Optimal Exclusion

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Motivation

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  3. Constant forgiveness lotteries
  4. One-time forgiveness lottery
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- What is the optimal shape of of exclusion in in a canonical model of lending with endogenous default?

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- It maximizes the the stationary volume of mutually beneficial transactions . . .
- . . . and the average welfare of the excluded
- We go on to characterize how the optimal length of punishment depends on fundamentals such as agent patience and the direct consequences of default, if any

Literature

- Kehoe and Levine (1993), Kocherlakota (1996), Alvarez and Jerman (2000), Chatterjee et al. (2007), Tertilt et. al (2007) Liu and Skrzypacz (2013)
- Bond and Krishnamurthy (2004), Elul and Gottardi (2015), Bethune et al. (2017)
- Dubey et al. (2005), Quintin (2013),

Other related papers

The environment

- Time is discrete and infinite
- One, non-storable good
- Infinitely-lived investors can activate a project each period which delivers $y > 0$ with probability π, nothing otherwise
- Requires one unit of the good at the start of the period
- Investors are risk-neutral and discount future payoffs at rate β
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- Large mass of lenders are born each period with a unit of the good
- Can store their endowment for return $R \in [0, y)$

Loan markets

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- They can choose to report 0 and make no payment even when outcome is positive (= *strategic default*) in which case they experience disutility $\tau \sim F$
- Absent other forms of punishment, this imposes $m_t \leq \tau$
- Lending takes place if a solution $m_t \leq \tau$ to

$$\pi(1 - F(m_t))m_t = R.$$

- Existence: Dubey et al. (2005), Quintin (2013)

Exogenous exclusion

- Exclusion technology is a sequence $\{\phi_s\}_{s=0}^{+\infty}$ of *forgiveness* probabilities
- An agent is going to be excluded for exactly n periods with probability
$$\phi_{n+1} \prod_{s=0}^{n} (1 - \phi_s).$$

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- $V^E(n)$: expected lifetime utility of agent who has been excluded n periods
- V^N: same for non excluded agents

Bellman equations

$$
\begin{aligned}
V^N &= (1-\pi)\beta V^E(0) + \\
&\qquad \pi E_\tau \max\left\{y - m + \beta V^N, y - \tau + \beta V^E(0)\right\}
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Bellman equations

$$V^N = (1-\pi)\beta V^E(0) + \pi E_\tau \max\left\{y - m + \beta V^N, y - \tau + \beta V^E(0)\right\}$$

$$V^E(n) = \phi_n V^N + (1-\phi_n)\beta V^E(n+1)$$

In particular, $V^E(n) \leq V^N$ for all n

Default

Investors choose to pay if

$$m \leq \tau + \beta \left[V^N - V^E(0) \right]$$

making the lender's break-even condition

$$\pi \left(1 - F \left(m - \beta \left[V^N - V^E(0) \right] \right) \right) m = R.$$

Stationary distributions

$$
\begin{aligned}
\mu^N &= \mu^N(1-\delta^D) + \sum_{n=0}^{+\infty} \mu^E(n)\phi_n \\
\mu^E(0) &= \left[\mu^N + \sum_{n=0}^{+\infty} \mu^E(n)\phi_n\right]\delta^D \\
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\end{aligned}$$

Lemma

A stationary equilibrium with $\mu^N > 0$ exists only if

$$\sum_{n=0}^{+\infty} \Pi_{s=0}^{n}(1-\phi_n) < +\infty.$$

Existence

Proposition

A stationary equilibrium with strictly positive investment exists if and only if

1. $\sum_{n=0}^{+\infty} \Pi_{s=0}^{n} (1 - \phi_n) < +\infty$,
2. *A solution $m \leq y$ exists to the lender's break-even condition.*

Optimal Exclusion

Consider a social planner who designs the exclusion policy to maximize stationary welfare:

$$\mu^N V^N + \sum_{s=0}^{+\infty} \mu^E(s) V^E(s)$$

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Proposition

In any stationary equilibrium that maximizes average welfare, the forgiveness policy must be such that for some s^**,*

1. $\phi_s = 0$ *for all* $s < s^*$*;*
2. $\phi_{s^*} \in (0, 1]$*;*
3. $\phi_{s^*+1} = 1$ *and* $\phi_s \in [0, 1]$ *for all* $s > s^* + 1$.

Step 1: Maximize the welfare of the excluded

- Given $V^E(0)$, what policy maximizes the welfare of the excluded?
- Knowing $V^E(0)$ pins down m, V^N, and δ^D
- Conditional on $V^E(0)$ assume the planner maximizes

$$\frac{\sum_{s=0}^{+\infty} \mu^E(s) V^E(s)}{\sum_{s=0}^{+\infty} \mu^E(s)}$$

subject to:

$$V^E(0) = \phi_0 V^N + (1-\phi_0)\phi_1 \beta V^N + (1-\phi_0)(1-\phi_1)\phi_2 \beta^2 V^N + \dots$$

- That's best done by front-loading punishment (we show)

Step 2: Maximize the number of transactions

Lemma

$$\mu^N = \frac{1}{1 + \frac{\delta^D}{1-\delta^D}(1+\zeta)}$$

where ζ *is negatively related to the welfare of the excluded*

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Maximizing the welfare of the excluded also maximizes the volume of transactions!

Exclusion length

Proposition

When default costs are homogenous at given value τ, the optimal exclusion discount solves

$$\kappa = max\left(\pi^2(y-\tau) - \frac{1}{\beta}(R-\tau\pi), 0\right).$$

In particular, exclusion length falls with investor patience (β), project size (y), project quality (π), and with the direct punishment (τ) associated with default.

Exclusion length with strategic default

Assume that ex-post default costs are low at $\tau_L = \tau - \epsilon$ or high at $\tau_H = \tau + \epsilon$

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Proposition

A mean-preserving spread in default costs raises exclusion length for ϵ small enough but must eventually drives exclusion length to zero as ϵ becomes large

Extensions

1. Risk-averse investors
2. Exogenous punishment while excluded
3. Exogenous exit
4. Observable income
5. Non-stationary forgiveness policies

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