

Asset Quality Dynamics

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- ▶ Debt tends to be pro-cyclical, except for the largest firms
- ▶ Safe corporate debt, in fact, is acyclical, at best
- ▶ We propose a model that is quantitatively consistent with this fact ...
- ▶ ... and use it to quantify the impact of safe corporate debt markets on the US business cycle

Methodological approach

- ▶ We lay out a macroeconomic model that is standard on the real side but where the security space responds endogenously to changes in fundamentals
- ▶ We do so by embedding Allen and Gale's 1988 "Optimal Security Design" model into a dynamic environment

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- ▶ Fixed point problem:
 1. Taking the contingent path of financial structures as given, agents choose an optimal consumption policy
 2. This consumption path, in turn, determines agents' willingness to pay for different securities
 3. Taking this willingness to pay as given, producers issue menus of securities that maximize their profits
 4. The resulting financial structure must coincide with the guess agents made in the first place

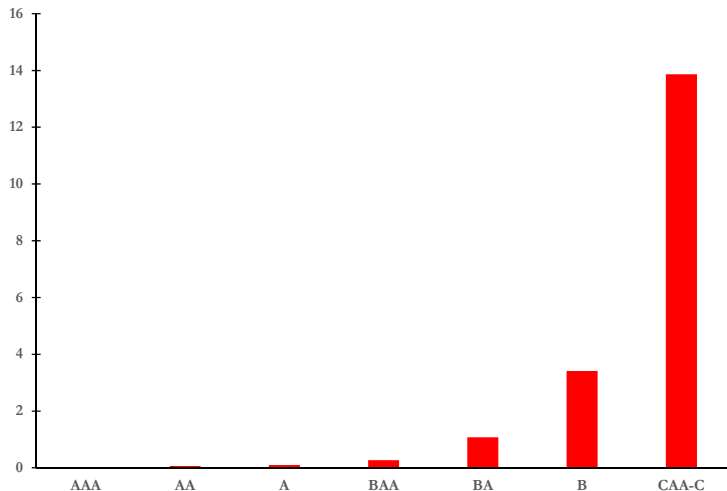
Data

- ▶ Firms in Compustat, 1985-2014 (366,317 firm-year)
- ▶ Exclude foreign, utility and financial firms (186,818)
- ▶ Exclude missing or bad data (96,668)

Firm count details

	AAA	AA	A	BBB	>BBB	No rating
1985	13	58	125	75	163	1,680
1986	13	64	159	108	258	2,040
1987	13	59	137	117	316	2,023
1988	13	60	136	100	291	1,996
1989	14	51	140	102	280	1,980
1990	14	54	129	107	255	2,098
1991	13	55	130	114	222	2,292
1992	13	55	131	117	225	2,526
1993	13	54	134	132	270	2,697
1994	13	49	136	141	283	2,863
1995	12	48	135	155	289	3,251
1996	12	45	146	176	331	3,282
1997	12	43	149	186	336	3,175
1998	10	35	158	208	387	3,244
1999	9	32	144	218	383	3,065
2000	9	27	136	223	406	2,739
2001	8	22	124	223	425	2,522
2002	7	20	115	219	465	2,516
2003	6	15	113	220	482	2,531
2004	6	13	115	210	504	2,466
2005	6	12	113	205	509	2,466
2006	6	11	113	201	495	2,399
2007	6	11	101	203	478	2,287
2008	6	12	93	200	454	2,226
2009	6	13	86	194	461	2,190
2010	5	14	83	197	448	2,166
2011	4	13	89	209	441	2,131
2012	4	13	87	205	453	2,198
2013	4	13	88	212	472	2,232
2014	4	15	90	211	488	2,095

Frequency of default by rating, 1920-2010



Cyclicality of safe corporate debt

Firm rating	AAA	AAA-AA	AAA-A	\geq BBB	$<$ BBB
$\text{mean}(D^S/Y)$	0.81	2.93	7.66	11.8	4.16
$\rho(D^S, Y)$	-0.14	0.03	0.23	0.19	0.47
$\rho\left(\frac{D^S}{Y}, Y\right)$	-0.20	-0.09	0.06	0.00	0.38
$\rho\left(\frac{D^S}{D+E}, Y\right)$	-0.25	-0.17	-0.04	-0.15	0.35

▶ BP filter

▶ CF filter

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Cyclicality of safe corporate yields

	1985-2006	1985-2014	1947-2014
$\rho(\text{AAA yield}, Y)$	-0.0401	-0.2491	-0.3048
$\rho(\text{AAA-BAA spread}, Y)$	-0.2991	-0.3774	-0.6028

Related work

1. Allen and Gale (1988, 1991)
2. Quadrini and Jerman (2011), Covas and Den Haan (2011), Karabarbounis, Macnamara and McCord (2014), Eler et. al (2012), Khale and Stulz (2011)
3. Bernanke et. al. (2011)
4. Brunnermeier and Sannikov (2012)

The model

- ▶ Time is discrete and infinite, one good
- ▶ Mass 1 of households who value consumption only, time-separable CRRA preferences
- ▶ Large mass of producers characterized by talent/productivity $z \sim \mu$

Producers

- ▶ Can activate a project by investing 1 unit of capital at the start of the period
- ▶ Gross output is

$$A_t z^{1-\alpha} n_t^\alpha + (1 - \delta)$$

- ▶ A_t is aggregate TFP and n_t is labor input
- ▶ TFP follows a first-order markov process with transition G_A

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- ▶ A_t is aggregate TFP and n_t is labor input
- ▶ TFP follows a first-order markov process with transition G_A
- ▶ Net operating income is:

$$\Pi(A_t, w_t; z) \equiv \max_{n>0} A_t z^{1-\alpha} n^\alpha - n w_t$$

- ▶ Labor demand is:

$$n^*(A_t, w_t; z) \equiv \arg \max_{n>0} A_t z^{1-\alpha} n^\alpha - n w_t$$

Security markets

- ▶ Producers sell claims to households and to world markets
- ▶ Take as given the households' willingness to pay $q_t^H(A)$

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- ▶ World markets pay q_t^W per unit of risk-free claims at date t

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- ▶ q^W is measurable with respect to the history of aggregate shocks

Producer problem

$$MV_t(z) \equiv \max_{b^s \geq 0} \quad b^s q_t^W + \int_{\mathcal{A}} q_t^H(A) \left[(\Pi(A, w_t; z) + 1 - \delta) - b^s \right] dA \\ - (1 + \mathbf{1}_{\{b^s > 0\}} \kappa),$$

subject to:

$$b^s \leq \Pi(\underline{A}_t, w; z)$$

► Debt vs. EBITDA

Risky security prices and returns

- ▶ Securities sold by producers of type z pay

$$\Pi(A, w_t; z) + 1 - \delta - b_t^S(z)$$

and sell for:

$$\int_{\mathcal{A}} q_t^H(A) \left[(\Pi(A, w_t; z) + 1 - \delta) - b_t^S(z) \right] dA$$

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- ▶ Stochastic return on the same security is

$$r_t(A; z) = \frac{\Pi(A, w_t; z) + 1 - \delta - b_t^S(z)}{\int_{\mathcal{A}} q_t^H(A) \left[(\Pi(A, w_t; z) + 1 - \delta) - b_t^S(z) \right] dA}.$$

Household problem

- ▶ \mathcal{H}_t : possible histories of aggregate TFP shocks up to date t
- ▶ Households assume a mapping

$$S_t : \mathcal{H}_t \mapsto \mathcal{S}$$

- ▶ ...where $S_t(h_t) = \{q_t^W, r_t(\bullet, z) : z \geq 0\}$

Household problem

$$\max_{b^d \geq 0, e^d \geq 0} E \sum_{t=0}^{+\infty} \beta^t U(c_t)$$

subject to:

$$q_t^W b_t^d + \int e_t^d(z) d\mu(z) + c_t = a_t(h_t) + \int \max\{MV_t(z), 0\} d\mu(z),$$

$$\begin{aligned} a_{t+1}(h_t, A) &= \int e_t^d(z) r_t(A, z) d\mu(z) \\ &+ b_t^d + w_t(h_t, A), \text{ for all } A \in \mathcal{A} \end{aligned}$$

where:

$$\{q_t^W, r_t(\bullet, z) : z \geq 0\} = S_t(h_t),$$

Equilibrium

Prices, security menus and and decisions such that:

1. Decision plans are optimal given prices;
2. $\int_{\{z: MV_t(z) \geq 0\}} n^*(A, w_t; z) = 1$ for all $A \in \mathcal{A}$;
3. $\int_{\{z: MV_t(z) \geq 0\}} b_t^s(z) d\mu \geq b_t^d$;
4. $e_t^d(z) r_t(A, z) = \Pi(A, w_t; z) + 1 - \delta - b^s(z)$ for all $A \in \mathcal{A}$;
5. $MV_t(z) = b_t^s(z) q_t^W + \int_{\mathcal{A}} q_t^H(A) \left[(\Pi(A, w_t; z) + 1 - \delta) - b_t^s(z) \right] dA - \left(1 + 1_{\{b_t^s > 0\}} \kappa \right)$.

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6. $q_t^H(A_t) = \frac{\beta G_A(A_t | A_{t-1}) U'(c_{t+1}(h_t, A_t))}{U'(c_t)}$;

Aggregation

Given capital K , labor N and exogenous TFP A , aggregate output is:

$$F(A, K, N) = AE [z|z \geq \underline{z}(K)]^{1-\alpha} K^{1-\alpha} N^\alpha.$$

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GDP accounting:

$$c_{t+1} + K_{t+1} - (1-\delta)K_t + \int_{b_t^s > 0} \kappa d\mu + b_t^W - q_{t+1}^W b_{t+1}^W = F(A_t, K_t, N_t).$$

Security markets

Proposition

The solution to the producer security design problem at a given date t is fully described by two thresholds $0 \leq \underline{z}_t \leq \bar{z}_t$ such that:

- 1. Producers issue securities if and only if $z \geq \underline{z}_t$;*
- 2. $b_t^s(z) = 0$ if $z < \bar{z}_t$;*
- 3. $b_t^s(z) = \Pi(\underline{A}_t, w_t; z)$ if $z > \bar{z}_t$.*

Household portfolio

Effectively, households invest in one security/portfolio whose stochastic payoff is

$$F(A_t, K_t, 1) + (1 - \delta)K_t - b_t^W,$$

and whose price at the start of the period is

$$\int_{\mathcal{A}} q_t^H(A) \left[F(A_t, K_t, 1) + (1 - \delta)K_t - b_t^W \right] dA,$$

hence whose return, for all possible values of A_t is

$$r^H(A_t) = \frac{F(A_t, K_t, 1) + (1 - \delta)K_t - b_t^W}{\int_{\mathcal{A}} q_t^H(A) \left[F(A_t, K_t, 1) + (1 - \delta)K_t - b_t^W \right] dA}.$$

Recursive equilibrium

- ▶ Aggregate state is: $\theta = (\bar{a}, A_{-1}) \in R_+ \times \mathcal{A}$,
- ▶ An equilibrium consists of the following objects:
 1. $g : \Theta \times \mathcal{A} \mapsto \Theta$
 2. $K : \Theta \mapsto R_+$
 3. $\underline{z} \times \bar{z} : \Theta \mapsto R_+^2$
 4. $q^H : \Theta \times \mathcal{A} \mapsto R_+$
 5. $r^H : \Theta \times \mathcal{A} \mapsto R_+$
 6. $e^H : \Theta \times R_+ \mapsto R_+$
 7. $c : \Theta \times R_+ \mapsto R_+$
 8. $w : \Theta \times \mathcal{A} \mapsto R_+$
 9. $b^W : \Theta \mapsto R_+$
 10. $MV : \Theta \times R_+ \mapsto R_+$
 11. $V^H : \Theta \times R_+ \mapsto R$

Household value function

$$\begin{aligned} V^H(\theta, a) &= \max_{e^H > 0, b > 0} U\left(a + \int_{z \geq \underline{z}} MV(z) dz - e^H - q^W b\right) \\ &+ \beta \int_{\mathcal{A}} V^H(g(\theta, A), a'(A)) dG(A|A_{-1}) \end{aligned}$$

where, for all $A \in \mathcal{A}$

$$a'(A) = b + e^H r^H(\theta, A) + w(\theta, A).$$

Allen-Gale condition

Proposition

The household value function V^H is concave and differentiable in assets. Furthermore, for all possible values $\theta \in \Theta$ of the aggregate state,

$$\frac{\partial V(\theta, a)}{\partial a} = U'(c(\theta, a)).$$

RCE conditions

1. $K(\theta) = \int_{z \geq \underline{z}} d\mu$
2. $e^H(\theta, a) = \int_{\mathcal{A}} q^H(\theta, A) \left[F(A, K(\theta), 1) + (1 - \delta)K(\theta) - b^W(\theta) \right] dA$
3. $\int_{z \geq \underline{z}} MV(\theta, z) dz = q^W(\theta) b^W(\theta) + \int_{\mathcal{A}} q_t^H(A) \left[(\Pi(A, w(\theta); z) + 1 - \delta) - b^S(\theta, z) \right] dA$
4. $c(\theta, a) = a + \int_{z \geq \underline{z}} MV(\theta, z) dz - e^H(\theta, a)$
5. $a'(\theta, A) = e^H(\theta, A) r^H(\theta, A) + w(\theta, A)$
6. $q^H(\theta, A) = \frac{\beta G(A|A_{-1}) U'(c^i(g(\theta, A), a'(\theta, A)))}{U'(c(\theta, a))}$ for $A \in \mathcal{A}$
7. Financial structure solves producer problems

Mapping from model to data

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1. $\int \max \{MV_t(z), 0\} d\mu(z) \Rightarrow$ Income of top managers and proprietors
2. $Y \equiv F(A, K, N) \Rightarrow$ Value added by the private sector minus the compensation of top managers and proprietors

Calibration

Parameter	Description	Value
β	Discount rate	0.95
σ	Utility curvature	2.00
δ	Depreciation rate	0.10
α	Labor share	0.65
κ	Tranching cost	0.0115

Calibration (2)

- ▶ Producer talent (z) is log-normally distributed with location parameter 0 and dispersion parameter 0.5

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- ▶ Aggregate TFP has support $\{0.97, 1.00, 1.03\}$ and transition matrix

$$G_A = \begin{bmatrix} 0.50 & 0.50 & 0.00 \\ 0.24 & 0.50 & 0.26 \\ 0.00 & 0.56 & 0.44 \end{bmatrix}$$

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- ▶ q^W has support $\{0.972, 0.968, 0.976\}$

Key moments

Moment	Data	Model
Ratio of safe debt to Y	3.24%	3.18%
Ratio of management rents to Y	11.00%	11.72%
$std(\log(Y))$	2.15%	2.28%
$\rho\left(\frac{1}{q^W} - 1, Y\right)$	-0.25	-0.28

Basic horse race

Two sources of procyclicality for AAA debt use:

1. Constraint on safe debt issues is looser during good times
2. q^W is procyclical

Basic horse race

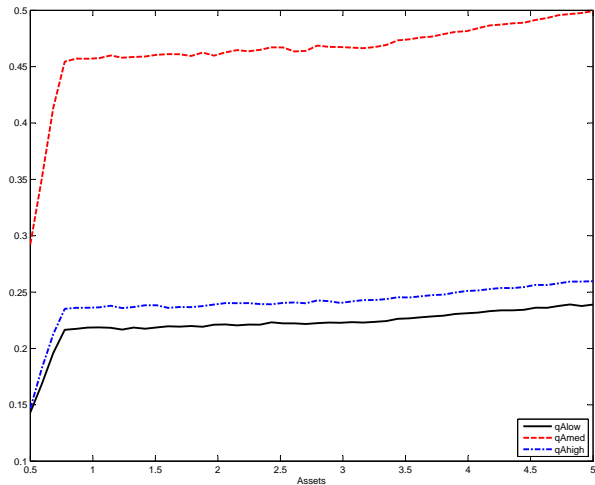
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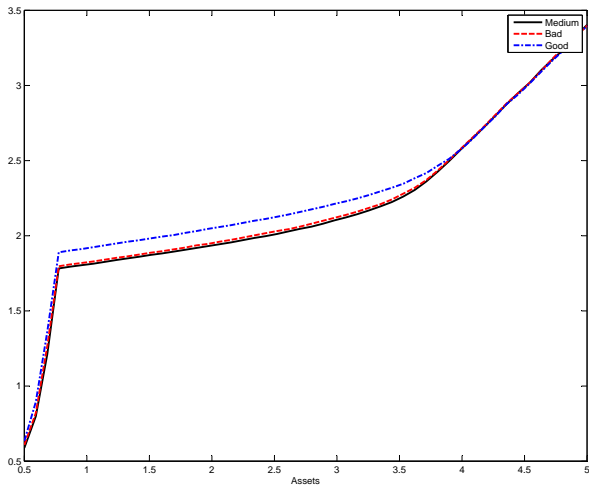
One source of countercyclicality:

1. q^H is procyclical

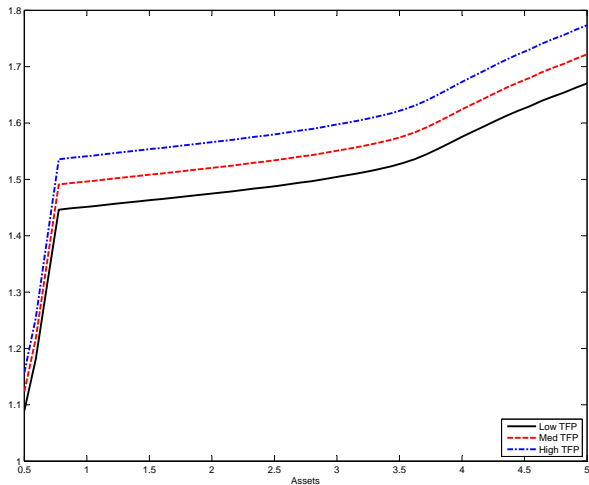
Household willingness to pay vs. assets



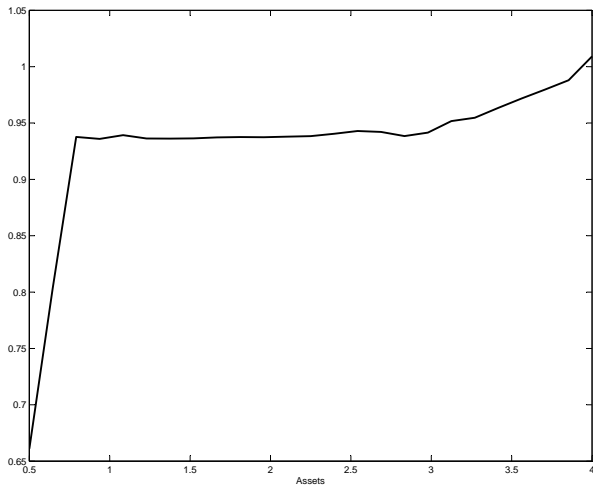
Capital stock vs household assets



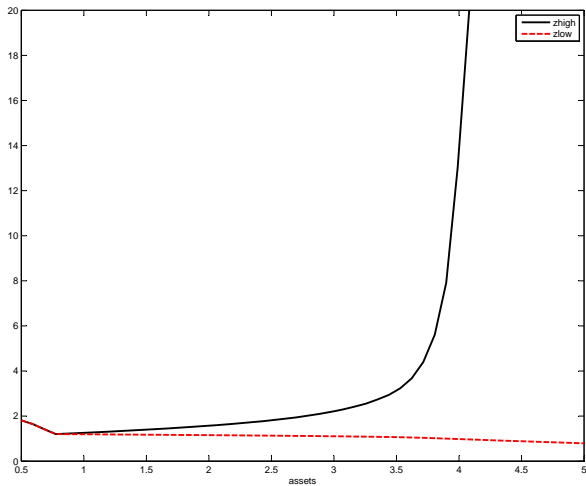
GDP vs household assets



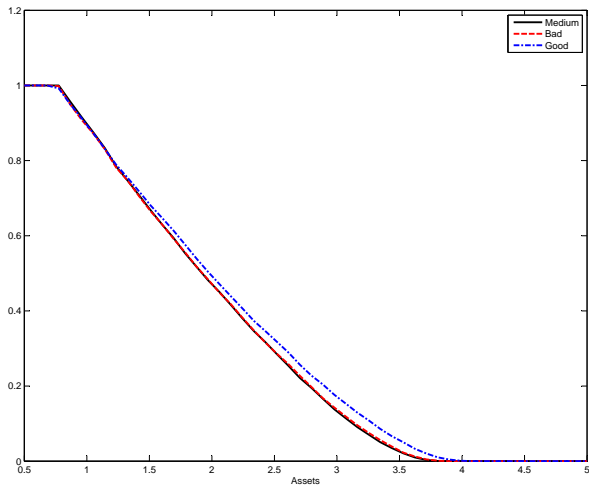
Household's WTP for safe assets



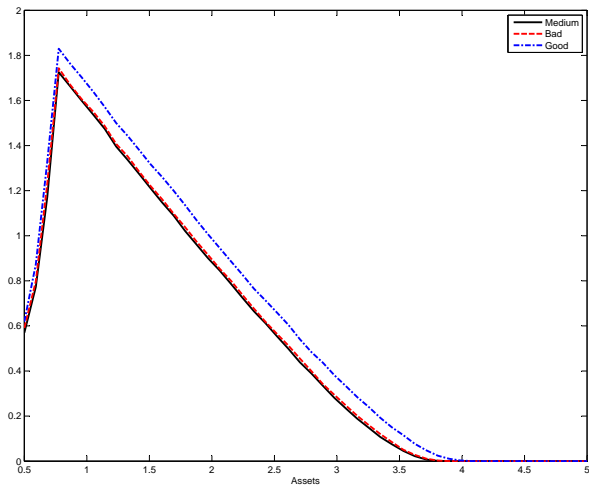
Securitization cutoffs



Fraction of producers that issue safe debt



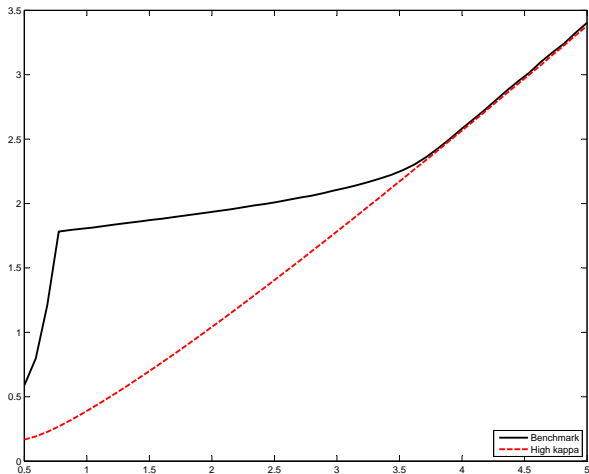
Volume of safe debt



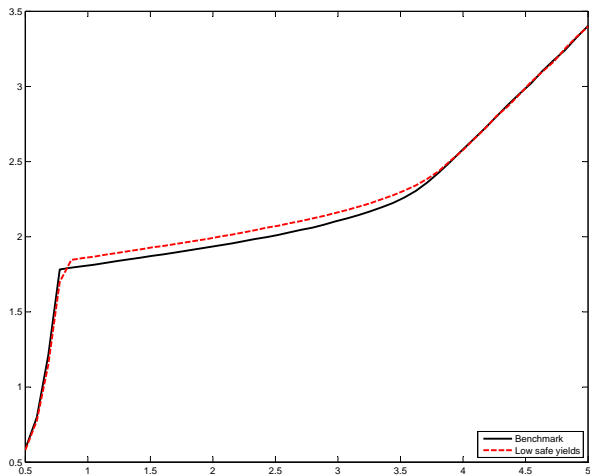
Cyclical properties of key model variables

Moment	Data	Model
$\rho(D^S, Y)$	0.03	-0.10
$\rho\left(\frac{D^S}{Y}, GDP\right)$	-0.09	-0.13
$\rho\left(\frac{D^S}{D+E}, GDP\right)$	-0.17	-0.12
$\rho(\text{spread}, Y)$	-0.25	-0.13

Tranching and capital formation



Low safe yields



Safe debt markets and the business cycle

	Benchmark	High κ	Low safe yields
Mean GDP	1.5894	-0.93%	+0.04%
$std(\log(GDP))$	0.0236	-0.42%	-0.85%
Mean Cons	1.3532	-0.27%	-0.27%
$std(\log(C))$	0.0134	+8.21%	-2.99%
Mean Assets	3.6482	-1.34%	-1.98%
$std(\log(a))$	0.0269	-15.24%	+6.69%
Mean K	2.3426	-4.02%	+0.17%
$std(\log(K))$	0.0276	+0.72%	-10.51%

Cyclical properties of key model variables

Moment	AAA-AA	Model	AAA-A	Model
$\rho(D^S, Y)$	0.03	-0.10	0.23	-0.07
$\rho\left(\frac{D^S}{Y}, GDP\right)$	-0.09	-0.13	0.06	-0.09
$\rho\left(\frac{D^S}{D+E}, GDP\right)$	-0.17	-0.12	-0.04	-0.11

Safe debt markets and the business cycle

	(AAA-AA)	High κ	(AAA-A)	High κ
Mean GDP	1.5894	-0.93%	1.5909	-1.01%
$std(\log(GDP))$	0.0236	-0.42%	0.0235	-0.00%
Mean Cons	1.3532	-0.27%	1.3510	-0.09%
$std(\log(C))$	0.0134	+8.21%	0.0128	+13.28%
Mean Assets	3.6482	-1.34%	3.5792	-1.98%
$std(\log(a))$	0.0269	-15.24%	0.0302	+24.50%
Mean K	2.3426	-4.02%	2.3533	-4.39%
$std(\log(K))$	0.0276	-10.51%	0.0267	+4.49%

Summary

1. A dynamic model of costly security creation can account for the acyclicity of safe corporate debt issues
2. Access to safe debt markets raise the level of output and consumption but also increases volatility
3. Exogenous, permanent reductions in safe yields have limited effects on the level of GDP and consumption but large effects on the volatility of both

Cyclicality of safe corporate debt (BK filter)

Firm rating	AAA	AAA-AA	AAA-A	\geq BBB	$<$ BBB
$\text{mean}(D^S/Y)$	0.81	2.93	7.66	11.8	4.16
$\rho(D^S, Y)$	-0.26	0.10	0.30	0.07	0.22
$\rho\left(\frac{D^S}{Y}, Y\right)$	-0.30	0.06	0.26	-0.01	0.19
$\rho\left(\frac{D^S}{D+E}, Y\right)$	-0.27	0.16	0.44	0.25	0.32

▶ Back

Cyclicality of safe corporate debt (CF filter)

Firm rating	AAA	AAA-AA	AAA-A	\geq BBB	$<$ BBB
$\text{mean}(D^S/Y)$	0.81	2.93	7.66	11.8	4.16
$\rho(D^S, Y)$	-0.26	0.07	0.28	0.02	0.17
$\rho\left(\frac{D^S}{Y}, Y\right)$	-0.29	0.06	0.27	-0.03	0.16
$\rho\left(\frac{D^S}{D+E}, Y\right)$	-0.26	0.16	0.46	0.26	0.29

▶ Back

