $\frac{1}{\eta}\left(\frac{1}{\pi_1} - \frac{1}{\lambda\eta}\right)$.

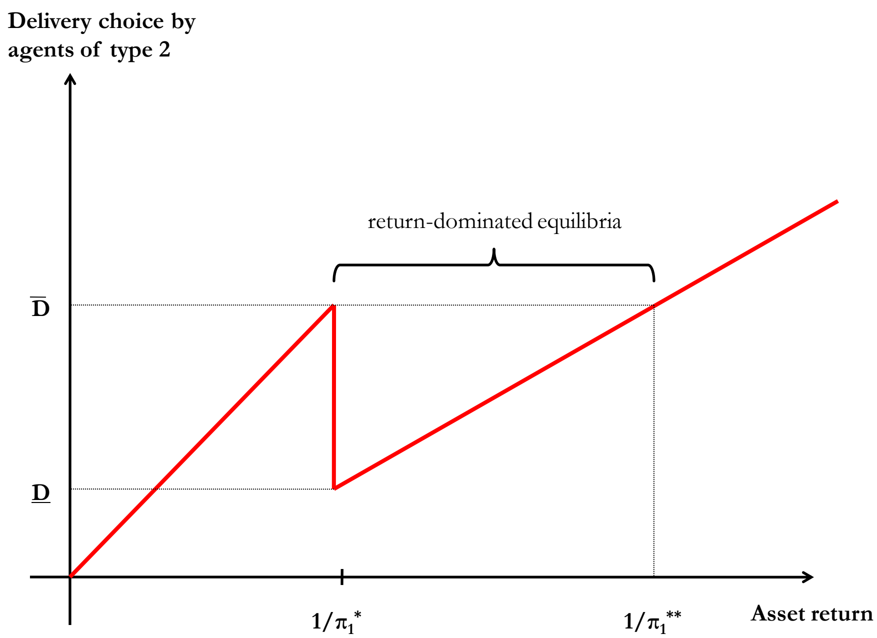
These equilibria are depicted in figure 2. When the asset return reaches $\frac{1}{\pi_1^*}$, agents of type 2 begin defaulting hence the amount received by agents of type 1 falls. Only when the return becomes large enough, at $\frac{1}{\pi_1^{**}}$ on the figure, do delivery rates reach the same level as at $\frac{1}{\pi_1^*}$. Equilibria in the $(\frac{1}{\pi_1^*}, \frac{1}{\pi_1^*}]$ interval are pathological because lenders (agents of type 1) could choose to lower the returns they demand on the assets they purchase, which agents of type 2 would welcome, and <u>increase</u> delivery rates.

One aspect of this example that may seem important at first glance is the fact that agents of type 1 (the lenders) only face one type of agents hence do not have to worry about selection issues. However, this plays no role in generating return-dominated equilibria. Recognizing and taking advantage of the profit opportunity that exists when $\frac{1}{\pi_1} \in (\frac{1}{\pi_1^*}, \frac{1}{\pi_1^*}]$ does not require that agents of type 1 know the identity or type of the agents with whom they are dealing. It only requires that they understand the structure of the economy in which they are lending. A pool of agents of type 1 (or an intermediary representing them) who would offer a lower interest rate than the equilibrium rate would attract the interest of <u>all</u> agents of type 2. Any random selection device would then yield a representative set of agents of that type. Anonymity, in other words, cannot possibly preclude lenders from lowering stated returns in order to increase effective returns.

Assuming that lenders face a continuum of borrower types is useful in fact if one wishes to generate a hill-shaped relationship between stated returns and effective returns as drawn by Stiglitz and Weiss (1981) or in figure 1 instead of the stark break displayed in figure 2, using the fact that different borrowers would then default at different thresholds. Quintin (2012) provides a specific example. Arnold and Riley (2009) argue that the textbook hill-shape case is not a robust outcome in the model of Stiglitz and Weiss (1981). That issue does not arise in the DGS model. In particular, while the examples I have presented in this section rely on discrete deviations from the equilibrium set of contracts, it is easy to generate examples where marginally profitable deviations exist as well. In those cases, exploiting the profitable deviations the DGS concept fosters only involves recognizing the local shape of the price-default relationship.

In summary, the DGS concept produces equilibria where lenders ignore a glaring profit opportunity. This possibility makes assuming price-taking behavior on the part of lenders

Figure 2: A continuum of equilibria




in the DGS environment impossible to defend. Return-dominated equilibria such as those depicted in figure 2 could not possibly persist in the economic environment described by DGS (1990, 2000, 2005.)

# 5 An exclusion procedure

One way to rule out the class of equilibria described in the previous section is to relax the assumption that asset buyers take prices as given and make asset price decisions choice variables. Implementing this solution requires a drastic departure from the canonical general equilibrium model with incomplete markets. Another approach, which this section pursues, is to select equilibria in the DGS set that give buyers no incentives to depart from equilibrium prices.

This approach requires a systematic way to describe and exclude return-dominated equilibria. To that end, for $(\pi, K) \in I\!R^J_+ \times I\!R^{S\times J}_+$, define:

$$\mathcal{K}(\pi, K) = \left\{ \left( \frac{\int_i D^i_{sj} di}{\int_i R_{sj}\psi^i_j di} : s \in S, j \in J \right) : (D, \psi) \text{ solves agents' problem given } (\pi, K) \right\},$$

with the convention that the delivery rate is set to zero when the denominator of the integral is zero.

Note that in equilibrium, consistency requires that $K \in \mathcal{K}(\pi, K)$. In general, $\mathcal{K}$ lists all the delivery rates compatible with optimal behavior on the part of agents given $(\pi, K)$. Standard arguments show that $\mathcal{K}$ is non-empty and convex valued, and that it has a closed graph. I can now state:

**Definition 5.1.** *A DGS equilibrium $\left(\pi, K, \{x^i, \theta^i, \psi^i, D^i\}_{i\in[0,1]}\right)$ is return-dominated if there exists $\hat{\pi} \in I\!R^J_+$ such that:*

1. *$\hat{\pi}_{j^*} > \pi_{j^*}$ for some $j^* \in J$ such that $\int \psi^i_{j^*} di > 0$ while $\hat{\pi}_j = \pi_j$ if $j \neq j^*$ and,*
2. *there exists $\hat{K} \in \mathcal{K}(\hat{\pi}, K)$ such that $\frac{\hat{K}_{sj^*} R_{sj^*}}{\hat{\pi}_{j^*}} > \frac{K_{sj^*} R_{sj^*}}{\pi_{j^*}}$ for all $s \in S$.*

In such an equilibrium, a set of agents who are buying asset $j^*$ could lower the return they require on that asset, select a representative set of agents currently selling the asset with whom to trade, and strictly raise their income in the second period by offering these borrowers terms they strictly prefer. Since all buyers of asset $j^*$ should recognize this profit opportunity and no friction in the DGS environment precludes them from taking advantage of it, a return-dominated equilibrium could not persist in a competitive world.

This argument presumes that asset $j^*$ in definition 5.1 remains sold by borrowers once its price rises. That this has to be the case is immediate since the delivery rate is zero by convention when nobody sells the asset. Since delivery rates rise strictly at the counterproposal price vector $\hat{\pi}$, they must be positive. But even absent this convention, a more fundamental argument can be used to demonstrate that asset $j^*$ must remain sold by borrowers following the price change. To see this, take any agent $i$ for whom $\psi^i_{j^*} > 0$ at the original equilibrium. When the price of asset $j^*$ rises, their welfare must rise strictly. Indeed, these agents could simply choose to increase their consumption at date 0 and leave all other plans unchanged. So assume by way of contradiction that following the price change, these agents choose to set $\psi^i_{j^*} = 0$ . Since $0 \times \pi_{j*} = 0 \times \hat{\pi}_{j^*}$ the new plan was feasible at the original price vector hence cannot raise the welfare of these agents, which is the contradiction we sought.

To illustrate the exclusion procedure further, notice that in figure 2, the equilibria that meet the two criteria stated in definition 5.1 are precisely the equilibria I called return-dominated in the previous section. To see this, denote by $D(\pi_1)$ the delivery level chosen by agents of type 2 when the equilibrium price is $\pi_1$, making their delivery rate $K(\pi_1) = \frac{D(\pi_1)}{\frac{1}{\pi_1} \times 1}$ since in equilibrium agents of type 2 sell quantity $\frac{1}{\pi_1}$ of the asset. Then, an equilibrium at price $\pi_1$ is return-dominated if another price $\pi_1'$ exists such that

$$\pi_1' > \pi_1 \tag{3}$$

<u>and</u>:

$$\begin{aligned} \frac{K(\pi_1')}{\pi_1'} &> \frac{K(\pi_1)}{\pi_1} \\ \iff \frac{\frac{D(\pi_1')}{\frac{1}{\pi_1'}}}{\pi_1'} &> \frac{\frac{D(\pi_1)}{\frac{1}{\pi_1}}}{\pi_1} \\ \iff D(\pi_1') &> D(\pi_1) \end{aligned} \tag{4}$$

The first line of this string of inequalities is the return-domination criterion in this specific case since the return on the asset is 1 so that delivery per unit of asset bought when the price is $\pi_1$ is simply $K(\pi_1)$. Therefore, an equilibrium at price $\pi_1$ is return-dominated and excluded under my procedure if and only if both condition (3) and (4) hold. On figure 2, these are the equilibria in the interval $[\frac{1}{\pi_1^*}, \frac{1}{\pi_1^{**}})$, with the exception of the equilibrium with asset price $\pi_1^*$ where all agents choose to deliver $\bar{D}$.

The most natural way to deal with the issues raised in the previous section seems to be selecting equilibria that are not return-dominated. The question is whether there is such an equilibrium in the DGS set and I will now show that the answer, in general, may be negative.

# 6 Existence

The main point of DGS (2005) is that default is not incompatible with the "orderly functioning of markets" in the sense that introducing a default option in an otherwise standard general equilibrium model with incomplete markets does not jeopardize existence. I have argued that the DGS concept may generate equilibria that hardly seem compatible with price taking on the part of lenders. A natural question is whether there always exists at least one equilibrium that is not return-dominated.

Notice that on figure 2, there is a continuum of equilibria that are not return dominated and these happen to coincide with the set of DGS equilibria that are constrained efficient in the sense of Geanakoplos and Polemarchakis (1986). These equilibria, that is, are such that no reallocation of asset holdings alone could improve every agent's utility. I will now show that constrained-efficiency is in fact sufficient for an equilibrium to survive my exclusion procedure in the special case where $S = 1$. Specifically, the following result establishes the fact that constrained efficient equilibria are not return-dominated in that case.[10]

**Remark 6.1.** *Return-dominated DGS equilibria are constrained-inefficient when $S = 1$.*

*Proof.* Take a DGS equilibrium $\left(\pi, K, \{x^i, \theta^i, \psi^i, D^i\}_{i\in[0,1]}\right)$ that is return-dominated and let $j^* \in J$ be an asset on which it is possible to raise effective returns by lowering stated returns. Since delivery rates are bounded above by 1, for $q > 0$ high enough, $\frac{\bar{K}_{j^*} R_{j^*}}{\bar{\pi}_{j^*}} < \frac{K_{j^*} R_{j^*}}{\pi_{j^*}}$ for any $\bar{K} \in \mathcal{K}(\bar{\pi}, K)$ where $\bar{\pi}_j = \pi_j$ for $j \neq j^*$ while $\bar{\pi}_{j^*} = q$ if $j = j^*$. Since $\mathcal{K}$ is non-empty, convex-valued and has a closed graph, it follows that there exist $\tilde{\pi} \in I\!R^J_+$ such that $\tilde{\pi} \geq \pi$ with

[10] The result is trivial when $J = 1$ as well, but since punishment and short-sale constraints are individual and asset specific, several different assets may be traded even when $S = 1$.

a strict inequality only at coordinate $j^*$ such that $\frac{\tilde{K}_{j^*} R_{j^*}}{\tilde{\pi}_{j^*}} = \frac{K_{j^*} R_{j^*}}{\pi_{j^*}}$ for some $\tilde{K} \in \mathcal{K}(\tilde{\pi}, K)$. At this new set of prices, it is budget feasible for all agents to choose the same consumption and delivery plan as before by setting $\tilde{\theta}^i_{j^*} \equiv \theta^i_{j^*} \times \frac{\pi_{j^*}}{\tilde{\pi}_{j^*}}$ and $\tilde{\psi}^i_{j^*} \equiv \psi^i_{j^*} \times \frac{\pi_{j^*}}{\tilde{\pi}_{j^*}}$. At this new trading plan, however, the punishment for agents that underdeliver on asset $j^*$ falls strictly, while other agents are as well off as in the original equilibrium. This is the constrained Pareto improvement we sought. □

As the proof makes clear, return-dominated equilibria are inefficient in this case in an obvious sense. The exact same consumption allocation for all agents and the same delivery plans could be supported at asset prices and assets holdings that make punishment less severe on defaulting borrowers. All agents need to do is contract at a lower stated rate of return on some assets and deliver as before, which lowers the intensity of punishment, without altering anyone's consumption plans.[11]

DGS equilibria that are not return-dominated must therefore exist whenever efficient equilibria exist when $S = 1$. Outside of this very special case however, existence can fail. For one thing, it is known that even in the case without default (see Geanakoplos and Polemarchakis, 1986), equilibria are generically suboptimal when there are several goods and/or several assets. What's more, a glance at the proof above should make it clear that the return-dominated equilibria need no longer be inefficient when $S > 1$.

Much more specifically, a simple variation on the example presented in the previous section shows that DGS economies may generate nothing but return-dominated equilibria. Take an economy populated by the same two types of agents as in the previous section, but now add a positive mass of a third type of agent ($h = 3$) who never default ($g^3 = +\infty$, whenever $D < \psi R$), have no endowment in the first period, and have a positive endowment in the second period. These agents have linear preferences with discount rate $\beta > 0$ between the two periods. Then, in any equilibrium, $\frac{1}{\pi_1} > \frac{1}{\beta}$ since otherwise these agents would sell the

[11] While this result applies only to a rather special case, it conveys an important message. I am about to prove that universal existence arguments cannot be produced when default is a possibility and lenders behave competitively. That is not to say that existence arguments cannot be produced in models of default. Instead, the message is that existence arguments must be context-specific.

asset which would cause supply of the asset to exceed demand. Indeed, the other two types of agents have preferences such that their combined net demand for the asset is zero regardless of the price. One can then choose $\frac{1}{\beta} \in (\frac{1}{\pi_1^*}, \frac{1}{\pi_1^{**}})$ which lops off all equilibria to the left of the default threshold.

Next, assume that punishment is capped above for agents of type 2 at some upper-level $\bar{P} > \frac{1}{\pi_1^*}$. Agents of type 2 can now opt to deliver nothing and take that maximum punishment leaving them with utility $A + B - \bar{P}$. Since $\bar{P} > \frac{1}{\pi_1^*}$, they are better off making full delivery if the asset return is below $\frac{1}{\pi_1^*}$, just like in the economy described in the previous section. Assume now however that

$$A + \left(B - \frac{1}{\eta}\left(\frac{1}{\pi_1^{**}} - \frac{1}{\lambda\eta}\right)\right) - \tau - \frac{\lambda}{2}\left(\frac{1}{\lambda\eta}\right)^2 < A + B - \bar{P}. \tag{5}$$

That is, assume that at required return $\frac{1}{\pi_1^{**}}$, agents of type 2 are strictly better off making no delivery on the asset.

This construction presumes that a value for $\bar{P}$ exists such that both $\bar{P} > \frac{1}{\pi_1^*}$ and (5) holds. This is the case since at $\frac{1}{\pi_1^{**}}$ agents of type 2 deliver the same amount as $\frac{1}{\pi_1^*}$ but get punished for defaulting so that their utility at the first threshold is strictly higher than at the second threshold. Equivalenty, a revealed preference argument shows that utility at the first threshold must be higher. It follows that for $\bar{P}$ sufficiently close to $\frac{1}{\pi_1^*}$, (5) must hold.

Then, there is a threshold return in $(\frac{1}{\pi_1^*}, \frac{1}{\pi_1^{**}})$ past which agents of type 2 become better off opting for zero delivery. Now simply choose that threshold in $(\frac{1}{\beta}, \frac{1}{\pi_1^{**}})$ which caps the maximum delivery level past $\frac{1}{\pi_1^*}$ to a level $D_{max}$ strictly below $\bar{D}$.[12]

The construction is depicted in figure 3. The dotted line traces the equilibria from the previous section that become ruled out because of the presence of the new agents and because punishment is capped above. The solid line shows the DGS equilibria that remain. At any of those equilibria, lenders would be better off charging, say, $\frac{1}{\pi_1^*}$ on their loan and getting full delivery, making all borrowers better off as well. No friction in the model could explain why

[12]To implement this formally, choose the desired $D_{max}$ and simply define $\bar{P} \equiv D_{max} + \tau + \frac{\lambda}{2}\left(\frac{1}{\lambda\eta}\right)^2$.

lenders choose to forego this profit opportunity.

Naturally, if all lenders deviate to this new price, there is excess supply of the asset as agents of type 3 now want to sell it, so that assets markets can no longer clear. In other words, no equilibrium exists in this example other than return-dominated equilibria.

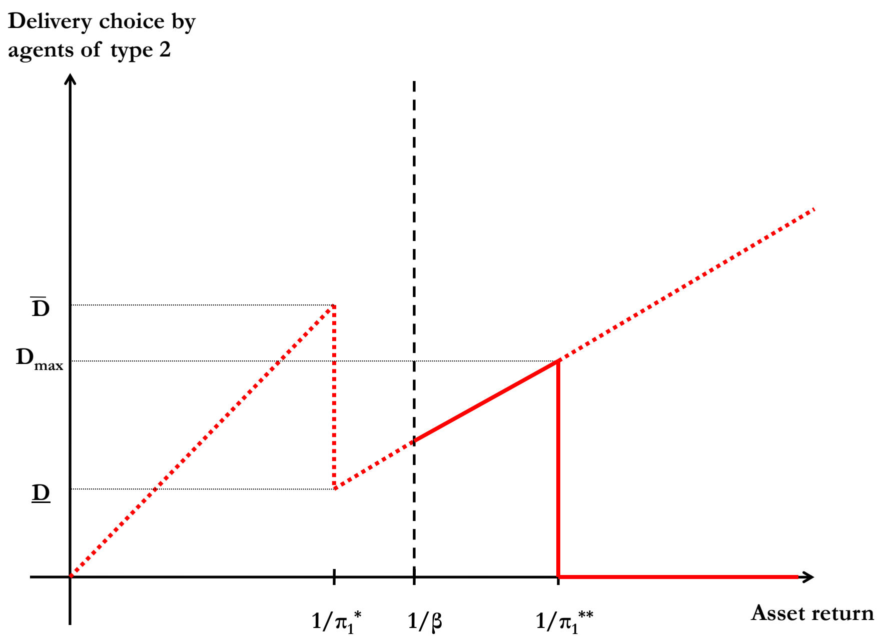
The simplicity of the construction should also make it clear that a large class of punishment technologies $g^2$ can lead to the very situation depicted in figure 3. Rather than capping $g^2$ above, one could specify it so that past the default threshold delivery rates are bounded below $\bar{D}$. Among many other possibilities, writing the quadratic part of $g^2$ as $\frac{\lambda}{2}\left((\psi^* R^*)^2 - \eta D^2\right)$ where $\psi^* R^*$ is a positive parameter – separating the quadratic term in $D$ from the size of the obligation, that is – makes $D$ constant past the default threshold. Adding a linear part such as $\epsilon\,(\psi R - D)$ where $\epsilon < 1$ preserves that property while making punishment unbounded as before.

Note also that this example does not rely on any sort of asymmetric information. All agents are fully informed about the types and opportunities of other agents. With asymmetric information however, producing economies whose DGS equilibrium set looks like the set depicted in figure 3 becomes even easier. Simply assume that the set of type-2 agents actually comprises two subsets of agents who only incur a fixed cost when they default. The first subset of agent has a low cost of defaulting while the other has a higher cost of defaulting, and costs are private information. One can then choose these two privately observed costs so as to produce the same thresholds as in figure 2 and produce the exact same outcome: an economy that only produces return-dominated equilibria. An existence argument would have to exclude this trivial and natural specification of punishment technologies.

What outcome should we expect to observe in such an economy? One clearly stable outcome has lenders charge $\frac{1}{\pi_1^*}$, which maximizes delivery per unit of the asset purchased.[13] At that rate however, there is excess supply of the asset as agents of type 3 are strictly better

[13] Strictly speaking at $\frac{1}{\pi_1^*}$, delivery could be anywhere in the $[\underline{D}, \bar{D}]$ interval but for simplicity in this discussion I focus on the equilibrium where all borrowers choose to deliver in full a that point, as is weakly optimal for them. Alternatively, one could resort to a limiting argument and argue that lenders can approximate that outcome with arbitrary precision by approaching $\frac{1}{\pi_1^*}$ from the left.

Figure 3: An economy where all DGS equilibria are return-dominated




off selling it. Note in addition that all potential asset sellers would be willing to pay a higher rate to asset buyers. However, asset buyers, as discussed above, would anticipate the adverse consequences of so doing on expected delivery rates.

A natural outcome in this environment, therefore, is credit rationing in the sense of Stiglitz and Weiss (1981). It is important to recognize, however, that the economics of my example are very different from those that underly Stiglitz and Weiss' example. At their rationing equilibrium, raising rates reduces lenders expected returns by changing the composition of the pool of borrowers. In my example, the pool of potential borrowers does not change at all: all agents of type 2 or 3 continue to want to borrow even if rates rise.[14] What happens is that the delivery behavior of existing or potential borrowers changes drastically. Type 2 borrowers, specifically, choose to default following the rate increase when they chose to deliver

[14]In this case and to be precise, this is true until the rate reaches agent 3's participation threshold.

in full before. The economics of this example are thus both trivial and compelling: raising payments increases the likelihood of default on the part of any given borrower by making the benefits of default higher. It is possible in fact and as happens in this example that this likelihood increases so much as to lower net returns.

# 7 Conclusion

Equilibria where all asset markets clear may not exist in environments where asset payoffs depend on endogenous default decisions on the part of asset sellers. Credit rationing is a distinct possibility in these economies. The concept proposed by DGS (2005) only delivers universal existence in the standard sense because it allows for equilibria that are inconsistent with assuming price taking on the part of lenders.

Existence in the standard sense may in fact obtain for certain subsets of this class of environments, but establishing it in a given economy requires context-specific arguments. One example of this is Quintin (2011). In that specific context, the exclusion procedure this paper develops not only remains compatible with existence of an equilibrium where all markets clear but in fact guarantees uniqueness, paving the way for meaningful comparative statics.[15]

A different strand of the general equilibrium literature on default (see Kehoe and Levine, 1993) studies environments where a complete set of securities are traded but default endogenously limits the positions agents can take in those various securities. In those models, both welfare theorems hold – equilibria are constrained efficient, and constrained equilibria can be

[15] I use this result to take on a policy question of current importance: would making punishment tougher in the event of mortgage default reduce average default rates? Since the recent foreclosure crisis has been particularly severe in states such as California or Arizona where recourse is severely limited, it is natural to conjecture that tougher statutes could have mitigated the crisis in those states. It turns out however that the theoretical implications of recourse are deeply ambiguous because changing recourse statutes can cause borrowers that are prone to default to enter mortgage markets. Simply put, knowing that they can collect more in some states can cause intermediaries to tolerate riskier borrowers. This implies that meaningful econometric tests of whether recourse matters cannot rely on aggregate variables but must instead rely on detailed microeconomic information on borrowers at contract origination.

supported as equilibria with transfers – and existence holds with great generality. Therefore and in sharp contrast with the model discussed in this paper, default in those models cannot improve welfare and, in equilibrium, no contract is written where agents have an incentive to default in some state.[16] The points this paper makes pertain exclusively to models where markets are exogenously incomplete.

Finally, one should point out that the fact that equilibria with incomplete markets and endogenous defaults can lead to several lending rates compatible with a given opportunity cost of funds has been known in the literature that studies the impact of bankruptcy and recourse design in general equilibrium using quantitative methods. Part of that literature studies the effects of bankruptcy reform on credit and default.[17] Another part of that literature (see e.g. Corbae and Quintin, 2010) explores quantitatively the consequences of mortgage recourse for equilibrium foreclosure rates in the residential mortgage market. These papers usually argue that among the rates that are compatible with zero expected net profits on the part of the lender, the most favorable rate to the borrower should prevail. As in this paper, the fundamental argument for this selection procedure is that other rates that give the intermediary zero profits should be driven out by competitive forces.

Computationally, locating this equilibrium requires searching on a grid starting from a rate under which net lender profits must be negative. Traditional, faster approaches such as bisection run the risk of producing returns that are return-dominated precisely in the sense I have made precise in this paper, hence economically implausible. Furthermore, tolerating these equilibria in all their multiplicity would make asking the comparative statics questions these quantitative papers seek to address difficult.

[16] At the same time, as Kehoe and Levine (2006) discuss, it is possible to implement the equilibrium allocation in a model with bankruptcy and collateral.

[17] See e.g. Athreya (1999), Chatterjee et al. (2007).

## Bibliography


Araujo, A., Monteiro, P., and M. R. Pascoa "Incomplete Markets, Continuum of States and Default," *Economic Theory*, 11, pp. 205–213, 1998.

Araujo, A., Fajardo, J. and M. R. Pascoa, "Endogenous collateral," *Journal of Mathematical Economics*, 41, pp. 439–462, 2005.

Arnold, L. G., and J. G. Riley, "On the Possibility of Credit Rationing in the Stiglitz-Weiss Model," *American Economic Review*, 99, pp. 2012–2021, 2009.

Athreya, K., "Welfare Implications of the Bankruptcy Reform Act of 1999," *Journal of Monetary Economics*, 49, pp. 1567–1595, 2002.

Bisin, A., Geanakoplos, J., Gottardi, P., Minelli, E., and H. Polemarchakis, "Markets and contracts," Economics Working Papers ECO2010/29, European University Institute, 2010.

Chatterjee, S., D. Corbae, M. Nakajima, and V. Rios-Rull, "A Quantitative Theory of Unsecured Consumer Credit with Risk of Default,"*Econometrica*, 75, pp. 1525–1589, 2007.

Corbae, D and E. Quintin, "Mortgage Innovation and the Foreclosure Boom," mimeo, 2010.

Dubey P. and and J. Geanakoplos, "Competitive Pooling: Rothschild-Stiglitz Reconsidered," *Quarterly Journal of Economics* 117, 2002.

Dubey P., Geanakoplos, J. and M. Shubik, "Default and Efficiency in a General Equilibrium Model with Incomplete Markets," Cowles Foundation Discussion Paper No. 773, 1990.

Dubey P., Geanakoplos, J. and M. Shubik, "Default and Efficiency in a General Equilibrium Model with Incomplete Markets," Cowles Foundation Discussion Paper N0. 1247, 2000.

Dubey P., Geanakoplos, J. and M. Shubik, "Default and Punishment in General Equilibrium," *Econometrica*, vol 73, pp. 1 - 37, 2005.

Geanakoplos, J and H. Polemarchakis, "Existence, Regularity and Constrained Suboptimality of Competitive Allocations when the Asset Market is Incomplete," in Heller et al., editors, *Uncertainty, Information and Communication*, 1986.

Geanakoplos, J., and W., Zame, "Collateral and the Enforcement of Intertemporal Contracts," Yale University Working Paper, 1995.

Gerardi, K., A. Shapiro, and P. Willen, "Subprime outcomes: Risky mortgages, homeownership experiences, and foreclosures," Federal Reserve Bank of Boston Working Paper 07-15, 2007.

Gerardi, K., A. Shapiro, and P. Willen, "Decomposing the Foreclosure Crisis: House Price Depreciation versus Bad Underwriting," Federal Reserve Bank of Atlanta Working Paper 2009-25, 2009

Gerardi, K., A. Lehnert, S. M. Sherland and P. Willen, "Making Sense of the Subprime Crisis," Federal Reserve Bank of Boston Public Policy Discussion Paper 07-15, 09-1, 2009.

Kehoe, T., and Levine, D. K., "Debt Constrained Asset Markets," *Review of Economic Studies*, 60, pp. 865-888.

Kehoe, T., and Levine, D. K., "Bankruptcy and Collateral in Debt Constrained Markets," Federal Reserve Bank of Minneapolis Research Department Staff Report 380, 2006.

Quintin, E., "More Punishment, Less Default?," *Annals of Finance* (forthcoming), 2012.

Rothschild, M., and J. Stiglitz, "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," *Quarterly Journal of Economics*, 90, pp. 629–649, 1976.

Stiglitz, J., and A. Weiss, "Credit Rationing in Markets with Imperfect Information," *American Economic Review*, 71, pp. 393–410, 1981.