More Punishment, Less Default?

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Abstract

The extent of lender recourse following contractual default varies greatly across economies. Intuitively, one would expect these differences to matter for default behavior at the micro-economic level and for equilibrium quantities. The objective of this paper is to study an equilibrium model in the spirit of Dubey, Geanakoplos and Shubik (2004) where the implications of recourse for default patterns can be characterized. Under plausible conditions, broader recourse causes yields at origination and default rates to fall for a given set of observable borrower characteristics. On the other hand, the effect of broader recourse on average default rates and the quantity of loans issued is deeply ambiguous because the composition of the pool of borrowers can change. Raising the fraction of assets subject to recourse can well increase equilibrium default rates. I discuss the implications of these results for how one should test empirically whether recourse statutes matter for loss severity rates and the frequency of default in secured loan markets.

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1 Introduction

Different economies punish contractual default very differently. In the residential mortgage market for instance, the formal extent of lender recourse differs greatly across US states and even more significantly between the United States and Western Europe. Intuitively, we would expect the intensity and the form of punishment to matter for default patterns and for the quantity of financial intermediation. In particular, intuition suggests that more punishment should lead to less default in equilibrium. The purpose of this paper is to take this intuition to task, and, more generally, to look for robust theoretical implications of lender recourse.

Among other benefits, characterizing the effects of recourse statutes will enhance our understanding of – hence our ability to deal with – the foreclosure crisis that began in the second half of 2006. States such as Arizona and California where lender recourse is highly limited have been at the forefront of the crisis. A natural question to ask then is whether different recourse statutes could have mitigated the unprecedented flare-up in mortgage default in those states. The first step towards answering this question is to understand how these statutes affect home-ownership and default decisions at the household level. While the partial equilibrium effects of toughening recourse accord well with simple intuition, this paper will show that intuition becomes a much more unreliable guide once general equilibrium effects are taken into account.

Studying the theoretical implications of recourse requires a general equilibrium model where borrowers have the option to renege on their financial promises and take advantage of this option with positive probability in equilibrium. As has been well known at least since the work of Stiglitz and Weiss (1981), existence can be problematic in this class of models.[1] In their seminal paper, Dubey, Geanakoplos and Shubik (DGS, 2004) propose an equilibrium concept that resolves this problem but, as Quintin (2011) points out, this solution raises fundamental issues that make applying it to competitive loan markets unreasonable. Specifically, DGS equilibria include outcomes where lenders could choose to lower the rates

[1] See DGS (2004) for a discussion of this issue.

they charge on their loans and *increase* their profits. To make matters worse for the purpose of asking comparative statics questions, multiplicity is endemic under that concept. This paper overcomes these difficulties by applying the selection procedure described in Quintin (2011). In the environment laid out in this paper, this procedure guarantees both existence and uniqueness which, in turn, makes establishing sharp theoretical predictions possible.

In the specific model of secured lending described in this paper, raising the extent of deficiency is formally equivalent to taxing capital income more intensively for agents that eventually choose to default. Much like increasing capital taxation can decrease capital income tax revenues in theory provided savings are sufficiently interest-rate elastic, broadening statutory recourse can in fact *lower* effective household liability. This trivially makes the theoretical effects of recourse statutes on mortgage yields and average delivery rates ambiguous.

When the interest rate elasticity of savings is sufficiently low in a sense this paper makes precise, effective recourse does rise when statutory recourse broadens. In that case and under the plausible assumption that housing is a normal good, recourse unambiguously causes default rates and yields at origination to fall for each possible set of default-relevant borrower characteristics. At the same time, the pool of borrowers changes deeply. The size of the pool can rise or fall and, likewise, average default rates can rise or fall. In other words, the aggregate implications of broadening recourse are fundamentally ambiguous in general equilibrium, even if one assumes that statutory and effective recourse co-move.

These results have immediate implications for the empirical literature that seeks to test whether recourse statutes matter. In the mortgage context, several papers have studied the empirical link between recourse statutes and default rates, as the next section discusses. The results of these investigations have been mixed. Since a significant fraction of these studies rely on aggregate data, this is hardly surprising: tests carried out on aggregate data can tell us nothing about whether or not recourse matters for default behavior since the theoretical effects of recourse on aggregate variables are ambiguous. Instead, meaningful tests of the hypothesis that recourse matters must rely on detailed microeconomic evidence on borrower and loan characteristics at origination.

The model described in this paper builds on a large theoretical literature devoted to mortgage default. Option-based models (see Kau et al. 1994, Deng et al. 2000) predict that households should default when the value of the home falls below the value of keeping the mortgage, where the value of continuing to make payments incorporates the fact that the borrower may be able to take advantage of improved price or financing conditions in the future. These models highlight the option aspect of default decisions, but predict (counterfactually) that household characteristics such as current wealth and income have no bearing on the default decision. Gerardi et al. (2009) propose a model where borrowing and lending rates differ so that the present value of keeping one's home depends on their income profile.

This paper relies on a model of mortgage default where incentives to repay are a simple function of the household's resources. This collateral approach to credit constraints is related to the class of models inspired by Kyotaki and Moore (1997).[2] A key feature of these models is that they allow default to occur in equilibrium. Chatterjee et al. (2007) use this type of framework to investigate the quantitative effects of bankruptcy rules on default and credit, but focus their attention on unsecured credit. This paper look for robust theoretical implications of broadening recourse in a model of secured credit.

As discussed above, my findings have important consequences for the potential role of deficiency policies in preventing future foreclosure crises.[3] Tougher recourse statutes should help curb default and loss severity rates on given borrower types, but because they change the pool of borrowers, their effects on overall default rates are ambiguous, at best. Beyond the mortgage markets, the ideas this paper discusses are also relevant for thinking about the effects of bankruptcy reform on credit and default.[4] The 2005 bankruptcy reform in the United States, for instance, made bankruptcy more costly for US households. My model predicts that such a reform should have a deep impact on the composition of borrowers and

[2]A version of the DGS concept has also been used to study environments where asset purchases must be collateralized. The pioneering paper in this strand is Geanakoplos and Zame (1995). See also Araujo et al. (1998) and Araujo et al. (2005).

[3]The fast growing literature on the foreclosure crisis includes Chatterjee and Eyigungor, 2010, Foote et al. (2007, 2008a,b), Garriga and Shlagenhauf (2009), Gerardi et al. (2007, 2008, forthcoming).

[4]See e.g. Athreya (1999), Chatterjee et al. (2007) and Li and White (2009).

the nature of borrowing, and, as a result, has theoretically ambiguous effects on average default rates and the equilibrium amount of borrowing.

The next section motivates the question this paper raises by briefly describing some of they key differences in recourse statutes in mortgage markets across US states and across countries. A natural question to ask in that context is whether these differences matter for the frequency of default, among other equilibrium variables. One of the contributions of this paper is to guide empirical investigations of this question.

2 Recourse in the Residential Mortgage market

The legal consequences of default on mortgage obligations vary significantly across jurisdictions. Within the United States, both the foreclosure process and the extent of lender recourse when foreclosure proceeds fall short of the mortgage balance differ markedly across states. Key differences include whether or not states force foreclosure proceedings to be administered by judicial authorities, how long a period the mortgagor is granted following the sale to redeem the property by making good on unpaid obligations and costs, and, finally, whether the borrower is allowed to claim the borrower's ex-housing assets when foreclosure proceeds fall short of the mortgage's outstanding balance.[5] Non-judicial options and shorter redemption periods potentially reduce the cost associated with the foreclosure process while deficiency judgments raise potential proceeds and, at least in theory, deter default in the first place.

Across countries, differences in recourse statutes are even more significant. Even in US states where recourse is at its broadest, wage garnishment is either precluded entirely or so restricted as to be practically useless as a remedy to unpaid mortgage claims. In sharp contrast, most Western European nations do allow for and enforce strict wage garnishment rules[6] and the perception is that these differences cause default rates to be markedly lower in Europe than they are in the United States during housing downturns.

[5]See Ghent and Kudlyak, 2010, for a state-by-state description of foreclosure statutes or, Kahn and Yavas (1994).

[6]See European Mortgage Federation, 2007, for a review.

While recourse differences between the US and Europe are perceived to matter for default behavior,[7] the conventional wisdom is that recourse differences across US states have little effect on default behavior. This conventional wisdom is founded on the following two arguments. First, in states where deficiency judgments is an option in principle, restrictions on their use are such that suing borrowers is too onerous for lenders. Second, mortgagors who default often have little liquid assets to go after. Even when they do have significant assets, they have ample opportunities to protect or dispose of them before a deficiency judgment is pronounced.

The ideal way to test the validity of this perception would be to study data on the frequency and outcomes of deficiency judgments. Unfortunately, direct data of this sort have yet to be produced and analyzed. Instead then, the literature has tested the hypothesis that recourse matters by asking if default probabilities are higher in states where recourse is narrow than in states with broader recourse laws. Clauretie (1987) finds for instance that state default rates show no significant correlation with deficiency statutes after controlling for various aggregate state characteristics. Clauretie (1987) and Clauretie and Herzog (1994) find on the other hand that loss severity rates are higher states that disallow nonjudicial procedures and have longer redemption periods. Ghent and Kuldyak (2010) study the effects of recourse statutes on default rate but, importantly, rely on microeconomic evidence on loan and borrower characteristics at origination rather than aggregate data. In contrast to the aggregate-level results of Clauretie (1987), they do find that recourse significantly decreases the probability of default when the borrower is estimated to have negative equity in their home.

As this paper will show, the fact that the empirical literature on the effects of recourse has

[7] In a 2008 Wall Street Journal editorial, Martin Feldstein summarizes the conventional wisdom thus: "The no-recourse mortgage is virtually unique to the United States. That's why falling house prices in Europe do not trigger defaults. The creditors' ability to go beyond the house to other assets or even future salary is a deterrent." See http://online.wsj.com/article/SB122697004441035727.html. This informal consensus notwithstanding and to my knowledge, that hypothesis has yet to be formally tested. Duygan-Bump and Grant (2008) present some evidence that institutional features that broadly affect the "cost of default" matter for repayment behavior within the European Community.

yielded mixed results is not surprising since the theoretical effects of recourse are themselves ambiguous in key respects. Some robust predictions do emanate from standard models of mortgage choice, but detailed evidence on loan and borrower characteristics at origination are necessary to construct informative tests of these predictions.

3 A model of secured borrowing with default

Consider an economy where time is discrete and infinite. The economy is populated by overlapping generations of two-period-lived households and a continuum of intermediaries. Households are endowed with different quantities $(\omega_0, \omega_1) \in \mathbb{R}^2_+$ of the consumption good in each of the two periods of their life. They value both consumption and housing services $(c_0, c_1, h) \in \mathbb{R}^3_+$ over their lifetime according to a continuous utility function U which is strictly concave and increasing in all arguments and satisfies Inada conditions with respect to both consumption goods. Furthermore, U is twice continuoulsy differentiable and the two consumption goods are complementary in the sense that the cross derivative U_{12} is positive everywhere.[8]

To keep the analysis simple, housing choices are constrained to the set $\{0, H\}$ where $H > 0$. I will think of households who choose $h = H$ as home-owners and of households who select $h = 0$ as renters. The difference in utility between these choices should then be interpreted as the gain in housing services associated with owning a home. Those gains could stem from the fact that houses are larger than rental units, or, more directly, from the fact that owners enjoy a utility premium over renters. Likewise, the cost of opting for $h = H$ should be interpreted as the additional cost of owning versus renting.

Financial markets are exogenously incomplete. Specifically, households can invest at a risk-free (gross) rate $R > 0$, but they cannot borrow at that rate. On the other hand, they must borrow the cost qh of their housing investment in the first period from an intermediary at a gross rate $R^h(\omega_0) > 0$ which may depend on the household's first-period wealth. Here q

[8] This complementarity assumption is invoked in the proof of proposition 5.5.

denotes the price of the housing good relative to first-period consumption. Below I will make assumptions on the rate of transformation between the housing and the consumption good such that this relative price is effectively exogenous and constant.

Constraining down-payments to be zero rather than, say, 20%, is a normalization that has no impact on my results. A critical assumption for my purposes is that down-payments on mortgages are exogenously specified rather than chosen by borrowers. In particular, agents cannot attempt to signal their full type by selecting different down-payments. This assumption sidesteps the high-dimensional signaling game that would result from introducing a down-payment menu. Section 6 discusses the consequences of relaxing this assumption.

Houses must be purchased at date 0 and must be sold back to the intermediary at the end of date 1. Intermediaries sell the resulting housing stock to young people. Providing a natural interpretation for this transaction is the only point of embedding the household problem into a dynamic framework. Aside from this technicality, this paper's results could be derived in a standard two-date framework with a exogenous demand schedule for the housing stock at the end of the second period.

Houses can depreciate or appreciate in real terms before they are sold, so that the quantity $h' \geq 0$ of housing capital old households sell to the intermediary may differ from h. Possible interpretations for depreciation include neighborhood shocks and aggregate taste shocks. Appreciation could stem from positive shocks of either type. These shocks have no bearings on the mathematics this paper employs but they serve an important purpose: they generate default situations where home equity is negative, as is the case in the data in most foreclosure filings.[9]

For notational simplicity and without loss of generality, assume that both ω_1 and h' are known with certainty by the household. Results would not change if both objects were subject to some uncertainty but notation would become heavier. Denote by Φ the distribution of (ω_0, ω_1, h') among households. For each ω_0, the conditional distribution $\Phi(\omega_0, \bullet)$ of (ω_1, h') has continuous marginal distributions and these two characteristics are independent from one

[9] See e.g. Gerardi et al. (2007, 2009, forthcoming.)

another conditional on ω_0, the borrower's observable type.

As in DGS (2004), households can choose to only pay fraction $\delta \in [0, 1]$ of their obligation $qhR^h(\omega_0)$ in the second period of their existence, where $R^h(\omega_0)$ is the mortgage rate available to households of observable first-period type ω_0. When they choose to default, households incur a fixed cost a fixed cost $\tau > 0$ which stands in for any cost beyond the settlement with the intermediary, including non-monetary costs such as the impact on the household's credit history and ability to borrow and become home-owners again. Reasonable models of mortgages default must be consistent with the fact that even in the midst of the most severe housing downturn on record, many households with negative equity choose to continue meeting their financial obligations.[10] A fixed cost of default trivially delivers that prediction. Importantly however, the comparative statics results this paper produces hold whether or not $\tau > 0$.

In the event of default (whenever $\delta < 1$), intermediaries can claim the value qh' of the home as collateral as well as fraction $\eta \in [0, 1]$ of the household's ex-housing wealth in the second period of their life. That is, the household liability is limited to

$$L(a_0, h', \omega_1; \eta) \equiv qh' + \max\{\eta(a_0 R + \omega_1 - \tau), 0\},$$

where a_0 is the household's saving choice in the first period. Note that the direct cost of default τ is applied before deficiency proceeds are calculated. Assuming that it is applied after deficiency losses would complicate the notation in the comparative statics section but would not change any of the analysis.

Another simplification implicit in this specification of household liability is the assumption that no assets are exempt from deficiency. In reality of course, the legal system typically provides mechanisms for households to protect some of their assets against deficiency claims. In most US states for instance, retirement accounts are shielded from recourse. Introducing a level $\underline{a}$ below which assets are exempt from recourse would not change any of my results.

[10] See, e.g., Gerardi et al. (2007, 2009, forthcoming.)

This specification of default costs implies that given mortgage rate R^h, agents who choose to default set δ to solve:

$$qhR^h\delta = L(a_0, h', \omega_1; \eta). \tag{3.1}$$

What's more, conditional on a savings choice $a_0 \geq 0$, agents are strictly better off defaulting if and only if:

$$qhR^h - L(a_0, h', \omega_1; \eta) > (1 - \eta)\tau. \tag{3.2}$$

Given these observations, households of type (ω_0, ω_1, h') solve:

$$\max_{(c_0, c_1, h, \delta) \in \mathbb{R}^4_+} U(c_0, c_1, h) \qquad \text{s. t.:}$$

$$\begin{aligned}
c_0 + a_0 &\leq \omega_0, \\
c_1 &\leq \omega_1 + a_0 R + qh' - qhR^h + 1_{\{\delta<1\}} \left[qhR^h - L(a_0, h', \omega_1; \eta) - (1 - \eta)\tau\right], \\
a_0 &\geq 0, \\
h &\in \{0, H\}, \\
\delta &\in \left[0, \max\left\{\frac{\omega_1 + a_0 R + qh'}{qhR^h}, 1\right\}\right].
\end{aligned}$$

The final constraint reflects the fact that agents cannot deliver an amount that exceeds the gross value of their assets in period 2. Note that even given h and even if $\tau = 0$, the household's choice set may not be convex because the extent of liability increases with a_0, which is a choice variable. I will establish below that the problem has a finite set of solutions, and, furthermore, that this potential multiplicity does not cause average default rates for each observable borrower type to be multiple-valued.

Intermediaries are risk-neutral agents who perform a number of tasks on behalf of agents. First, they store part of household deposits at return $R > 0$. Second, they maximize returns from transforming the consumption good into the housing good at constant and exogenous rate $\frac{1}{q}$, and from doing the opposite transformation. In equilibrium, the relative price of housing will be q, making intermediaries indifferent between any two transformation plans. Whether intermediaries produce or retire housing capital in each period in the aggregate depends only on whether houses tend to appreciate or depreciate on average.

Third, intermediaries seek to maximize the profits from their mortgage activities. They observe the household's characteristics at origination (ω_0) and make mortgage terms dependent on it, but other default-relevant characteristics are known only to the household. By allowing mortgage terms to depend on ω_0, I am departing from the strict anonymity assumption DGS (2004) choose to make. Assuming away heterogeneity in terms across households would make interpreting this model as capturing the basic workings of mortgage markets and the determinants of default decisions unreasonable. Furthermore, a key point in this paper is that allowing mortgage terms to vary across households leads to predictions regarding the impact of recourse conditional on observable borrower characteristics which, unlike aggregate predictions, can be tested.

Since intermediaries are many, it must be the case in equilibrium that the market rate gives intermediaries zero profit, i.e. exactly cover the opportunity cost $R > 0$ of funds. At the same time, it should be equally clear that, in general, several rates may meet this zero-profit condition, which means that multiple equilibria cannot be ruled out, at least under the standard DGS concept. That possibility would make asking comparative statics questions difficult. Below I will use the refinement proposed by Quintin (2011) to fully resolve this issue.

Let $\Delta(R^h; \omega_0)$ be the average delivery rate for households of first period type ω_0 when they are offered a mortgage rate R^h. Before proceeding, it will be convenient to establish that this average rate is well-defined as a function, and continuous, even though each household's choice set is not convex. For one thing, this will enable me to write intermediary profits on mortgages to observable borrower type ω_0 explicitly as:

$$H \times \left\{ \Delta(R^h(\omega_0); \omega_0) R^h(\omega_0) - R \right\}$$

where $\Delta(R^h(\omega_0);\omega_0)$ is *the* average rate of delivery by borrowers of type $\omega_0 \in I\!R_+$.

For any (ω_0, ω_1, h'), let $D(\omega_0, \omega_1, h'; R^h)$ be the set of optimal delivery rates for an household of type (ω_0, ω_1, h') when R^h is the mortgage rate where, by convention, $\delta = 0$ for renters. In the course of proving the next result, I will show that $D(\omega_0, \omega_1, h'; R^h)$ contains a finite number of values. Now, define the average delivery rate on households of (observable) first period type ω_0 for all possible $R^h > 0$ as:

$$\Delta(R^h;\omega_0) = \left\{ \frac{\int \delta(\omega_0, \bullet) d\Phi(\omega_0, \bullet)}{\int 1_{\{h(\omega_0,\bullet)=H\}} d\Phi(\omega_0, \bullet)} : \delta(\omega_0, \omega_1, h') \in D(\omega_0, \omega_1, h'; R^h) \text{ for all } (\omega_1, h') \in I\!R^2_+ \right\}.$$

The average delivery rate is thus computed by taking the subset of each household type that choose to purchase a home in equilibrium and averaging the ratio of their payment to their obligation.[11] This average rate is single-valued and continuous:

Lemma 3.1. *Generically, the average delivery rate* $\Delta(\bullet;\omega_0) : [R, +\infty] \mapsto [0,1]$ *is a continuous function.*

Agents face a choice between renting, owning and paying their mortgages in full, and owning but defaulting on their mortgage. A tie between any two of these three choices is a zero probability event since Φ has continuous marginal distributions. Delivery rates thus vary continuously almost everywhere with unobserved characteristics, and the result follows. The formal proof of this result and the proof of all upcoming results are in the appendix.

This result makes it possible to define an equilibrium according to the concept introduced by DGS (2004) in the generic case where delivery rates are functions for each type. Under that concept, a steady state equilibrium is a mortgage rate schedule $R^h : I\!R_+ \mapsto [R, +\infty]$, decision rules $(c_0, c_1, h, a_0, \delta) : I\!R^3_+ \mapsto I\!R^2_+ \times \{0, H\} \times I\!R_+ \times [0,1]$, average delivery rates for each observable type $K : I\!R^+ \mapsto [0,1]$, a quantity k of new housing capital such that:

[11] By (3.1),

$$\delta((\omega_0, \omega_1, h') \equiv \frac{L(a_0, h', \omega_1; \eta)}{qhR^h(\omega_0)}$$

for defaulting agents.

1. Taking rates and average delivery rates as given, lenders cannot earn strictly positive profits on any mortgage contracts, and earn zero expected profits on any mortgage contract that receive positive funding. In other words, for all $\omega_0 \in \mathbb{R}_+$,

$$K(\omega_0)R^h(\omega_0) \leq R,$$

   with equality when

$$\int_{\{(\omega_0,\omega_1,h'):h(\omega_0,\omega_1,h')=H\}} d\Phi > 0;$$

2. Given R^h, the price q of housing capital, and the returns R to storage, $\{c_0(\omega_0, \omega_1, h')$, $c_1(\omega_0, \omega_1, h')$, $h(\omega_0, \omega_1, h')$, $a_0(\omega_0, \omega_1, h')$, $\delta(\omega_0, \omega_1, h')\}$ solves the problem of households of type (ω_0, ω_1, h') for almost all $(\omega_0, \omega_1, h') \in \mathbb{R}^3_+$;

3. The market for the housing good clears: $\int_{\{(\omega_0,\omega_1,h'):h(\omega_0,\omega_1,h')=H\}}[h(\omega_0, \omega_1, h') - h']d\Phi = k$;

4. The delivery rates assumed by lenders are borne out in equilibrium:

$$K(\omega_0) = \Delta(R^h; \omega_0)$$

   for all ω_0 such that

$$\int_{\{(\omega_0,\omega_1,h'):h(\omega_0,\omega_1,h')=H\}} d\Phi > 0.$$

The housing market clearing condition says that the housing capital lost to depreciation must be replaced in each period. When there is appreciation on average among home-owners, k is negative in which case intermediaries are retiring some housing capital each period. In that case, the intermediary transforms some of the housing capital it bought back into the consumption good and no net addition to the housing stock need be made.

Since all investment activities yield zero profits in equilibrium, the intermediary is willing to meet any demand level for mortgages. Therefore, the mortgage market trivially clears provided aggregate deposits exceed loan demand in steady state. By not stating this last condition explicitly, the equilibrium notion above assumes either that parameters are such that deposits do suffice to serve all loans or, alternatively, that the intermediary has access to outside sources of funds should it need to borrow at rate R beyond the available quantity of deposits. One could think of such an economy as a small open economy. While this would complicate the analysis, allowing the deposit rate to rise when there is an excess demand for loans at R would not change any of my results.

The final condition is the main element of the DGS equilibrium concept. The delivery rates intermediaries assume when making their mortgage decisions must be borne out in equilibrium. DGS (2004) show that under that concept, existence boils down to a standard fixed point problem and can be guaranteed in full generality.

4 Existence and uniqueness

The key aspects of the DGS equilibrium concept are, 1) that lender beliefs about delivery rates have to be borne out in equilibrium, and, 2) that lenders take delivery rates as given and independent of their mortgage pricing choices. The first requirement leads to a standard fixed point problem with, DGS (2004) show, satisfies standard conditions and is well behaved. Equilibria thus always exist in this environment, as in DGS (2004.) There is, for instance, an equilibrium where all mortgage rates are so high that all agents default on nearly their entire obligations, so that no loans are extended, and at the same time, demand is zero. Now observe that the economy without housing satisfies standard conditions so that the usual arguments imply that a unique no-housing equilibrium exists.

Typically however, and as a result of DGS' strong delivery-rate-taking assumption, there are also many equilibria with positive housing choices. To see this, note that for any given ω_0, setting $R^h(\omega_0) = R$ yields zero or negative net profits. As R^h rises, returns may rise (continuously, by lemma 3.1) for a while but need not be monotonic as rising terms drive more agents into default territory. At best therefore, the mapping from mortgage rates to net intermediary profits on a given type is hill-shaped. It follows that many zero profit rates are possible for each household type, making multiple equilibria a distinct possibility. The following example provides a concrete illustration.

Multiplicity example: Assume that there is only one observable type with $\omega_0 = 0$ so that, conveniently, $a_0 = 0$. Normalize $H = 1$ and assume to simplify notation that $h' \equiv 0$ while ω_1 is uniformly distributed over $[\underline{\omega}, \bar{\omega}]$. Finally, assume that the utility premium associated with housing is high enough that all agents choose to buy a house if offered a mortgage. Given mortgage rate R^h, agents default if and only if

$$R^h > \eta\omega_1 + (1-\eta)\tau.$$

As long as $\underline{\omega} < \frac{R^h-(1-\eta)\tau}{\eta} < \bar{\omega}$ then, the rate of return net of default given R^h satisfies:

$$\Delta(R^h)R^h = \frac{1}{\bar{\omega}-\underline{\omega}}\left\{\int_{\underline{\omega}}^{\frac{R^h-(1-\eta)\tau}{\eta}} \eta\omega_1 d\omega_1 + \int_{\frac{R^h-(1-\eta)\tau}{\eta}}^{\bar{\omega}} R^h d\omega_1\right\}.$$

Differentiating this expression with respect to R^h yields

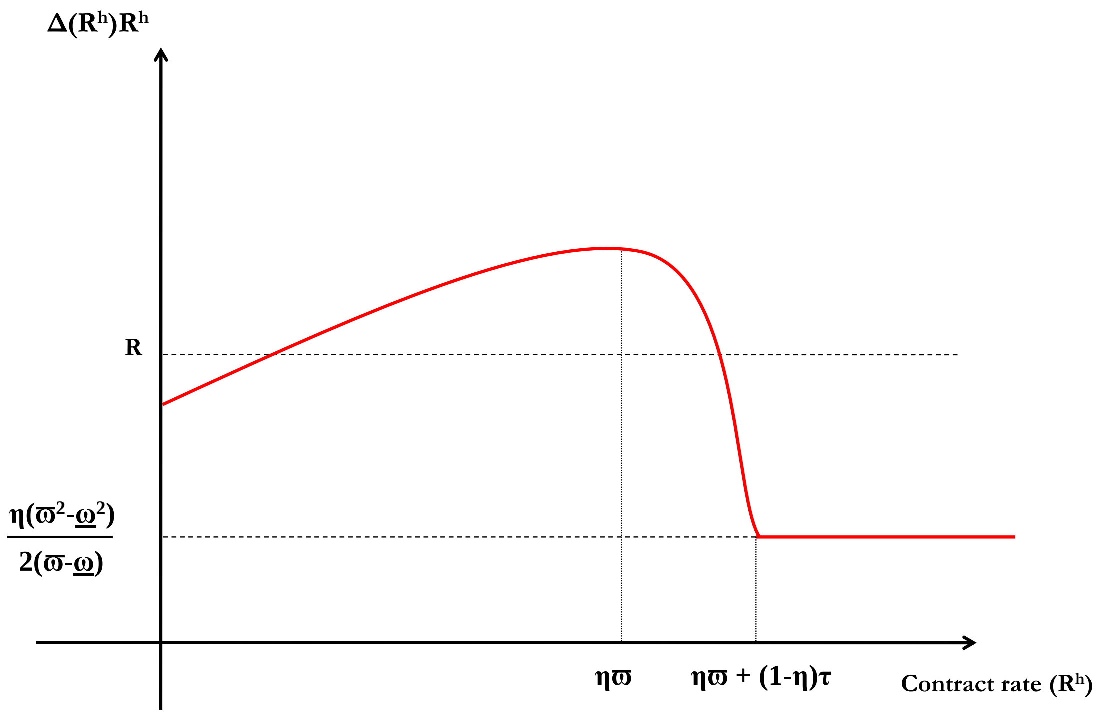
$$\begin{aligned}\frac{d\Delta(R^h)R^h}{dR^h} &\propto \frac{R^h-(1-\eta)\tau}{\eta} + \bar{\omega} - \frac{R^h-(1-\eta)\tau}{\eta} - \frac{R^h}{\eta} \\ &= \bar{\omega} - \frac{R^h}{\eta}\end{aligned}$$

It folllows that the return net of default rises in this example as long as $R^h < \eta\bar{\omega}$ but falls when $R^h \in [\eta\bar{\omega}, \eta\bar{\omega} + (1-\eta)\tau]$. Figure 1 depicts this result. The return net of default begins falling at $\eta\bar{\omega}$ until all agents choose to default in which case the net return becomes constant at

$$\frac{1}{\bar{\omega}-\underline{\omega}}\int_{\underline{\omega}}^{\bar{\omega}} \eta\omega_1 d\omega_1 = \eta\left(\frac{\bar{\omega}^2-\underline{\omega}^2}{2(\bar{\omega}-\underline{\omega})}\right)$$

Now recall that issuing mortgages yield zero expected profits for the intermediary provided $\Delta(R^h)R^h = R$. The figure shows a case where two contract rates yield zero profits. This is a case, therefore, where two DGS equilibria with strictly positive lending exist.

Figure 1: Multiple equilibria under the standard DGS equilibrium concept



It should be clear however that this source of multiplicity is fragile. When several rates give lenders zero profits, lenders are indifferent across the associated contracts. Borrowers, on the other hand, are better off when the lowest rate compatible with zero profits prevails. In fact, it is easy to show that when the prevailing rate is not the lowest rate, rates that are lower than the prevailing rates and give intermediaries strictly positive profits exist, at least generically, as figure 1 illustrates. That situation could not possibly prevail in a competitive environment. If instead of taking all rates as given lenders are given a chance to compete on terms, free-entry will drive out all suboptimal equilibria.[12]

To guarantee uniqueness, instead of assuming in the spirit of DSG (2004) that lenders take prices and delivery rates as given and optimize over quantities alone, one simply needs to assume that lenders offer borrowers a menu of rates compatible with non-negative expected profits. Then it is natural to let borrowers choose the rate most advantageous to them, namely the lowest one. It should be clear that the resulting equilibrium maximizes welfare among all equilibria. This discussion leads to the following definition:

Definition 4.1. *A steady-state equilibrium is return-dominated if, for a positive mass of agents, there exist mortgage rates below R^h that imply positive profits for the intermediary. In other words, an equilibrium mortgage schedule R^h is return dominated if for a positive mass of agent observable types $\omega_0 \in \mathbb{R}_+$,*

$$\Delta(\tilde{R}^h; \omega_0)\tilde{R}^h > \Delta(R^h(\omega_0); \omega_0)R^h(\omega_0),$$

for some $\tilde{R}^h < R^h(\omega_0)$.

The definition simply adapts the return-domination refinement proposed by Quintin (2011) to the specific context of the model described in this paper. In equilibrium, there may be some agent types such that the intermediary earns negative economic profits at any mortgage rate. These types are excluded from mortgage markets and the mortgage rates they face are indeterminate. To simplify the statement of the next result,[13] Adopt the convention that $R^h = +\infty$ and $\Delta(R^h)R^h = 0$ for those agents. The following fact paves the way for meaningful comparative statics results:

[12] When default is not an option for borrowers, price taking can be justified on trivial, compelling grounds: in equilibrium, no borrower would accept a rate higher than the equilibrium rate and lenders have no incentive to fund loans at rate below the equilibrium rate. With default, the first part of the standard argument remains correct, but the second part may no longer hold because lenders can increase their profits by lowering their rates provided delivery rates rise sufficiently. Nothing prevents lenders from taking advantage of this option when it is present. Competition should eliminate this glaring profit opportunity.

[13] This convention also simplifies the statement of comparative statics results in the next section. Indeed, it is is trivially the case that observable types that go from renting to owning with positive probability when η rises see their mortgage rates fall.

Proposition 4.2. *A steady-state that is not return-dominated exists and is generically unique in terms of mortgage rate schedules, and, for each observable borrower type, in terms of delivery rates and home-ownership rates.*

The unique steady-state that is not return-dominated simply gives each potential borrower the lowest rate that yields expected return R, the exogenous opportunity cost of funds. Since the solution to this problem is unique for all possible observable borrower types, there is only one possible equilibrium mortgage rate schedule.

In sum, not only does excluding return-dominated equilibria preserve existence,[14] it brings uniqueness to a model in which multiplicity would be endemic otherwise. With this result in hand, it becomes possible to take on the comparative statics questions this paper seeks to answer.

5 The implications of recourse differences

Consider two versions of the economy laid out in the previous section that differ only in the extent of recourse. How do equilibrium default patterns differ? To tackle this question, it will be useful to make the dependence of variables on η explicit. Write, for instance, $R^h(\omega_0; \eta)$ for the equilibrium mortgage rate available to households of first-period type ω_0 when the recourse level is η. The notation for decision rules is generalized in the same fashion.

5.1 Statutory recourse vs. effective recourse

Holding savings constant, increasing η raises the liability of households in the event of default. Naturally however, households do respond to institutional changes in part by altering their savings choice. Broadening recourse is equivalent to reducing the returns on savings for

[14] As Quintin (2011) explains, excluding return-dominated equilibria can jeopardize existence in general. One feature of this model that helps guarantee existence is the presence of a storage technology with exogenous returns. In more general financial economies, there can be an excess supply of funds at all equilibria that survive the return-domination exclusion procedure. See Quintin (2011) for specific examples.

defaulters. In principle, if savings are highly elastic, some households could respond to the increase in punishment by lowering their savings to the point where $L(a_0, h', \omega_1, \eta)$ falls. This means that statutory recourse could be made tougher and yet cause effective recourse to *fall* because agents carry fewer resources in the second period of their life.

When statutory and effective recourse move in opposite directions for positive masses of agents, comparative statics become trivially ambiguous. Depending on the distribution of unobserved characteristics for any given observable type ω_0, toughening recourse could well cause delivery rates to fall, *even conditional on borrower characteristics at origination.*

The question of whether deficiency proceeds rise when the fraction of qualified assets subject to deficiency increases is formally equivalent to the question of whether capital income taxes rise when capital income is taxed at a higher rate. The answer to either question is positive provided the uncompensated interest rate elasticity of savings is small enough.

To formally state conditions under which statutory and effective recourse do in fact co-move, consider the following maximization problem, for given values of $\hat{R}$ and Y:

$$(CS) \qquad \max_{(c_0, c_1, a_0) \in \mathbb{R}^3_+} U(c_0, c_1, H)$$

subject to:

$$\begin{aligned} c_0 + a_0 &= Y \\ c_1 &= a_0 \hat{R} \end{aligned}$$

Home-owners in this environment solve a version of this canonical consumption-saving (CS) problem for values of $\hat{R}$ and Y that depend both on their characteristics and on the extent η of recourse. Changes in recourse parameters will affect both the effective interest rate agents face and their income level. The effect of these changes on savings a_0 and, more importantly, on effective deficiency proceeds L, will therefore depend on the income and substitution effect implied by the specification U of preferences. A set of assumptions that guarantees that statutory and effective recourse co-move can now be stated:

Assumption 5.1. *U is such that in problem (CS), the elasticity of savings (a_0) with respect to $\hat{R}$ is non-negative but less than $\frac{1-\eta}{\eta}$ for any $(Y, \hat{R}) \geq (0, 0)$.*

Note that this set of assumptions is met for instance when U has a Cobb-Douglas (log) representation.[15] Generally, the requirement is that the elasticity of savings with respect to the interest rate be sufficiently low so that increases in η cause $\eta a_0 R$ to rise.[16] As long as $\eta < 50\%$, the second requirement is met provided the elasticity of savings is below unity, which seems reasonable given the available empirical evidence on this issue. In fact, under the commonly held belief that recovery rates associated with deficiency judgments are small, assumption 5.1 describes the more reasonable case.[17] Under this assumption, the model can in fact yield a robust, testable empirical implications.

5.2 Implications of broadening recourse on given borrower types

This section produces comparative statics results conditional on observable characteristics at origination. The argument is standard: an increase in η causes intermediary profits to rise given observable type ω_0 and given a mortgage rate R^h. Starting from an equilibrium in which the intermediary makes zero profits on a given type, an increase in η would then cause profits to become strictly positive at the original rate. Since by lemma 3.1 delivery rates vary continuously with mortgage rates, a standard appeal to the intermediate value theorem then implies that a lower rate compatible with zero profits exists. As this section will discuss, this fall in yields at origination then generates several testable predictions conditional on a given borrower type.

[15] That is, it holds for instance when U is as described in remark 5.4.

[16] See proof of proposition 5.5 for details.

[17] The dominant interpretation of currently available estimates is that this elasticity is in fact small. See Bernheim, 2003, for a review of the relevant evidence. The assumption that the interest rate elasticity of savings is non-negative is not necessary to guarantee that effective and statutory recourse move together, but it simplifies the proof of my main comparative statics result below.

The central part of the argument, then, is to show that the average delivery rate for a given type rises for a given mortgage rate R^h when η increases. Establishing this, in turn, requires two steps. First, it is necessary to argue that agents who are home-owners in the original equilibrium and remain home-owners deliver at least as much following the increase in η. Second, we need to show that entry into or exit from home-ownership cannot cause delivery rates to fall holding R^h constant. The following result takes care of the first step. The proof provided in the appendix centers around showing that assumption 5.1 guarantees that effective recourse rises for defaulters when η rises.

Lemma 5.2. *Consider a household of observable type $\omega_0 \in \mathbb{R}_+$ that chooses to own a home at a given mortgage rate R^h and recourse intensity η. Under assumption 5.1 and holding R^h the same, an increase in η can only increase the household's delivery rate.*

There only remains to sign the possible effect of entry into or exit from home-ownership when η rises, holding R^h fixed. Given R^h raising η cannot cause any entry. Agents who choose to pay in full do not exit since they are unaffected by the change in η. Defaulting agents, on the other hand can choose to exit when η rises. We need to argue, then, that these exiting agents deliver at a rate that is below the average delivery rate in the original equilibrium.

While this cannot be guaranteed in full generality, it is enough[18] to assume that housing and both consumption goods are normal goods in the following, precise sense.

Assumption 5.3. *U is such that:*

1. *$h(\omega_0, \omega_1, h'; \eta)$ rises with ω_1;*

2. *Consumption when young and when old are both normal goods in problem (CS).*

In words, the first and key part of the assumption is that, all else equal, an increase in lifetime income makes home-ownership more likely. Assuming that an increase in wealth

[18]See proof of proposition 5.5. It is in fact enough that preferences guarantee this for agents who expect no net capital gains from housing because the only agents that could potentially cause delivery rates to fall following an increase in η are defaulting agents. Agents who can increase their second period resources by owning have no incentive to default.

boosts home-ownership prospects certainly seems reasonable and should hold in a satisfactory model of housing choice.[19] It holds for instance for the class of utility functions that are linear homothetic in consumption and multiplicative in housing, as demonstrated in the appendix:

Remark 5.4. *Assume that*

$$U(c_0, c_1, h) = g(h) c_0^{\alpha} c_1^{1-\alpha}$$

for all $(c_0, c_1, h) \in I\!R_+^2 \times 0, H$ *where* g *is a strictly increasing function and* $\alpha \in (0, 1)$. *Then assumption 5.3 holds.*

The main result of this subsection can now be stated:

Proposition 5.5. *Consider a household of observable type* $\omega_0 \in I\!R_+$. *Under assumptions (5.1) and (5.3), in steady state equilibrium:*

1. $R^h(\omega_0; \eta)$ *decreases with* η;
2. *As* η *rises,* $h(\omega_0, \omega_1, h'; \eta)$
   (a) *can only rise for agents who deliver in full or are renters in the original equilibrium, and,*
   (b) *can only fall for agents who are strictly better off defaulting.*

The proof provided in the appendix is lengthy because signing the possible impact of exit from mortgage markets when η rises requires showing that delivery rates among incumbent home-owners increase with ω_1. This in turn requires considering a number of distinct cases.

The second item of the proposition could suggest that the implications of recourse for delivery rates at the household level are ambiguous. Yet, there is no ambiguity in this respect in equilibrium, as the following remark points out.

[19]See Green and Malpezzi, 2003, for a discussion of the relevant empirical evidence on this issue.

Corollary 5.6. *Consider a household of observable type $\omega_0 \in I\!R_+$. Under assumptions (5.1) and (5.3), expected delivery rates rise when η rises. That is:*

$$\frac{\int_{\{\omega_0,\omega_1,h':h(\omega_0,\omega_1,h';\eta)=H\}} \delta(\omega_0,\omega_1,h';\eta)d\Phi(\omega_0,\bullet)}{\int_{\{\omega_0,\omega_1,h':h(\omega_0,\omega_1,h';\eta)=H\}} d\Phi(\omega_0,\bullet)}$$

rises with η.

Proof. Since $R^h(\omega_0;\eta)$ decreases with η, average delivery rates must rise for non-negative profits to continue holding. □

In the parlance of the real estate finance industry, corollary 5.6 says that average loss severity rates fall when recourse broadens for a given set of borrower characteristics at origination. As section 5.4 will discuss however, most empirical tests of the importance of recourse focus on the frequency of default rather than average losses. From that point of view, the key consequence of proposition 5.5 is:

Corollary 5.7. *Under assumption (5.1) and (5.3), the fraction of borrowers of a given observable type $\omega_0 \in I\!R_+$ that default falls when η rises.*

Proof. The fall in R^h that follows the increase in η can induce entry, but only by agents that pay in full. Indeed, the value of defaulting does not depend on R^h and can only fall when η rises for agents of a given ω_0 type. For the same reason, only defaulters of type ω_0 can exit mortgage markets following the increase in η. Borrower who paid in full before the increase in η have an even greater incentive to do so following the drop in rates and the increase in punishment. The result follows. □

It is important to recognize that without conditions that imply the first part of proposition 5.5, the apparently intuitive impact of broadening recourse on default rates would be ambiguous. Indeed, were it possible for rates to increase following an increase in η, borrowers who pay in full could choose to default or exit mortgage markets which, in and of itself, would increase the frequency of default. Assumptions (5.1) and (5.3) or other assumptions that make the effect of recourse on rates the intuitive one are necessary to produce robust implications for the frequency of default.

This section has established, in summary, that under plausible assumptions delivery rates rise while mortgage rates and default rates fall when recourse is made broader conditional on observable borrower characteristics at origination. A natural question to ask is whether similarly plausible assumptions can lead to robust comparative statics predictions at the aggregate level. The answer is negative, as the next section establishes.

5.3 Aggregate Implications

Proposition 5.5 makes it clear that the implications of higher recourse for home-ownership rates are ambiguous, since they are ambiguous at the household level. As η rises, the borrowing cost falls which entices households of low future wealth or high future wealth into home-ownership. Some agents (among those who were better off defaulting before the increase in η), on the other hand, may drop out. On the delivery rate front however, one could think that the sharpness of corollary 5.6 carries through to equilibrium quantities. It does not, which is one of this paper's main point.

Remark 5.8. *As η rises, equilibrium delivery rates may rise or fall.*

A simple example detailed below will suffice to prove that this is in fact a possibility. The construction is simple: consider in economy where all borrowers deliver in full before recourse becomes tougher. Toughening recourse can make dealing with riskier agents profitable for the intermediary, hence can lower average default rates. The simplicity of this argument makes it clear that no reasonable assumptions can guarantee that an increase in punishment will lower default.

This ambiguity may seem surprising given that proposition 5.5 says that defaulters are more likely to leave mortgage markets once recourse broadens while other agents either remain or become home-owners. The ambiguity is caused entirely by entrants, the agents who go from renting to owning. These entrants may have higher-than-average propensities to default.

Part of the proof of proposition 5.5 involved signing the impact of entry and exit on average delivery rate but 1) holding mortgage rates fixed and 2) for agents who are home-owners in the original equilibrium. Here, an increase in recourse can allow previously excluded agents to enter mortgage markets.

For a concrete example, consider an economy with exactly two observable ω_0 types. Each of these two observable types comprise agents of different house appreciation and future income prospects but those differences within ω_0 types are unobservable by the intermediary.

Let the original value of η be $\eta_{low} > 0$. At η_{low}, all type 1 agents deliver in full because, say, their endowment in the second period of their life or their home appreciation prospects are high. Type 2 on the other hand always default with positive probability. A set of assumptions that guarantees this is

$$\omega_0 = \tau = 0$$

for all agents of type 2 while, for a positive mass of agents of type 2,

$$h' = 0 \text{ and } \omega_1 < qHR.$$

Furthermore, η_{low} is sufficiently low that the intermediary cannot cover its opportunity cost (R) on these agents on average so that, in the original equilibrium, agents of type 2 are renters.

Now raise η to $\eta_{high} > \eta_{low}$. Type 1 agents continue to deliver in full. Assume further that η_{high} is sufficiently high that the intermediary can now cover its opportunity cost if it issues a mortgage to agents of type 2. To guarantee this, assume that:

$$\int \eta_{high}\omega_1 d\Phi > qHR$$

for agents of type 2. In that case, setting $R^h = \eta_{high}\bar{\omega}_1$ (where $\bar{\omega}_1$ is the maximum endowment of an agent of type 2) yields enough to cover the opportunity cost of lending for the intermediary provided all agents of type 2 participate. This, in turn, can be guaranteed by assuming that the utility premium associated with housing is high enough for these agents. Then, after raising η default rates become positive, when they were zero before. In particular, they rise following a broadening of recourse.

The intuition for this possibility is trivial. While delivery rates must rise among household types that are active before the increase, the fraction of each type that participates (that is, the distribution of borrower types) in mortgage markets changes as well. The net result can be anything one wants depending on the shape of Φ.

5.4 Empirical tests

The empirical literature on mortgage recourse has focused on two broad hypotheses. The first class of papers rely on aggregate data (see e.g. Clauretie, 1987) to test the hypothesis that economies where recourse is broader should have higher average default rates than economies with weaker recourse, holding other aggregate characteristics constant. The previous section show unequivocally that the outcome of that type of test can tell us nothing about whether recourse statutes matter. Even when rates do respond to recourse changes, average default rates could well rise when recourse become tougher, and participation (the quantity of intermediation) could increase. Since this result obtains holding all other fundamental characteristics of the economy the same, controlling for every default relevant aggregate variables, assuming this were possible, would not solve the issue. Aggregate statistics can tell us nothing about whether or not recourse matters.

A second branch of the literature (see e.g. Ghent and Kudlyak, 2010) tests similar hypotheses but on the basis of microeconomic data. It asks, in substance: do observably similar borrowers with observably similar loan characteristics default at a higher frequency in economies where recourse is weaker? An obvious difficulty with this approach is that, in practice, it is unlikely that one observes all default-relevant borrower and loan characteristics. Besides detailed borrower data, testing this hypothesis carefully would require detailed information on all the mortgages and secured lines of credit the borrower takes on at purchase time. But at least in theory, proposition 5.5 says that this microeconomic approach does yield a meaningful test of whether recourse statutes matter.

The aggregate implications of recourse changes discussed above do point to potential pitfalls associated with the microeconomic approach. Given borrower data from economies with different recourse statutes, a standard empirical approach consists of running a pooled regression of default outcomes on borrower and loan characteristics and a variable that measures the intensity of recourse. The impact of recourse differences on mortgage market participation means that the coefficient on the recourse variable is likely biased by selection effects. In the example laid out in the previous section for instance, economies with tougher recourse allow riskier borrowers to participate. The significance of the recourse variable could capture the resulting sample composition effects.

This suggests that a safer econometric approach would limit outcome comparisons to pairs of observable borrowers instead of fitting a default equation across a pooled sample of borrowers who hold mortgages in economies with different recourse rules. One could for instance code recourse as a binary variable (see, e.g., Ghent and Kudlyak, 2010) and evaluate the treatment effect associated with belonging to a recourse economy through a matching estimator (see e.g. Rosenbaum, P. and D.B. Rubin, 1983.) That approach would effectively deal with the potential effects of selection on the basis of observable borrower characteristics and no default equation need be specified. There would only remain to deal with the possibility of selection on the basis of unobserved characteristics, a daunting but standard issue in empirical work on treatment effects.

6 Discussion of key assumptions

Two strong assumptions greatly simplify the analysis in this paper: the effective exogeneity of house prices and the exogeneity of down-payments. This section argues that relaxing these two assumptions cannot affect the fundamental ambiguity of the aggregate consequences of default.

What makes prices effectively exogenous in the model laid out in this paper is the assumption that the rate of transformation between the consumption good and the housing good is independent of the scale of production. Consider relaxing this assumption by assuming instead that returns to housing transformation are decreasing. Specifically, assume that devoting quantity $k \geq 0$ of the consumption good to housing production yields a quantity $g(k)$ of housing capital where $g(0) = 0$ and g is strictly concave.

Given these assumptions, there are now positive profits associated with housing production. One could interpret the profits associated with the production of housing capital as returns to a fixed factor (land, presumably.) The allocation of those profits could in principle complicate the analysis since price changes would have an impact of the distribution of income.

Rather than consider the many ways in which one could allocate those profits and for the purpose of this discussion, assume for concreteness that they do not accrue to borrowers. There are at least two stories one can tell to justify this: one could posit the existence of some risk neutral agents who are endowed with this fixed factor, and consume the associated rents in each period. Alternatively, one could simply think of $qg(k) - kR$ as a production cost. My point in this section is to argue that the ambiguity of the effect of punishment on default rates is robust to endogenizing house prices and this simplifying assumption will enable me to accomplish that without having to deal with tangential issues.

Existence of a steady state equilibrium with positive housing demand becomes a more difficult proposition in this case because a constant positive relative price of housing is only compatible with positive depreciation in the aggregate. To make the analysis interesting, assume that the economy's parameterization is such that such an equilibrium exist at the starting level of punishment. It is easy to see that an equilibrium then continues to exist for a marginal increase in η. That marginal increase, however, typically causes a change in q, in either direction, since home-ownership rates typically change. Yet, as $\underline{\eta}$ rises, it remains the case that equilibrium delivery rates may rise or fall. To see this, return to the simple example the previous section used to establish the ambiguity of the aggregate consequences of default.

To reach the same conclusion, one simply needs to assume that agents value housing enough that even if prices rise in equilibrium, high-risk borrowers choose to enter markets when a mortgage is issued to them while recourse remains sufficiently profitable for the bank to break even.

What of the assumption that down-payment are exogenous? A wider menu of contracts would greatly complicate the analysis by giving agents potential ways to signal their type to borrowers. It is easy to see however that those options cannot eliminate the aggregate ambiguity discussed in this paper. Indeed, take the extreme case where types are known, say because the distribution of second period characteristics is degenerate for all possible ω_0. Even in that case one can obviously write a simple example where agents who default with positive probability are able to participate only when recourse is sufficiently broad. It is equally easy to see that conditional on participating in mortgage market before and after the increase in η, a given borrower will see her mortgage terms improve following the increase in punishment and will default less frequently.

In sum therefore, while exogenous down-payments and home prices have greatly simplified the analysis in this paper, the fundamental nature of my results does not depend critically on these assumptions.

7 Conclusion

The aggregate implications of broadening recourse on equilibrium default rates are fundamentally ambiguous. In the specific context of mortgage markets, a change in policy towards more recourse tends to lower mortgage rates offered to specific borrowers, quite intuitively. By doing so, it typically changes the pool of borrowers, potentially causing default-prone agents to enter mortgage markets. The impact of this change on equilibrium quantities then depends on the distribution of borrower fundamentals, and can go in any direction.

This simple observation has direct consequences for one we should test whether recourse statutes matter. Whether or not deficiency statutes and economy-wide default rates (or for that matter the quantity of loans) are correlated can tell us nothing about whether these differences do in fact matter for borrower behavior and the allocation of credit. This is true – critically – even if one compare two economies with similar distributions of borrower fundamentals. This goes well beyond the more standard caveat in cross-states studies, namely the fact that different states have very different distributions of socio-economic characteristics. Even if states were identical in terms of that distribution, aggregate default statistics and their correlation to deficiency statutes would still tell us nothing about whether recourse matters.

Instead, tests of whether deficiency matters can only be done meaningfully using detailed micro-evidence on borrower characteristics. For instance, my model predicts quite robustly that toughening deficiency should cause yield at originations to fall conditional on initial borrower characteristics.

These theoretical results suggest that anticipating the aggregate consequences of changes in recourse policy will require a quantitative approach along the lines of Chatterjee et al., 2007. In a version of their model adapted to housing markets, one could ascertain what specification choices and parameter combinations cause specific policy changes to have quantitatively meaningful impact on aggregate outcomes of interest. One could ask for instance whether tougher deficiency statutes would have mitigated the financial crisis in states such as California or Arizona.[20] The answer is anything but obvious: in theory, tougher statutes could well have amplified the crisis.

[20]Corbae and Quintin (2011) calibrate a model of mortgage and housing choice to US data and find that allowing lenders to claim liquid assets in the event of default causes default rate to fall markedly while interest rates and home-ownership rates rise. Importantly, these calculations abstract from transaction costs associated with deficiency hence give deficiency its best chance to matter. Calibrating the effective extent of recourse instead of its formal extent is bound to be a challenging task.

A Proof of lemma 3.1

Proof. Take a household of a given first-period type ω_0. Given (ω_0, ω_1, h') and given R^h, the household's problem has at least one solution since the choice set is compact and the objective function is continuous. Now consider adding the following constraints to the household's problem, one at a time:

1. rent: $h = 0$,
2. buy and pay in full: $h = H$, $\eta(a_0 R + \omega_1) > qhR^h - qh' - (1-\eta)\tau$,
3. buy and default: $h = H$, $\eta(a_0 R + \omega_1) \leq qhR^h - qh' - (1-\eta)\tau$.

Solutions to the original problem must solve one of these 3 artificially constrained problems. Furthermore, the artificially constrained choice sets are convex hence the resulting sub-problems have exactly one solution.

The possibility of a tie across any two of the resulting solutions is a zero measure event, since Φ has continuous marginal distributions. Specifically, any change in h' breaks ties between renting (1) and owning an paying in full (2) and between defaulting (3) and paying in full (2). Ties between defaulting and renting are generically broken by changes in ω_1.[21]

Generically then, solutions to the household problem are unique, and average delivery rates are therefore uniquely defined, and continuous. □

B Proof of proposition 4.2

Proof. It is enough to prove that a best zero-profit rate exists for each household type, and that this unique rate implies unique ownership and delivery rates. The first part follows from the fact that the infimum of all rates that guarantee non-negative profits is well-defined for each ω_0. As for the second part, lemma 3.1 established that the non-convexity associated with the recourse rule does not result in multiple delivery rates. The same intuition holds for home-ownership rates, i.e. the fraction of households of a given first-period type who opt for $h = H$. Note that uniqueness is only generic because it is possible to build cases where the intermediary's profit function is exactly tangent to the zero-profit line for a positive mass of agents. The slightest parameter perturbation breaks these ties. □

C Proof of lemma 5.2

Proof. Generically (for almost all households of a given type) and holding R^h constant, changes in η cause a continuous change in a_0. A marginal increase in η does not change

[21]At values of ω_1 where agents are indifferent between renting and owning, a change in ω_1 breaks that tie unless the derivative of the utility associated with renting with respect to ω_1 happens to exactly coincide with the derivative of the utility associated with defaulting with respect to ω_1.

the delivery rate or default choice of agents who were strictly better off not defaulting. As for agents who are strictly better off defaulting, the marginal change in η does not affect the saving decision of agents whose borrowing constraint is binding. For all other defaulting agents, simple algebra shows that they solve a consumption-saving problem of type (CS) with

$$\hat{R} = R(1-\eta)$$

and

$$Y = \omega_0 + \frac{\omega_1 - \tau}{R}.$$

Now,

$$\begin{aligned}
\frac{d(a_0\eta)}{d\eta} &= a_0 + \eta\frac{da_0}{d\eta} \\
&= a_0 + \eta a_0 \epsilon_{\hat{R}}(a_0)\frac{d\hat{R}}{\hat{R}} \\
&= a_0 - \eta a_0 \epsilon_{\hat{R}}(a_0)\frac{1}{1-\eta} \\
&> a_0 - a_0 \\
&= 0
\end{aligned}$$

Here $\epsilon_{\hat{R}}(a_0)$ denotes the elasticity of a_0 with respect to $\hat{R}$. By assumption it is capped above by $\frac{1-\eta}{\eta}$ hence $\eta\epsilon_{\hat{R}}(a_0)\frac{1}{1-\eta} < 1$ which explains the strict inequality in the derivation. This establishes that for defaulting agents, ηa_0, hence L, rise at least weakly. Therefore, the delivery rate of agents that are home-owners in the original equilibrium can only go up if they remain home-owners, as claimed. □

D Proof of remark 5.4

Proof. That consumption is normal for this utility function in (CS) is obvious. As for the first part of assumption 5.3, Let $V^R(\omega_1)$ be the log utility associated with renting given $\omega_1 \geq 0$ while $V^D(\omega_1)$ is the log utility associated with owning and defaulting in the second period. Some algebra shows that V^R rises with $(1-\alpha)\log(\omega_1)$ when ω_1 is high enough to insure that the no borrowing constraint $a_0 \geq 0$ is binding. When ω_1 is low and the borrowing constraint is not binding, V^R rises with $\log(\omega_0 + \frac{\omega_1}{R})$. The same holds for defaulters except that in both cases ω_1 is replaced with $\omega_1 - \tau$.

Now note that with this utility function, consumption in the first period is $\max\left\{\alpha(\omega_0 + \frac{\omega_1}{R}), \omega_0\right\}$ for renters and the same for defaulters with $\omega_1 - \tau$ replacing ω_1. If follows that the borrowing constraint starts binding earlier for renters than for defaulters.

The derivative of V^R with respect to ω_1 is $\frac{1}{\omega_0 R + \omega_1}$ for ω_1 low enough and then jumps down[22] to $\frac{1-\alpha}{\omega_1}$ where the constraint binds. Furthermore, $\frac{1-\alpha}{\omega_1}$ remains everywhere below

[22]The jump is downward because V^R is concave in ω_1.

$\frac{1}{\omega_0 R+\omega_1}$ past the jump since $\frac{\omega_0 R+\omega_1}{\omega_1}$ decreases with ω_1. For defaulters, the same is true with $\omega_1 - \tau$ replacing ω_1. It follows that the derivative of V^D with respect to ω_1 is everywhere higher than the derivative of V^R with respect ω_1. Therefore, if an agent is better off owning and defaulting than renting at ω_1 the same holds when ω_1 rises.

The same argument can be applied to establish the same for owning and paying in full versus renting. The result follows. Indeed, note first that if $qh' \geq qHR^H$ all agents choose to own and pay in full and the result holds trivially. If $qh' < qHR^H$ then one can replicate the argument above exactly after replacing τ with $qHR^H - qh' > 0$. $\square$

E Proof of proposition 5.5

Proof. Hold R^h fixed and raise η. Lemma 5.2 guarantees that home-owners in the original equilibrium who remain home-owners in the new equilibrium increase their delivery rates, at least weakly. As pointed out in the text, raising η cannot cause any entry holding R^h fixed. Agents who pay in full are unaffected by the change in η. There only remains to consider defaulting agents who chose to exit.

If an agent of type (ω_0, ω_1, h') chooses to default at the original equilibrium, this is true for agents with the same (ω_0, ω_1) but a lower h' since second period income does not depend on h' for defaulters. For h' high enough, agents with the same (ω_0, ω_1) choose to pay in full instead of defaulting.

Now take a given ω_1 and pick h' so that $V^D(\omega_1) = V^F(\omega_1)$ where the first value function gives the utility associated with defaulting while the second gives the utility associated with paying in full. In other words, h' is the threshold past which agents are better off paying in full than defaulting. I will now show that if ω_1 increases marginally, $V^D(\omega_1) \leq V^F(\omega_1)$ so that the h' threshold falls when ω_1 rises.

Recall that when the borrowing constraint does not bind, the problem that defines V^D is a (CS) problem with $Y = \omega_0 + \frac{\omega_1 - \tau}{R}$ and $\hat{R} = R(1-\eta)$ while the problem that defines V^F is a (CS) problem with $Y = \omega_0 + \frac{\omega_1 + qh' - qHR^h}{R}$ and $\hat{R} = R$. The borrowing constraint binds when the optimal c_0 in those artificial problems exceeds ω_0. Note that we must have

$$(1-\eta)(\omega_1 - \tau) \geq \omega_1 + qh' - qHR^h$$

since otherwise defaulting would be strictly dominated by paying in full. Since c_0 is a normal good and the effective interest rate is lower for defaulters, it follows that at the h' where $V^D(\omega_1) = V^F(\omega_1)$, defaulters save less than agents who pay in full hence consume more in the first period. For $V^D(\omega_1) = V^F(\omega_1)$ to hold, they must then consume less in the second period. That is, letting (c_0^D, c_1^D) and (c_0^F, c_1^F) be the consumption profiles of defaulters and agents who pay in full at h', respectively, we must have $c_0^D \geq c_0^F$ while $c_1^D \leq c_1^F$.

Note now that if the borrowing constraint binds for agents that pay in full, the constraint must also bind for defaulters. But since the income of defaulters is strictly higher in the second period while first period income is held constant, this would contradict the fact that

$V^D(\omega_1) = V^F(\omega_1)$. That leaves two cases to consider. If the borrowing constraint is loose for both types of agents, then the envelope theorem implies that:

$$\begin{aligned}
\frac{\partial V^F(\omega_1)}{\partial \omega_1} &= U_1\left(c_0^F, c_1^F, H\right)\frac{1}{R} \\
&\geq U_1\left(c_0^D, c_1^D, H\right)\frac{1}{R} \\
&= \frac{\partial V^D(\omega_1)}{\partial \omega_1}
\end{aligned}$$

The inequality uses the facts that the two consumption goods are complementary, that U is strictly concave and that $c_0^D \geq c_0^F$ while $c_1^D \leq c_1^F$.

If the borrowing constraint is binding for defaulters, then:

$$\begin{aligned}
\frac{\partial V^D(\omega_1)}{\partial \omega_1} &= U_2\left(\omega_0, (1-\eta)(\omega_1-\tau), H\right)(1-\eta) \\
&\leq U_1\left(\omega_0, (1-\eta)(\omega_1-\tau), H\right)\frac{1}{(1-\eta)R}(1-\eta) \\
&< U_1\left(c_0^F, c_1^F, H\right)\frac{1}{R} \\
&= \frac{\partial V^F(\omega_1)}{\partial \omega_1}
\end{aligned}$$

The first inequality is necessary for the borrowing constraint to bind for defaulters. The second inequality uses the fact that $c_0^F < \omega_0$, that $c_1^F \geq c_1^D$, the strict concavity of U and the fact that both goods are complementary.

Given these observations, take two agents whose second period endowment is such they choose to default for some values of h'. If at a given h' they both choose to default, deficiency proceeds are higher for the agent with the higher ω_1 since consumption when old is a normal good. If the lower ω_1 agent chooses to pay in full, so a fortiori does the agent with the higher ω_1. Since the marginal distribution of h' does not depend on ω_1 by assumption, it follows that among agents of income profile (ω_0, ω_1) who default for some values of h', average delivery rates fall with ω_1. Agents that exit home-ownership following an increase in η are therefore the agents with the lowest possible delivery rates. Therefore, exit from home-ownership can only cause average delivery rates to rise.

This establishes that intermediary profits can only rise when η rises holding R^h fixed and, therefore, the lowest mortgage rate compatible with zero-profits must fall at the new equilibrium.

The second item of the proposition states that for households who are strictly better off making full delivery on their mortgage promises or rent in the original equilibrium, the increase in η can only make them more likely to take on a mortgage and purchase a home. On the other hand, for agents who are better off defaulting, the increase in deficiency acts as a tax on second period income when they choose to purchase a home. $\square$

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