Efficient Contract Enforcement

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September 25, 2012

Abstract

We study how the efficient choice of contract enforcement levels interacts with the efficient allocation of capital in a simple production economy. Contract enforcement makes trade possible but requires an aggregate investment of capital that is no longer available for production. In such an economy, more dispersion in ex-ante marginal products makes it optimal to invest more resources in enforcement. Furthermore, implementing the optimal allocation requires a specific distribution of the institutional cost across agents that is not monotonic and results in a redistribution of endowments. At the efficient solution, agents at the bottom of the endowment distribution benefit the most from institutional investments and these investments lead to a reduction in consumption and income inequality.

Keywords: Enforcement; Institutions; Inequality; Human and Physical Capital.

JEL Classification: D31; D52; O11; O43.

*E-mail: thor@econ.queensu.ca, Tel: (613) 533-2271, FAX: (613) 533-6668. We would like to thank Russ Cooper, Per Krusell, Huw Lloyd-Ellis, Elias Papaioanou, B Ravikumar, Richard Rogerson, Christian Zimmermann and seminar and conference participants at the Federal Reserve Bank of Richmond, the University of Montreal, Rice University, UQAM, the University of Western Ontario, the University of Iowa, Ryerson University, Ente Enaudi, the University of Calgary, the SED meetings, the Midwest Theory Meetings, the CMSG meetings and the Vienna Workshop in Macroeconomics for many helpful comments.

1 Introduction

An efficient allocation of resources requires institutions that enforce contracts and property rights, as pointed out most notably by North (1990) and Hurwicz (1994). However, such institutions are costly to set up. In this paper, we provide a tractable way to model endogenous contract enforcement as an efficient institutional choice.

The basic idea we formalize is that building institutions is akin to the adoption of a new technology. Like new technologies, institutions require the investment of resources that cannot be used for other productive purposes. Although investing in institutions is not directly productive, it makes a better allocation of resources possible. It follows that potential efficiency gains should be a key determinant of institutional investments.[1]

We build directly on the framework that Benabou (1996) and Aghion (1998) have developed to study the interplay between inequality, institutions, and growth. All agents operate the same production technology characterized by decreasing returns to scale but capital endowments vary across agents. Therefore, agents can in principle benefit from trading capital with one another in exchange for post-production transfers. The enforcement of post-production payments is limited, however, which constrains capital exchanges, leads to more dispersion in marginal products, hence less output. It may therefore be desirable to invest resources in raising the level of enforcement before production begins. A trade-off thus arises as capital can be used either as an input in production or to improve enforcement.[2] The benefits of enforcement depend on the potential gains from trade and hence on the initial discrepancy in marginal products across agents. As pre-trade inequality in marginal products increases, benefits from investing in enforcement also rise.

Who should pay for setting up contract enforcement? Until enforcement is introduced, its financing cannot be imposed on agents and contributions have to be made on a voluntary basis. We show that a market mechanism can achieve the efficient social investment in the enforcement technology. The efficient investment is financed by fees that grant agents access to the market. These fees depend on individual endowments but we show that they cannot be monotonic in endowments, let alone linear. Effectively then, the optimal arrangement

[1]Precisely how these investments make trade possible matters little for our results, and it is mostly for concreteness that we focus on limited enforceability as the friction that better institutions can mitigate. Modeling other aspects of institutions that increase the size of the set of contracts which agents can write with one another would produce similar insights.

[2]Our set-up follows Koeppl (2007) who studies a similar trade-off in the context of intertemporal risk sharing.

combines some pre-production redistribution of endowments with institutional investments.

Endogenizing market imperfections thus changes the policy implications that emanate from the canonical Aghion/Benabou framework in fundamental ways. When enforcement frictions are taken as exogenous, a mean-preserving contraction in endowment levels optimally raises output and growth. When investing in better enforcement is an option, it becomes optimal to combine such investments and redistribution. More importantly, the optimal redistribution policy no longer takes the form of a simple mean-preserving contraction. The intuition is straightforward. The maximum contribution an agent can be required to make depends on how much he benefits from borrowing or lending capital on the market. Because these gains vary non-monotonically with endowments, contributions must be non-monotonic. It follows that achieving better institutions and redistributing endowments are intrinsically linked policy instruments.

We then study two straightforward extensions of our model. First, we consider a version of our economy with two types of productive resources we label human and physical capital. If human and physical capital are complementary in production, a sufficiently high positive correlation between human and physical capital endowment leads to small differences in marginal products across agents, which reduces incentives to invest in enforcement. This simple extension of our model shows that a high degree of wealth inequality may or may not imply large potential gains from institutional investments. Instead, the key determinant of those potential gains is the ex-ante dispersion in marginal products.

In the second extension, we introduce simple intergenerational linkages to study the dynamic relationship between inequality, redistribution and the level of contract enforcement. Agents with little or no capital reap the benefits of more enforcement, as they can accumulate more resources. Therefore investment in enforcement reduces endowment and income inequality over time. As the dispersion in endowment diminishes, so do differences in marginal products across agents, and the optimal investment in enforcement also declines. Differences in marginal products, however, never fall below a threshold that would make investment in enforcement unnecessary.

To conclude, our theory implies that in order to understand the relationship between contract enforcement, inequality and economic performance, one must consider where the gains from adopting different degrees of contract enforcement arise, and how these gains can be redistributed throughout the economy. Fundamentally, an inadequate degree of contract enforcement shold persist only if additional enforcement does not generate enough benefits to compensate those who pay for it.

2 The Model

We will first study the determinants of contract enforcement in a static environment populated by a continuum of agents indexed by $i \in [0,1]$. Each agent is endowed with a quantity $a_i \geq 0$ of capital, where a_i increases with i on $[0,1]$. This distribution of endowments is non-degenerate so that there is some measurable inequality among agents and is sufficiently smooth to allow us to use standard variational arguments.

Agents are also endowed with a technology that transforms an input $k \geq 0$ of capital into a quantity k^α of a consumption good where $\alpha \in (0,1)$. We adopt this specification of the production function simply for concreteness. Assuming some form of decreasing returns to scale suffices to derive our results. All agents seek to maximize their end-of-period income which is equal to their consumption of the single consumption good.

Since the individual production technology exhibits decreasing returns to scale and endowments differ across agents, marginal products also differ across agents. Therefore, agents have an incentive to trade capital before production starts: agents with low capital endowments would like to borrow some capital for production from other agents in exchange for transfers of the consumption good once production has taken place. We assume, however, that enforcement is limited. Agents can default on any transfer they owe after production, in which case they incur a fixed cost $\eta > 0$ denominated in units of consumption. This formulation of the default option follows Sappington (1983) and Banerjee and Newman (1993), among others. We interpret η as an absconding cost. By paying this fixed cost, agents erase any payment obligation they face.[3]

Our main goal is to endogenize the level of enforcement. In this context, a natural trade-off arises between devoting resources to improve enforcement and to produce the consumption good. Studying this trade-off will enable us to characterize the determinants of institutional quality. We assume therefore that establishing an economy-wide enforcement level $\eta \geq 0$ requires an aggregate capital cost $g(\eta) \geq 0$ that must be borne before production begins. The cost function g is strictly convex, strictly increasing and twice differentiable on $(0, +\infty)$, and

[3]Our results can easily be generalized to a broader class of default cost specifications. One could assume for instance that default costs rise with the quantity of capital a given agent chooses to use in production (or, equivalently, with output) according to a schedule $D(\eta, k)$ for all $\eta, k \geq 0$. The tools we use in this paper would simply require that D be jointly concave.

that $g(0) = 0$. To allow for fixed costs associated with a positive level of enforcement, we only require that $\lim_{\eta \searrow 0} g(\eta) \geq 0$.

3 Institutional Choice and the Allocation of Capital

3.1 The Social Planning Problem

In our economy, returns to scale are decreasing by assumption. Given unequal capital endowments, capital exchanges can in principle improve productive efficiency. Capital trade, however, require post-production transfers hence positive enforcement levels. In turn, establishing a positive level of enforcement requires resources that are no longer available for production. We will now study a social planning problem that formalizes the resulting trade-off. We assume that the planner seeks to maximize aggregate consumption. This focuses the analysis on production efficiency, since such a planner has no direct interest in reducing inequality.[4]

Given a distribution of endowments, the planner chooses a capital allocation $k = \{k_i \geq 0 | i \in [0,1]\}$, a schedule of post-production transfers $t = \{t_i \in \mathbb{R} | i \in [0,1]\}$ and a level of enforcement, $\eta \geq 0$. The planner's choice is restricted in two ways. First, agents can choose to stay in autarky rather than participate in the proposed allocation. Second, agents can decide to default on the transfers stipulated by the planner in which case they incur the punishment of η in equivalent units of consumption. Therefore, the planner solves

$$\max_{(k,t,\eta)} \int (k_i^\alpha - t_i)\, di \tag{3.1}$$

[4] It should be clear that this entails no loss of generality. Assuming instead that the planner maximizes a strictly concave welfare functional over agents' end-of-period consumption does not change the nature of the optimal allocation. The inclusion of ex-ante participation and ex-post enforcement constraints implies that there is no room for redistributing income at the end of the period once production efficiency has been taken into account.

subject to

$$
\begin{aligned}
\int k_i di + g(\eta) &= \int a_i di && (3.2)\\
k_i^\alpha - t_i &\geq a_i^\alpha \quad \text{for almost all } i && (3.3)\\
t_i &\leq \eta \text{ for almost all } i && (3.4)\\
\int t_i di &\geq 0. && (3.5)
\end{aligned}
$$

The first constraint is a resource feasibility constraint. The second set of constraints states that agents have to be willing to participate in the proposed allocation given that they can always opt for autarky. The third set of conditions expresses the fact that the enforcement of transfers is limited. The final constraint states that the planner cannot distribute additional resources at the end of the period: aggregate consumption cannot exceed aggregate output. Naturally, this constraint will bind at the optimal allocation, so that maximizing aggregate consumption is equivalent to maximizing aggregate output. In the appendix, we show that, generically, a unique solution to this social planning problem exists.

As a benchmark for the upcoming analysis, consider first how the planner would allocate resources if she faced neither participation nor enforceability constraints. In that first-best case, no investment in enforcement needs to be made, so that $\eta = g(\eta) = 0$. Furthermore, output is maximized by having all agents operate at $k^{FB} \equiv \int k_i di$.[5] We will now establish that absent *either* the participation *or* the enforcement constraints, the planner can still implement the first-best allocation of capital.

Proposition 3.1. *If either the participation constraints (3.4) or the enforcement constraints (3.5) are dropped, the solution of the social planning problem is the first-best allocation of capital.*

Proof. Assume that the planner faces enforcement constraints, but no participation constraints. Then, setting $\eta = 0$, $k_i = k^{FB}$ and $t_i = 0$ for almost all i delivers the maximum level of aggregate output and aggregate consumption. Likewise, when the planner only faces participation constraints, set $\eta = 0$, $k_i = k^{FB}$ and define $\hat{t}_i \equiv \left(k^{FB}\right)^\alpha - a_i^\alpha$ for almost all i. Since $\alpha < 1$ and since the distribution of endowments is not degenerate, Jensen's inequality implies that $\int \hat{t}_i di > 0$ so that the first-best allocation of capital is feasible and, hence, the

[5] Maximizing aggregate consumption only requires choosing a set of transfers such that the ex-post resource constraint (3.5) binds. Indeed, the optimal transfer scheme and consumption allocations are indeterminate when the planner faces neither participation nor enforcement constraints.

maximum aggregate output is attained. To maximize aggregate consumption as well, we can simply increase transfers to any positive measure of agents, since we have used less than aggregate output to satisfy all participation constraints: $\square$

3.2 The Optimal Level of Enforcement

When the social planner faces both participation and enforcement constraints, the maximum level of output and/or aggregate consumption can no longer be attained. This is the case for two reasons. First, establishing positive enforcement is costly, so that whenever it is used, there is less capital input available for production. Second, because raising the enforcement level implies a first-order cost increase, the planner never chooses to eliminate all inequality in marginal products.

Proposition 3.2. *The optimal allocation with endogenous enforcement is such that k_i is not almost everywhere equal.*

Proof. Denote the non-negative multipliers associated with the constraints (3.2) - (3.5) by θ, $\{\lambda_i | i \in [0,1]\}$, $\{\mu_i | i \in [0,1]\}$ and τ, respectively. If the planner chooses not to invest in enforcement ($\eta = 0$), then $t_i = 0$ for almost all i, so that, by the participation constraint, $k_i = a_i$ almost everywhere and the result holds trivially. Assume then that $\eta > 0$. Necessary conditions for an interior solution to the planner's problem are given by[6]

$$\theta g'(\eta) - \int \mu_i di = 0 \tag{3.6}$$

$$\alpha k_i^{\alpha-1}(1+\lambda_i) - \theta = 0 \text{ for almost all } i \tag{3.7}$$

$$-1 - \lambda_i - \mu_i + \tau = 0 \text{ for almost all } i. \tag{3.8}$$

Also note that, together with the usual slackness conditions, these conditions are sufficient.

Assume now, by way of contradiction, that k_i is constant a.e. Then, λ_i is constant a.e. as well by condition (3.7) as is then μ_i by condition (3.8). But since $\eta > 0$, resource feasibility requires that $a_i > k_i$ for a non-negligible set of i. This implies that $t_i < \eta$ and $\mu_i = 0$ for that set. Hence, we need $\mu_i = 0$ for almost all i, which cannot be the case by condition (3.6), because $\eta > 0$ and $\theta > 0$ by condition (3.7). $\square$

The intuition for this result is straightforward. When capital use is equated across agents, marginal products are equated as well. It follows that small deviations from such an allocation

[6] See Section 4.8 in Cesari (1983). Our set of constraints satisfies a standard constraint qualification.

have a negligible impact on output. On the other hand, reducing the need for enforcement (for instance by directing more capital towards constrained agents) has a first-order effect on the resources available for production. With this result in hand, we can now characterize the optimal allocation of capital more precisely.

Proposition 3.3. *The optimal allocation of capital is characterized by two endowment thresholds* $0 \leq \underline{a} \leq \bar{a}$ *and two bounds on capital use* $0 \leq \underline{k} \leq \bar{k}$ *that determine the optimal capital allocation for almost all* i *according to*

$$k_i = \begin{cases} \underline{k} = (\underline{a}^{\alpha} + \eta)^{\frac{1}{\alpha}} & \textit{if } a_i \leq \underline{a} \\ (a_i^{\alpha} + \eta)^{\frac{1}{\alpha}} & \textit{if } a_i \in [\underline{a}, \bar{a}] \\ \bar{k} = (\bar{a}^{\alpha} + \eta)^{\frac{1}{\alpha}} & \textit{if } a_i \geq \bar{a}, \end{cases}$$

where η *is the optimal level of enforcement.*

Proof. As $\eta = 0$ implies autarky in which case our characterization holds trivially, assume that the optimal solution has $\eta > 0$. Conditions (3.6)-(3.8) together with the associated slackness conditions describe the optimal solution. We first establish that for almost all agents, either the participation or the enforcement constraint holds with equality. Assume to the contrary that for some non-negligible set of agents this is not the case. Then, by condition (3.8), this is true for all agents. But this contradicts condition (3.6) whenever $\eta > 0$ since $\theta > 0$ by (3.7).

Consider next the set of agents with non-binding enforcement constraints ($\mu_i = 0$). For these agents, $1 + \lambda_i = \tau$ which implies that $k_i = \bar{k} \equiv \left(\frac{\alpha\tau}{\theta}\right)^{\frac{1}{1-\alpha}}$. On the other hand, agents whose participation constraint is slack ($\lambda_i = 0$) employ capital $\underline{k} \equiv \left(\frac{\alpha}{\theta}\right)^{\frac{1}{1-\alpha}} < \bar{k}$, where the inequality follows from the fact that $\tau > 1$ by (3.8), since $\mu_i + \lambda_i > 0$ for almost all agents. Finally, agents for which both constraints bind employ

$$k_i = \left(\frac{\alpha(\tau - \mu_i)}{\theta}\right)^{\frac{1}{1-\alpha}} = (a_i^{\alpha} + \eta)^{\frac{1}{\alpha}} \in [\underline{k}, \bar{k}].$$

There only remains to be shown that these three groups of agents are separated by certain endowment thresholds. If agent $i \in [0, 1]$ is in the group with non-binding enforcement constraints, $\bar{k}^{\alpha} - a_i^{\alpha} < \eta$. Similarly, being in the group with non-binding participation constraints implies $\underline{k}^{\alpha} - a_i^{\alpha} > \eta$. Finally, a necessary condition for being in the group where both constraints bind is given by $k_i^{\alpha} - a_i^{\alpha} = \eta$ for some $k_i \in [\underline{k}, \bar{k}]$. These three conditions are mutually exclusive and define the thresholds we need. □

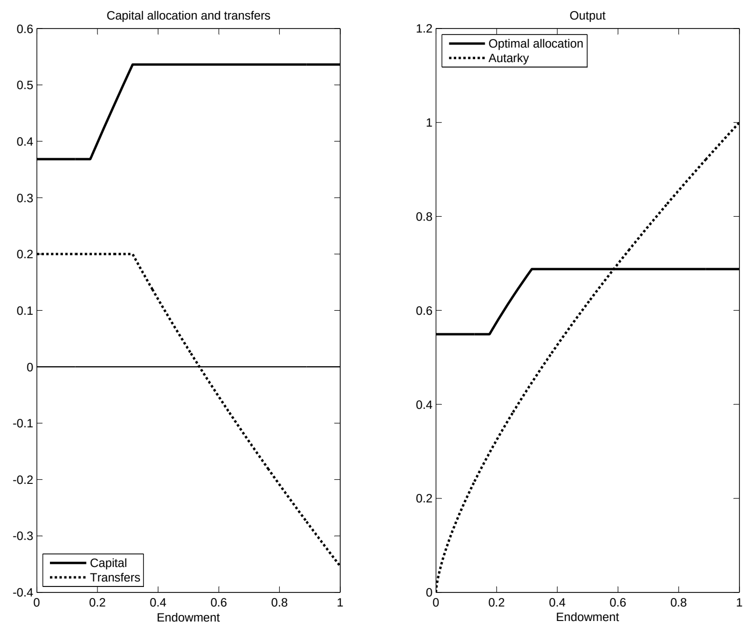

Figure 1: Solution to the Social Planner Problem

Figure 1 illustrates this result by showing the optimal solution under the parametric assumptions that $\alpha = 0.7$, that endowments are uniformly distributed on $[0, 1]$ (that is, that $a_i = i$ for almost all $i \in [0, 1]$) and that $g(\eta) = 0.01$ for $\eta \in (0, 0.2]$ and becomes arbitrarily large thereafter.[7] Agents with low endowments have a binding enforcement constraint, but strictly prefer to participate in the optimal arrangement. As the first panel of figure 1 shows, all these agents employ the same level $\underline{k}$ of capital. Conversely, agents with high endowments have a binding participation constraint, but a loose enforcement constraint. These agents operate at the highest level of capital $\bar{k}$. Agents in the middle have both constraints binding and operate with the capital stock such that their income level $k_i^\alpha - \eta$ exactly matches their autarky income.

The corresponding transfers are also shown on the first panel of the figure. Up to endowment level $\bar{a}$ all agents pay the maximum transfer $t_i = \eta = 0.2$ compatible with limited enforcement. Then transfers decline and eventually become negative. The optimal solution is such that agents with negative transfers have lower capital input than their initial endowment hence produce less than they would under autarky, as the second panel of figure 1 shows. But as the figure also shows, all other agents produce at a higher level than under autarky. For the specific parameters we used to produce these pictures, aggregate output at the optimal solution exceeds aggregate autarky output by over 10%.

The optimal allocation can be described by three equations that pin down the optimal level of enforcement η and the optimal allocation of capital by determining the two cut-off points $\underline{a}$ and $\bar{a}$,

$$g'(\eta) = \int_{\{i|a_i \leq \bar{a}\}} \frac{1}{\alpha \bar{k}^{\alpha-1}} di - \int_{\{i|a_i \leq \bar{a}\}} \frac{1}{\alpha k_i^{\alpha-1}} di \tag{3.9}$$

$$g(\eta) = \int a_i di - \int k_i di \tag{3.10}$$

$$\eta = \int_{\{i|a_i \geq \bar{a}\}} (a_i^\alpha - \bar{a}^\alpha)\, di. \tag{3.11}$$

The first condition equates the marginal costs and the marginal benefits of enforcement expressed in units of capital input. More enforcement enables a better allocation of capital across agents. Hence, one can achieve the same output with a lower aggregate input of capi-

[7] While, strictly speaking, this violates the assumption that g is differentiable and convex, it is easy to show that our results extend to this step-function case. Conveniently in this case, the planner obviously opts for $\eta = 0.2$ and the optimal allocation becomes easy to compute.

tal. Interestingly, marginal benefits can then be expressed as the wedge between the inverse of the marginal product of capital of unconstrained and constrained agents.[8] The other two equations describe the feasibility of allocating capital and of transfers in terms of the two endowment cut-offs.

The optimal allocation leads to a more equal income and consumption distribution than under autarky. In particular, all agents above the lower endowment threshold $\underline{a}$ receive their autarkic income, while all other agents receive a fixed income higher than autarky. Hence, improving institutions by investing into costly enforcement benefits agents at the lower end of the endowment distribution.

Proposition 3.4. *The optimal income distribution is a left-censored version of the income distribution under autarky. Specifically,*

$$k_i^\alpha - t_i = \begin{cases} \underline{a}^\alpha & \text{if } a_i \leq \underline{a} \\ a_i^\alpha & \text{if } a_i \geq \underline{a}. \end{cases}$$

Proof. Agent i's end-of-period income is $k_i^\alpha - t_i$ for all $i \in [0,1]$. All agents whose endowments exceed $\underline{a}$ have a binding participation constraint. Hence, they have the same income as under autarky. Agents with endowments under $\underline{a}$ all realize income $\underline{k}^\alpha - \eta = \underline{a}^\alpha$. □

3.3 Implementing the Optimal Allocation

One natural question to ask in light of the last result is how an allocation that equates consumption at the bottom of the distribution can be decentralized when agents in the interval $[0, \underline{a}]$ start the period with uneven resources. This section shows that the optimal capital allocation can in fact be implemented via decentralized, competitive markets. The solution involves distributing the cost of enforcement unevenly across these agents. The specific nature of the unique implementation scheme makes it explicit that the optimal social arrangement involves a combination of investment in the enforcement technology and some redistribution of endowments *before* production begins.

Observe, first, that in order to implement the optimal allocation, it is necessary to pay for the introduction of the generically unique level η of enforcement that emanates from the social planner's problem. Denote agent i's contribution of capital to the aggregate enforcement cost

[8] This is reminiscent of an inverse Euler equation describing efficiency in the literature on Mirleesian taxation in dynamic economies. See e.g. Rogerson (1985) or Kocherlakota (2005).

by κ_i. This contribution can – and, we argue below, must – vary across agents. Furthermore, since institutions must be established before production begins, contributions must come from endowments.

The planner then invests these contributions into establishing an enforcement level $\eta = g\left(\int \kappa_i di\right)$. Once the enforcement technology is in place, a competitive market for borrowing and lending capital opens where total repayments of borrowed capital cannot exceed η. Agent i having paid his individual fee κ_i enters capital markets with his new endowment $\hat{a}_i = a_i - \kappa_i$ of capital. He then goes on to trade capital at a competitively determined rate R subject to a borrowing constraint given by

$$(k_i - \hat{a}_i)R \leq \eta. \tag{3.12}$$

For the planner, one can interpret κ_i as a tax schedule to finance the establishment of a capital market. On the agent's side, κ_i serves as an entry fee for participating in the market for capital. The following result describes how the cost of establishing enforcement must be distributed in order for markets to deliver the optimal allocation.

Proposition 3.5. *Let (η, k, t) be the solution to the social planner's problem. Competitive markets implement the optimal allocation if and only if, for almost all i,*

$$\kappa_i = a_i - \left(k_i - \frac{t_i}{R}\right)$$

where $R \equiv \alpha\bar{k}^{\alpha-1}$ is the market-clearing gross interest rate.

Proof. Take the sufficiency claim first. Let $\hat{a}_i = a_i - \kappa_i$ for all i so that the total supply of capital available for production is $\int \hat{a}_i di$. Then, the candidate allocation clears the capital market. Indeed,

$$\int \hat{a}_i di = \int k_i di - \frac{1}{R}\int t_i di = \int k_i di$$

since $\int t_i di = 0$ at the planner's solution. In addition, we have that

$$\int \kappa_i di = \int (\hat{a}_i - a_i)\, di = \int k_i di - \int a_i di = g(\eta)$$

where the last equality uses the feasibility constraint (3.2) in the planner's problem. In other words, the contribution schedule κ_i covers the enforcement cost $g(\eta)$, as needed.

We only have to verify that for almost all $i \in [0, 1]$, agent i chooses the optimal capital input, k_i, at interest rate R. If agents are unconstrained, they will choose a capital level such that the marginal product is equal to R. Constrained agents, however, will choose a capital level that satisfies the constraint (3.12). Consider an agent with $a_i > \bar{a}$ who therefore has $k_i = \bar{k}$ and $t_i < \eta$ in the optimal allocation. In that case, $(\bar{k} - \hat{a}_i)R = t_i < \eta$ so that, given R, the agent chooses to operate at $k_i = \bar{k}$. Agents with $a_i \leq \bar{k}$ are constrained in the optimal allocation so that $t_i = \eta$. By the definition of κ_i, this implies that $\hat{a}_i = k_i - \frac{\eta}{R}$. Hence, the agent chooses $k_i < \bar{k}$ given his borrowing constraint (3.12). This shows sufficiency.

As for necessity, note first that the sum of contributions must be $g(\eta)$ for any implementation. Next, at the market stage, some agents must be lenders and, consequently, are not constrained in their borrowing. Furthermore, being unconstrained they will choose $\bar{k}$, their level of optimal capital input, only if the market clearing interest rate is $R = \alpha\bar{k}^{\alpha-1}$. Indeed, if the interest rate were below R, these agents would choose a capital level that exceeds $\bar{k}$, which cannot be at the optimal allocation. On the other hand, if the interest rate were above R, no agent would operate at $\bar{k}$, which cannot be either at the optimal allocation.

Next, since agents whose $a_i \geq \underline{a}$ receive exactly their autarky consumption, their contribution must be exactly as stated in the proposition. As for agents below that lower threshold, the borrowing constraint must bind at exactly the right level which also pins down uniquely their endowment after paying their contribution κ_i. This completes the proof. □

Proposition 3.5 points out that there is a unique tax schedule that achieves the efficient allocation. We can characterize the schedule further. In particular, as we will now argue, the optimal contribution schedule cannot be monotonic, let alone linear.

Corollary 3.6. *Assume that $\eta > 0$. The contribution schedule $\{\kappa_i | i \in [0,1]\}$ rises monotonically with endowments on $[0, \underline{a}]$, decreases on $[\underline{a}, \bar{k}]$, and rises once again past $\bar{k}$. Furthermore, the schedule's local maximum at $\underline{a}$ is strictly positive, while its local minimum at $\bar{k}$ is zero. In particular, $\kappa_i > 0$ whenever $k_i < a_i$.*

Proof. For an agent i such that $a_i < \underline{a}$, we have $\kappa_i = a_i - (\underline{k} - \frac{\eta}{R})$ implying that κ_i rises with i since a_i does. If an agent has endowment $a_i \in [\underline{a}, \bar{a}]$, we have

$$\kappa_i = (a_i - k_i) + \frac{\eta}{R} = a_i - (a_i^\alpha + \eta)^{\frac{1}{\alpha}} + \frac{\eta}{R} \,.$$

Differentiating this expression with respect to a_i shows that the schedule decreases with endowments in this region. Finally, if agent i has endowment $a_i > \bar{a}$, then $\kappa_i = a_i - (\bar{k} - \frac{t_i}{R})$. Since the participation constraint binds for such agents, it is the case that $t_i = \bar{k}^\alpha - a_i^\alpha$. Some algebra then shows that

$$\kappa_i R = \bar{k}^{\alpha} - \bar{k}R - (a_i^{\alpha} - a_i R).$$

As $\bar{k}$ is the value of k that maximizes $k^{\alpha} - kR$, it follows that κ_i falls with a_i until $a_i = \bar{k}$ – where it is zero – and then rises again. Since $\kappa_i = 0$ when $a_i = \bar{k}$ and the compensation schedule decreases on $[\underline{a}, \bar{k}]$, it follows immediately that the schedule's local maximum at $\underline{a}$ is strictly positive. $\square$

Hence, the implementation scheme must look as depicted in Figure 2 under the same parametric assumptions we used in the previous section. The intuition is simple. At the stage of trading in competitive markets, some agents are lending, others are borrowing. The agent whose endowment is $a_i = \bar{k}$ neither borrows nor lends. Hence, he does not benefit from the establishment of markets, so that his contribution must be non-positive. Agents in the neighborhood of that agent do benefit and, at least locally, cannot be enforcement-constrained in any solution where η is strictly positive. Their participation constraint, therefore must bind, by Proposition 3.3. Hence, it must be that their contribution κ_i to establishing the enforcement technology is positive. The transfer schedule must therefore decline over $[\underline{a}, \bar{k}]$ and must increase strictly thereafter.

Below the lower-endowment threshold agents are all borrowing-constrained and the optimal allocation calls for them to all operate with the same quantity $\underline{k}$ of capital. The only way to achieve this is to have κ_i rise with endowments in that range. In fact, agents whose endowment is low receive a subsidy. Indeed, for achieving an optimal allocation of capital in competitive markets all agents in $[0, \underline{a}]$ must enter markets with the same resources, so that they operate with the same quantity of capital. Agents with no resources must therefore receive a subsidy at the optimal solution.[9]

To summarize, since the contribution scheme is non-monotonic, the planner alters the distribution of endowments before letting markets operate. The planner thus implements the desired allocation first by altering the distribution and then by introducing an enforcement technology that allows agents to enter into mutually beneficial financial contracts. A key aspect of this implementation scheme is that it solves the simultaneity issue inherent to the institutional investment problem. Enforcement is necessary to begin collecting taxes, but, at the same time, taxes are necessary to compensate the agents who finance the cost

[9]In general then and for any set of parameters, widening the distribution of endowments sufficiently always causes negative contributions to become optimal at the bottom of the distribution.

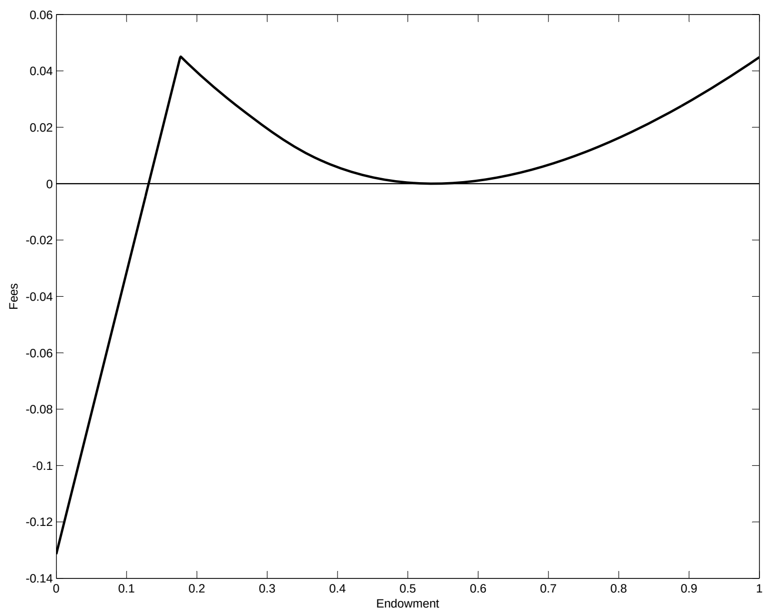

Figure 2: Optimal Fee Schedule

of enforcement. The social planner problem solves this issue by imposing participation constraints on the proposed allocation. Therefore, agents contribute voluntarily to the cost of enforcementsince they know that they will be compensated once production is complete.

4 Inequality and Optimal Contract Enforcement

4.1 Enforcement Choice and Dispersion in Marginal Products

Whenever endowments are more unequally distributed, the ex-ante dispersion in marginal products increases. Hence, one would expect that economies with more endowment inequality should invest more resources in institutions that provide better enforcement. To formalize this, we model the notion of rising inequality as follows. Let $E(a)$ denote the average endowment. We say that the endowment schedule $\hat{a} = a + \delta(a - E(a))$ is more *unequal* than the distribution a when $\delta > 0$. Throughout this section, we assume that for δ small enough, endowments remain almost surely non-negative. Alternatively, we could produce symmetric results for mean-preserving contractions rather than spreads.

We first look at the case where there are only two agents with different endowments. In that simple case, the impact of greater inequality on returns to enforcement is transparent. The nature of the optimal allocation is easy to describe, as the two cut-off points determine the allocation. When the spread in endowments increases, at the old enforcement level the spread in capital inputs must also increase. But then the marginal benefit of enforcement exceeds the marginal costs. Hence, it is optimal to invest more in enforcement.

Proposition 4.1. *Assume that there are two agents. When the distribution of endowments becomes more unequal, the optimal level of enforcement η increases.*

Proof. Write initial endowments in this case as $(a - \delta, a + \delta)$ where $a > 0$ and $\delta \in [0, a)$. Denote the production function as f and its inverse as h. Since f is strictly concave, h is strictly convex. We also denote capital use by k_1 for the agent with low endowment, while k_2 denotes capital use by the agent with high endowment.

Suppose the enforcement level is given by $\eta > 0$. Then the capital allocation must solve

$$k_2 = h(f(a + \delta) - \eta)$$

where $k_2 \leq 2a$. Indeed, the enforcement constraint has to be binding for the poor agent at the optimal choice since otherwise enforcement costs could be reduced. This means that the rich agent receives a transfer of exactly η at the optimal arrangement.

The agent with the low endowment then operates with capital $k_1 = 2a - h(f(a+\delta)-\eta) - g(\eta)$. Hence, total output is given by

$$\Phi(\eta,\delta) \equiv f(a+\delta) - \eta + f\left(2a - h(f(a+\delta)-\eta) - g(\eta)\right).$$

Note that the function Φ is strictly concave in η. The envelope theorem then implies that at the optimal level of enforcement

$$\frac{\partial\eta}{\partial\delta} = -\frac{\Phi_{12}(\eta(\delta),\delta)}{\Phi_{11}(\eta(\delta),\delta)}.$$

Differentiating the function Φ with respect to η we obtain

$$\Phi_1(\eta(\delta),\delta) = -1 + f'(k_1)(h'(f(a+\delta)-\eta) - g'(\eta))$$

where $k_1 = 2a - h(f(a+\delta)-\eta) - g(\eta)$ is the capital allocated to the low endowment agent.

We then have that

$$\Phi_{11}(\eta(\delta),\delta) = f''(k_1)(h'(\cdot) - g'(\cdot))^2 - f'(k_1)(h''(\cdot) + g''(\cdot)) < 0,$$

since h is convex and f is strictly increasing and concave. Furthermore, both $h(f(a+\delta)-\eta)$ and $h'(f(a+\delta)-\eta)$ are increasing in δ, as $f(a+\delta)$ rises with δ. This implies that k_1 decreases in δ. The concavity of f then implies that $f'(k_1)$ rises with δ too. Thus we obtain that

$$\Phi_{12}(\eta(\delta),\delta) > 0$$

which completes the proof. □

Returning to the case with a continuum of agents, suppose that the planner chooses to bear the fixed cost $\lim_{\eta\searrow 0} g(\eta)$ and to invest in strictly positive enforcement for a given distribution of endowments. We now show that this remains the case if the distribution of endowments becomes more unequal.

Proposition 4.2. *Assume that the planner opts for strictly positive enforcement for a given endowment distribution. This remains true when the endowment distribution becomes more* *unequal.*

Proof. Let k be the optimal capital allocation in the first economy while $\eta > 0$ is the chosen degree of enforcement. We must have that $\int k_i^\alpha di \geq \int a_i^\alpha di$.

Consider now the more unequal distribution of endowments described by $\hat{a}_i = a_i + \delta(a_i - a^*)$. We will show that holding η fixed a feasible capital allocation $\hat{k}$ exists in the more unequal economy such that $\int \hat{k}_i^\alpha di \geq \int \hat{a}_i^\alpha di$. Hence, strictly positive enforcement remains optimal for a more unequal distribution of endowments.

For all i, let

$$\hat{k}_i^\alpha = k_i^\alpha + \hat{a}_i^\alpha - a_i^\alpha$$

where it is assumed that δ is small enough that $\hat{k}_i \geq 0$ for all i. This is without loss of generality as the argument we use below is local. Leaving transfers unchanged, participation is ensured since it was in the original economy. The new allocation dominates autarky (at least weakly), since

$$\int \hat{k}_i^\alpha di = \int \hat{a}_i^\alpha di + \int k_i^\alpha di - \int a_i^\alpha di \geq \int \hat{a}_i^\alpha di.$$

We need to show that the new allocation satisfies the resource constraint. First, note that the total capital employed is given by

$$\int \hat{k}_i di = \int \left((k_i^\alpha - a_i^\alpha) + \hat{a}_i^\alpha\right)^{1/\alpha} di.$$

Differentiating the integrand with respect to δ gives (up to multiplying constants)

$$\left((k_i^\alpha - a_i^\alpha) + \hat{a}_i^\alpha\right)^{1/\alpha - 1} (a_i + \delta(a_i - a^*))^{\alpha-1}(a_i - a^*).$$

Hence, evaluating this expression at $\delta = 0$ shows that small changes to δ do not increase the total capital employed provided

$$\int \left(\frac{k_i}{a_i}\right)^{1-\alpha} (a_i - a^*) di$$

is weakly negative. We know that the original optimal allocation is such that $\left(\frac{k_i}{a_i}\right)^{1-\alpha}$ decreases as i rises. By Chebyshev's integral inequality, we have

$$\int \left(\frac{k_i}{a_i}\right)^{1-\alpha} (a_i - a^*)di \leq \int \left(\frac{k_i}{a_i}\right)^{1-\alpha} di \times \int (a_i - a^*)di = 0$$

which completes the proof. □

Intuition suggests that monotonicity holds in this environment in a more general sense. Increases in the dispersion of marginal products should lead the planner to increase η. Looking at equations (3.9) through (3.11), notice that for any *given* enforcement level η, the last two equations alone pin down the cut-off levels for the optimal capital allocation. Hence, one can find this allocation for any value of η *independently* of the enforcement choice. In other words, one can solve for the optimal level of enforcement by first determining the optimal capital allocation as a function of η and, then, compare the marginal cost and benefits to find the efficient level of enforcement. Hence, it is optimal to raise the enforcement level for any marginal change in the distribution of endowments that causes more agents to be constrained or the wedge between the two cut-off points to increase.

4.2 Inequality in Human and Physical Capital

Our framework predicts that economies with an endowment distribution that implies a high dispersion of marginal products should be quick to invest in institutions that support trade. When endowments are one-dimensional, more inequality in endowments implies more inequality in marginal products. An apparent prediction of our framework, therefore, is that more initial wealth or income inequality should be conducive to the development of institutions.

If one considers, however, human and physical capital as determining jointly marginal products, what matters is not inequality in each factor, but the correlation between the two. To derive this argument formally, we augment our static model to include heterogeneity in both physical and human capital. Agents are now endowed with a quantity $a_i > 0$ of physical capital and a level $h_i > 0$ of human capital for all $i \in [0, 1]$ with the joint distribution of human and physical capital described by G. Agents are also endowed with a technology that transfers physical capital into consumption goods according to a Cobb-Douglas production function $h^{1-\alpha}k^{\alpha}$, where h is the human capital of the agent and $\alpha \in (0, 1)$. Since human capital cannot be traded across agents, the endowment of human capital acts like an agent-specific productivity parameter that is fixed.

For simplicity, we consider only the case where the choice is between autarky and full enforcement. If there is no investment in enforcement, default cannot be punished. Hence all transfers are zero and autarky prevails. Aggregate output is then given by

$$y^A = \int h^{1-\alpha} a^\alpha dG = E(h^{1-\alpha} a^\alpha) = E(h^{1-\alpha})E(a^\alpha) + COV(h^{1-\alpha}, a^\alpha). \tag{4.1}$$

Alternatively, the planner can achieve full enforcement at an aggregate fixed cost of $C > 0$ units of capital. Markets are then complete and the planner is able to equate marginal products across agents. Denoting the total endowment of human and physical capital by $\bar{h}$ and $\bar{k}$ respectively, we obtain, for all $i \in [0, 1]$ that

$$\frac{k_i}{h_i} = \frac{\bar{k}}{\bar{h}}. \tag{4.2}$$

Since with full enforcement all transfers can be enforced, participation constraints for all agents can be met if and only if aggregate output increases after the enforcement cost C has been incurred.[10] Using aggregate resource feasibility, it follows directly that full enforcement leads to aggregate output equal to

$$y^E = \int h_i \left(\frac{\bar{k}}{\bar{h}}\right)^\alpha dG = \left(\int hdG\right)^{1-\alpha} \left(\int adG - C\right)^\alpha = E(h)^{1-\alpha}(E(a) - C)^\alpha. \tag{4.3}$$

The planner will choose to invest in enforcement whenever $y^E > y^A$.

Holding the endowment distribution of the other factor fixed, a mean preserving spread in either human capital *or* physical capital endowments lowers output under autarky without affecting the outcome under complete markets. Hence, as in the analysis with only one input, more inequality can lead to more investment in institutions, as the benefits from trade have increased. However, in this two-dimensional setting, for any given marginal distribution of physical and human capital, the correlation in the endowments of both factors of production also determines institutional investment. If endowments in human and physical capital are sufficiently positively correlated, aggregate output is higher under autarky, and there is no institutional investment.

Proposition 4.3. *Introducing complete markets with full enforcement at a fixed cost C leads*

[10] An optimal allocation with enforcement can once again be implemented via decentralized markets with a specific schedule of entry fees and subsidies, where the fee schedule has to satisfy all agents' participation constraints.

to higher output than under autarky if and only if

$$E(h)^{1-\alpha}(E(a)-C)^{\alpha} - E(h^{1-\alpha})E(a^{\alpha}) > COV(h^{1-\alpha}, a^{\alpha}). \tag{4.4}$$

In economies where both human and physical capital are highly concentrated, the gains from introducing institutions are small, as the marginal products of capital are not very unequally distributed. In fact, when h and are a are perfectly correlated, we have

$$COV(h^{1-\alpha}, a^{\alpha}) = E(h)^{1-\alpha}E(a)^{\alpha} - E(h^{1-\alpha})E(a^{\alpha}),$$

so that inequality (4.4) can never be met.

This result underscores the fact that it is inequality in marginal products before trade that matters for returns to institutional investments, *not* endowment inequality per se. In economies where physical and human capital endowments are highly correlated, institutions conducive to trading physical resources may not have much effect on output and growth, unless poor individuals are able to acquire more human capital.

5 The Dynamics of Enforcement and Inequality

How do institutions and inequality evolve over time given initial conditions? This section provides some answers by introducing the same simple inter-generational linkages as in Aghion and Bolton (1997), Banerjee and Newman (1993) or Benabou (1996). Time is discrete and denoted by $t \in \{0, 1, \ldots\}$. In every period $t \geq 0$, a single member of each family $i \in [0, 1]$ is alive and inherits as endowment a given fraction $\gamma_i \in [0, 1]$ of their parent's income, where γ_i is strictly increasing in i. This endowment can be used in production exactly as in the static model, or as investment in the enforcement technology. We also assume that investments in enforcement fully depreciate across periods, although this could be relaxed with little effect on our results. For simplicity, we will assume that the current enforcement choice is myopic in the sense that each generation makes their institutional choice without taking into account the effects of the current choice on future generations' welfare.[11]

[11] There are at least two interpretations for such transfers: "warm-glow" altruism in the sense of Andreoni (1989) and intergenerational spillovers. The second interpretation is best understood if one thinks of productive resources in part as human capital. It has the advantage of side-stepping an obvious weakness of the warm-glow interpretation, namely the fact that lineages fail to internalize the consequences of their transfers on the welfare of their offspring. That concern is particularly strong in environments with redistribution

If there is no enforcement technology, the only feasible allocation of capital is autarky in all periods. It follows directly that for this case the endowment of members of lineage i converges geometrically to $\gamma_i^{\frac{1}{1-\alpha}}$ over time and the endowment distribution to the corresponding invariant distribution. Hence, having different bequest fractions across agents ensures persistent endowment inequality. Were the fractions the same across agents, the endowment distribution would converge at a geometric rate to a single mass point and investment in enforcement would matter only during the transition, but not in the long run. We finally assume that in period 0, the distribution of endowments is the invariant distribution without enforcement.

With the possibility to invest in enforcement, we assume that this invariant distribution is sufficiently unequal so that it is optimal to bear the fixed cost in period 0. Otherwise, no investment in enforcement is ever made, and the economy remains forever at the initial invariant distribution. Under this assumption, bearing the fixed cost remains optimal in all subsequent periods, and introducing enforcement leads to a progressive reduction of endowment inequality. This in turn reduces the benefits of investing in enforcement and successively reduces such investment.

Proposition 5.1. *If it is optimal to invest in enforcement at date $t = 0$, then the optimal allocation features a positive enforcement level for all periods $t \geq 0$ that decreases over time. Furthermore, the economy converges monotonically to a long-run invariant distribution of income and endowments with progressively less inequality and higher output.*

Proof. Let a^t be the endowment function and η_t the optimal enforcement level in period t. We proceed in two steps. First, we show that enforcement decreases in period 1, $0 < \eta_1 < \eta_0$. Then, we establish that endowments increase over time, i.e. that $a_i^2 \geq a_i^1$ for all i. The desired result will then follow by induction.

Given the initial endowment distribution $a_i^0 = \gamma_i^{\frac{\alpha}{1-\alpha}}$, the optimal allocation at $t = 0$ is described by equations (3.9)-(3.11). There are two cut-off points $\underline{a}^0 = \underline{\gamma}_0^{\frac{1}{1-\alpha}}$ and $\bar{a}^0 = \bar{\gamma}_0^{\frac{1}{1-\alpha}}$ determining capital and transfers given the optimal level η_0. The new endowment function is then given for all $i \in [0, 1]$ by

$$a_i^1 = \begin{cases} \gamma_i \underline{\gamma}_0^{\frac{\alpha}{1-\alpha}} & \text{if } \gamma_i \leq \underline{\gamma_0} \\ \gamma_i^{\frac{1}{1-\alpha}} & \text{if } \gamma_i \geq \underline{\gamma}_0. \end{cases}$$

policies. We thus emphasize the spillover interpretation and assume that generations are not linked in any other way.

In particular, because $\gamma_i \underline{\gamma}_0^{\frac{\alpha}{1-\alpha}} > \gamma_i^{\frac{1}{1-\alpha}} = a_i^0$ whenever $\gamma_i < \underline{\gamma}_0$, we have that $E(a^1) > E(a^0)$. Suppose first that we constrain the planner to continue opting for enforcement level η_0 in period 1. It is straightforward to verify that the optimal allocation of capital is still described by equations (3.10) and (3.11). In particular, the new upper endowment threshold $\bar{a}^1(\eta_0)$ is the same as in period 0, as the enforcement level has not changed. The lower threshold $\underline{a}^1(\eta_0)$ has to increase from its period 0 value, as the aggregate endowment $E(a_1)$ has gone up. Otherwise, some capital would not be used in production which cannot be optimal. These new thresholds pin down the new optimal allocation conditional on keeping the enforcement level at η_0.

We will first argue that this candidate allocation yields an average income level that exceeds its autarky counterpart at endowment distribution a^1, so that, $\eta_1 > 0$. First note that the optimal allocation of capital $\{k_i^0 : i \in [0,1]\}$ in period 0 is still feasible in period 1 given enforcement level η_0. Indeed, for any $i \in [0,1]$ such that $a_i^0 \geq \underline{\gamma}_0^{\frac{1}{1-\alpha}}$, $a_i^1 = a_i^0$ so the value of autarky is the same in period 0 and 1. For any i such that $a_i^0 < \underline{\gamma}_0^{\frac{1}{1-\alpha}}$, we have

$$\left(\underline{k}_i^0\right)^\alpha - \eta = \underline{\gamma}_0^{\frac{\alpha}{1-\alpha}} > \gamma_i \underline{\gamma}_0^{\frac{\alpha}{1-\alpha}} = a_i^1 (\gamma_i)^\alpha,$$

as $\gamma_i < \underline{\gamma}_0$. This implies directly that in period 1 income is higher for everyone with the optimal allocation of period 0. As transfers sum to zero, aggregate output is also higher with enforcement than with autarky. Hence, it remains optimal to invest enforcement in period 1.

We will show next that $\eta_1 < \eta_0$. At η_0, we have for the new thresholds $\underline{a}^1(\eta_0)$ and $\bar{a}^1(\eta_0)$ so that

$$g'(\eta_0) > \int_{\{i|a_i \leq \bar{a}\}} \frac{1}{\alpha \left[\bar{k}(\eta_0)\right]^{\alpha-1}} di - \int_{\{i|a_i < \bar{a}\}} \frac{1}{\alpha \left[k_i(\eta_0)\right]^{\alpha-1}} di,$$

as the corresponding cut-off point for capital $\underline{k}_i(\eta_0)$ has increased. From the concavity of the objective function and the strict convexity of the constraint set – which is ensured by our assumptions on g conditional on incurring the fixed cost – we have a unique optimal value of enforcement η_1 that satisfies the first-order condition (3.9). Furthermore, for $\eta \to 0$ marginal benefits exceed marginal costs and for $\eta \to \infty$ the opposite is true. Hence, the optimal level of enforcement in period 1 must decrease, i.e., $\eta_1 < \eta_0$.

Finally, we will show that endowments increase over time for almost all i. Note that the endowment distribution has not changed above the cut-off point $\underline{a}_0$. As $\eta_1 < \eta_0$, by equation (3.11) it must then be the case that $\bar{a}^1(\eta_1)$ increases relative to its period 0 value. Suppose now that the lower cut-off point decreases, i.e., $\underline{a}^1(\eta_1) < \underline{a}^0(\eta_0)$. Since all agents with $a_i^1 \geq \underline{a}^0(\eta_0)$ have the same endowment level as in period 0, their binding participation constraint implies that they have the same income level. All other agents have a strictly lower income level than $\underline{a}^0(\eta_0)$. Since transfers sum to zero with the new threshold $\bar{a}^1(\eta_1)$, it must be the case that total output has declined relative to its period 0 value. Since the old enforcement level η_0 is still feasible, this allocation cannot be optimal. A contradiction.

Hence, $\underline{a}_1 > \underline{a}_0$. A simple recursive argument then shows that there is positive, but declining enforcement in all subsequent periods. This implies that average income rises over time, rising the lower income threshold as well. We have then shown that the sequence of endowment distributions is a monotonically increasing sequence of distribution functions on $[0, 1]$. Furthermore, we can bound the values of each distribution of the sequence below by 0 and above by $\gamma_{\max}^{\frac{1}{1-\alpha}}$. It then follows that the sequence of endowment distributions converges to some distribution as $t \to \infty$. Along this sequence, we have successively less inequality and lower, but strictly positive enforcement. $\square$

This result implies that differences in institutional choices caused by differences in initial conditions can persist indefinitely, for reasons not unlike those explored in another context by Monnet and Quintin (2007). Institutional investments, in turn, allow for inequality to become reduced over time in a very specific sense. The endowment distribution is a censored version of the autarky distribution, with an ever higher censoring point. While there is always a strictly positive investment in enforcement, the fact that the ex-ante dispersion of marginal products falls over time causes the optimal level of enforcement to fall over time as well.

6 Concluding Remarks

We conclude with a brief discussion of what our theory implies for economic development. In much of the recent literature on inequality and growth, it is the combination of endowment inequality and market imperfections that leads to bad economic outcomes and creates a rationale for redistribution. In this paper, we have pointed out that as long as it is possible to invest in better functioning markets, the optimal social arrangement calls for a combination of redistribution and institutional investment.

It is critical to recognize that it is the ex-ante inequality in marginal products that matters for our results, and not endowment inequality per se. One implication of our theory is that societies where physical and human capital endowments are highly correlated do not necessarily have an incentive to invest into costly institutions that improve market exchange. This could help to support the claim put forward by Engerman and Solokoff (2000) that initial inequality could explain why some nations in the western hemisphere were much faster to develop institutions conducive to trade than others. As a case study, Engerman and Sokoloff (2002) describe 19th century Latin America as an area where both human and physical capital were highly concentrated. The United States and Canada, however, had less inequality in both forms of capital and developed better market-supporting institutions.[12]

Our insights also matter for the policy debate on how best to promote development. Commonly, one mentions a lack of enforcement – both, of contracts and property rights – as a major impediment for economic growth. As pointed out by some contribution to this literature an unequal distribution of resources compounds that problem justifying a relationship between inequality, lack of investment and, hence, growth (see for example, Perotti (1994) among others). One could then argue quickly that *either* improving enforcement *or* alleviating inequality are key measure for development. We have shown here, however, that the two measures need to be seen as complementary. Indeed, as we have shown, delays in introducing better institutions are likely to arise whenever the benefits of these investments cannot be distributed across agents in a way that makes financing initial costs feasible.

Of course, our findings rely on some strong assumptions.In particular, we have assumed that endowments are publicly observable for the planner. Suppose instead that endowments were private information and that agents could hide their output from the planner. The planner would then need to give agents incentives to reveal their private information. We conjecture that an agent's pay-off $\Pi(a)$ needs to be strictly increasing at a rate that is at least equal to the marginal product of capital employed in the agent's production, or

$$d\Pi/da \geq \alpha k(a)^{\alpha-1},$$

where $k(a)$ is the capital stock as a function of endowment in the solution to the planning problem. Furthermore, the relationship should hold with equality, since the planner would like to redistribute capital as much as possible for productive efficiency.

The intuition is straightforward. Were this not the case, an agent could (locally) lie downwards, retain some endowment and employ it in his own production after receiving capital inputs from the planner. This would make him better off as the marginal product in

[12]Doepke and Eisfeldt (2007) offer a different explanation for this hypothesis that combines economic and strategic considerations.

his own production exceeds the marginal change in the payoff offered by the planner. The optimal allocation would therefore change once endowments are private information. For the bottom end of the distribution, pay-offs would become constant, while for the upper end of the distribution they would increase at a slower pace. This should restrict how much the planner can redistribute resources and, therefore, the degree of contract enforcement needed to sustain such redistribution. We leave a full fledged analysis of private information in our context for future work.

7 Appendix

7.1 Existence and generic uniqueness

Proposition 7.1. *A solution to the social planning problem exists. The solution is generically unique.*

Proof. The planner's problem is a Mayer problem with integral (isoperimetric) constraints (see Section 4.8 in Cesari (1983), for a discussion). In order to apply Filippov's existence theorem (Theorem 4.3.i in Cesari, 1983), we need to restrict transfers and capital choices to a compact set. One can impose arbitrary bounds on both objects that are large enough not to bind at any solution. This ensures existence.

To establish generic uniqueness, note that given the fixed cost associated with implementing the enforcement technology, we need to compare the value of the problem when $\eta = 0$ – i.e., the value of the problem at autarky – and the value of the problem when the planner chooses to bear the fixed enforcement cost $\lim_{\eta \searrow 0} g(\eta)$.

This second problem corresponds to solving the problem assuming that $g(0) = \lim_{\eta \searrow 0} g(\eta)$. We will argue that the solution under that assumption is unique, so that the only case in which multiple solutions exist is when that solution happens to give exactly the same value as autarky. Generically, there is then at most one solution.

Under the assumption that $g(0) = \lim_{\eta \searrow 0} g(\eta)$ and upon weakening resource constraint (3.6) without any loss of generality, the planner's choice set is convex in (η, k, t). Since the planner's objective function is strictly concave in k, there is at most one optimal capital allocation in that case. The resource constraint then implies that η must be unique as well. The transfer scheme is also unique because, as we argue below, either the participation or the enforcement constraint must bind for all agents. If the enforcement constraint binds, we have $t_i = \eta$. If the participation constraint of the agent is binding, transfers are given by $t_i = k_i^\alpha - a_i^\alpha$. $\square$

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